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Pre-service teachers' understandings of exploratory task design in mathematics: from GeoGebra task design to the 8th grade student

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ABSTRACT
The aim of the study informing this paper was to investigate pre-service teachers' (PSTs') approaches to task design, posing the research question: How do pre-service teachers respond when they are asked to design an exploratory task using GeoGebra for 8th grade students? The research study and this paper focus on the PSTs' design stories rather than their task formulations. Drawing on multiple case study design, the study's sample consisted of 54 PSTs from a Norwegian teacher education program in mathematics for lower secondary school. Once an initial analysis and coding of the tasks was complete, a group of 13 PSTs were interviewed about their own task design, with some of these PSTs taking part in a focus group discussion as well. In this paper, six representative cases of PST task design are presented and analyzed together with additional support from the rest of the interviews and focus groups. Findings revealed four key themes in their design stories: the knowledge of 8th graders, guiding or exploring, use of context or not, and the role of GeoGebra. Each of these themes is discussed with an aim of highlighting key implications for PST task design and the education of new mathematics teachers.

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Introduction

Inquiring and exploring in mathematics are vital approaches in the teaching and learning of school mathematics, in addition to being a recommended focus in the education of school mathematics teachers (Jaworski, 2006, 2019). In the Norwegian context of the study described in this paper, clear definitions for what explore means (in general and in mathematics) can be found in Norway curriculum guides as follows:

> Exploring is about experiencing and experimenting and can maintain curiosity and wonder. Exploring can mean sensing, searching, discovering, observing, and scrutinizing. In some cases, this means examining different aspects of an issue through open and critical discussion. Exploring can also mean testing or testing and evaluating working methods, products or equipment (Author's translation) (Kunnskapsdepartementet, 2018)

> Exploring in mathematics involves students searching according to patterns, finding connections and discussing their way to a common understanding. The students should place more emphasis on the strategies and approaches rather than on the solutions. (Author's translation) (Kunnskapsdepartementet, 2019)

Yet, these curriculum intentions around exploring are not always well-understood by teachers or teacher education students (pre-service teachers, PSTs) regarding what exploring looks like in mathematics classrooms, within given contexts, and when provided with various tools (Chapman, 2017; Paolucci & Wessels, 2017). In this paper, we report on a research study conducted with a group of PSTs (who were enrolled in a Norwegian university teacher education program) as they shared their thinking about how

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and why they created specific tasks when engaged in a design activity to explore linear functions. We chose a multiple case study design to address the study's research question of: How do pre-service teachers respond when they are asked to design an exploratory task using GeoGebra for 8th grade students? Here, we first set the context for the study by describing the task design activity assigned to PSTs and the process for how their responses (i.e. the tasks they designed) were analysed and coded. Then, based on the tasks and codes, we present the four phases of our multiple case study, eventually narrowing to the cases of six participant PSTs and the stories behind the tasks they chose to design. Inspired by their stories, we suggest several themes which point to important implications for the role of PST problem posing and exploring in mathematics teacher education programs. Thus, the significance of the study is in how it highlights the need to ask PSTs to *design*, not only *solve*, mathematical tasks as a means to developing and reflecting on their own conceptual understanding and their use of dynamic geometry tools as both learners and future teachers of mathematics (Hartmann, et al., 2021; Yao et al., 2021).

Background and context

An inspiration for this study was the first author's (Hovik's) review of Norwegian PST responses to a specific question on the 2021 National Exam. Through a national exam, PSTs in all of Norway's teacher education institutions are tested to find out what level of knowledge they possess about algebra and algebraic thinking.[1] Knowledge about functions is a part of this, and dynamic geometry learning environments (DGE) such as GeoGebra are seen to develop such knowledge. The 2021 National Exam task design question was given as follows:

> 8th grade students work to explore graphs of linear functions.
>
> Design a task where the students are supposed to use a dynamic geometric program to explore the gradient and the constant term of linear functions. Give a solution. The suggested solution must include pictures from the exploration in a dynamic geometric program. (Author's translation)[2]

Hovik was involved in grading the 199 submissions in this 2021 National Exam. She found the grading results on this special task to be a surprise and a curiosity, primarily because so few PSTs had produced an acceptable response (that is, an acceptable task design). In reading and analyzing each of the 199 responses and based on her experience working with the national exam grading guidelines from the exam committee, Hovik developed seven codes (A- G as shown in Table 1) which were used to describe the main categories of solutions. To be clear, these codes relate to the type of tasks PSTs chose to *design*, not their suggested *solutions* to the tasks. According to the grading guidelines, supplied to Hovik when serving on the exam committee, two main strategies were to receive a score (points on the exam): Either by asking the 8th grade students to look at some given functions with a rich variation in values of a and b (code B) or asking them to use the slider tool in GeoGebra and then explore what happens varying the values of a and b (code A). Table 1 provides all seven codes (A-G) developed by

Table 1. The main categories (assigned codes) from the 199 PSTs taking the 2021 National Exam.

Categories (codes)	Explanation	Number	Percentage
A	Task uses sliders with a general equation: $y = ax + b$, exploring what happens when varying the values of a and b.	42	21 %
B	Task gives a selection of functional equations with different values of a and b, using GeoGebra for drawing.	20	10 %
C	Task gives a word problem with a constant (y-intercept) and a variable (gradient) context, using GeoGebra for drawing, either directly from the word problem text or asking for the equation first.	70	35 %
D	Task gives a specific equation, finds the gradient and the constant term before or after using GeoGebra for drawing the graph.	14	7 %
E	Task gives some points, draws a linear graph in GeoGebra, finds the constant term (y-intercept) and the gradient, eventually comparing with the use of a formula for calculating the gradient and the constant term.	3	2 %
F	Others	22	11 %
G	No answer	28	14 %
		199	

Hovik, along with a brief explanation of each code; this is followed by the number and percentage of PST solutions receiving that code.

The results from this national exam solution analysis showed that, of these 199 responses, only 31% were coded as A or B combined, while 35% were coded as C. To investigate why so many PSTs designed tasks that are word problems based on real-life problems (especially, as it turned out, based on the context of taxi-trips), we designed a study on which this paper is based. The study was guided by the following research question, and sub-questions:

How do pre-service teachers (PSTs) respond when they are asked to design an exploratory task using GeoGebra for 8th grade students?

- Do they use GeoGebra to explore or to confirm?
- What do they understand exploring to mean?
- How do they explain their own responses (tasks) in retrospect?

Literature

Based on our research question and sub-questions above, we focus our attention on a review of the literature on task design research—research in general through to task design using a dynamic geometric environment (DGE) (such as GeoGebra)—in the field of mathematics teacher education. We divide our review of the literature into four distinct areas: mathematical task design research; tasks designed by practicing teachers; tasks designed by pre-service teachers; and tasks designed using GeoGebra.

Before delving into each of these four areas, it is important to first clarify that 'task design' is defined in different ways across the research, with Jones and Pepin (2016) offering one comprehensive and inclusive definition. These authors use the term task design to refer to:

> … mathematical tasks (including tasks in the form of digital resources and tools) that are developed and designed in, or for, mathematics teaching, or in, or for, mathematics teacher education. Hence, task design could include designing tasks for teaching specific mathematical topics to specific learners, designing tasks for textbooks (including digital platforms and e-books), designing learning sequences, and designing tasks for the professional learning of mathematics teachers. (p. 107)

For our study and review of the literature, we consider the terms of task design and problem-posing to be synonymous. Although we fully understand the many nuances distinguishing these two terms from one another, both are generally understood to involve the generation and/or modification of new problems from provided information and contexts (Chapman, 2012; Silver, 1994). In other words, 'problem posers have to appropriately combine problem contexts with key concepts and structures in solutions along with constraints and requirements in the task' (Chapman, 2012, p. 137). Zhang and Cai (2021) offer that 'mathematical problem posing is the process of formulating and expressing a problem within the domain of mathematics … the focus in problem posing is on the generation of problems based on situations, and problems are the objects of study' (p. 962). These authors draw on Cai and Jiang (2017), which 'identified four common types of problem-posing tasks, as follows: posing a problem that matches the given or specific kinds of arithmetic operations; posing variations on a question with the same mathematical relationship or structure; posing additional questions based on the given information and a sample question; and posing questions based on given information' (Zhang & Cai, 2021, p. 964). In other research, Chapman (2012) identifies nine categories of problem-posing tasks including, for example, posing problems which are open-ended, similar to another problem, or reformulating a given problem. We return to these ideas in a later section.

Mathematical task design research

Task or classroom design research is a genre of qualitative research that, generally, gathers data in classroom settings, often via a collaboration of researchers and classroom teachers. The primary aim of task design research is to improve both teaching practice and student learning by creating a stable instructional sequence for a topic which will help students learn that topic with meaning (Stephan, 2015). The instructional sequence is designed, tried and revised in a classroom, with strong potential for it to be adapted by other teachers. According to Watson and Ohtani (2015):

> Attention to task design is important from several perspectives in mathematics education research and practice. From a cognitive perspective, the detail and content of tasks have a significant effect on learning; from a cultural perspective, tasks shape the learners' experience of the subject and their understanding of the nature of mathematical activity; from a practical perspective, tasks are the bedrock of classroom life, the 'things to do'. (p. 3)

Jones and Pepin (2016) point to research that attempts to define and elaborate on design research in mathematics education, also referred to in some contexts as didactical design (Ruthven & Hofmann, 2013; Sensevy, 2012). Commonly, in design research, mathematical tasks are designed by researchers and/or curriculum developers and are then implemented by teachers in classrooms with their students. However, research also points to more creative ways to partner with teachers on task design so that the acts of task design and teaching are not viewed as separate, but instead provide opportunities where 'the teachers develop their *design capacity* and, in turn, their *agency* by working with mathematical curriculum materials/tasks' (Jones & Pepin, 2016, p. 109). One way to feature these kinds of partnerships is through offering professional development programs for teachers (Lee & Özgün-Koca, 2016; Stephan, 2015; Watson & Mason, 2007), though it should be noted that a good number of such professional development programs engage the teacher in discussing, implementing, revising, and reflecting on *already developed* tasks. In the next section, we focus explicitly on research where teachers play more active roles in designing the task or creating/posing the problem.

As can be noted in the definition of task design presented above, Jones and Pepin (2016) refer to task design in the context of both mathematics teaching in schools and mathematics teacher education. The next two sections address each of these contexts separately by outlining relevant research in each.

Mathematical tasks designed/posed by practicing teachers

According to research, the kinds of tasks teachers design and use with their students is strongly related to teachers' perceptions of mathematics teaching and learning (Jones & Pepin, 2016), as well as being 'influenced by the teacher's goals, subject matter knowledge, and knowledge of students and student thinking and understanding' (Lee & Özgün-Koca, 2016, p. 13). In research by Jones and Pepin (2016), the authors 'argue that 'task design' needs to pay particular attention to *what* to design, *which* tools are necessary or beneficial, and under *what* conditions; digital tools and task resources offer particular affordances that traditional resources cannot provide' (p. 105).

Even though our study was conducted with PSTs, we note here how Zhang and Cai's (2021) study with mathematics teachers is also relevant for the context of PSTs in mathematics teacher education programs. Zhang and Cai (2021) advocate for further research focused on developing 'instructional strategies for handling posed problems in problem-posing lessons' (p. 970). Such research, they claim, would help teachers understand, process, and assess the relevance and level of posed problems. In turn, educating PSTs using these instructional strategies could make a difference in their own problem-posing capabilities as well as with their own (future) students

Mathematical tasks designed/posed by pre-service teachers

Pre-service teacher (PST) problem posing is not a well-researched area to date (Chapman, 2012; Leavy & Hourigan, 2022). Research which does exist suggests that problem posing, as compared with problem solving, offers benefits such as increased autonomy, agency, confidence, critical thinking, and enjoyment (Dempsey & O'Shea, 2020; Leavy & Hourigan, 2022; Voica et al., 2020) as well as enhancing and extending one's problem-solving skills, reasoning abilities, and general mathematical knowledge (Cervantes-Barraza & Araujo, 2023; Chapman, 2012). Most relevant to our study is understanding the extent to which research has been conducted on the types of tasks designed by PSTs while they are in their teacher education program. Just as problem posing is valuable in school classrooms for revealing and understanding an elementary or high school student's thinking (Cai & Jiang, 2017), having PSTs pose, not just solve, problems is an important learning activity in mathematics teacher education (Hartmann et al., 2021; Sari et al., 2023). Problem posing in a university teacher education program, with mathematics PSTs, can provide important insights into their conceptual understandings (and misunderstandings) about the material they are required to teach (Chapman, 2012; Yao et al., 2021). Yao et al. (2021), in their study of problem-posing and problem-solving with PSTs, proposed that 'preservice teachers tend to be much less familiar with problem posing than with problem solving in mathematics classes' (p. 946). Their research showed, however, that 'the activity of problem posing is conducive to preservice teachers' development of their conceptual understanding or their inclination to make use of that understanding' (p. 945).

In relation to posing (and then solving) mathematical problem-solving tasks, some research focuses on the skills, abilities, and creative potential of PSTs. In a study with ninth- and tenth- graders, Hartmann and colleagues (2021) sought to understand the relationship between real-life modelling and problem posing by exploring the 'types of problems students pose when they are asked to pose problems that are based on given descriptions of real-world situations and how they solve their self-generated problems' (p. 919). Their results indicated that 'students showed a strong tendency to generate closed problems that did not require problem solvers to make assumption[s] or to structure the real-world situation' (p. 932). These authors support their findings with previous problem posing research, claiming that students' [PSTs'] previous experiences solving mostly closed and overly scaffolded problems had a strong influence on the kinds of problems they in turn posed. We believe this finding has relevance to our study since, in our study, PSTs were asked to pose 'exploratory' (open, real-world) problems based on what they think an eighth grader knows and can do.

In research specifically designed to support the problem-posing capabilities of PSTs, Leavy and Hourigan (2022) share a 'Framework for Posing Elementary Mathematics Problems (F-PosE)' (p. 147), which was 'developed to focus prospective teacher noticing on desirable features of mathematics problems and inform decision-making processes around the selection of problems for use in elementary classrooms' (p. 147). Their framework incorporates eight indicators (including such items as clarity in language, curriculum coherence, engaging context, and so on), which they claim 'should be considered when selecting, designing, or modifying an elementary mathematics problem' (p. 158).

Problem-posing poses many challenges to PSTs. Not only is problem posing cognitively demanding (Cai & Hwang, 2002) but additional challenges 'are evidenced in reports of irrelevant, nonmathematical, ill-formulated and sometimes unsolvable problems posed by prospective and novice teachers' (Leavy & Hourigan, 2022, p. 150). These findings open the field for further exploration.

Task design in dynamic Geometric Environments

The PSTs in our study were asked to design a task where the 8th grade students are expected to use dynamic geometric software in the process of solving the task. In Norway, GeoGebra is the preferred geometric software in lower and upper secondary school. Developed by Markus Hohenwarter in 2002, GeoGebra is a tool for dynamic and interactive exploration of mathematical objects. Task design using GeoGebra (or other DGE) requires that teachers and PSTs have some knowledge about the opportunities this technology offers in addition to more general task design knowledge. Leung (2017) suggests an integration of four dimensions in a model he refers to as Mathematical Digital Task Design Knowledge (MDTDK), with the four dimensions being: knowledge about the artifact used, digital technological knowledge, mathematical content knowledge, and pedagogical knowledge. Leung claims that teachers should possess MDTDK for digital tool-based mathematics task design.

The ways in which PSTs implement GeoGebra in their task design are relevant for our study. Laborde (2001) distinguishes between four types of roles the designers of tasks attribute to Cabri (another DGE). We claim that GeoGebra may play the same role as Cabri and, as such, we find two of the roles relevant for our study. The first role is about tasks for which GeoGebra facilitates the material aspects; that is, using the drawing facilities offered without changing solution strategies much. The second role is when GeoGebra is used as a facilitator of the task; that is, when GeoGebra is used as a visual amplifier, helping to make conjectures using, for example, the drag mode. When Fatimah (2019) observed PSTs making mathematical tasks using GeoGebra, this researcher found that PSTs took advantage of the feedback generated by GeoGebra in the form of graphics visualization and algebra, but they did not feature GeoGebra in their initial design.

The task that the PSTs in our study were asked to design was intended to be exploratory. Leung (2011) writes that a meaningful exploratory task should involve conjecturing and explaining activities. One significant advantage with using GeoGebra is that the software makes it possible to 'create' mathematical objects and then manipulate them and observe what happens. Olsson (2018) considers GeoGebra as a learning milieu that will provide feedback at the task level to the student, which may be used as verification or elaboration. The drag-mode in GeoGebra may be considered a way to provide quick feedback. In fact, Leung (2011) views the drag-mode in DGE as a pedagogical tool that offers many possibilities to 'explore'.

The ways in which PSTs formulate their instructions and questions using DGE as reported in the research are interesting for our study (Bozkurt & Yiğit Koyunkaya, 2022; Hıdıroğlu, 2022; Hollebrands & Lee, 2016; Olsher & Lavie, 2023). For instance, Hollebrands and Lee (2016) studied PSTs' characteristic questions when implementing dynamic geometry tasks. They found that PSTs tended to ask broad questions like 'what do you notice?' and asked students to record what they observed without pushing them to explain why (p. 162). Drawing on his previous work, Hähkiöniemi (2017) considered three types of questions which could be used for mathematics teaching when GeoGebra is involved: factual, guiding and probing. Factual questions request knowing a fact, guiding questions give hints and scaffold a solution, and probing questions ask for elaboration, explanation, or justification. He investigated PSTs' uses of different kinds of probing questions in GeoGebra-enriched lessons compared with other lessons. In general, this researcher (and others) considers that PSTs are in the process of learning how to use technology tools themselves and that this will have an impact on the questions posed, including how inquiry-focused and 'probing' they are.

Olsson and Granberg (2019) investigated the difference in practice performance and learning outcomes for students solving guided or unguided non-routine tasks supported by dynamic software. They claim that direct instructions could be seen as promoting 'unproductive success', referring to 'a fair amount of research looking into learning effects comparing a didactical design of construction-before- instruction or instruction- before- practice in favor of the former' (p. 434). Other researchers like DeCaro and Rittle-Johnson (2012) found something similar, that exploring problems before instruction improved understanding as compared to a more conventional 'instruct- then- practice' sequence.

Method

Our study was designed as a multiple case study with four phases (Table 2). According to Grima-Farrell (2017), multiple case studies 'involve a number of cases being investigated at the same time as part of one overall study to generate the detailed description within and between cases' (p. 74) According to Mills et al. (2010), multiple case study 'is more powerful than single-case designs as it provides more extensive descriptions and explanations of the phenomenon or issue' (p. 584). The aim of this multiple case study was to find out more about PSTs' justifications for their designed tasks when they were given the 2021 Norwegian National Exam task. In Phase 1 of the case study, a convenient sample of 5th to 10th grade PSTs (54 in total), not previously known to the researchers, were asked to solve the same task from the 2021 National Exam (task presented on page 2). In Phase 2, a smaller group of these

Table 2. Phases 1 to 4: Our participating PSTs.

Phase 1	P1 – P54:	As part of a course activity, 54 PSTs contributed a task toward this research task design study; all 54 tasks were analyzed and coded, producing Table 3.
Phase 2	P1 – P13:	Of the 54 original participants, 13 agreed to be interviewed, allowing researchers to gather their perspectives and reflections on the design of their tasks.
Phase 3	8 participants from P1 – P13	Of the 13 participants interviewed, 8 agreed to participate in one of two focus group discussions, designed to discuss their own and others' tasks.
Phase 4	6 individual case study participants (P1-P4, P9, P10)	Based on the analysis of the 54 tasks, the 13 interviews and the 2 focus groups, 6 participants from P1 – P13 were selected since they were representative of typical tasks coded A – E: P1 (Ada), P2 (Cora), P3 (Ben), P4 (Edmund), P9 (Diana) and P10 (Celia).

Table 3. The main categories for all 54 PSTs.

Categories	Explanation	Number	Percentage
A	Task uses sliders with a general equation: $y = ax + b$, exploring what happens when varying the values of a and b.	6	11,1 %
B	Task gives a selection of functional equations with different values of a and b, using GeoGebra for drawing.	4	7,4 %
C	Task gives a word problem with a constant (y-intercept) and a variable (gradient) context, using GeoGebra for drawing, either directly from the word problem text or asking for the equation first.	26	48,1 %
D	Task gives a specific equation, find the gradient and the constant term before or after using GeoGebra for drawing the graph.	16	29,6 %
E	Task gives some points, draws a linear graph in GeoGebra, finds the constant term (y-intercept) and the gradient, eventually comparing with the use of a formula for calculating the gradient and the constant term.	1	1,9 %
F	Others	1	1,9 %
G	No answer	0	
		54	

PSTs (13) were interviewed individually. Following this, in Phase 3 of the multiple case study, eight of these Phase 2 PSTs agreed to participate in focus groups. Finally, based on an initial analysis of the tasks as well as the interview data, six representative cases were selected for Phase 4 of the multiple case study. Table 2 provides an overview of the four phases of this multiple case study, describing participating PSTs and the progression in our data collection and analysis processes.

Participants for the study were selected from a group of PST students who were expected to take the National Exam in Autumn 2022. Overall, 70 PSTs across two university courses were invited to participate in the study, with 54 PSTs (P1 - P54) responding favourably and agreeing to participate. They shared with the researcher that their motivation for agreeing to participate was in how it presented an opportunity to solve the exam task as a form of preparation for their own exam that they would take a month or two later. Hovik (the first author) gave them individual feedback on their solutions (i.e. their designed tasks) and, for those who participated in an interview, an opportunity was provided to discuss their own designed task with Hovik.

The group of PSTs for this study were in their first semester of a Norwegian university teacher education program. By the time they designed the tasks, they had not yet practiced or observed teaching as part of their teacher education program. While their courses included some algebra and functions during the semester, the PSTs had not yet, at the time of the study, taken courses explicitly in GeoGebra at university. In Norwegian schools, GeoGebra is the preferred DGE in lower and upper secondary schools, even though the extent to which it had been used with these PSTs depended on the specific teacher and the course they took at upper secondary school.

The first step in the study's analysis was Phase 1 of the study's multiple case study and involved analysing the 54 PSTs' designed tasks using the same codes as described in Table 1. That is, in the same way as for the National Exam, the tasks designed by the 54 PSTs were assigned the codes A, B, C, D, E or F (Table 3). The 2021 National Exam grading guidelines provided by the exam committee were also important in this study since they served as a measure for what is expected of the PSTs' knowledge about how to design an exploring task using a dynamic geometric program. Although most PSTs also constructed solutions for their designed task, their suggested solutions were not referred to in the coding process, not for this study group of PSTs or for the original 199 National Exam PSTs. Admittedly, the solutions could be seen as an important part of the exam since PSTs may have been drawn to design a task that they can solve themselves.

Once the 54 tasks had been analyzed and coded, all 54 PSTs were invited to be interviewed to gather deeper information about their choices. While 29 PSTs accepted this invitation to be interviewed, due to time and resource constraints, only 13 were individually interviewed (Phase 2), followed by two focus groups created from the interviewed PSTs (Phase 3). All six codes in Table 3 were represented in the tasks designed by these 13 PSTs.

During Phase 2 of the multiple case study, the semi-structured interviews with the 13 PSTs began by the interviewer (Hovik) presenting them with their own designed task. The interview questions, influenced by the analyses of the designed tasks, are presented below. Each interview, however, was formulated different for each PST participant since it was important to have a relaxed conversation, one that emerged from a discussion of their own task:

- From where did you get your ideas, inspiration for your choice of task? Any associations and personal experience?
- What was your intention about the student's learning outcome?
- What was it in the exam question that you noticed and emphasized? Was there something you did not consider?
- What associations do 'explore' give?
- Why did you not use this word 'explore' in your task? (no one in this group did)
- Did the use of GeoGebra have any impact on your designed task?
- The fact that you should make a solution proposal, did it have any influence?
- Did you have other ideas in mind which you chose not to use?

As noted previously, once all 13 interviews were completed, Phase 3 of the study began with eight of these PSTs taking part in two focus groups (a group of five and a group of three). In these focus groups, the PSTs did not work with their own designed tasks but instead they were presented with tasks designed by PSTs from the National Exam. In addition, these National Exam tasks were distributed to PSTs in the focus groups in such a way that each PST received a task which had been assigned a code different from the code assigned to their own designed task. Each focus group member was asked to present the given National Exam task(s) to the others in the group and, in a discussion, to compare this given task with their own designed task. After this presentation, a discussion about different task designs and other issues related to the interviews took place.

The second step in the study's analysis, carried out for Phases 2 and 3 of the multiple case study, involved analysing all 13 PSTs' responses to the main questions from the interview guide as well as further discussions on the same topics in the focus groups. The method for analyzing the data from the interviews was a form of thematic analysis (Braun & Clarke, 2006). The interviews and focus group conversations took place in Norwegian and were audio recorded, then transcribed into Norwegian text. The transcripts, together with the audio recordings, were analyzed primarily by the first author in order to uncover patterns of meaning and issues of interest. Following this, substantial portions of the transcript text were translated into English, at which point both researchers collaborated on the analysis process. Due to the second author not being fluent in Norwegian, this approach to analysis may be perceived as a limitation to the study and its findings; that is, since it was not possible for both researchers to independently code all the data and then to engage in collaborative conversations to arrive at consensus for all data. However, the researchers consulted with each other extensively on interpretations and applications of the coding frame (Table 3), establishing 'the systematicity, communicability, and transparency of the coding process' (O'Connor & Joffe, 2020, p. 1). In addition, while data analysis was underway, an overview of the results and some suggested case examples were presented and discussed with colleagues in the first author's research group at her university. Their contributions to the analysis process offered further credibility and trustworthiness to the findings.

Together with the suggested designed tasks, the full analysis process led to the selection and presentation of the final phase of our multiple case study. Phase 4 consisted of six representative cases along with a thematic map consisting of four main findings, abbreviated in the results section with the headings: the knowledge of 8th graders, guiding or exploring, use of context or not, and the role of GeoGebra.

Data from the six cases of Phase 4 are presented and discussed in this paper. As can be viewed in Table 2, four PSTs from the selected six cases were focus group members, while two were not. As noted earlier, the six case study participants were chosen from the 13 interviewed PSTs because their designed tasks served to illustrate typical tasks for each of codes A-E. In addition, the six PST cases selected provided strong support for the themes emerging from the interviews and focus groups. Two tasks coded C were selected since this was the most frequent code appearing in the data (and in the national exam tasks). Code F was not addressed because it did not represent a distinct task design.

Data from the interviewed PSTs who were not selected as one of the six cases and data from the two focus groups are presented and discussed in this paper only insofar as the data provide additional support for the themes emerging from analysis of the individual cases. To avoid confusion with our primary PST cases (the six who have been named), we present the additional supportive interview and focus group data in an anonymous manner. To summarize, in this paper our analysis focuses primarily on the six case interviews, supported by the seven other interviews and focus-group conversations, with a goal of understanding what task design stories the PSTs tell about their task. The aim here is not to deeply analyse each task formulation.

Analysis

To begin, we present the results from coding all 54 PST designed tasks, which showed an even stronger tendency than the results from the National Exam (Table 1) to design a task with a word problem that involves a context with a constant and a variable (Table 3). As can be noticed in Table 3, nearly half (48%) of the PSTs chose to design a word problem task (code C). On the other hand, almost 30% of the PSTs provided an equation without a context, asking the students to find the gradient and the constant term or the y-intercept before or after they drew the graph in GeoGebra (code D). Only 11% of the PSTs made use of a slider in their designed task (code A), whereas 21% of the National Exam PST tasks were coded A. This difference between the 54 PSTs and the 199 PSTs might be due to the fact that the PSTs involved in this study had less experience in their university teacher education program than the group who took the National Exam.

Phase 4: the six cases

Based on the results from analyzing the 54 PSTs' tasks (Phase 1), 13 interviews (Phase 2), and two focus groups (Phase 3), six cases, selected from the 13 interviewed PSTs, are presented here as Phase 4 of the case study to highlight 'typical' student task design responses for each of codes A-E. It is reasonable that our analysis draws only on select participants in a manner similar to Chapman (2012), who justifies such a selection by stating that the six selected participants' 'thinking seemed representative of different ways of making sense of problem posing' (p. 139). In her study, after selecting six representative participants, she went on to interview only these participants to further explore and unpack their thinking for her analysis. In a similar manner, after carefully examining the 54 designed tasks, along with data from the 13 interviews, we decided on our six case study participants (Ada, Ben, Celia, Cora, Diana, and Edmund) whose tasks, we claim, are strong representatives for the previously generated categories of problem types (codes A-E). As a reminder, we generated these initial categories of problem types (A-G) according to a coding process of the National Exam results, a process which involved identifying the nature and distinguishing features of the problems posed by the PSTs who wrote the exam.

In what follows, we present and discuss the six cases, to illustrate the tasks representative of codes A-E across all designed tasks. For each case, we include a rationale for the assigned code and the details of the task designed by the PST. Then, the task is discussed and supported through a presentation of key ideas offered by the PST with respect to their design stories.

Case 1: Ada

Ada's task is considered a **code A** task because it uses sliders to investigate what happens when varying the values of a and b. It also serves as an example of a task which guides the students by providing instructions through the posing of individual sub-questions (a-e).

a. Make a slider for a in GeoGebra.
   Make a slider for b in Geogebra.
   Write the function $y = ax + b$.
b. What happens when you move a? How does the line y look when a are positive? What about negative? What about 0?

c. What happens when you move b? What is the impact on the graph?
d. A function goes through the points (1,1) and (3,5) Find the gradient and constant term.
e. Can you make a function with gradient 2 which goes through the point (2,6). What will be the formula?

In the interview about her task, Ada expressed concern over how extensive the task was supposed to be. She shared that she wanted to design a task which 'allows students to learn something or gain a better understanding of the subject'. Her first thought was to use sliders which was a tool she had become aware of in her own work with students. However, she noted that she had not previously seen this tool at lower and upper secondary school or in teacher education. She argued that the slider tool makes it easy to see connections and look for different values of constant terms and gradients. Ada highlighted the importance of giving the students 'power' by allowing them to change the parameters: 'It might be easier to see the connection when you in a way have the power to change the numbers yourselves'. She shared how she wanted the students to see connections for themselves including how the gradient and constant term could impact a linear function. Also, she reported wanting them to formulate the mathematics themselves (refers to question b). That said, she chose a list of guiding questions because she thought that 8th grade students need concrete and clear instructions to get through the task. She chose not to use the word explore explicitly in her task however because she did not want to scare students into believing that they had to write a lot. She herself associates the verb explore with not only solving a task but trying to understand and explain why. Ada noted: 'More why than how if you understand?' Ada emphasized the use of GeoGebra because drawing graphs by hand would take too long and, she commented, GeoGebra is important to learn by itself. In relation to task design, she shared that it is important for a teacher to be able to modify a task, not always follow the book.

Case 2: Ben

Ben's task was assigned **code B** since, in his task, the students are given different functional equations with different values of a and b, for example, positive and negative parameters.

Use GeoGebra to show these formulas:

1. a.$f(x) = 3x + 1$ b.$f(x) = x - 7$ c.$(x) = -2x + 10$ d.$f(x) = 5$ e.$x = 5$
2. Look at how the graphs look, and describe the relationship between
    a. The values for x and how the graph look.
    b. The value of the constant term and the graphs intersection with the $y-$ axis.

In Ben's case, he was concerned about the 8th grade student, that this would be one of their first meetings with GeoGebra and that the task didn't have to be difficult. The choice of functions was not random; that is, the vertical and the horizontal lines were deliberately chosen. He shared that he wanted the students to experience and see the different possible functions and what happens when different values are given for x and y. He noted: 'I noticed that it was a long time since I had used GeoGebra so I explored when I wrote down the tasks too'. The absence of the word explore in Ben's task was, he noted, not conscious; he just chose another formulation. He addressed that GeoGebra makes it possible to try and fail and that drawing by hand might have the impact that some drawings would be wrong. For Ben, making the solution of his task was a part of feeling confident in his capability to solve the task himself.

Case 3: Celia

Celia's task is a good example of a task assigned **code C** since it is a word problem. The task is also about the most frequently used context among PSTs both in this study and in the national exam: the taxi trip. Celia's task is also an example of a task that strongly guides (or scaffolds) the students. It can be noticed that the constant and the gradient are explicitly asked for *after* drawing the graph in GeoGebra, suggesting that the input needed for drawing the three graphs is expected to be construed directly from the text. Celia's designed task is:

Eva is taking a taxi from home to the sports club. It is 22 km away. She can choose between three different taxi companies: Taxi A: initial cost 100 kr and 7 kr per minute. Taxi B: Initial cost 200 kr and 5 kr per minute. Taxi C: Initial cost 70 kr and 15 kr per minute.

a. Draw the graphs for the costs of the different taxi companies. Mark each graph with its formula in the grid.
b. What is the constant and the gradient in the three functional expressions you can make for the taxi companies?
c. Which of the taxi companies ought Eva choose and why?
d. Which should she have chosen if the trip was half as long?
e. What would have the greatest impact on the cost if the trip had been twice as long?

Celia experienced the exam task as an inspiring opportunity to focus on connections, didactics, and mathematics. She reported in the interview that she thought it was fun to design the task. She wanted it to be practical, but not so much so that the math would disappear. She emphasized use of a context, assuming that her choice is a situation many are familiar with. She wanted the students to get a wide knowledge of gradient and the constant term and when each of the parameters has the most impact, asking: 'What is this about, what is the constant term, what is the gradient, and what is the difference between them?' She never did consider designing a non-context task as an opportunity because she supposed that the task should be a problem-solving task. Celia chose taxi-trips because she had seen something like that before. For her, the word explore was important. She sees 'exploring' as a possibility for the students to work with a task where the solution is not immediately obvious, an unknown landscape. To inquire is something else: 'then you know where to go in a way and you collect information that brings you there'. However, she did not find it natural to use explore in the task text, explaining it was because she assumed that the designed task should be a kind of test task and not part of a teaching session. She did not see the dynamic property of GeoGebra as important, but that that the program makes it easier to draw. Her idea was that a student could respond to the task even if not using GeoGebra. If not GeoGebra, she would have used other numbers (easier to draw) and equal units on both axes (in her suggested solution).

Case 4: Cora

As in Celia's case, Cora's task was also assigned **code C** since it is a word problem, and it is also about the most frequently used context of a taxi trip. The functional expression, the gradient, and the constant term are asked for before drawing in GeoGebra, demanding a transition from situation to formula and then from formula to graph.

You are taking a taxi which costs 50 kr per kilometer, and the standard price is 70 kr.

a. Write an expression for the function. What is the constant term and what is the gradient?
b. Write the function in GeoGebra, as $f(x)$. On Saturdays the standard price for the taxi changes to 100 kr. Write this in as a new function ($t(x)$) in GeoGebra. How does the graph change? What if the gradient increases to 60?

Cora was concerned about what she herself knew when she was an 8th grade student, and therefore, how difficult might the task be? What do they know about the constant term and gradient? She shared that this was her first experience designing a task, and she commented that she is more familiar with 'explain what this student might have thought' type of tasks. She chose a taxi trip context because that was what she had experienced as a task context earlier in her own learning, both in school and at university. Cora indicated that she wanted the students to know more about the impact that the gradient has on the slope of the graph and how only changing the constant term gives a parallel graph. She wanted them to be able to explore this. In retrospect, Cora shared that she would have reversed parts a. and b. of the task to enable students to see the changes that occur when plotting different numbers, prior to explaining it to them. She did not consider using the word explore in designing her task and shared that she associates the word with looking for similarities between different tasks; to experience, for example, y-intercepts. For her, the use of GeoGebra in the task design was as an aid to see the graphs and compare different graphs. Without GeoGebra, she would have formulated task b differently. She emphasized use of context and the students making their own function.

Case 5: Diana

Diana's task was assigned **code D** because in it she provides a specific equation with no context. This is also a typical example of a task where the gradient and the constant term, and their impact on the graph, are asked for before using GeoGebra for drawing the graph.

You have $f(x) = 2x + 8$

a. What is the gradient and what is the constant term?
b. What do you think the graph will look like?
c. Draw the graph in GeoGebra.
d. Create a functional expression $g(x)$ which has an intersection with $f(x)$. What do you have to think about when you choose gradient and constant term for $g(x)$?
e. Draw $g(x)$ in GeoGebra. If they do not have an intersection, what do you have to change?

Since the exam task was different from what Diana expected—in that she had to design, not solve, a task—she noted that she had to 'realign the brain'. Diana was also concerned about the 8th graders' initial knowledge about the topic, so she chose to start with an investigation of their initial knowledge about gradient and the constant term by asking question a. She shared: 'Because if you're already uncertain, then it makes sense for maybe the rest of the task'. Diana explained that she wanted the students to create their own function based on the assumption that they know what the two parameters mean: 'Then you can imagine what the graph looks like'. She wanted them to explore the gradient and the constant term, and she believed that the use of GeoGebra should help the students see for themselves before answering what they had assumed. She wanted them to show some understanding about the intersection between the graphs.

Diana admitted that she associates the word explore with try and fail in looking for connections or properties of the parts of a functional expression. In this regard, she chose not to use the word explore because it could then be difficult for the students to know what exactly to do. She felt that to use inquire might be looking for something more specific. She also noted that she might have designed a different task if she had been more confident in GeoGebra, but she claimed that she was 'a bit rusty'.

Case 6: Edmund

Finally, for our sixth case, Edmunds' task was assigned **code E** because it focuses on the table representation by giving some points and asking students to plot them into GeoGebra before finding the functional expression.

In Edmund's task, the students are first presented with the graph and the table (with 3 points) for $f(x) = x$. (This part is not presented here)

Then they are given the table for a new unknown function $g(x)$

x	1	2	3	4
y	2	4	6	8

Followed by the questions:

a. Use the values in the table to find an expression for $g(x)$ by the use of GeoGebra. Also find the gradient.
b. Imagine a function intersecting the y- axis at (0,3) instead of (0,0). Draw this new graph in GeoGebra.

Edmund expressed deep concern about how much 8th graders know about GeoGebra. He assumed that many of them will struggle with the program. As he explained, it would have been something else if it was 10th grade. He wanted it to be much easier than a task he had once seen given to upper secondary (vocational) which was working with 2*x* plus 3. He chose what he saw as the simplest approach: to start with a table. Edmund reasoned: 'I remember that the first thing we learned was to draw points, so I assumed that they know how to draw points and that will be easier than drawing a function'. In his task, Edmund wanted students to use GeoGebra to draw a line through the given points and he mentioned that GeoGebra then gives you the function, clearly and distinctly. For him, the word explore means to experience new 'things' or experience it again together with trial and error. However, he did not think about using the word explore explicitly in his task. As he noted, he would not have known how to word it. When asked about the lack of context in his task, Edmund admitted that it was due to a lack of creativity, but he also thought that it would be a distraction for the students anyway.

Findings

As we reflect on the six cases, while drawing support from the focus group data and the seven other interviews, we propose that four key themes are prominent in our analysis and important to consider in the broader context of mathematics teacher education. We name these themes as: the knowledge of 8th graders, guiding or exploring, use of context or not, and the role of GeoGebra. Next, we discuss each of these themes in turn.

The knowledge of 8th graders

When this study was conducted, the PSTs were in their first semester of a five-year teacher education program. As a result, it is interesting to notice how there is a clear tendency for the PSTs to be concerned, in some way, about the level of the students. Given that the PSTs in this study were early in their teacher education program, the student knowledge and thinking they can draw on is primarily their own, when *they* were 8th grade students. In one of the focus group discussions for example, one participant offered that the question 'what did I know at 8th grade?' was the most important part of the exam task for them. For Cora, attempting to remember what she herself knew about the topic of linear equations when she was in grade 8 served as important information in designing her task. Similarly, Edmund shared that he chose a table of points because he remembers that this was the first thing he learned during his own school experience, and he thought this approach would be familiar to 8th graders. In many ways, these PST recollections of their own 8th grade experience should not come as a surprise since, as mentioned earlier in our literature review, the kinds of tasks that teachers use are influenced by 'knowledge of students and student thinking and understanding' (Lee & Özgün-Koca, 2016, p. 13).

Some PST participants assumed that 8th graders do not know much about linear functions and properties like gradient and y-intercept, and so they approached their task design with this in mind. Also, they expressed concern with what 8th grade students know about using GeoGebra, suggesting that this could be one of their first meetings with the software. To explain why taxi-trips were commonly drawn on in the design of tasks, some PSTs explained the use of this context as matching well with the level and experience of the students. Some others, like Cora, referred to her own experience with this context at school and university, suggesting that it was an appropriate choice of context for a grade 8 mathematical task on linear functions.

In one focus group, a participant mentioned that 8th grade students would not know what to do if asked to *explore* linear functions, claiming that the students would not understand what it means to explore. In fact, the PSTs themselves admitted to not fully understanding the concept of explore either (as seen in their own designed tasks) so, they questioned, how could 8th graders do this. For instance, Ada expressed her view that 8th graders would need concrete and clear instructions that did not leave things too open, or it might 'scare them'. The next section deals more explicitly with this theme of exploring.

Guiding or exploring

For some case study participants, using the word explore as part of their task was not even considered. For Celia, however, she saw plenty of 'possibility' in the word explore since, as she stated, it is 'where the solution is not immediately obvious, an unknown landscape'. In the end, however, she chose not to use the verb explore in her designed task.

In a focus group conversation, time was spent discussing how 'explore' is one of the most frequently used verbs in the Norwegian curriculum. To begin that conversation, the definition of explore from the verb explanation list in the curriculum (presented in the introduction part of this paper) was read for the PSTs. Some PSTs reported associating the word explore with trying and failing, understanding, and explaining, finding connections, looking for similarities. Also, some mentioned that exploring means that the solution is not obvious. One participant in a focus group offered that exploring is about 'What happens if I do this?' When PSTs attempted to explain the difference between 'to explore' and 'to inquire', they emphasized that exploring is more of a try-and-fail approach to finding connections, whereas inquiry is more about knowing where to go—something more specific, as Diana offered. Diana noted she did not think that a 13-year-old student (typical age for an 8th grader) would know what to do if he or she was asked to explore linear functions; the student would not understand what it means. Interestingly, even though the word explore appeared twice in the problem task that was given to the PSTs, only two of the 54 PSTs in the study used the word explore explicitly in their own designed task. Since they shared that they themselves do not fully understand the word, they proposed that it may stress the students to use the word. The tendency to not push students to make decisions and explain why, but instead ask students to focus on what they observe, is also seen in literature (Hollebrands & Lee, 2016).

PSTs who included the use of the GeoGebra slider tool in their task (those who were coded A) or those who were presented with a task using sliders in the focus group, recognized this tool as one for exploring. As one PST offered: 'Instead of making different graphs, you can just change the gradient value and then *y*-intercept by using a slider'. Drawing on the work of Leung (2011), we refer to the slider as an example of drag-mode in DGE: 'In particular, the drag-mode in Digital geometry environments (DGE) is regarded as a DGE tool to empower learners with amplified abilities to explore' (Leung, 2011, p 329). As mentioned earlier, in her interview Ada spoke about giving the students power by allowing them to change the parameters, though, in contrast to what exploring entails, she emphasized giving students 'concrete and clear instructions'.

Given that guiding questions are generally questions which provide hints and scaffold a solution (Hähkiöniemi, 2017), it was clear that some PSTs used many sub-questions to scaffold the students toward their solution. In fact, many of the designed tasks shared this property of scaffolding through guided questions, with the PSTs emphasizing their conviction that making a list of concrete questions (a, b, c and so on) might help avoid making the task too difficult for the 8th grade students. For instance, in one of the focus groups, PST participants shared their views on these guiding questions, offering statements such as: 'So it leads you to explore in a way, because they have not only to find out themselves, but you give things they can find out and then try it' and 'It is many more students that will learn from those kinds of tasks than tasks [which] ask for explore negative gradient'. During a focus group, one PST argued that many sub-questions are compatible with the idea of exploring, and she criticized the modern open tasks which are part of the proposed exams (the new Norwegian curriculum, LK 20). Her view was that there can easily be too many wrong answers to those tasks:

> There are … too few clues given to what students can do and then a lot of the potential falls away because the students kind of stay seated[3], they spend their time thinking about okay, what does that really mean? What am I supposed to do because the task becomes too open?

It is interesting to compare these PSTs' reasons for choosing closed tasks with many sub- questions to what has been reported in the literature. For example, Olsson and Granberg (2019), who investigated the difference in performance and learning outcomes for students solving guided or unguided non-routine tasks supported by dynamic software, found that more open-ended tasks with fewer instructions resulted in students remembering a rule for a longer period. On the other hand, students working with strict guidelines were successful in finding a rule, but they did not remember the rules or processes in a post-test at a later point in time. Other researchers like DeCaro and Rittle-Johnson (2012) reported something similar, that exploring problems before instruction improved understanding compared to a more conventional 'instruct- then- practice' sequence. Since these findings are supported with previous problem posing research, it is reasonable to surmise that PSTs' previous experiences as students solving mostly closed and overly scaffolded problems had a strong influence on the kinds of problems they in turn posed as becoming teachers (Chapman, 2012; Hartmann et al., 2021).

Use of context or not

Overall, of the 54 PST tasks analyzed for this study, 48% were coded C and many of them chose the taxi trip as the context to focus on in the task. The two case study PSTs using taxi trips as their context (Celia and Cora) explained their choice as a way of making meaning for the students since, they believed, the taxi-trip problem is typical, and they have actually met a task about that context themselves. Celia and Cora were both concerned with the impact of the gradient and the constant term and relating it to a context to gain wide knowledge. It seems reasonable to infer that, for these PSTs, linear functions are associated with word problems. As offered by a focus group member: 'I think that it's often because it's tied to a real situation which you can relate to and everyone knows about a taxi ride and all know that sometimes you have to give an amount first and then yes, an x afterwards'.

As with their reflections on the knowledge of students in 8^{th} grade, many PSTs referred to their own experience with context (or not). One of the interviewed PSTs (not one of the selected 6 cases) shared that all the linear function examples they have encountered at university have been about taxi trips and comparing two taxi companies. However, this PST reported that he deliberately chose something different (the context of bowling, with shoe rental and cost per game) because of the age of the students, saying that they 'can't relate to taxis'. Another PST shared how she made a point of asking her students to create an equation for the linear function from a specific situation, to challenge the students. While we do not, in this paper, conduct a deep analysis of the tasks themselves, following from the research of Zhang and Cai (2021), we can notice that the use of the taxi trip context illustrates one common type of problem-posing task: 'posing variations on a question with the same mathematical relationship and structure' (Zhang & Cai, 2021, p. 964). For PSTs in this study, the taxi trip provided a context which PSTs can closely relate to because they have experienced problems very much like this in their own prior learning about linear functions.

Interestingly, those PSTs who chose to design a task without a specific context explained their choices in different ways. For example, one PST (with a code A task), drawn from the other seven interviewed participants, reported that he finds it more interesting to make the student see what changing the values of the terms (*a* and *b*) means for the graph and the function rather than their correspondence with real world. Similarly, in one of the focus groups, it was mentioned that a context might make it more difficult to, for example, change the gradient from positive to negative. Another PST, who suggested that including a practical situation which students could imagine could be a good idea, also criticized one concrete context task (coded C) for not being exploratory at all since the gradient and the constant term (or, *y*-intercept) were asked for before giving the instruction to draw the graph in GeoGebra. As she claimed, at this stage, drawing the graph would be unnecessary in a way.

The role of GeoGebra

The impact of using GeoGebra was seen in very different ways by the PSTs involved in this study. Some PSTs focused on the dynamic properties of GeoGebra, such as the sliders and the possibilities for try-and- fail approaches to student learning. They used GeoGebra as a visual amplifier, helping to make conjectures about the role of the gradient and the constant term. One PST (coded A) suggested that his task would not have been possible to solve without the software because 'then you don't have the possibility to explore, and just put in new numbers there and there and see what happens'. It was also obvious, however, that other PSTs in the study thought about GeoGebra as an easier way to draw graphs properly, and to confirm something they already have assumed. This view is more consistent with what Laborde (2001) refers to as using technology as 'facilitating material aspects of the task while not changing it conceptually' (p. 293). In fact, this latter approach is the way in which the PSTs claimed they were trained to use GeoGebra.

In taking a closer look at the six tasks in the cases presented in this paper, one can notice that, for the tasks coded C and D (Celia, Cora and Diana), the PSTs did not use GeoGebra to find the gradient and the constant term; instead, their tasks merely used GeoGebra to confirm what was already constructed or calculated in earlier parts of the guided instructions. It was only Edmund who used the software to find the algebraic formula. The rest tended to imitate tasks first and then integrate them with GeoGebra, an approach also reported in Fatimah (2019); that is, they were using GeoGebra for drawing after creating a function. Other research, for example that by Gulkilik (2023), also showed how 'PSTs did not utilize the potential of the DGE in terms of exploration, re-construction, predicting, conjecturing, or proving, which are particularly emphasized for mathematical knowledge acquisition in DGE tasks by different researchers' (p. 34). Similarly, Volk et al. (2017), in their tablet-specific research on integrating technology into problem tasks, highlighted the importance of technological tools in supporting important aspects of cognitive development in mathematics and cross-curricular achievement.

Even though some PSTs reported that their design was totally dependant on the use of GeoGebra, most PSTs shared that either they themselves are not so familiar with GeoGebra or that GeoGebra did not feature prominently in their design. For example, Diana shared that she was a bit 'rusty' with the use of the software and Celia noted that without GeoGebra, she would have made a similar task anyway, but maybe with simpler parameters.

Discussion

To introduce this discussion, it is important to clarify that the 54 PSTs who participated in the task design activity and who agreed to be involved in Phase 1 of this multiple case study were not completing this task as an exam. Instead, many PSTs shared that they viewed this task design activity as a form of preparation for the exam. Given this, it is possible that they did not put forth as much effort in designing their tasks as they would have done if it was an exam, making this situation a possible limitation of the study and of the data analyzed. On the other hand, our analysis has shown that their tasks are quite similar to the 199 tasks designed for the National Exam (see Table 1). The aim of the task design question was for PSTs to combine their own knowledge about linear functions with the tools that GeoGebra offers to design an exploratory task for 8th grade students. Admittedly, problem-posing is challenging and cognitively demanding (Cai & Hwang, 2002) since there are many elements to master all at the same time. Thus, another possible limitation of the study is that the number of elements involved in such a task design activity (e.g. PSTs' knowledge of mathematics, of grade 8 curriculum, and of GeoGebra) make it challenging for us, as researchers and teacher educators, to identify what, precisely, could be the barriers or limitations that PSTs encounter as they learn about and attempt task design activities.

The analysis of our data, reflected in the stories of these fresh PSTs, shows that they are already beginning to relate to their future profession as teachers. As our data have shown, the PSTs are concerned about 8th graders' initial knowledge, while simultaneously expecting students to be familiar with more complicated mathematical ideas such as the transition from situation directly to formula (Celia and Diana, for example) or the values for the gradient and the constant term (or *y*-intercept) prior to drawing the graph (Diana). Since the exam task formulation included the words gradient and constant term, we think this might have been associated with the functional equation rather than the graph for the PSTs. One of the indicators in F-PosE (Leavy & Hourigan, 2022) is the use of a motivating and engaging context. Our data show an awareness on the part of PSTs to make mathematics meaningful by using students' personal experiences when designing word problem tasks. This might be an explanation, together with a tendency to associate linear functions with word problems, for the fact that so many of the PSTs designed real-life tasks. Gaining a deeper understanding of PSTs' reasoning behind featuring real-life contexts in their tasks, in relation to their own experiences as learners and doers of mathematics, is identified as a future area of study and reflection for mathematics teacher education programs and curricula.

In many of the designed tasks, GeoGebra was not fully integrated with mathematics concepts, demonstrating that the PSTs still require additional learning experiences to put the many tools GeoGebra has to offer to good pedagogical use. According to Hollebrands and Lee (2016), further development and experience will have an impact on the questions posed by PSTs, and teachers in general. As an example, consider the way that several PSTs asked the students, in their guided sub-questions, to elicit the gradient and constant term directly from a word-problem *before* asking the students to use GeoGebra as a tool. Related to this, Fatimah (2019) found that students imitated routine tasks before integrating them with GeoGebra. In many cases, the PSTs in our study appeared to use the software only for visualization of the graph(s) and not for exploring and locating the algebraic terms. Only a few PSTs incorporated the slider tool in their task design as a way for students to manipulate the coefficients in the functional equation while looking for changes in the graph. We expect, however, that such an approach might promote deeper understanding between algebraic equations and graphs and, ideally, the students would then be in a dragging dialogue with DGE, empowered with 'amplified abilities to explore' (Leung, 2011) as well. Leung refers to a *dragging dialogue* with DGE as the feedback patterns between a learner's dragging behavior and the DGE environment.

As noted above with regard to teasing out the elements involved, the association between effective use of GeoGebra and clear instructions with sub-questions is not possible to assess based on our data. In fact, a limitation of our study is that we can only surmise that the associations PSTs made may be based on their own experiences with mathematical tasks in general. An especially interesting finding in the research interviews is that PSTs argued for the importance and necessity of clear instructions when students are asked to explore, even though clear instructions would not necessarily 'maintain curiosity and wonder' (Kunnskapsdepartementet, 2018). Since explore is one of the most frequently used verbs in the curriculum and it is a core element in the mathematics curriculum as well, it is expected that students would know how it looks, and what it means, to explore. However, it is worth offering some insight into the differences between explore and inquire, according to the PSTs, to help interpret this finding. The word 'undersøke' is the Norwegian translation of inquire and it is more of an 'everyday language' verb, meaning to find out or look into. On the other hand, the Norwegian word 'utforsk' is more closely associated with the word explore and is focused on looking for something you do not yet know enough about. As noted previously, only two of the 54 PSTs used 'explore' in their designed task, with its absence explained in many instances as being because they did not want to frighten the students. However, if one takes a closer look at the tasks themselves, it seems that some of the actions they ask the students to complete do fit the meaning of explore in one way or another. Also, during the interviews, many PSTs argued for a kind of guided exploring. It would be worth focusing future research on studying even more explicitly than we have here the nuanced meanings of these words explore and inquire (among other action verbs used in mathematical problem solving).

Regarding the tasks themselves, we plan to build on this study by conducting a deeper analysis of the tasks, including the 54 PST tasks from this study as well as the 199 National Exam tasks. Such an analysis, we believe, will offer a deeper understanding of PSTs' problem-posing tendencies in Digital Geometric Environments with linear functions as the subject. As an important reminder, the PSTs in this study were in their first semester. As such, the findings presented in this paper offer insights for teacher educators into some aspects of 'fresh' PSTs' mathematical knowledge and experiences with posing mathematical tasks, with and without GeoGebra. Laborde (2001) found that integrating technology into teaching is a long process which 'requires the teachers to know enough about possible strategies and uses of technology by the students' (p. 312). In any case, the study reported on in this paper could prove valuable to mathematics teacher educators as they plan problem-posing activities for their PSTs, while keeping in mind the wide diversity of experiences and understandings most PSTs bring in this regard to their university programs (Chapman, 2012; Yao et al., 2021).

Conclusions

As noted in the introduction and throughout this paper, the multiple case study (Phases 1 - 4) reported on here highlights the need for PSTs to be actively involved in *designing*, not only *solving*, mathematical tasks as a means to developing and reflecting on their own conceptual understanding as well as their use of dynamic geometry tools (Hartmann, et al., 2021; Yao et al., 2021). In other words, task design is a valuable activity for PSTs as both learners and future teachers of mathematics. To date, research studying the types of tasks designed by PSTs while in their teacher education program is still not a well-represented area (Chapman, 2012; Leavy & Hourigan, 2022). Yet the research which does exist describes noticeable benefits of PST task design, such as increased autonomy, agency, confidence, enjoyment, and mathematical knowledge (Chapman, 2012; Leavy & Hourigan, 2022; Sari et al., 2023; Voica et al., 2020). We claim that, although the themes described in this study (the knowledge of 8^{th} graders; guiding or exploring; use of context or not; and the role of GeoGebra) may be quite recognizable at the level of the practicing teacher, this study makes an important contribution by pointing to their importance for new, first year 'fresh' PSTs. The findings provide insight into the powerful influence of their own previous experience on the kinds of problems they pose as becoming teachers (Hartmann et al., 2021). Throughout this study, the PSTs expressed this influence when they reported designing a task in relation to what *they* knew in 8^{th} grade; to what *they* preferred in terms of scaffolding and guiding versus open exploration; to what *they* had experienced as learners as a context for problem-solving; and to what *they* knew about GeoGebra as primarily a visual aid tool to confirm what they had already done algebraically. We claim that these are important insights for mathematics teacher educators as they work with PSTs in their teacher education courses. In closing, we recommend that the provocative perspectives presented in this paper on what PSTs know and do with regard to exploratory task design using GeoGebra be further explored and acted upon in teacher education programs.

Notes

1. The National Exam is administered annually in all Norwegian teacher education institutions and counts for 5 credits. Each institution manages the remaining 55 credits through local exams.
2. Translation from Norwegian to English presented some challenges according to concepts and names. In Norway, general linear functions are given by the equation: $y = ax + b$. In Norwegian schools, a is referred to as "value of the slope" which might be translated as "gradient," not so often as just slope, whereas b is referred to as the constant term, but sometimes also as the y-intercept. The distinction in naming between the two representations (algebraic formula and visual graph) is not very precisely applied in schools. In this paper, we have chosen to use the term gradient for 'a' and constant term for 'b', except when pointing directly to a graph.
3. This is directly translated from Norwegian.

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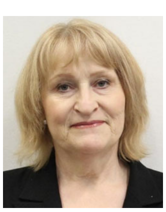
No potential conflict of interest was reported by the author(s).

About the authors



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Data availability statement

The data that support the findings of this study are not publicly available for confidentiality reasons, as they include information that may disclose participants' identity. The parts of the data that do not include personal information are available from the corresponding author upon request.

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