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Does tax reduction have an effect on gross domestic product? An empirical investigation.

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Does tax reduction have an effect on gross domestic product? An empirical investigation

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Abstract

Tax reduction shocks in US economy: 1964, 1979-81 and 2002 increased gross domestic product, GDP, in the short run (≈ 3 years) so that 1% reduction increased the detrended GDP with 0.48 – 0.77 %. Following tax reductions, tax series became a leading variable to GDP for 9 to 15 years completing 1 to 2 cycles. However, in the long run, ≈ 10 years, 1 % tax reduction decreased the detrended GDP with about 0.25 %. However, tax, as Government recipts, and GDP are both composite measures so it is not unlikely that the effects may be attributable to specific components of the tax or GDP. I used a novel technique that identifies running leading relationships between time series, extracts common cycle lengths from the series and estimates lag times.

Key words: Tax rate, Gross domestic product, GDP; monetary supply M2; Federal funds rate

JEL : C15, E02, E62, H21

1. INTRODUCTION

What are the effects of tax changes on the economy? Some studies suggest that they are either small or negative, Gale and Samwick (2014), whereas others find significant effects, e.g., Mountford and Uhlig (2009), Romer and Romer (2010). Mountford and Uhlig calculate effects over 6 years periods whereas Romer and Romer (2010) calculate over 3 years periods.

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I here use a technique that is novel in the present context, the leading –lagging, LL- method, to identify the period in which tax changes have a potential impact on GDP. When the changes are causal for changes in GDP, the changes have to come before changes in GDP. With cyclic variations a causal variable can be envisaged to show a peak (or a through) before the peak (or through) of the effect variable. In the method section, it is shown that if the variable x leads the variable y, then with the x-variable plotted on the x-axis of a phase plot and the y- variable plotted on the y-axis, the trajectories for the pair rotate clock-wise. This means that the rotational direction can be used to identify which variable comes first, and thus which variable is a candidate causal variable for the other. However, being leading is a necessary, but not sufficient, requirement for causation. The method allows extraction of running average common cycle lengths for paired time series even if the series are superpositions of underlying series with different cycle lengths. The method also identifies phase shifts between the paired series.

Tax types. Several authors have examined subgroups of taxes, distinguishing for example between personal and corporate taxes, Mertens and Ravn (2013), or express the taxes as average marginal taxes, e.g., Arin et al. (2013). Taxes may also have complicated structures, like taxes related to the Economic recovery act of 1981 (ERTA) or they may be financed by increased deficit or reduced government spending, Gale and Samwick (2014). Here, the total current tax receipts are used as a variable.

Confounding factors. Effects of tax changes may be confounded with several other monetary variables, like changes or shocks in business cycles, monetary policy, government revenues and government spending, (Mountford and Uhlig 2009). Other factors that may impact the steady state level of GDP iare credit and stock market developments, (Durusu-Ciftci et al. 2016). Most conspicuous are probably changes in the Federal funds rate, FF, or changes in monetary supply, measured e.g., by M2. Federal funds rate, FF, and M2 may be changed at the same time as tax rates are changed, for example when the growth in GDP is slowing down or unemployment is increasing (Seip and McNown

2013). Romer and Romer (2010) control for monetary policy by including monetary shocks derived from changes in Federal Funds rate and government spending. These authors find that although there are significant effects of the control variables, their effects were moderate.

For the study of leading – lagging relationship, it is important for the interpretation of the results if it is a decrease or an increase in the candidate causal variable that gives an increase in the target variable. Here, decreases in both tax rate and FF are hypothesized to increase GDP. Thus, the sign of both the tax rate and FF are changed so that peaks and troughs in both the tax rate and FF will precede peaks and troughs in GDP if the negative values are causal variables to GDP.

1.1   Hypotheses.

I *first* hypothesize that the time windows where changes in tax rate area a potential cause for changes in GDP, tax changes are consistently leading GDP, and there will be a tax shock associated with the beginning of the time window. It does not have to initiate the leading relations exactly, because the tax shock can have been anticipated, or the reaction to the tax shock takes some time to manifest itself, Mertens and Ravn (2013) p. 1220. *Secondly*, I hypothesize that a negative tax change will increase GDP and a positive tax change will decrease GDP sometimes after it has taken effect, and the effects will last as long as tax rates are a leading variable to GDP. *Thirdly*, I hypothesize that tax changes will initiate cyclic relationships between tax rate and GDP and the two movements will have a common cycle time.

To my knowledge, the present study distinguishes itself from other studies in that i) the domains of interest are restricted to the portion of the paired time series where tax changes are leading GDP. ii) I examine if these domains are initiated by a tax shock, and tax shocks are identified as extreme events in the tax rate changes; iii) the long term reaction to tax shocks are calculated over the period where the tax rate is a leading variable to GDP. Since the effect of tax shocks may be confounded with effects of changes in FF and changes in M2, I also examine if - FF and M2 are leading variables to GDP within the time domains defined by the tax shock/ GDP pair. v) I use the β- coefficients for GDP

as a function of the independent variables: tax reductions, -FF and M2 over a short initial period (3 years) and over the full LL- periods as a measure of the reaction of GDP to the three policy variables. (The time series have been normalized to unit standard deviation.) iv) Since the tax change / GDP time series both are cyclic, their common cycle times are also reported. (They have roughly common cycle times since one variable is a leading variable to the other.)

I find that tax shocks make tax series leading variables to the economy, GDP. Three identified large tax reductions initially increase GDP over a 3 years period, but then set up regular cycles where changes in tax rates are followed by changes in GDP. For the period where the tax rate is a leading variable to GDP and therefore is a candidate causal variable, the net effect is to decrease GDP over decadal time scales, 9 to 15 years. Note that the series have been detrended to avoid the effects of growth over multidecadal time scales.

The rest of the paper is organized as follows. In Section 2 the material is presented, in Section 3 the method is presented with emphasis on the leading- lagging method, in Section 4 the results are presented. Then I discuss and compare the results to findings in the literature in Section 5. Section 6 conclude.

2. MATERIALS

I use the gross domestic product, GDP, as response variable. The fiscal and monetary variables that are used as policy instruments are the Federal funds rate, FF, monetary supply, M2 and Tax returns (Current tax receipts, CR: Government Current Receipts and Expenditures: Billions of dollars (Annual)). The tax returns increased dramatically after 1940. Tax returns are divided with GDP to avoid including changes in the tax base. I also include variables that are regarded as automatic movements of variables in response to other types of shocks, such as the consumer price index, CPI, and the unemployment rate, U. Figure 1a shows the original variables multiplied with 10^n and shifted vertically to ease visualization. All time series end in 2014. CR, GDP and CPI starts in 1947, UE starts in 1948, FF

starts in 1955, and M2 starts in 1959. Figure 1b shows the same variables detrended with a second

order polynomial function, if significant, and then normalized to unit standard deviation.

Figure 1 in here (data and PCA plots)

Tax rate shocks. I use three sources to identify years that are associated with tax rate shocks.

Mertens and Ravn (2013) identify narratively 3 positive shocks and 7 negative shocks during the pe-

riod 1950 to 2006, see Table 1. These data are compared to shocks "that receive more than inci-

dental mention" identified by Romer and Romer (2010) and thirdly, to abrupt changes in the tax rate

that are at the tails of a normal distribution fitted to the annual tax rate changes. Table 1 shows tax

rate shocks and corresponding shocks in FF and M2. The distribution of tax shocks is compared to

their normal distribution in Supplementary material 1.

Table 1 in here (shocks in CR, FF and M2)

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3. METHOD

I used a method that allows us to calculate running averages of leading and lagging relations, and

also running averages of cycle lengths and leading or lagging times (if significant). The variance does

no have to be stabilized in this study, because the methods are locally restricted on the full data set.

All calculations are presented in Excel and are available from the author. The method is explained in

more detail in Seip and McNown (2007) and Seip and Grøn (2015). The method is distinct from other

cross correlation methods, or causality identification methods, in that it does not require the time

series to be (piecewise) stationary, (Granger 1969; Sugihara and May 1990; Kestin et al. 1998; Liang 2014; Deyle et al. 2016), and can thus identify short time windows where leading relations fail.

3.1 Tax shocks.

I identify tax shocks in the time series by first taking the derivative of the tax rates, then normalizing to unit standard deviation, and lastly I compare the distribution of the resulting time series to a Gaussian distribution. Tax shocks are then defined as changes in tax rates that are at the tails of the distribution. The same procedure is used for FF and M2, see Supplementary material 1.

3.2 Detrending and normalizing.

The variables GDP, CR and M2 were detrended by maintaining the residuals after subtracting a 2nd order polynomial regression against time. All data were then normalized to unit standard deviation. Figure 1b shows the detrended and normalized series. Detrending may distort the short term and decadal patterns to some extent, but detrending over multidecadal times removes some of the variability in the series that is caused by long-term factors, like innovations and technical and societal developments.

3.3 Leading and lagging relations.

The method utilizes the dual representation of paired time series x and y as series depicted along a time axis, x(t) and y(t) and as series depicted in a phase plot, y = f(x). For the paired variables, one variable is depicted along the x-axis and the other variable along the y-axis of a phase plot, Figure 2. In this study, the policy variables, -CR/GDP, - FF or M2, are depicted on the x-axis and GDP on the y-axis. In the example, sine functions are used as proxies for the tax rate and the GDP series. For the latter series, a small amount of random noise is added to make the illustration more realistic, Figure 2a. If the variables show cyclic patterns that are shifted in time relative to each other, the trajectories for the (x,y) pair in a phase plot will rotate relative persistently in one or the other direction as suggested in Figure 1b for the two sine functions. If the trajectory rotates positively (counter-clock-

wise per definition) then the y-axis variable lags the x-axis variable. Figure 2c shows consecutive an-gles between successive trajectories in the phase plot (black bars). The shaded bars show trajectories that rotate in one direction with a certain probability. That is, some exceptions are allowed to a per-sistent rotational direction.

Figure 2 in here (method)

The rotational patterns are quantified in phase plot by[2]:

$$(1) \qquad V = sign(\overline{v_1} \times \overline{v_2}) \cdot A\cos\left(\frac{\overline{v_1} \cdot \overline{v_2}}{\left|\overline{v_1}\right| \cdot \left|\overline{v_2}\right|}\right)$$

where $\overline{v_1}$ and $\overline{v_2}$ are two vectors formed by two sequential trajectories between three sequential points in the phase plots. From these angles, a leading – lagging, LL- strength can be identified. It can be formulated as a the ratio of the number of positive angles minus the number of negative an-gles to the sum of the absolute value of both positive and negative angles over a certain time span, n, (here 9 years).

$$(2) \quad LL = (N_{pos} - N_{neg}) / (N_{pos} + N_{neg})$$

The variable LL ranges between -1 (y- variable leads x- variable) to +1 (y-variable lags x- variable). With LL = 0, there is no leading- lagging relationship and within a domain around the zero value, LL - relations are non- significant ($p > 0.05$).

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[2]With x- coordinates in A1 to A3 and y-coordinates in B1 to B3 the angle is calculated by pasting the following Excel expression into C2: =SIGN((A2-A1)*(B3-B2)-(B2-B1)*(A3-A2))*ACOS(((A2-A1)*(A3-A2) + (B2-B1)*(B3-B2))/(SQRT((A2-A1)^2+(B2-B1)^2)*SQRT((A3-A2)^2+(B3-B2)^2))).

Cycle lengths. Since one full rotation of the trajectories in phase space corresponds to a closed cycle, running averages for cycle lengths, CL (n ≥ 3), is calculated by estimating what full rotations would be if the angles continued to be like the angles encountered. This would be correct if the phase plot trajectories formed an exact circle, corresponding to two perfect sines displaced ¼ of a cycle length relative to each other.

$$(3) \qquad CL = 2\pi n / |\textstyle\sum V|$$

I also calculate cycle length as the number of time steps used for the angles to close a full circle. Results for the two methods are shown in Figure 2d. The saw – toothed line counts time steps as angles are added. When the sum reaches 2π counting starts again with zero. In Supplementary material 2 it is shown that for time series consisting of two superimposed sine functions, sin(ωt + φ), with ω = 0.1 and 0.2 respectively, simple shifted sine functions with φ = 0.785 and ω = 0.1 and ω = 0.2 respectively extract long and short cycles from the superimposed sine functions.

Phase shifts between paired series. Lead or lag times, PS, are estimated from the correlation coefficient, r, for sequences of 5 observations, PS (5). If the two series co-vary exactly, their regression coefficient will be 1, and the time lag zero. If they are displaced half a cycle length, the correlation coefficient is r = -1 and the series are counter cyclic. The phase shift between two cyclic series can be approximated by:

$$(4) \qquad PS \approx \lambda/2 \times (\pi/2\text{- Arcsine (r))}$$

Slopes and volatility. I calculate running averages of slopes as β – coefficients (n = 5) for a regression between the two variables, and volatility for each series as running average of standard deviation (n = 11).

An expanded explanation of the method is given in Seip and Grøn (2017). The method is implemented in Excel and requires only the pasting of new datasets into two columns. It is available from the authors.

3.4 Uncertainty estimates.

Using Monte Carlo technique, confidence estimates were made for leading lagging relations and for cycle times. The 5% confidence intervals were identified as LL < - 0.32 or LL > +0.32, the relationships are significant for these values if n > 9. The running average of LL was calculated over 9 successive observations (years in this study). To estimate confidence intervals for cycle lengths, the distribution of cycle lengths for two paired uniformly random series was calculated and then the calculations were repeated 100 times. The probability distribution for cycle lengths is shown in Supplementary material 3 and shows that cycles > 7 years occur in less than 5% of the cases, corresponding to a p-value of about 0.05. Thus, cycle lengths longer than 7 time steps probably reflect a persistent cyclic pattern in the time series.

3.5 Smoothing.

To see the medium term trends for our resulting variables, I smoothed the running average values using the 2D smoothing algorithm of Sigma Plot©. The algorithm is a locally weighted polynomial smoothing function. I used the parameter values f = 0.2 and f = 0.4 to define local domains (20% and 40 % of the full series respectively) and a second order polynomial function, p = 2. To calculate the effects of a 1% change in the tax rate, ordinary linear regression, OLR, were applied to the time domain where the tax rate was a leading variable to GDP.

3.6 Principal component analysis, PCA.

To obtain a graphic picture of the relationship between the running variables: LL- relationship, cycle length, phase shift, β- coefficient, volatilities and recession periods, I used principal component analysis, PCA. The PCA produces two plots, the score plot that shows how samples are related (here observations for each year) and the loading plot that shows how variables are related (here our seven variables). Variables that are in the same direction from the origin are associated. Variables that are at a right angel relative to a line from the origin are either unrelated, or shifted in time. I used the PCA to obtain an overview of the relationships between variables.

4. RESULTS

In the present study, I examined both the immediate effects of tax changes, that is, the changes in GDP that follows tax changes that have been announced or implemented, and I examined the long-term effects of tax changes. Long term effects may be important, Mountford and Uhlig (2009 p. 986), for example, stated that the long-term consequences of an unanticipated deficit-financed tax cut may be far worse than the benefit of short-term increase in GDP.

4.1 Long term results, 1945-2013.

In this section, the term "determinant" is used for a variable that appears to contribute a causative effect on another variable because it co-varies, and current theories suggest that the candidate causative variable contributes to changes in the target variable. However, the relationship may be spurious. Figure 1c showed the loading plot for a PCA applied to the detrended data series normalized to unit standard deviation. The lines connecting the origin to GDP and CR respectively point in the same direction showing that tax returns increase with GDP. The anticipated inverse relation between unemployment and GDP is also shown. M2 and CPI are both at almost right angles to the GDP – UE line suggesting that the either are unrelated to GDP and UE, or that their time series are shifted ¼ of a cycle length relative the time series for GDP and UE. (Sine - like time series that are shifted ¼ of a cycle length relative to each other will have zero correlation and show a circle when depicted in a phase plot.)

From Figure 1d it appears that in terms of the key variables in US macroeconomics, the year's 1985 to 1995 are clustering, suggesting that there were no great changes in the economy during this decennial. It corresponds to the period called "The great moderation" in US economy (Fang and Miller 2008).

4.2 Policy variables and GDP responses

The three policy variables, tax returns, Federal funds rate and monetary supply all appear as leading variables to GDP during most of the period 1947 to 2014, Figures 3, 4 and 5. There are two periods where the three variables tend to be lagging variables to GDP, before about 1960 and during the period 1995 to 2000. The first period corresponds approximately to "The soaring 60s", Volcker (1978), and the second corresponds to the end of "The great moderation", (Canarella et al. 2009), (McNown and Seip 2011). The three policy variables and their impact on GDP are discussed below. The same type of figures are used for all three variables: the paired time series, their leading lagging relations, their common cycles, and examples of phase plots showing trajectories during periods where the policy variable leads GDP. For the phase plot of - CR/GDP versus GDP, the regression lines for the full periods and for the three years following the tax shock are shown. For -FF and M2 I calculate the effects of the policy variables on GDP for the three periods where the tax – variable is leading GDP.

4.2.1 Tax changes

The tax shocks defined by Romer and Romer (2010), Mertens and Ravn (2013) and those found by comparing the residual tax changes to the tails of a Gaussian distribution are somewhat different. However, there appear to be high tax "activities" during the periods 1964, 1979- 1983, 1990 -1993 and 2002 - 2003.

Leading- lagging relations. I examined if tax reduction would be followed by a change in GDP by calculating leading- lagging relationships between (minus) –CR/GDP and GDP. The minus sign is introduced because it is anticipate that a tax reduction would affect GDP positively. Results are shown in six panels in Figure 3.

Figure 3 in here (tax return vs. GDP)

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The first panel, Figure 3a, shows the variables GDP, Tax returns; CR and -CR/GDP detrended and normalized to unit standard deviation. Figure 3b shows the leading – lagging relations between (minus) – CR/GDP and GDP. Positive bars shows that (minus) - CR /GDP (x- variable) comes before the GDP (y variable). There are three periods where tax reduction, as -CR/GDP, leads GDP particularly pronounced: from 1964 to about 1975, from 1980 to about 1992 and from 2000 to about 2008. The lower curves in the graph show significant tax reduction or tax increase events. The tax decrease in 1964 initiates a period where tax changes leads GDP. Common cycle characteristics for -CR/GDP and GDP is shown in Figure 3c. Filled circles show running average (9 years) cycle times of around 10 years, and open circles show phase shifts of around 3 – 4 years. As angles in the phase plot are added starting in 1964, the saw tooth line counts time steps. When the angles close a full circle (6. 28 radians) counting restarts. Comparing the saw toot pattern with the time series in Figure 3a, the results seems reasonable, with a long cycle in the beginning and shorter cycles further on. (See also Supplementary material 2.) Phase plots of the time series for -CR/GDP (x-axis) and GDP (y axis) during the periods where tax changes lead GDP are shown in Figure 3d (1964-75), 3e (1980 -92), and 3f (2002-08). It is seen that the trajectories mainly rotate counter-clockwise (positive per definition) showing the –CR/GDP is a leading variable to GDP. Two regression lines have been added. One for the first three years following a tax shock and a second for the full period. It is seen that in all three periods GDP increases with increasing tax reductions in the first three initial years (dashed lines). For the full periods GDP decreases with decreasing taxes. Characteristics for the three periods are shown in Table 2.

Table 2 (Period characteristics)

4.2.2   Federal funds rate

An inverted federal funds rate can be anticipated to be a leading variable in times with bad econo-mies and a lagging variable during times with good economies. During times with bad economies, the Federal reserve decreases the short-term rate and this is supposed to boost the economy, that is, to increase GDP sometimes later (Taylor 1979). The Federal Reserve is then a leader. Conversely, during good economies, the Federal reserve will only act if the economy becomes heated, (Greenspan 2007), thus, GDP, as an expression for the market, will be a leader, and the Federal reserve a fol-lower, (Seip and Gron 2016). The LL- relations found for - FF and GDP suggest that the Federal Re-serve normally is a leader and that the market is a follower. Neither the shocks in FF in 1975 nor in 1982-3 caused a change in LL – relation between FF and GDP. However, the tax shocks in 1964 and 2003 coincides with periods where -FF starts leading GDP. There are also periods where the Fed's rate is low or decreasing.   Cycle times vary between about 5 and 15 years. Results for relationships between - FF and GDP at the beginning of the three "tax- change" periods and for the full periods are shown in Table 2. The FF was raised during the 3 years following a tax shock.

Figure 4 in here (Federal funds rate)

4.2.3   Monetary supply

Monetary supply, M2, is a policy variable used to boost productivity. Figure 5 shows relationships between M2 and GDP. The shocks in 1980-81 do not change the LL – relations between M2 and GDP. The shock in 2009 is too close to the end of the series to allow an assessment. However, the tax shock in 1964 again seems to be associated with the beginning of a period where M2 leads GDP. However, after 2000 both - FF and M2 are leading variables to GDP. Cycle times are about 5 to 10 years in the beginning of the period, but increase to well above 15 years at the end of the period. Re-sults for relationships between M2 and GDP at the beginning of the "tax change" periods (three first

years) and for the full period are shown in Table 2. The detrended M2 was reduced 2 times and

raised 1 time during the 3 years following a tax shock.

Figure 5 in here (M2 and GDP)

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5. DISCUSSION

I first discuss our results on tax change effects and thereafter possible confounding variables. Since

the method is novel in the present context, I thereafter discuss details of the methods used in the

study.

5.1   Effects of tax rate changes

The empirical results reported are based on time series from 1947 to 2014. During this period there

are three major negative tax shocks (reducing the tax burden), and several minor shocks, both nega-

tive and positive. Thus, the empirical material is restricted. However, I try to corroborate the results

by bringing together additional information that may support or weaken the conclusions.

Tax shocks and tax changes as a leading variable. The tax rate level increased from 1964 as a propor-

tion of GDP and has been around 20 % in recent years, c.f., curves in Gale and Samwick (2014).

Thus, the tax rates are lower than the rates in most other modern economies (Wikipedia 2016).

However, the burden on people to pay for services that often are supported by the government

through taxation in other (rich) countries is not that different. (Garfinkel et al. 2006).   Associated

with the major negative shocks in 1964, 1979 and 2003 (reduction in taxes), tax rates became a lead-

ing variable to GDP. These finding supports our *first* hypothesis that tax shocks and leading roles for

the tax rates are associated.

Short time effects on GDP. The negative tax shocks increased GDP over the 3 first years, Figure 3, Table 3. Furthermore, following a negative tax rate shock, cycles are set up that last for 9 to 15 years and complete 1 to 2 cycles. Since tax rate is a leading variable to GDP during these cycles, at least one increase in tax change has to be followed by one decreasing response in GDP. Thus, tax changes have a potential effect on GDP.

Long time effects on GDP. However, calculating the overall effect over the time windows where tax changes are leading GDP, that is, 9 to 15 years, shows that the positive effects of tax reductions are reversed. Assuming that tax reduction is a dominating variable for all three time windows; tax reduction has a negative effect on decadal changes in GDP. (GDP has been adjusted for the multidecadal positive long-term trend 1947-2014.) Thus, our *second* hypothesis were only partially supported. Although tax reduction seems to increase GDP over the short run, over the long run GDP decreases.

The literature report different results with respect to the effects of tax changes and a summary is given in Gale and Samwick (2014). Most studies show that there are positive output effects in the short run (about 3 years), e.g., Mertens and Ravn (2013), Romer and Romer (2010), Hayo and Uhl (2014). However, there may be lower, or negative effects, in the long run (about 10 years), and Gale and Samwick (2014) p. 6 state that the higher deficits and the decline in national savings out weighted the positive effects of reduced marginal tax rates in 2001. Thus, our results on the short term effects correspond with numerical results and narratives reported elsewhere in the literature. Few authors address quantitatively effects, say 5 to 10 years into the future. However, tax changes are a leading variable to GDP over such time spans and probably also contribute a causal effect on monetary mechanisms in the economy.

Confounding factors. An important question is whether other policy variables affect GDP, either alone or in addition to tax changes. Both - FF and M2 may affect GDP over a short and a long time horizon. Although the Federal Funds rate becomes a leading variable to GDP in association with two of the three tax rate shocks changes in FF just after a tax shock do not seem to increase GDP, Table

2. Over a longer horizon, 9 to 15 years, changes in FF both increases and increase GDP. Changes in money supply, M2, seem to increase GDP over the 3 years short horizon, but have mixed effect over the 9 to 15 years long horizon. However, none of the effects are significant at the $p = 0.05$ level.

Multiple regressions. For the three periods where -CR/GDP leads GDP multiple regression were applied with GDP as response variable and -CR/GDP, -FF and M2 as independent variables. The independent variables were moved backward, corresponding to the lag time of GDP relative to the 3 policy variables. The regressions were non- significant, partially because the variables are cyclic and the number of observations are few.

Setting up cycles. In addition to the direct effects on GDP from tax shocks, the shocks also set up regular cycles between the policy variable and GDP. Cyclic periods are typically 9 to 15 years long and include one or two full cycles. With one cycle in a policy variable and GDP, the policy variable has been changed two times with a corresponding response in GDP. The lag time during the first periods were 1 to 2 years, increasing to 2 to 3 years during the recent cycles. Since regular cycles are set up between tax reductions and GDP, reactions to tax reductions may have been politically and economically unavoidable responses in US during the period 1947 to 2014. Thus, our *third* hypothesis was supported, tax shocks initiate cycles in the variables.

Corroboration of the results. The US data only allows identification of three major tax shock reductions, thus the database for making statistical interferences is small. The data are cyclic and not normally distributed, and there are serial correlation and covariances among the variables. There may be methods that can overcome these obstacles, e.g., as in Arin et al. (2013), but gathering information from several findings may be an additional approach for testing the robustness of the results. The association between tax shocks and a leading role for tax changes should support the results. Furthermore, the tax shocks set up persistent cycles with cycle lengths in excess of cycle length that could be anticipated from stochastic movements. Thirdly, it is often possible to "corroborate" the numerical calculation by comparing them visually to characteristics in the time series. Finally, our results are

consistent with narratives on important effects of tax shocks, although there are also contrasting results. The response in GDP for a 1 % change in tax rate is shown in Table 3 and compared to effects of tax rate changes found in the literature. However, tax rate studies often address selected tax rates so the results are not directly comparable.

Table 3 in here (GDP response to 1 %..)

The results should be regarded with caution. The statistical results are not significant ($p > 0.05$), but such non-significance is normal in many tax rate studies, (Barro and Redlick 2011 p. 96). In Mountford and Uhlig (2009), their Figures 2 to 9, the significance depends upon the time horizon for the results. The theories on how tax changes affect the economy through different mechanisms are many, but they are not established within reasonable doubt. The empirical material is scant and the possibility that confounding factors are responsible for changes in GDP that appear to be caused by tax rate changes can not be excluded.

5.2   Methods

I first discuss the multidecadal detrending applied to the data and then the LL –method.

5.2.1 Detrending

I detrended the variables that showed a significant 2nd order polynomial trend reflecting the increasing rate of GDP growth with time that may have been caused by e.g., technical and management innovations. This applies to GDP, Current recipes, CR, and monetary supply, M2. Detrending in this way is a normal procedure in global warming studies where decadal, multidecadal and centennial processes often are addressed independently, and with references to different physical mechanisms, e.g., Wu et al. (2011). There may also in economic time series be processes that act on different time scales.

5.2.2 LL- relations

The methods applied to the empirical material are partly novel in the present context. In particular, the LL- method for identifying running leading lagging relations are novel.

The present author and coworkers have applied the LL- method in several contexts, e.g. economics: Seip and McNown (2007), ecology, Seip (2015), global warming,.(Seip and Grøn 2017). Compared to these applications, the method to identify common cycle lengths by examining the number of time steps required to close cycles in phase plots is novel.

5.2.3   Policy implications

Our study add empirical evidence that reducing the overall tax rate increases GDP in the short run, but decreases GDP in the long run (≈ 10 years). However, since both the tax rate and GDP are aggregated measures, it could be interestring to apply the method to more specific components of both tax rates and GDP emphasizing different effects of taxation. Suggested pairs presented during the American economic association, San Francisco January 2016 may be tax rate on the foreign profits of U.S. multinational firms versus industrial production, Salvatore (2016), tax rates that affect incentives versus productivity growth, Taylor (2016), and taxes on the rich versus infrastructure, Stiglitz (2016). An added advantage of the method used here is that one get a measure of the probable duration of tax policy effects.

6. CONCLUSION

This study presents a new approach for distinguishing the effects of tax rate shocks on the gross domestic product, GDP. The method identifies leading – lagging relations between the policy variables, e.g., tax rates and GDP. A basic assumption is that tax rate shocks make the tax rate a leading variable to GDP. The study applies the method to the post –war data on the US economy.

I found that pronounced negative tax shocks (decrease in the tax rate) in 1964, 1980 and 2002 increased GDP for the first 3 years. Response to 1 % tax reduction were 0.48 to 0.77 % of GDP above a

positive, multidecadal trend. The tax rate shocks set up cycles where tax rates are a leading variable to GDP for 9 to 15 years, completing 1 to 2 cycles. For these periods, the average GDP responded to changes in the tax rate with a reduction of 0.23% to 0.50 %.

Like many studies on the effects of tax rates on the economy, it is difficult to obtain results at the p < 0.05 level, e.g., Barro and Redlick (2011). However, factors have been identified that could corroborate or detract from the result, such as the association between tax shocks and the leading relation of tax rates to GDP. In conclusion, the present study shows that tax rate shocks may have a slightly stimulating effect on the economy over a short time horizon (≈3 years), but a slightly negative effect in the long run (≈ 10 years).

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Tables

Table 1

Tax shocks reported in the literature and shocks calculated as "outliers" when the distribution of tax changes are compared to a normal distribution. The series are ordered approximately sequentially. Shocks in FF and in M2 are estimated as outliers relative to a normal distribution.

Time window	Tax rates, CR			FF	M2
	Mertens and Ravn (2013)	Romer and Romer (2010)	Outliers	Outliers	Outliers
1	1954 (-0.28)	1955 (-0.3)			
2	1964 (-2.5)	1964 (-1.2)			
3	1972 (-0.6)	1972 (-0.7)	1974 (-0.26),		
4	1976 (0.51)	-		1975 (-2.23)	
5	1977 (-0.43)	1977 (0.1)			
6	1979 (-1.33)	1979 (-0.4)	1979 (-0.51)	1982 (-1.9)	1980 (0.81)
7	1981 (-0.05)	1981 (0.6)	1980-83 (-0.81,-0.64, -0.25)	1983 (-1.5)	1981 (0.64)
8	1991 (0.87)	1991 (0.3)	1990 (- 0.30)		
9	1993 (1.15)	1994 (0.3)	1998 (+0.32)		
10	2003 (-1.88)	2002 (-0.8)	2002 (+0.33)		
11		2003 (-1.2)			
12			2009-10 (+0.85, +0.33)		2009 (0.85)
13			2013-14 (+0.39, + 0.35)		

Table 2 Characteristics of periods where tax changes as -CR /GDP, leads GDP in US economy from about 1950 to 2014. CR = Government Current recips, GDP = gross domestic product, FF = Federal funds rate, M2 = Monetary supply. The β – coefficient is defined by GDP = β (policy variable) + constant. All variables are normalized to unit standard deviation.

Periods	Parameter	Policy variable		
		-CR/GDP	-FF	M2
1964 –1975, n = 12; Tax shock 1964 (-2.5)	B - coeff. (64-66)	+3.35	-1.44	+1.42
	β –coeff. (trend 64-75)	-1.48	+0.82	-0.49
	Cycle length, years	9.3	5.5	5,5
	# cycles	2.1	0.8	2.0
1980-1992 n = 13; Tax shock 1979 (-1.3)	B - coeff. (79-81)	0.88	-0.47	+2.44
	B – coeff. (trend, 80-92)	-1.72	+0.21	+0.90
	Cycle length, years	9.3	9.9	12
	# cycles	2.5	1.3	0.9
2000 - 2008 n= 9; Tax shock 2003 (-1.88)	B - coeff. (02-04)	0.69	-2.24	+1.10
	B -coeff. (trend, 00-08)	-0.29	-0.59	-6.30
	Cycle length, years	14.7	13.6	13.9
	# cycles	1.0	0.8	0.9

Table 3. GDP response to 1 % reduction in tax rates. Response to changes in tax rates may be non-lin-ear; I therefore sometimes quote examples found in the texts that summarizes results. The time series in the studies are, with the exception of Arin et al. (2013), from US economy about 1950 to 2010. Arine et al refer to several economies, e.g., US, UK, Nordic countries.

Type tax rate change	Short term changes ≈ 3 years	Long term changes ≈ 10 years	Source
Average marginal personal tax rate (1)	-	0,6 %	Arin et al. (2013) p. 894 example
Average marginal income tax rates (2)	0,5 % (1 year), Multi-plier: -1.1	-	Barro and Redlick (2011)
Average personal income tax rate	1.4 % at Q1; 1.8 % at Q3	-	Mertens and Ravn (2013), p. 1227
Average corporate income tax rate, decrease	0,4 % at 1Q; 0.6 % at Q4	-	Mertens and Ravn (2013), p. 1228
Deficit financed tax cut (3)	Present value multiplier: 0.29 at Q1; 5.25 at Q12	Present value multiplier: -4.55 at Q20	Mountford and Uhlig (2009)
Legislated tax liabilities	0.5 % at Q1; 3 % at Q10		Romer and Romer (2010), their Fig 4
Current tax receipts(4)	1964:0.77%, 1980: 0.48 % 2002: 0.53 %	1964: 12 years:-0.23%, 1980:13 years: -0.23%, 2002: 9 years: -0.50%	present work

(1) Taxes are most distorting at relatively moderate tax rates (≈ 20 % tax rate)
(2) Including state taxes but excludes most forms of capital taxes
(3) Multiplier for GDP =(GDP response/Initial fiscal shock) / (Average fiscal variable share of GDP)
(4) Local changes in GDP following changes in (minus) current receipts divide with GDP, -CR/GDP. Series are detrended, thus multidecadal changes in GDP is not included.

Figures

*Figure 1 Time series used in the present study. a) Raw data; b) Data detrended and normal-
ized to unit standard deviation. c) PCA, loading plot for the variables. Long-term relations be-
tween GDP and fiscal policy variables. d) PCA score plot for the samples. CR is Tax returns, -
CR/GDP (minus sign) is tax reduction, GDP is gross domestic product, M2 is monetary supply,
FF is Federal funds rate, CPI is Consumer price index, and UE is unemployment rate. PC1 Ex-
plains 42 % of the variance and PC2 explains 24 % of the variance in the data set.*
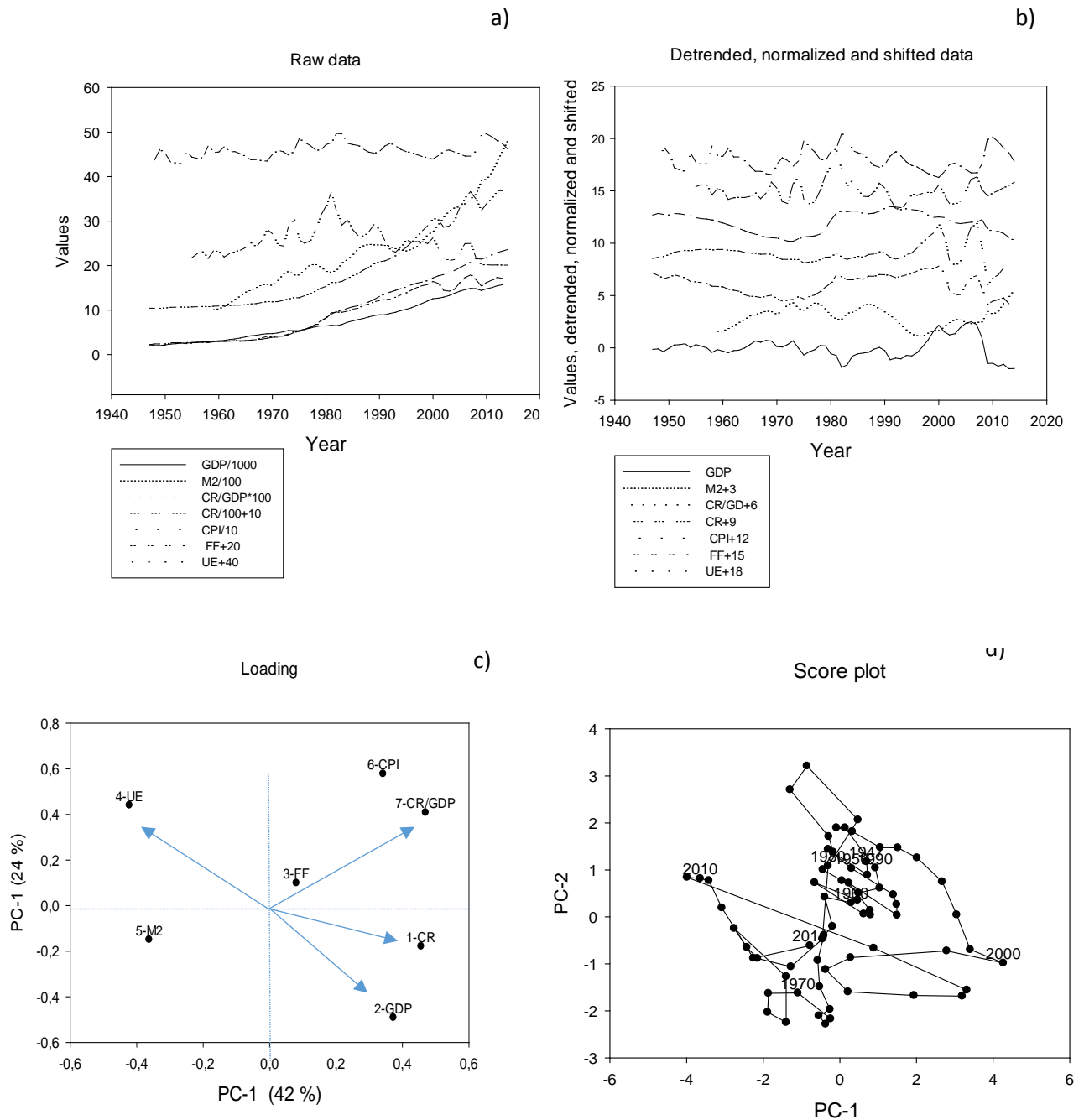
Figure 2 Tax reduction (x) and GDP (y) represented by two sines, the tax sine contains a random component. a) Sine x peaks roughly before sine y. b) Their phase representation with sine x on the x-axis and sine y on the y-axis. Their trajectories rotate roughly counter clockwise (or positively, per definition). c) The angles between two consecutive trajectories in phase space, (black columns) and LL –strength (gray columns). d) Calculation of cycle lengths with the "average" method (filled circles) and with the "closed cycle" method.

a)

b)

Sine (0,5 t) and sine (0.5 t+0.785* RAND())

Phase plot

x = Sin(0.5 t + 0.785*RAND()*0.1.5)

c)

d)

Pos. bars: x leading y
Neg. bars Y leading x

Cycle length
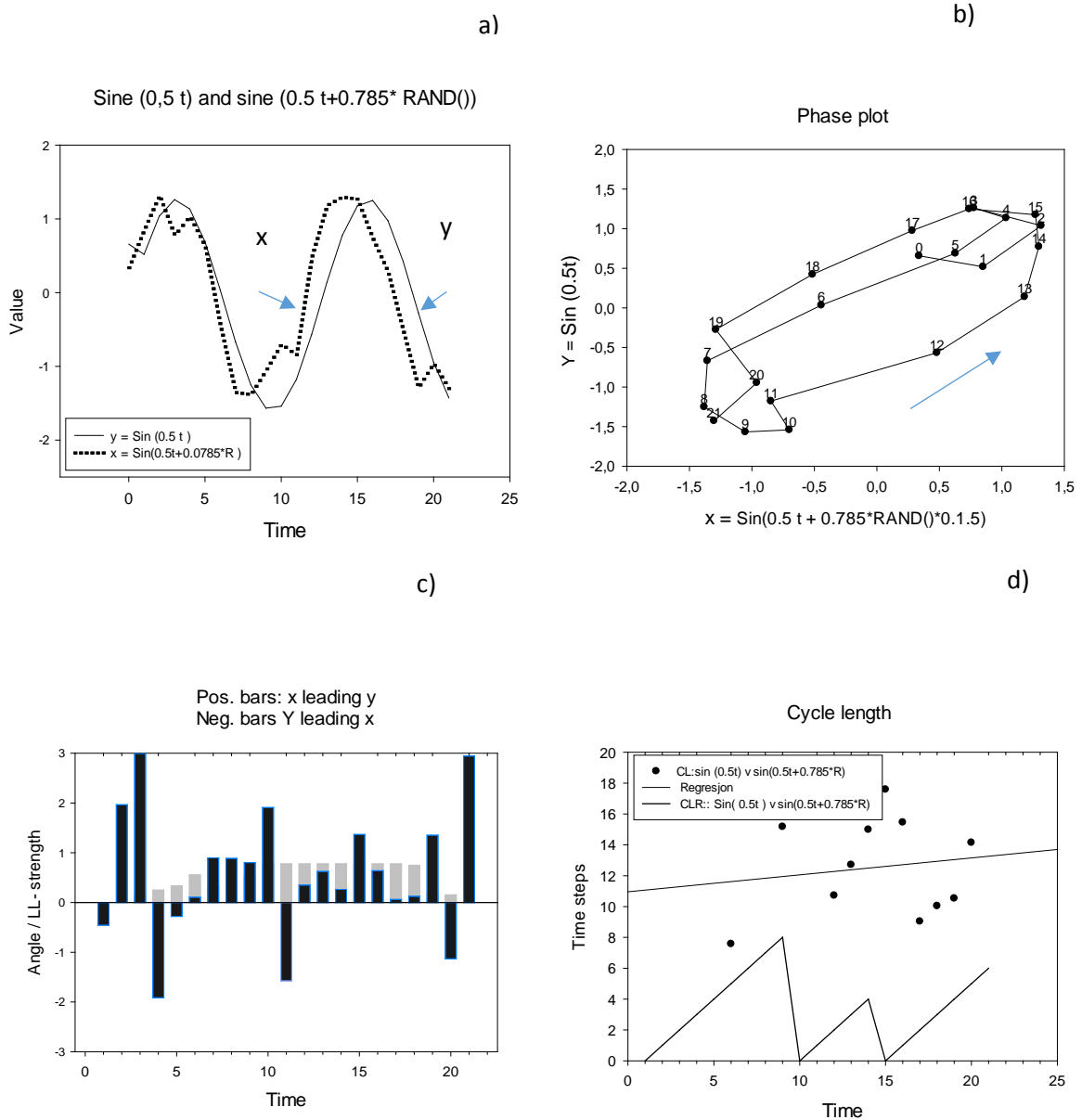
Figure 3 Tax changes and effects on GDP. a) GDP, Current tax recips, CR, and relative recips, - CR/GDP detrended and normalized to unit standard deviation. b) Leading- lagging relations between tax rates and GDP. Positive bars means that tax changes are leading GDP. Grey and black fillings show LL- strength and rotational angles respectively. Dashed lines shows confidence estimates. Lower curves show strong tax changes as reported by Romer and Romer (2010) full line and Mertens and Ravn (2013), dashed line. c). Cycle time and lag times. Saw toothed line is cumulative time steps until a circle (6.28 radians) are completed. Line at the bottom is β– coefficient for GDP = f(– CR/GDP). d) Phase plot for tax reductions (x-axis) and GDP (y-axis) during the period 1964-1975. Full line is regression line for scatter plot, dashed line is regression line for the 3 years starting with a tax shock. e) Phase plot for tax changes (x-axis) and GDP (y-axis) during the period 1980-1992. f) Phase plot for tax changes (x-axis) and GDP (y-axis) during the period 2000-2008.

a)

CR, GDP and (m) -CR/GDP

b)

**Pos. bars: -tax rates/GDP (x) leading GDP (y);
Neg. bars: -tax rates/GDP lagging GDP
Lines: tax shocks**

c)

**Cycle times and lead- lag times for (m)-CR/GDP (x) vs. GDP(y)
running ☐ - coefficient**

d)

-CR/GDP vs. GDP; 1964-75

e)

-CR/GDP vs. GDP; 1980 - 1992

f)

-CR/GDP vs. GDP; 2000 - 2008
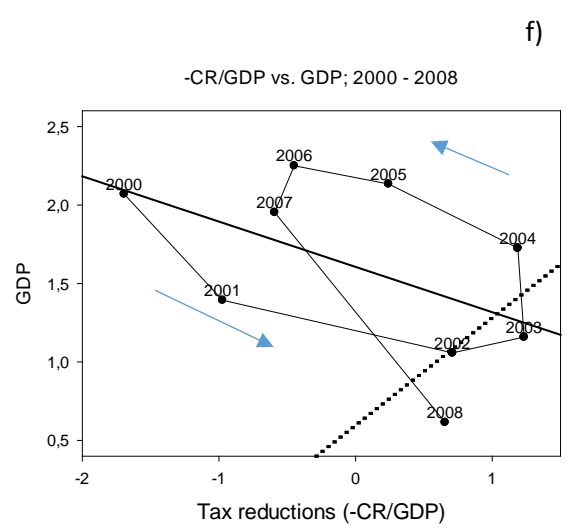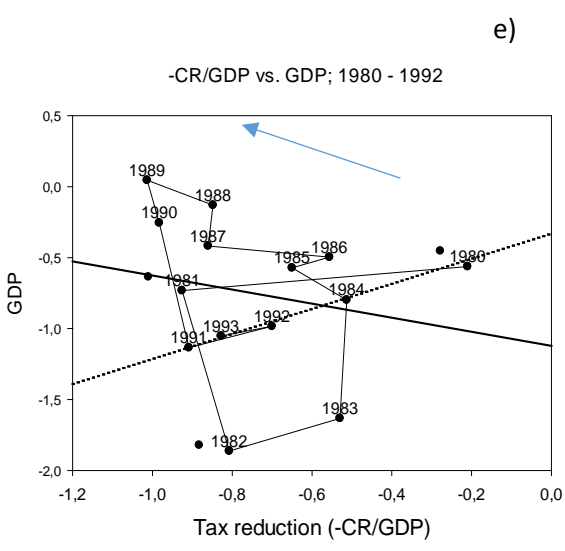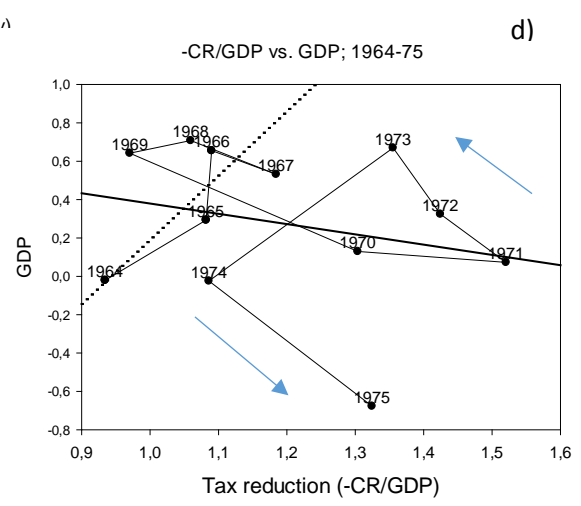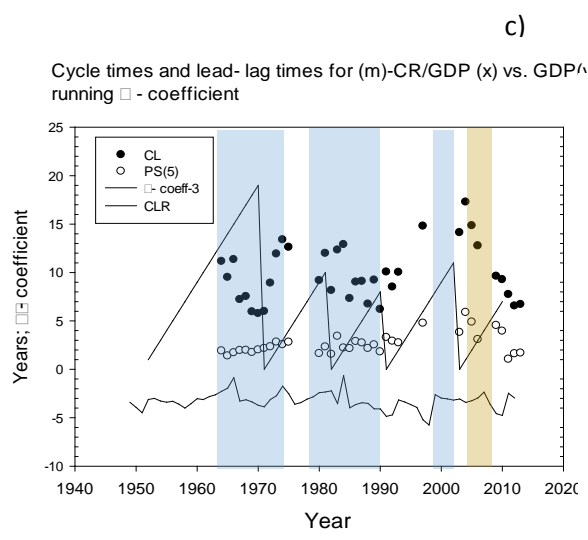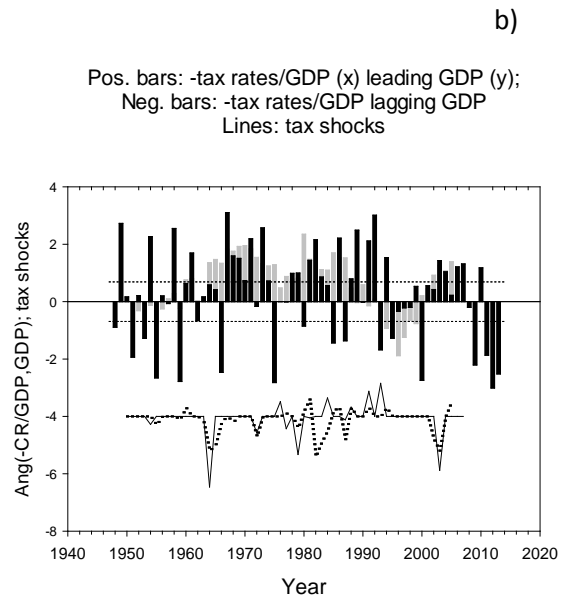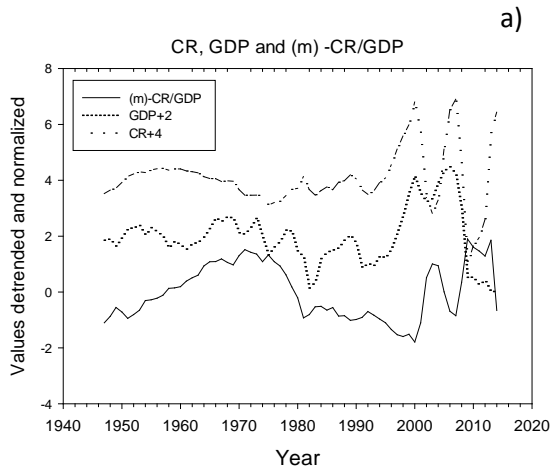
Figure 4 Federal funds rate and effects on GDP. a) Reductions in Federal funds rate, - FF and GDP.
GDP detrended and smoothed (LOWESS, 0.2,2), both variables centered and normalized to unit stand-
ard deviation. GDP shifted up. b) Leading – lagging relation between –FF and GDP. c) Cycle times and
β – coefficient GDP = f (FF). (Legends as in Figure 3) d) Phase plot for –FF and GDP for the periods
1970 -90 (thin line) and 1990 -93 (bold line). (Legends as in Figure 3)
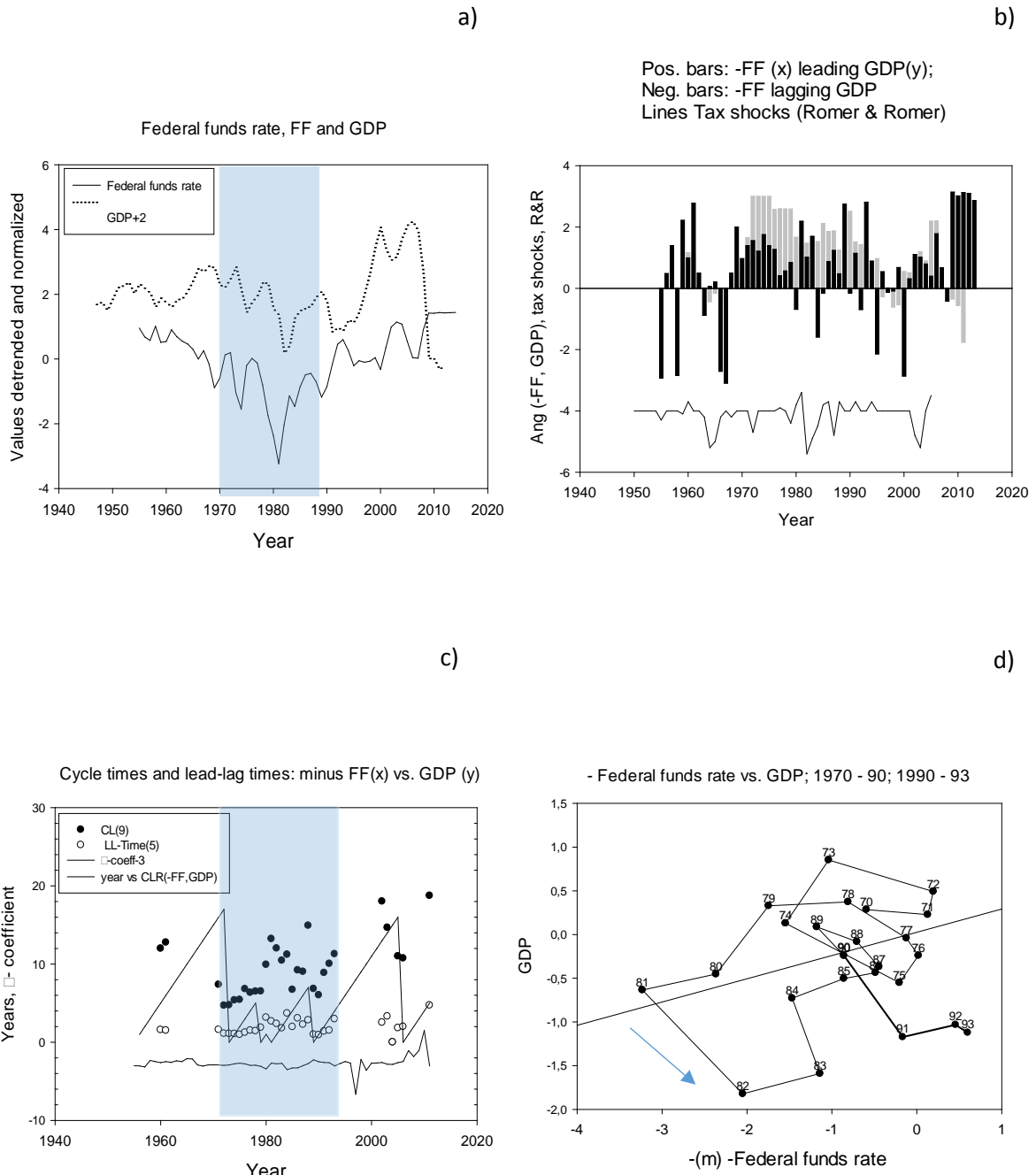
Figure 5 Monetary supply versus GDP. a) Monetary supply, M2 and GDP, detrended, smoothed and normalized to unit standard deviation. b) Leading – lagging relations between M2 and GDP. Lower line is tax rate changes after Romer and Romer (2010) . c) Cycle times, phase shifts and β- coefficient for GDP = f (M2). Saw toothed line is cumulative time steps until a circle (6.28 radians) are completed. d) Section of phase plot for M2 versus GDP during the period 1964 to 1986

a)

b)

c)

d)
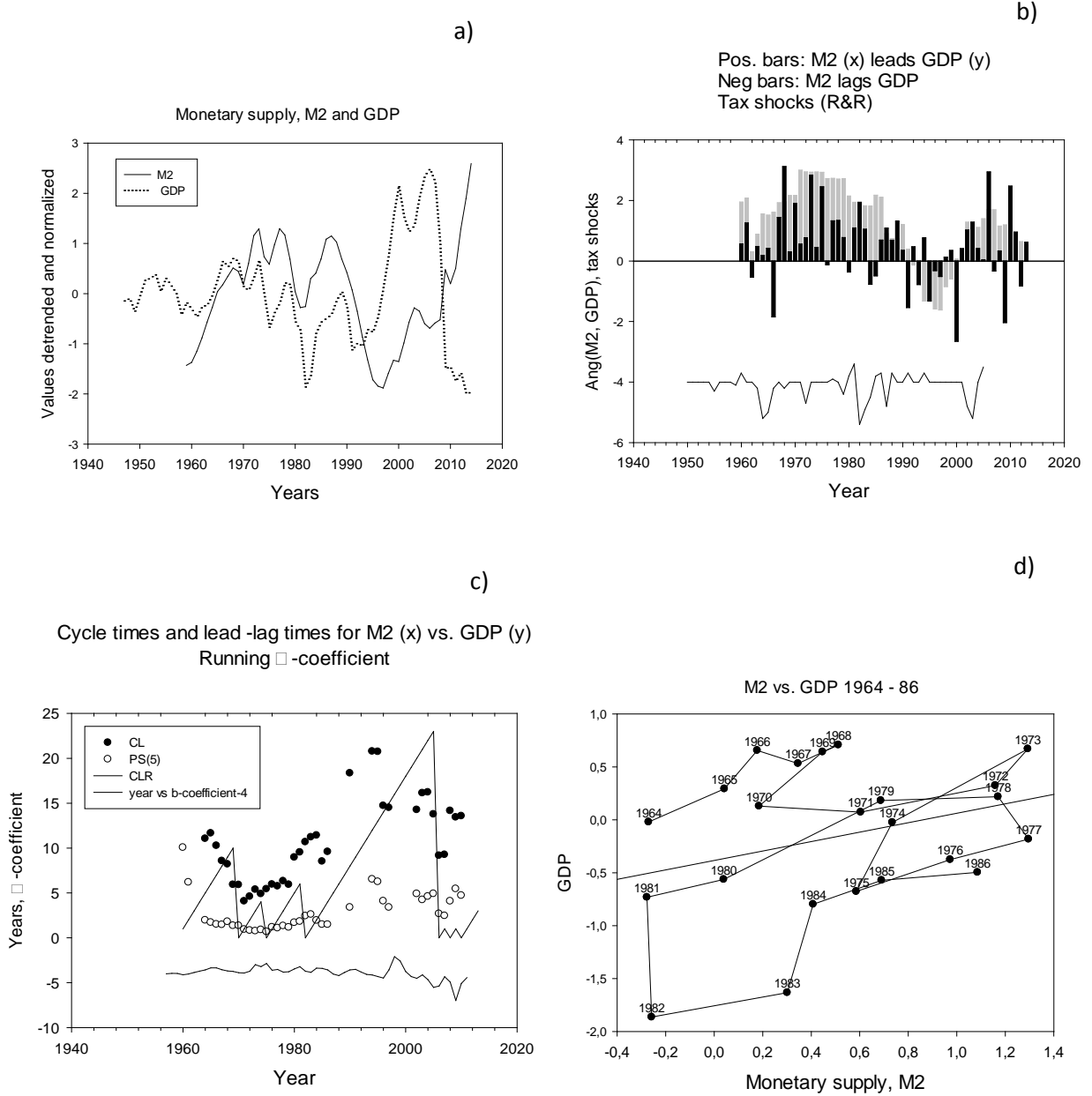
Supplementary material 1; Shocks in time series

*Figure S1. Shocks in time series. All series have been normalized to unit standard deviation. The derivative of the series are compared to a fitted normal distribution. **a)** Values less than -0.2 and values higher than + 0.3 are shocks in the (minus) tax rate series – CR/GDP, (CR and GDP both detrended with a second order polynomial function) 1974 (-0.26), 79-82 (-0.25—0.81), 90 (-0.3), 98 (+0.32), 2002 (+0.33), 2009-10 (+0.85), 2013-14 (0.35 - 0.38). **b)** There are two negative - FF shocks at -2 and one at -1: 1975 (-2.23), 1982 (-1.9), 1983 (-1.5). **c)** There are one negative M2 shock at -0.8 and two positive shocks at > 0.6: 2009 (-0.85), 1980 (0.81), 1981 (0.64).*
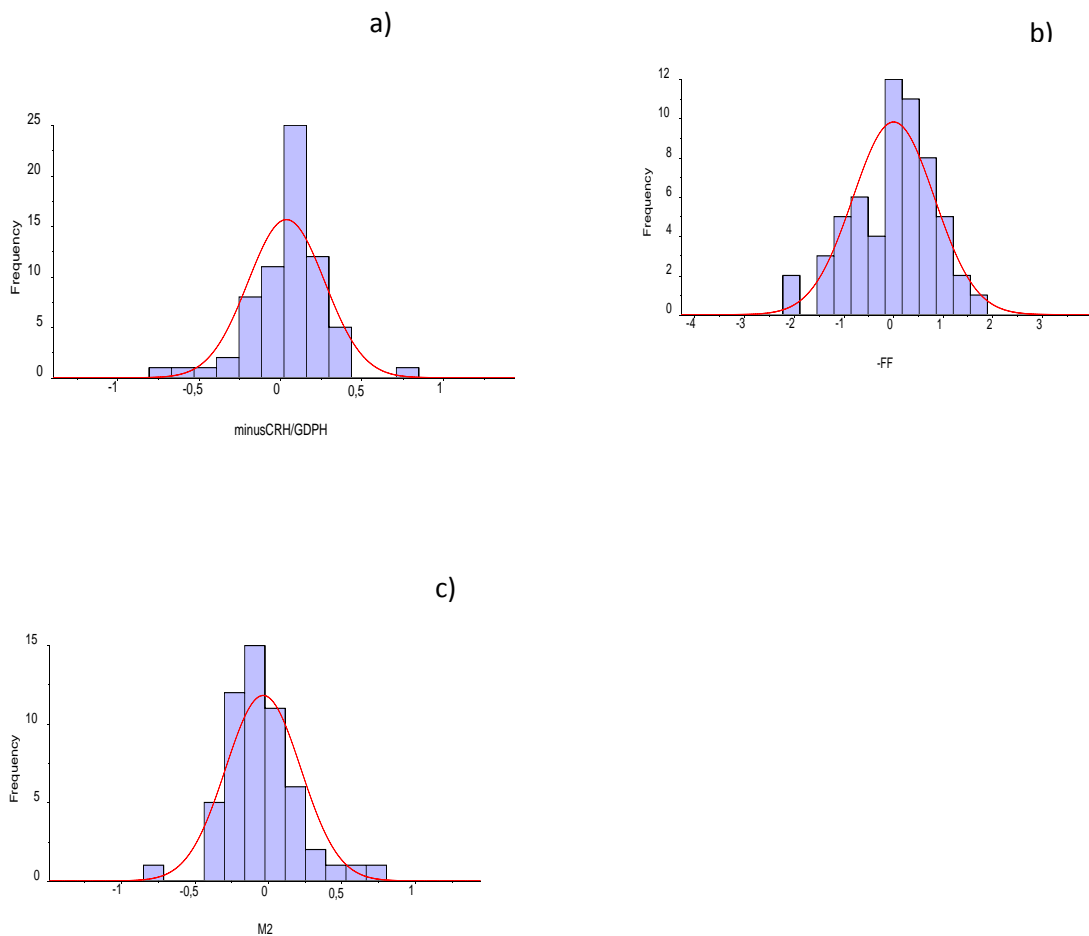
Supplementary material 2. Extracting common cycle times from synthetic time series

By pairing a complex time series (a superposition of two cyclic series with different cycle lengths) with a simple series that contain only one of the cycle lengths, but which is shifted relative to the same component in the complex series, I identify the cycle that correspond to the cycle of the simple series. The problem is characteristic for a situation where one cyclic mechanism contributes an effect on a response variable.

Figure S2 Additive sines. a) The sine functions, Upper sine = sin (0.2t), middle sine = (0.1t), lowere sine (sin(0.1t)+sin(0.2t). Extracting with sine (0.1t), "the long cycles". b) The sine functions as in a. Extracting with sin(0.2t). c) Leading lagging relations for the a - case, d) leading –lagging relations for the b –case. e) cycle lengths (average method gives ≈ 43, closed circle ≈ 64 (correct), f) Extracting with sine (0.2t) "the short cycles", average method gives ≈ 36, closed circle 32 (correct).
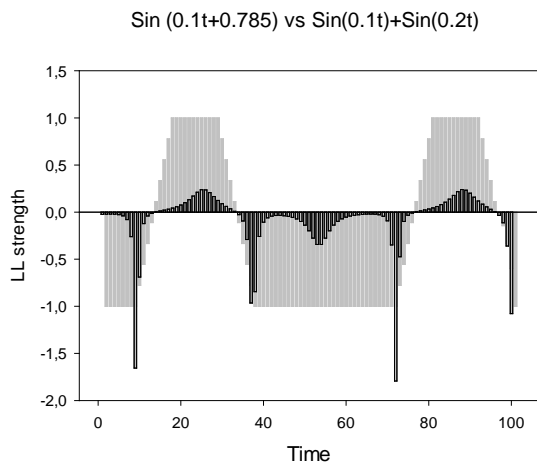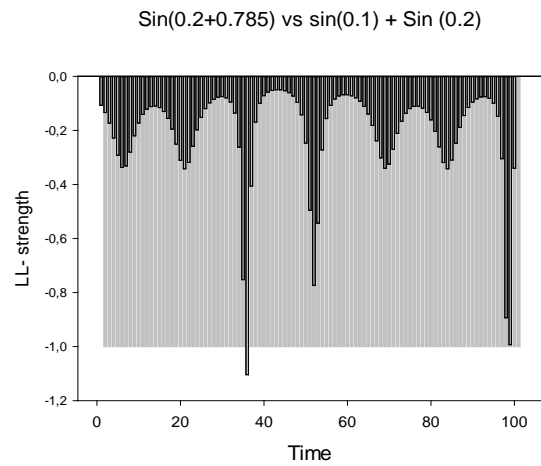
a)

Syntheic time series, sin(0.1 t) and sin (0.2t) added

b)

Synthetic time series, sin(0.1t) and sin(0.2t) added;
Extracting sin(0.1t)

c)

Sin (0.1t+0.785) vs Sin(0.1t)+Sin(0.2t)

d)

Sin(0.2+0.785) vs sin(0.1) + Sin (0.2)
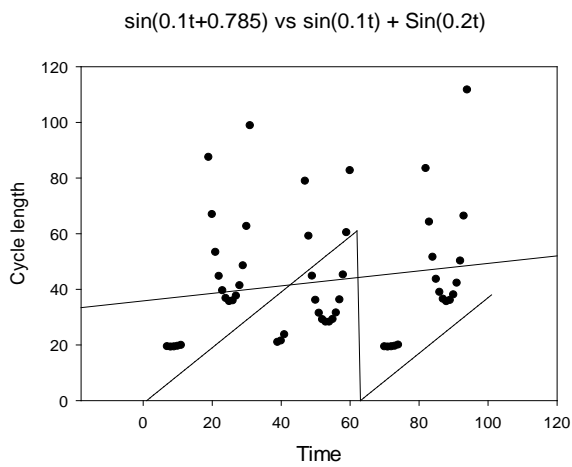
e)

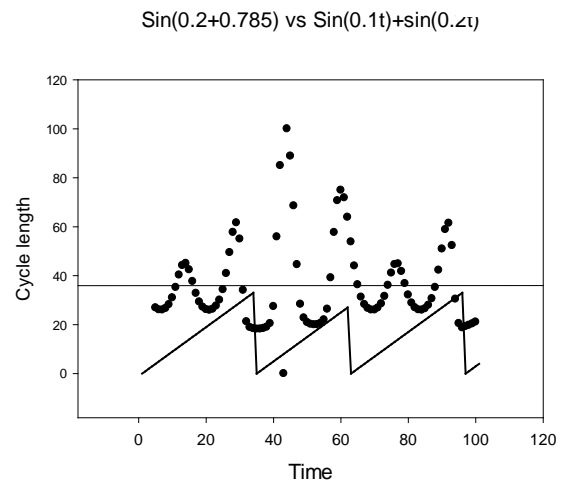sin(0.1t+0.785) vs sin(0.1t) + Sin(0.2t)
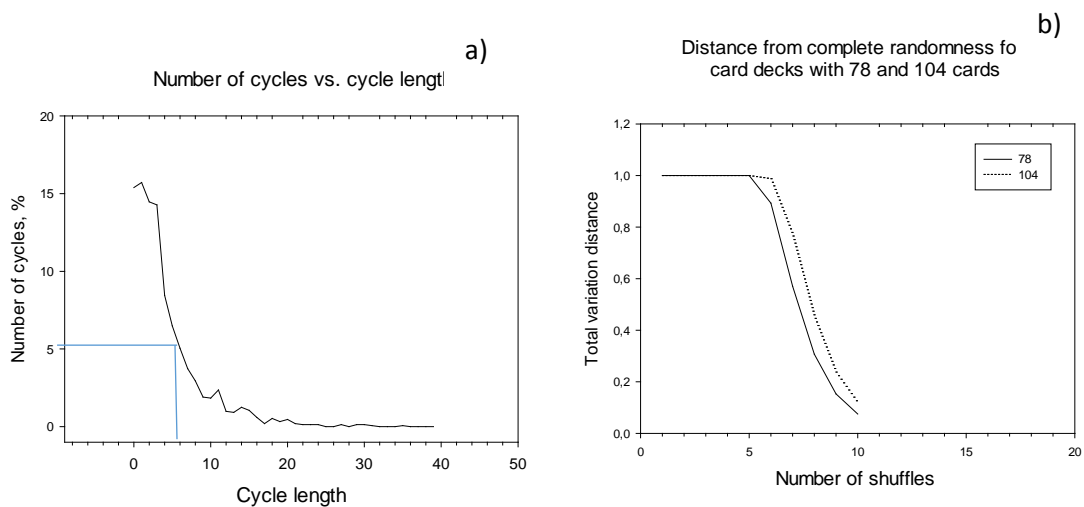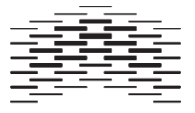
f)

Sin(0.2+0.785) vs Sin(0.1t)+sin(0.2t)
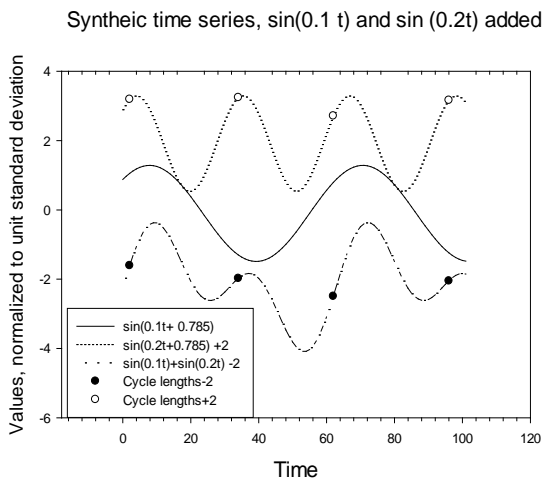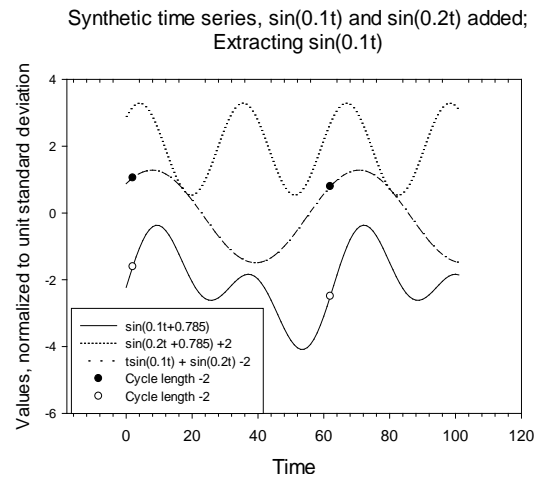
Supplementary material 3. The probability that two paired uniformly random series show leading lagging signature over a certain cycle length.

Figure S3 shows the percentage number of cycles as a function of cycle length in time series 100 time steps long. Calculations are repeated 100 times. The 5% confidence estimate suggests that cycle lengths > 7 occur less frequently than 5 % of the times. Shuffeling card decks may have the same characteristics, Mann (1994) since both processes involves uniform stochastic distributions.

Figure S3. a) Number of cycles of a given length found by pairing two uniformly random time series and calculating common cycle times, b) number of shuffles required to reach a certain distance from complete randomness. After Bayer and Diaconis (1992) p. 309