Mathematical insights into eye gaze dynamics of autistic children

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Abstract

Early detection and diagnosis of Autism Spectrum Disorder (ASD) is crucial for effective intervention and improved outcomes. Eye-tracking technology offers a non-invasive and objective method for detecting autism symptoms, as individuals with ASD often exhibit distinct gaze patterns. In this study, we aim to use eye-tracking data to distinguish individuals with ASD from those without, by analyzing the statistics of saccades and fixations. The study aims to go beyond simply identifying where individuals with ASD look and instead focuses on how they perceive images. By comparing our approach to other methods in the literature, we seek to improve the accuracy of autism diagnosis. Specifically, we use Hidden Markov Models (HMMs) to model gaze dynamics and apply power analysis to identify the most informative model parameters for future classification of individuals with and without autism. The use of HMMs and power analysis in this study provides an informative approach to understanding gaze dynamics in individuals with and without ASD, with potential implications for future research and clinical applications.

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Acronyms

ASD Autism Spectrum Disorder. 1

HMM Hidden Markov Model. 30

TD Typically developing. 17

Chapter 1

Introduction

Neurodevelopmental disorder (NDD) is a group of conditions characterized by impairments in cognition, communication, behavior, and/or motor skills resulting from abnormal brain development [1]. Even though all of these conditions originate in childhood, they persist into adulthood, posing severe challenges to individuals and society [2].

According to America's Children and the Environment (ACE), children with NDD may suffer from Attention-Deficit/Hyperactivity Disorder (ADHD), **Autism Spectrum Disorder (ASD)**, learning disabilities, intellectual disability (mental retardation), conduct disorders, cerebral palsy, hearing and visual impairments [3].

The term "autism" was first introduced by Swiss psychiatrist and psychoanalyst Eugen Bleuler in 1910 [4]. He used the term to describe a group of symptoms that included social withdrawal, difficulty communicating, and repetitive behaviors in his patients who had schizophrenia. The word "autism" comes from the Greek word "autos," meaning self, as Bleuler observed that his patients with this condition seemed to be living in their self-contained world. However, the formal diagnosis of autism did not enter the psychological realm until 1910; autism likely existed long before any clinical diagnosis was possible. In 1943, American child psychiatrist Leo Kanner published a paper describing a group of children with similar symptoms to those of Bleuler but with no evidence of schizophrenia [5]. The characteristics described in this article, especially "autistic aloneness" and "insistence on sameness," are still regarded as typical of autistic spectrum disorders.

According to the American Psychiatric Association's (APA) Diagnostic and Statistical Manual of Mental ASD is a complex developmental condition characterized by persistent difficulties with social communication and interaction across multiple contexts, restricted interests, and repetitive patterns of behavior, interests, or activities.

A variety of symptoms are associated with this disorder, which can vary in severity and presentation and can be described as a "spectrum disorder." APA decided to combine the terms "autistic disorder," "Asperger's syndrome," "Childhood disintegrative disorder (CDD)," and "Pervasive Developmental Disorder-Not Otherwise Specified (PDD-NOS)" into ASD and

discontinue the three earlier terms [6]. According to World Health Organization (WHO), ASD is frequently associated with co-occurring conditions, such as epilepsy, depression, anxiety, attention deficit hyperactivity disorder, and challenging behaviors, such as sleeping difficulties and self-harm [7]. Based on the National Institutes of Mental Health(NIMH), older children and adolescents may have difficulty understanding figures of speech, humor, or sarcasm. They might have also abnormal responses to sensory stimuli. It may also be difficult for them to form friendships with their peers since they cannot maintain consistent eye contact, resulting in repeated missed opportunities for social and emotional learning in early childhood, adversely impacting social cognitive development [8].

Early signs of autism may be detected in childhood (in the first year), but autism is often not diagnosed until much later. Some children develop normally in the first year and then begin to show signs of autism between 18 and 24 months of age. Based on NIMH and the Centers for Disease Control and Prevention (CDC), ASD can usually be reliably diagnosed by age two [8–10]. It is not uncommon for children to receive a final diagnosis much later in life. There is a risk that children with ASD may not receive the necessary assistance due to this delay. ASD should be diagnosed as early as possible so that treatment services may begin as soon as possible [9].

According to CDC [9], National Health Services (NHS) [11], and APA [12], there is **no cure** for autism. Treatments for ASD aim to reduce symptoms that interfere with daily functioning and quality of life [9]. Based on APA treatment concludes applied behavioral analysis, social skills training, speech and language therapy, occupational therapy, parent management training, special education services, treating co-occurring conditions, and medication [12].

Although people with ASD typically require services and support as they grow older, they may be able to work successfully and live independently or in a supportive environment, depending on the severity of their condition. Timely access to evidence-based psychosocial interventions, from early childhood and across the life span, can improve IQ, cognitive and adaptive behavior, communication and social skills, intellectual capacity, and educational support, enhancing the well-being and quality of life for autistic people and their caregivers, and reduce the severity of ASD. Therefore, detecting autism at an early stage of development is highly beneficial for both children and their families [7, 13, 14].

Young children are often diagnosed through a two-stage process [8]: General Developmental Screening during Well-Child Checkups and Additional Diagnostic Evaluation. According to American Pediatrics (AP), all children should undergo developmental screenings at their 9-, 18-, 24- or 30-month well-child visits [9]. A further examination may be conducted for children at high risks, such as those with a family member with ASD, who exhibit behaviors characteristic of ASD, who have older parents, who have certain genetic conditions, or who were born with a low birth weight [7–9]. Children who show developmental differences in behavior will go to the second stage. Diagnosis may include: "medical

and neurological examinations, assessment of the child's cognitive abilities, assessment of the child's language abilities, observation of the child's behavior, an in-depth conversation with the child's caregivers about the child's behavior and development, and assessment of age-appropriate skills needed to complete daily activities independently, such as eating, dressing, and toileting" [8]. Adults with ASD are often more difficult to diagnose than children with ASD. Adults with ASD may exhibit symptoms similar to anxiety disorders or ADHD. This should be discussed with a neuropsychologist, psychologist, or psychiatrist familiar with autism spectrum disorders. The diagnosis process involves a variety of cognitive tests, which usually require hours of intensive clinical examination. Furthermore, standardized tests require substantial time and effort, and the diversity of symptoms contributes to the difficulty of identifying an accurate classification [15].

1.1 Motivation

In recent years, the approximate prevalence of autism has increased over time and has varied greatly within and across sociodemographic groups. This may be due to changes in the clinical definition of autism spectrum disorder, differences in methodology and contexts among the prevalence studies, and improved efforts in diagnosing autism spectrum disorder.

An actual change in the number of people with ASD is possible and could be due to a combination of factors. And there are different statistics about the percentage of individuals with ASD [16]. For example, according to the WHO, approximately one child in 100 has autism worldwide, but there is yet no information about the prevalence of autism in many low- and middle-income countries [16]. The prevalence reported in this estimate is an average of various studies, and the prevalence rate varies considerably from study to study. And some well-controlled studies have reported significantly higher figures [7]. According to two systematic literature reviews conducted by Bougeard C. et al. for the period 2014–2019, in PubMed and Embase, the prevalence of ASD in U.S. children aged 4 and 8 years was 1.70% and 1.85%, respectively, as opposed to 0.38% and 1.55% in European children [17].

Another estimation by CDC shows that about 1 in 44 eight-year-old children has ASD. Moreover, ASD occurs almost four times more frequently in boys than in girls. These estimates are based on data collected by the Autism and Developmental Disabilities Monitoring (ADDM) Network in 2018 from 11 communities throughout the United States. This study collected information on 8-year-old children since previous research has shown that most children with ASD have been identified by this age [18].

In light of the increasing number of individuals with ASD that are being detected and the importance of early intervention, raising public awareness of ASD is critical in several respects. As previously mentioned, when ASD is detected at an early stage, early intervention can greatly improve the outcome of a child. In the early stages of a child's development,

therapy, support, and educational services can be provided to assist them in developing important skills, such as communication, social interaction, and self-management. Parents and caregivers will be taught how to communicate effectively with children with ASD. As a result, the child's ability to communicate and understand others is greatly improved and they are capable of developing positive relationships with others and forming friendships. Moreover, it might improve academic performance, employment opportunities, and independence as well [19].

Both individuals with ASD and the government can benefit from early detection and intervention for ASD. Among the advantages for the government is that it can help reduce the long-term costs associated with the disorder, such as special education, healthcare, and social services and the need for more expensive and intensive services can also be reduced [20].

Furthermore, individuals with ASD are more likely to become productive members of society and be less dependent on government assistance, which can ultimately benefit the economy. At the same time, the government can plan for future funding and support based on an accurate estimate of the number of individuals affected by the disorder, which can assist in the allocation of resources. Through advancing technologies and ongoing research, we may also gain a better understanding of ASD and improve diagnostic accuracy. This would benefit individuals with ASD, caregivers, and the government [20].

For instance, research on oculomotor function has played a significant role in studying various neurodevelopmental disorders such as autism, ADHD, and Tourette's syndrome. There are distinct patterns of deficits observed in each disorder that can shed light on the pathophysiology of these conditions. In addition, abnormal outcomes in oculomotor tasks may provide insights into the aberrant neural circuitry present in these populations. In recent years, eye movement studies have become increasingly useful for understanding the cognitive and neurophysiological aspects of neurodevelopmental disorders [21]. It is relatively easy to record eye-movement tasks with eye trackers, in contrast to other cognitive approaches that may require complex task instructions and cannot be used with individuals of all ages. A number of research studies have demonstrated that individuals with ASD exhibit unique eye movement patterns that differ from those of typically developing individuals. These works have utilized the use of eye-tracking technologies to ascertain what types of images catch ASD individuals' attention. As an example, individuals with ASD tend to focus more on the background of an image than the main subject when exposed to an area of interest in a setting filled with autism-related activities. This indicates difficulty selectively attending to relevant information during an exposure. In addition, they exhibit fewer saccadic movements, which are quick, ballistic eye movements between two fixed points. Reduced saccadic movements may be indicative of difficulty shifting attention quickly between different aspects of a scene. Furthermore, individuals with autism typically make saccadic errors, such as making a saccade to the wrong location or failing to make a saccade at all [21].

1.2 Objective

In this study, we aim to distinguish individuals with ASD based on gaze dynamics by evaluating unique aspects of gaze dynamics, namely the statistics of saccades and fixations. We intend to investigate whether the distribution parameters of fixations and saccades can be used to distinguish individuals with autism from neurotypical ones.

In contrast to most studies that focus on identifying *where* individuals with ASD look, our thesis aims to understand *how* they perceive images. Therefore, we believe that eye-tracking methods might be particularly useful because of their high resolution in detecting autism symptoms, even when participants are engaged in a non-autism-related task. In addition, we would like to compare our approach to other approaches in the literature to improve the accuracy of autism diagnosis, which is becoming increasingly crucial for many people.

This thesis will consist of five chapters. An overview of ASD is presented in the first chapter along with a concise statement of the motivation and purpose of the thesis.

In Chapter 2, we will discuss the current state of the art, including previous work, recent novel approaches, and fundamental concepts such as eye anatomy, different eye movements, eye tracking technology, and gaze dynamics in autistic individuals, and the theoretical aspect of Hidden Markov models and power analysis. These are the models we will focus on for modeling gaze dynamics.

Chapter 3 covers data description and an overview of the methods.

Chapter 4 includes a full description of the analysis and results highlighting.

Finally, in Chapter 5, we discuss our main results and put them in perspective with the state-of-the-art printed in Chapter 2.

We also address some possible future research directions which can be built from the outcome of the thesis.

Chapter 2

Background and state of the art

2.1 Eye anatomy and vision

The eye is an important sense organ that allows humans to see. Together with the optic nerves, it forms an image of the environment around us. Initially, light reflected off an object passes through the cornea, which helps focus the light by bending it. Through the pupil, some of this light enters the eye. The pupil is controlled by the iris and regulates the amount of light entering the eye. Along with the cornea, the lens facilitates the transmission of light to the retina and helps to focus it correctly. Photoreceptors in the retina convert light into electrical signals, which are then transmitted through the optic nerve to the brain. These signals are then processed and converted into images by the brain. As a result of this complex process, which involves the cornea, iris, lens, retina, optic nerve, and brain, we are able to see and perceive the outside world [22]. Figure 2.1 [23] illustrates the anatomy of the human eye.

The eye's outer coat is comprised of two main structures: the sclera and the cornea, both of which serve to protect the eye.

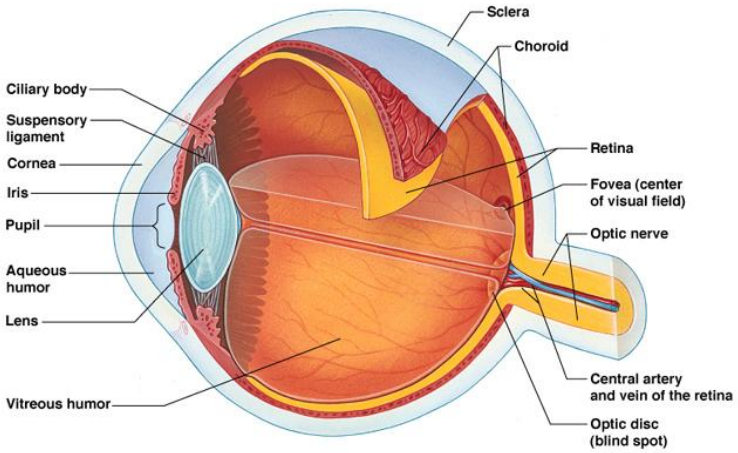


Figure 2.1: A brief sketch of the anatomy of the human eye, taken from Ref. [23].

The sclera is the white part of the eye that covers almost the entire surface of the eyeball. In this part of the eye, light is not permitted to enter, and six extraocular muscles are attached to it, allowing the eye to move in a wide range of directions when these muscles are contracted. In contrast, the cornea is a clear and transparent dome that covers the front part of the eye, including the pupil and iris. Its primary function is to focus incoming light on the retina, which is the inner coat of the eye, making objects appear sharp and clear. Together, the sclera and cornea provide essential protection for the eye, as well as play a crucial role in vision [22].

The middle coat of the eye, also known as the vascular and nutritional part, consists of the choroid, ciliary body, and iris. An eye's choroid is a thin tissue layer that covers the sclera. It becomes thicker as it reaches the ciliary body. The ciliary body is connected to suspensory ligaments that attach to the lens. The lens, which is a clear, transparent structure behind the iris, helps to focus light on the retina, allowing the eye to see both far and near objects. The lens may need to be more concave or flattened if you need to focus light for a longer distance and closer. Since the ciliary body is attached to suspensory ligaments, it influences the shape of the lens through its muscles. Iris, the colored portion of the eye, serves as a diaphragm to control the light entering the pupil. The pupil is a dark circle located at the center of the eye, and it is the opening in the center of the iris [22]. Adult pupils range in size from 2 to 4 mm in diameter in bright light to 4 to 8 mm in darkness. In response to bright light, the pupils shrink (constrict) to prevent too much light from entering the eye. However, in the dark, the pupils expand (dilate) to allow more light to enter, which improves night vision. The size of the pupil can be influenced by various factors in addition to light. As an example, emotional arousal can result in dilated pupils [24]. As people age, their pupils may dilate and constrict less [25]. Some medications, such as antidepressants, antihistamines, and opioids, can constrict or dilate the pupils.

The retina is the inner coat of the eye that lines the back of the eyeball and is composed of two layers: the pigment layer and the neural layer. This light-sensitive tissue contains photoreceptors that convert light into electrical signals. A retina's inner layer contains rods and cones, which are types of photoreceptor cells. The rods are sensitive to light and provide good vision in low-light conditions. Night vision and black-and-white perception are their main functions. A rod can be activated by only a few photons (bits of light) due to its sensitivity to light. And since rods cannot contribute to color vision, we see everything in grayscale at night.

Cone cells, on the other hand, are responsible for color perception and central vision. There are millions of cone cells located in the macula, which is the central region of the retina. The fovea, which is the central area of the macula, contains only cones.

In normal vision, the fovea is responsible for providing 100% sharp and high-resolution vision by focusing light precisely on it. So, the fovea is responsible for sharp central vision, which is also known as foveal vision. For activities requiring a high level of detail, such as reading and driving, this type of vision is essential. With foveal vision, one can clearly

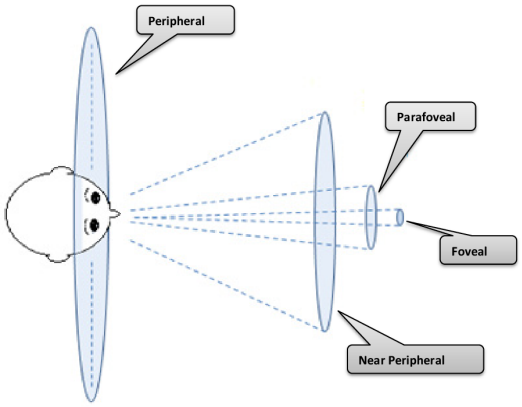


Figure 2.2: Illustration of the visual regions composing our visual field, taken from Ref. [26].

see details such as small text or the intricate features of an object. The fovea is able to provide sharp and detailed vision as a result of its high concentration of cone cells, which are responsible for color perception and high visual acuity. Thus, the fovea plays an important role in our daily lives by enabling us to see fine details clearly and accurately. As we see in Figure 2.2 [26], the visual field can be divided into three regions: foveal vision, parafoveal region, and peripheral region. In each eye, foveal vision covers only a small area of approximately 2° around its focal point. The parafoveal region extends up to five degrees of visual angle on either side of fixation and is capable of discerning some details (50% visual acuity) that the peripheral region cannot. In the peripheral region, however, visual acuity and color perception are significantly reduced, resulting in blurry images.

Motion detection is the primary function of the peripheral region. In spite of the fact that it provides less detailed information about the environment, it is crucial in detecting any potential threats or changes in the environment. From the center axis of the head, the most accurate vision (also known as the cone of vision) is 30% to either side of the direct center. Consequently, the visual acuity decreases as the angle from the center axis increases, which is why peripheral vision provides less information about the environment. The three regions of the visual field work together to provide us with a complete and accurate view of our surroundings [27]. As mentioned, the foveal vision is responsible for providing high-resolution and detailed information, but it only covers a small area of the visual field. Therefore, to capture different aspects of a scene in detail, our eyes **must** move and shift their gaze.

It is due to the fact that the visual information from the parafoveal and peripheral regions is less detailed and provides a broader overview of the environment. The high-resolution foveal vision can be directed to specific areas of interest by moving our eyes.

We move our eyes numerous times throughout the day without even realizing it. By scanning the environment continuously, our eyes are able to capture different aspects of a scene in detail, enabling us to gain a complete understanding of the surrounding environment.

2.2 Eye movements: a composition of saccades and fixations

Eye movements play a significant role in our visual system, as we have discussed previously. We use our eyes to navigate our world by focusing on different things. Nevertheless, eye movements aren't just for visual processing, they're also crucial for social communication. Eye contact and gaze direction convey important information about our emotions, intentions, and thoughts, making eye movements a key part of social interactions.

Human beings are capable of making eye contact and using gaze direction to communicate from a very young age. In the early stages of life, babies use eye contact as a means of establishing social bonds with their caregivers and seeking attention or comfort from them. Eye contact and gaze direction continue to be essential components of our social interactions as we age, allowing us to convey emotions, dominance or submissiveness, interest or disinterest, and other social signals.

Our ability to use eye movements to communicate is an essential aspect of our social cognition and contributes to our ability to form and maintain relationships with others [28].

Eye movements are always paired with head movements and locomotor patterns in nature. The study of eye movements is essential to understanding visual brain function, including perception, attention, memory, dynamic decision-making, and motor control. It's the brain's job to construct a stable and coherent visual perception from local discrete snapshots that are sampled through eye movements. The oculomotor system, which is a part of the central nervous system, is responsible for maintaining visual stability and controlling eye movements. The sensorimotor system must direct the eyes precisely to the object of interest and maintain a stable position to achieve sharp and clear vision. If the eyes are not aligned correctly, it can lead to blurred vision or diplopia. Therefore, eye movements are crucial to stabilizing images on the retina, enabling clear vision even when a subject or object in the environment is moving. There are two parts to the oculomotor system: the vestibular system, which maintains balance and spatial orientation, and the efferent limb of the visual system, which are neural pathways that control the movement of the eyes in response to visual stimuli [29].

In general, we can categorize eye movements into two types: those that maintain our gaze on a particular location and those that enable us to shift our gaze to another location. We maintain our gaze through Fixation, Optokinetic Nystagmus (OKN), and Vestibulo-Ocular Reflex (VOR). And, eye movements that help us switch our gaze to a new point include VOR

cancellation, smooth pursuit, vergence, and saccade [30].

Fixation, also known as visual fixation, is the act of maintaining a steady focus on an object for an extended period of time. It can take between 0.2 and 0.6 seconds for the cone to fully respond to a change in light. Fixation is the most common type of eye movement, and it has been used to explain cognitive and attentional processes. As a result of the high resolution of the fovea, fixation can gather as much information as possible about the object in focus. It is only during periods of fixation that the brain processes visual information, and the length of the fixation indicates the amount of effort required to process the visual information. As a result, the amount of detail that is taken in by the foveal "spotlight" is determined by the location and duration of fixations in the visual scene.

There are two well-known mechanisms that control overt visual attention: bottom-up and top-down mechanisms. Bottom-up mechanisms are characterized by the ability of an area to attract attention unconsciously and effortlessly based on low-level characteristics such as color, luminance, texture, and motion. Saliency maps serve as a source of bottom-up guidance by identifying the most visually interesting areas in our visual field. Top-down contributions to attention are influenced by an observer's goals, prior knowledge, motivation, mood, and experience [31].

Despite the impression that the eyes are completely still during fixation, they are never actually at rest, since doing so would result in a loss of vision. "Fixational eye movements" are involuntary, small, and often unconscious eye movements that constantly interrupt visual fixation. There are three types of fixational eye movements: drift (also called slow control), oculomotor microtremor (also called tremor), and microsaccades. Microsaccades are the largest of these and occur once or twice a second. Microsaccades may have a greater impact on perception and physiology than drifts or tremors since their amplitudes exceed 0.2° [32].

The OKN is repeated reflexive responses of the eyes to ongoing large-scale movements of the visual scene. The VOR stabilizes the eyes in relation to the external environment, compensating for the movement of the head by moving the same distance but in the opposite direction. By moving in the same direction, the VOR will be canceled (cancellation of VOR), and the gaze will switch. The Smooth Pursuit eye movement enables the eyes to smoothly track moving objects, such as a ball being thrown. During the motion of an object, it stabilizes images on the retina. Vergence is a movement of the eyes (convergence or divergence) that aligns the fovea of each eye with targets situated at different distances from the observer [30].

In 1879, Javal was the first to use the term saccade, derived from the French word 'jerk' or 'twitch', to describe a rapid eye movement. A **saccade** refers to the simultaneous ballistic movement (jumps of 2 deg longer, which continues for about 30–120 ms and reaches speeds up $700^{\circ}s^{-1}$ for large saccade [15]) of both eyes in the same direction between two or more fixation phases which is partially controlled by the posterior cerebellar vermis. So, Saccades shift the spotlight of attention. During a saccade, the movements are predetermined at the time of initiation, and the system generating the saccade cannot react to subsequent changes in the target's

position after saccade initiation, so saccades are called ballistic. Sequences of fixations and saccades guide our perception. Since the image on the retina is of poor quality during a saccade, most information is absorbed during the fixation phase (saccadic suppression) [33].

There are several terms that characterize saccades; we report them with a start and end time, as well as a list of metrics [30]:

- **Number:** The number of saccades occurring during an interval of time and within a target Area of Interest (AOI).
- **Amplitude(The saccade size):** The degree or minutes of arc between the centroid of the fixations preceding and following the saccade. A greater amplitude indicates that the eye has traveled a greater distance during the saccade.
- **Direction:** The angular distance between a straight line connecting the saccade's start to the saccade's end compared with a straight line connecting the start of the saccade to the horizontal axis of the screen (or active display area). The unit circle is used as the coordinate system, with 0 degrees located on the right and the angle increasing anticlockwise.
- **Gain:** The ratio of the actual saccade amplitude to the desired one (comparing the eye's displacement with the target stimulus' displacement). Gains greater than 1 indicate that the saccade was too large or hypermetric, whereas gains less than 1 indicate a saccade that was too small or hypometric.
- **Duration:** The time it takes for the saccade to be completed.
- **Velocity:** The amplitude of a saccade divided by its duration, reported in degrees per second.
- **Peak velocity:** The highest eye movement velocity reached during a saccade, reported in degrees per second.
- **Latency:** The time it takes for a saccade to occur between two fixations, measured in milliseconds (approximately 200 to 250 ms). Depending on the individual, it may differ. As the brain controls the muscles in the eye, it is the brain that commands the eye to move. We measure saccadic latency as the time it takes for the brain to instruct the eye to make a saccade. This means whatever grabbed our attention had to be extremely important if the saccade had a short delay (saccades are very fast, and the latency is short). However, if the latency for the saccade is long and is a high delay, then whatever caught our attention was less important.

There is a close correlation between saccade duration and saccade amplitude, with longer saccades taking a longer time to complete. Additionally, saccade velocity and amplitude exhibit a linear relationship, meaning that larger saccades have higher velocities. Saccades have an inverted u-shape

velocity profile, with the velocity increasing until the midpoint of the saccade, and then decreasing [30]. Smooth pursuit eye movements have much slower acceleration and velocity than saccades, and they require the presence of a moving visual stimulus to be performed accurately. Two common parameters studied in the smooth pursuit are gain and phase lead/lag.

Phase lead/lag refers to the distance between the eye's position and the target stimulus during the smooth pursuit phase. Specifically, it measures the mean displacement of the eye during a trial, comparing the position of the eye relative to the target stimulus.

Saccades can be categorized in several ways based on their characteristics. One method of classification is based on saccade amplitude, which can be classified as either small and rapid (such as during reading), or large ($> 20^{\circ}$) movements (such as scanning a room). Another method is to classify saccades by their direction, either horizontally or vertically. Saccades can also be voluntary or involuntary. Voluntary, endogenous saccades are self-directed eye movements that can be initiated by a command, while reflexive saccades are those that respond to a visual or auditory stimulus.

In terms of cognitive and neurological deficits, a study by Rommelse NNJ. et al. found that four types of saccades were the most significant. These include antisaccades (AS), visually guided saccades (VGS), memory-guided saccades (MGS), and smooth pursuit eye movements (SPEM). Understanding these different types of saccades and their associated deficits can provide valuable insights into the mechanisms of saccadic eye movements and how they are related to cognitive and neurological functioning [34].

AS involves intentionally shifting the gaze away from a visual stimulus, which requires inhibition of reflexive saccades to the target location and voluntary eye movements to the mirror location. Directional errors and saccade latency are typical measures of AS. In VGS, participants are required to make eye movements in response to a visual stimulus presented in their peripheral vision. The VGS is frequently used as a baseline condition to study the basic dynamics of eye movement. VGS measures include saccade latency, variability in latency, amplitude, undershoot versus overshoot, peak velocity, and duration. The MGS procedure involves moving the eye toward a remembered point based on a prior visual stimulus.

Generally, saccades help us explore and interact with the world. By using them, we can move our gaze quickly and accurately to new locations of interest, like people, objects, or events. Superior colliculi, frontal eye fields, and parietal cortex are involved in the control of saccades. Also, factors like attention, motivation, and emotion can influence saccades, and abnormalities in saccade production or control have been linked to various neurological and psychiatric problems [35].

A thorough examination is required before determining the dynamics of eye movements. Thus, it is crucial to develop techniques for recording eye movements. Additionally, oculomotor research development is closely related to these technologies. Eye tracking is one of these technologies. Eye tracking is an effective method for collecting information regarding

eye-related signals, such as gaze direction, pupil size, and blink rate. Data such as this can be used to generate accurate insights into visual attention, cognitive processes, behavioral responses, and mental states- enabling researchers to gain a deeper understanding of behavioral science [36].

2.3 Eye tracking technology

Eye tracking involves tracking eye movements, or the absolute point of gaze (POG: where the user's gaze is focused in a visual scene), in response to moving objects, lines of text, or other visual stimuli. There is a wide range of applications for eye tracking, from psychological research and medical diagnostics to usability studies and interactive, gaze-controlled applications. As early as 1879, French ophthalmologist Louis Émile Javal discovered for the first time that readers' eyes make quick movements mixed with short pauses while reading rather than skimming fluently through the text. Edmund Huey developed a device that tracked eye movements during reading in 1908. The first eye tracker was intrusive since readers had to wear a contact lens with a small opening for their pupils. Early in the 1900s, a non-invasive eye-tracking apparatus based on photography and light reflected from the cornea was developed as a breakthrough in eye-tracking technology. By developing unobtrusive camera-based systems and increasing computing power, eye-tracking data can be collected in real-time. As a result, eye trackers became less intrusive, more accurate, and could separate eye movements from head movements. In parallel, psychological theories began to examine the relationship between eye-tracking data and the cognitive function [37].

For eye tracking, there are many different approaches, some of which require additional hardware, while for others, an ordinary webcam will suffice. The following are various methods that can be used [38]:

- **Observation:** Before the 19th century, this was an early method of eye tracking.
- **Scleral Search Coil method:** The eye movement is measured using an annulus or modified contact lens with small coils of wire embedded within it. A local anesthetic is injected into the eye before the device is inserted into the eye. The coil's wire leaves the eye at the temporal canthus. There are two field coils located on either side of the head that produce the field. As a result, horizontal eye movements can be recorded. In order to monitor vertical eye movements as well, a second set of field coils is usually placed orthogonally to the first set. It is also possible to record torsional movements if the eye coil has been designed appropriately. The eye coils are often implanted surgically in experiments on eye movements in animals. This method has the advantage of having a very high spatial and temporal resolution, which makes it possible to study even small types of eye movements such as microsaccades. The disadvantage of this procedure is that it is an invasive procedure

requiring the placement of something in the eye. Clinically, this method is rarely used, but it is an invaluable research tool.

- **Electro-Oculography (EOG):** EOG is a technique used for measuring the resting potential of the retina. The cornea and retina have a permanent potential difference of approximately 1 mV, and as the eye position changes, small voltages can be recorded from the area around the eye. A pair of electrodes are typically placed above and below or to the left and right of the eye. Careful placement of the electrodes allows for separate recording of horizontal and vertical eye movements. When the eye moves from the center position towards one electrode, that electrode detects the positive side of the retina, while the opposite electrode detects the negative side, resulting in a potential difference between the electrodes. If the resting potential of the retina remains constant, the recorded potential can be used to measure the position of the eye. However, EOG is less reliable for measuring medium and large saccades quantitatively. Despite this limitation, clinicians often use this method extensively because it is simple, inexpensive, and non-invasive.

- **Infra-Red Oculography:** An eye's position determines how much light will be reflected back from a fixed light source to a fixed detector. Eye trackers based on this principle are commercially available. Infra-Red light is used because it is invisible to the eye and does not distract the subject. The ambient light level has no significant impact on infrared detector measurements since infrared detectors are not influenced by other light sources. With this technique, the size of the smallest movement can be reliably detected. The method is better suited to measuring horizontal eye movements than vertical ones. Blinks pose a problem since they not only cover the eye's surface but also cause the eye to retract slightly, which reduces the amount of light reflected after a blink.

- **Video Oculography (VOG):** The method uses a head-mounted mask with small cameras to measure horizontal, vertical, and torsional position components of the movements of both eyes.

- **Video-based Infra-Red (IR) Pupil-Corneal Reflex (PCR) tracking:** In IR-PCR, the eye is illuminated, and an image of the eye is captured. Using the eye image, it is possible to identify the pupil center and the reflection of the illuminators on the cornea. Calculating the participant's gaze is based on the pupil's position and the illuminators' reflections, as shown in Figures 2.3 and 2.4 [39].

A valuable tool for analyzing the gaze dynamics of individuals with ASD is eye-tracking technology. Several studies have demonstrated that individuals with autism have atypical eye movements and fixations, which can affect their ability to discern social cues and comprehend emotional expressions. Using eye-tracking technology, gaze patterns can be precisely

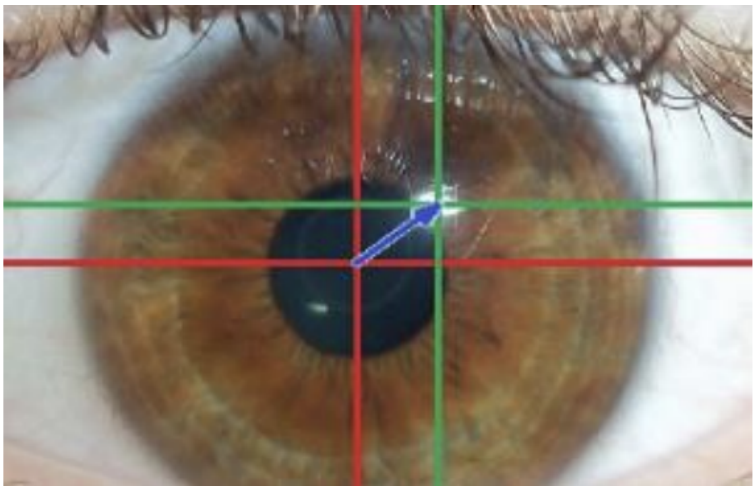


Figure 2.3: An illustration of the relative position of the pupil and corneal reflex, taken from Ref. [39].

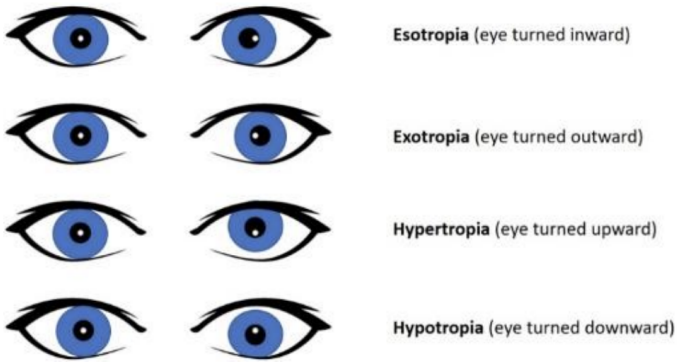


Figure 2.4: The Pupil position and corneal reflex, taken from Ref. [39].

measured and analyzed, providing insight into how individuals with autism process visual information. Eye-tracking technology offers a powerful tool for advancing our understanding of individuals with ASD and has the potential to lead to more effective treatments and interventions.

2.4 Gaze dynamics in autism

Studying eye-tracking data provides valuable information about spontaneously occurring priorities and patterns of attention. The regions in an image where a subject looks can be used to infer the subject's attention priorities. Autism is particularly well-suited to eye tracking because what an autistic individual chooses to pay attention to is different from typical controls - especially regarding socially relevant information and face perception, as demonstrated in the first eye-tracking study of individuals with autism conducted by researchers at the University of North Carolina in 2002 [40].

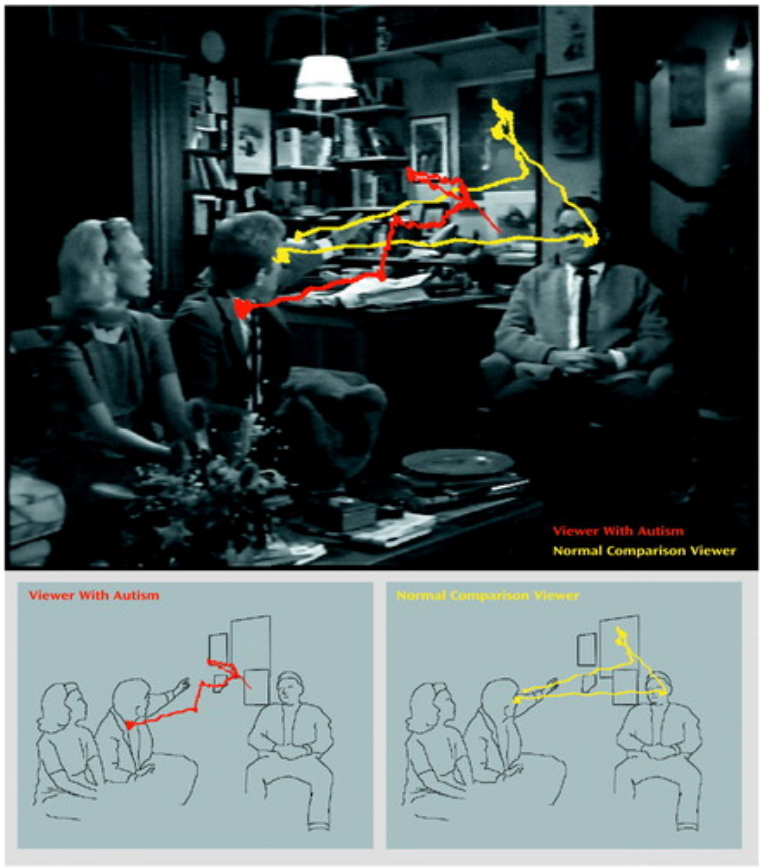


Figure 2.5: In this film clip with social-visual and verbal cues, the visual scanning patterns of an autistic man are compared with those of a normal comparison subject, taken from Ref. [41].

They used a technique called the visual scan path of infrared corneal reflection. This technique allowed them to observe the eye movements of both groups during an emotion recognition test using facial expression photographs. According to the study, participants with autism viewed non-featured areas of faces significantly more often than typically developing (TD) participants. In particular, core features such as the eyes, nose, and mouth were viewed significantly less frequently. This resulted in a deficit in emotion recognition, specifically in recognizing fear. Also, individuals with autism had fewer fixations on the core facial features based on the analysis of gaze data. The findings suggest a mechanism that may explain the deficits in social information processing associated with ASD [40].

In a notable study conducted by researchers [41], the fixation patterns of 15 young males with autism and 15 control individuals were tracked while they watched scenes from the movie "Who's afraid of Virginia Woolf." Unlike the previous study which used static stimuli i.e pictures, this study used dynamic stimuli i.e videos. A reason for selecting this movie was that it demonstrated the intense interaction between four protagonists

involved in a content-rich social situation. This helped to maximize viewers' awareness of everyone's socially expressive actions and reactions to those actions.

The study showed that individuals with autism view scenes differently than normal controls, as they tend to focus more on the actor's mouth or even movements in the periphery of a scene rather than the actor's eyes. This is demonstrated in Figure 2.5 [41]. Similar to the previous study, the autism group looked less frequently at the characters' eyes, suggesting a deficit in processing eye contact as a social cue.

Interestingly, in this study, the ability of an individual with autism to communicate socially was strongly predicted by their fixation of the mouth. The Area Of Interest (AOI)-based approaches used in this study measure the spatial distribution of visual attention across the scene, thus identifying which AOI attracts and holds participants' attention.

Similarly, in 2003, Robert M. Joseph and James Tanaka [42] conducted a study where they looked at a group of 22 people who had high-functioning autism and compared them to two other groups of typically developing individuals. The first group consisted of 9-year-olds, and there were 27 people in this group. The second group consisted of 11-year-olds, and there were 30 people in this group. According to them, the abnormalities in face recognition in autism may not fully be explained by impaired holistic face processing. They found that children with autism place unusual importance on the mouth region when interpreting the facial expressions of others. The study revealed that high-functioning autistic children had a significant advantage when it came to mouth recognition, however, they were significantly behind when it came to face recognition using the eyes. Children with autism may rely more heavily on the mouth region of the face for processing information. This could explain their difficulties in recognizing emotions and social cues conveyed by the eyes. Overall, this study provides valuable insights into the nature of face recognition deficits in autism and highlights the importance of considering the role of different facial features in social communication.

Autism is a disorder that affects an individual's social communication and interaction abilities. By understanding the role of early face-processing deficits in autism, we can develop more effective interventions and therapies that target these specific areas of difficulty. Dawsen G. et al. [43] suggested that individuals with autism may suffer from socio-communicational deficits because of a failure to orient and engage their attention to socially relevant stimuli such as faces at an early age. One of the crucial differentiation factors in early diagnosis of this disorder is the ability of the toddler to use facial information, such as gaze monitoring during joint attention. And a failure to process faces in a normal manner could be among the earliest measurable symptoms of autism, occurring as early as one year of age.

Studies have shed light on the mechanisms that underlie social communication difficulties experienced by individuals with autism. The identification of specific areas of difficulty, such as abnormal fixation patterns, can lead to more targeted interventions and therapies that can

improve the social functioning of individuals with autism.

The relationship between face gaze and social cognition in autism was investigated in a study conducted in 2007. In the study, the researchers measured both face gaze and how facial regions were used during judgments of emotion from faces. The study included nine high-functioning adults with autism who failed to use information from the eye region of faces, instead relying primarily on information from the mouth, which is a similar finding to previous studies.

The study found that subjects with autism made more fixations on the mouth, and they relied on information from the mouth to determine the emotion they were experiencing. The results of these studies have led to the first quantitative assessment of how people with autism make social judgments based on information from faces [44]. The findings suggest that autistic individuals have abnormal fixation patterns related to social stimuli.

Swettenham [45] conducted a novel experiment in 1998 involving 10 children with autism, 17 with developmental delays, and 16 normal children. The children were filmed during a sequence of free play for approximately five minutes each. The videotape was analyzed frame-by-frame using a timer accurate to a tenth of a second by two independent judges who were blind to the children's diagnoses. Three types of attention-shifting behavior were observed; between objects, between objects and people, and between persons. The study found that children with autism exhibited fewer attention shifts between objects and people and between persons to persons. Additionally, they looked at objects for longer durations and looked at people for a shorter period than the two control groups. The findings of this study suggest that individuals with ASD may have difficulty shifting their attention from one stimulus to another, particularly to social stimuli.

In 2019, a similar study was conducted [46] using a TobiiT120 eye tracker with 22 autistic and 22 typically developing children. The study found that ASD children had difficulty disengaging from neutral fixation to focus on peripheral stimuli. While they were slower to fixate on faces and common objects, they were faster to fixate on non-social peripheral stimuli related to their circumscribed interests. Comparatively to the other stimuli, typically developing children were quicker to fixate on faces.

In 2007, Speer et al. [47] conducted an experiment that replicated the study of Klin et al. [41] and Pelphrey et al. [40] using a different stimulus. The experiment involved 12 male autistic children and adolescents, aged 9 to 18, and 12 typically developing children of the same age group. The participants were shown four different types of stimuli: a social dynamic condition (video clip depicting highly emotional interactions between characters), an isolated dynamic condition (video clip showing nature with only one character), a social static condition (five images of two or more individuals), and an isolated static condition (five pictures depicting one individual). The researchers used eye-tracking techniques to measure gaze behavior in both groups of participants. Results indicated that autistic individuals had different fixation durations for social-dynamic stimuli

compared to their peers who were typically developing. In particular, their fixation durations were shorter for eye regions and longer for body regions. There was no difference between the two groups in the gaze patterns observed when viewing static photos of isolated individuals. As a result of these findings, it appears that the gaze patterns of autistic individuals differ from those of typically developing peers when viewing dynamic social scenes, but not when viewing static images of isolated individuals.

The literature regarding the nature of these fixation abnormalities is inconsistent, despite the fact that such abnormalities have been suggested as an explanation for difficulties in face recognition. There may be an explanation for these inconsistencies in part due to differences in sample sizes among studies and variability in fixation behavior within a given study. Variations in fixation patterns may also be attributed to aging.

Studies have shown that people with this disorder change their fixation behavior as they grow older [48] For example, toddlers around 21 months, young children under ten years, and adults over twenty years have different fixations. The results of experiments using image stimuli have demonstrated the importance of the image's content, particularly human faces, in distinguishing visual behavior among individuals with ASD at different stages of development.

Both young and older children with ASD display distinct differences in the duration of their fixations on human faces when compared to typically developing children of similar ages. According to these findings, individuals with autism may process visual information differently based on their age and the nature of the stimuli they are exposed to. Understanding the changes in fixation behavior across the lifespan may therefore be essential in developing effective interventions and treatment strategies for individuals with ASD.

In one study [49], children, adolescents, and adults (aged 7-30) with and without autism completed the Cambridge Face Memory Test while their gaze was recorded. In all age groups and groups of individuals, eye fixations were more frequent and longer than mouth fixations. A typical development child or adult, but not an adolescent, made more fixations to the eyes during face memorization compared with a typical development peer with autism. According to the results, there is a change in group differences in patterns of fixations to faces with age. Furthermore, the relationship between patterns of fixations and face recognition performance is different in typical development and autism, which suggests that these differences are at least partially responsible for the difficulties associated with autism. Overall, these findings highlight the importance of age-related changes in fixation behavior for the underlying mechanisms of face recognition deficits in people with autism.

An atypical oculomotor behavior may contribute to the difficulties individuals with autism experience in terms of orientation, exploration, and perception. Effective oculomotor behaviors, such as fixation and saccades, are essential for efficient visual perception. Individuals with autism, however, tend to display diminished gaze fixation, which may be due to abnormalities in their neural circuitry of affect.

Researchers tested emotion discrimination and facial recognition in two separate studies. In these studies, photographs of human faces were presented to participants with autism and controls while they were undergoing magnetic resonance imaging (MRI) scans. The activation of the fusiform gyrus, an inferior temporal cortex region responsible for recognizing objects, faces, and facial expressions, was positively correlated in both studies with the amount of time spent fixed on the eye region in the autistic group. Thus, diminished gaze fixation may be responsible for the fusiform hypoactivation that has been observed in autism when exposed to facial stimuli. In addition, both studies found a strong and positive association between variations in gaze fixation among autistic individuals and amygdala activation. As a result, gaze fixation may be associated with increased emotional response in individuals with autism [50]. These findings highlight the importance of gaze fixation in visual perception and emotional processing in individuals with autism.

Pretectal oculomotor structures can lead to involuntary saccades. A saccadic intrusion is an involuntary saccade that interrupts fixation. An involuntary, horizontal, saccadic intrusion that interrupts fixation is referred to as a square wave jerk (SWJ). SWJs are characterized by an initial saccade that moves the fovea away from the intended fixation position, followed by a second saccade that reorients the fovea back to the initial fixation position. Due to reduced inhibitory control of pretectal oculomotor structures, intrusive SWJs are more likely to occur during visual fixation. Both suppression of intrusive saccades (SWJ) and maintaining eccentric gaze require intact cerebellar function.

In a related study, Caralynn V. Nowinski et al. examined the impact of intrusive saccades and maintaining eccentric gaze on individuals with autism and healthy individuals. The experiment involved analyzing the suppression of intrusive saccades and maintaining eccentric gaze during the visual fixation of static central and peripheral targets using infrared reflection sensors to measure eye movements. The results showed that individuals with autism had an increased amplitude of intrusive saccades and reduced latency of target refixation after intrusive saccades. Possibly, this is due to faulty functional connectivity in the cortico-cerebellar system [51].

Two of the most common oculomotor assessments are those of saccades and visual smooth pursuit. In adults with ASD, saccade function is relatively normal, but the visual smooth pursuit is abnormal [52]. Assessing the oculomotor performance of children ages 6-12 years with high-functioning ASD revealed that they exhibited greater horizontal saccade latency (although no significant difference was observed between groups for vertical saccade latency) and greater phase lag during vertical smooth pursuit. This indicates an abnormal visual smooth pursuit phase in individuals with ASD. Individuals with ASD may experience abnormal smooth pursuit function due to local abnormalities in brain regions implicated in smooth pursuit (frontal eye fields, lateral intra-parietal area, superior temporal area, caudate, superior colliculus, cerebellar vermis, brainstem premotor nuclei, and vestibular nuclei) or abnormalities in long-

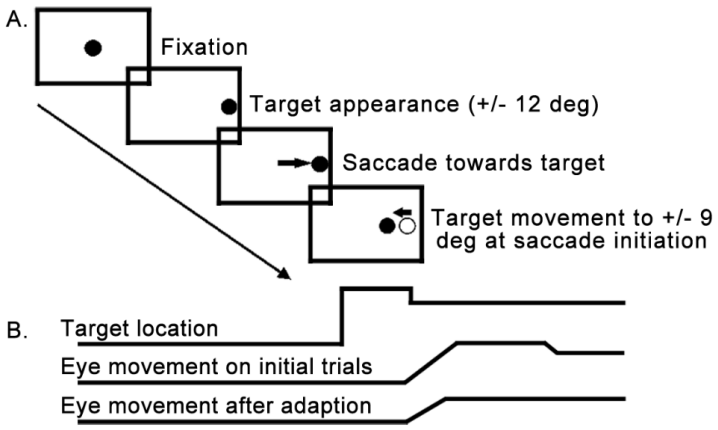


Figure 2.6: An illustration of the saccade adaptation test, taken from Ref. [53].

fiber tracts linking these regions [52].

The cerebellar vermis region plays a crucial role in maintaining the accuracy of saccadic eye movements. It plays a particularly important role in correcting systematic errors in saccade amplitudes induced by adaptation paradigms [53]. The term "saccadic adaptation" refers to the adjustment of the Oculomotor Transformation parameters when a saccade direction or size consistently fails to reach its target. This adjustment ensures that the direction and size of the saccade remain on target. Nevertheless, landing positions may still be error-prone due to intrasaccadic shifts in the target location after saccade initiation. With practice, subjects can gradually decrease the amplitude of their saccades to reduce movement errors (See Figure 2.6 [53]).

Individuals with autism have a deficit in the cerebellar vermis, which affects sensory-motor control and leads to slower adaptation and greater variability in saccade amplitude. [53]. This deficit might lead individuals with ASD to have difficulty adjusting their eye movements to induce retinal image position errors systematically.

Cerebellar and brainstem abnormalities can have an impact on sensory-motor control in individuals with ASD. Specifically, it is the cerebellum and brainstem that control saccade metrics. In a study conducted by Lauren M Schmitt et al. [54] in 2014, visually-guided saccades were examined in a relatively large sample of individuals with ASD (65 individuals with ASD matched with 43 healthy controls aged 6 to 44 years old). The study found deficits in saccade accuracy and saccade dynamics (using EOG) in individuals with ASD. The saccadic eye movements of participants with ASD were less accurate and more variable across trials compared to healthy controls.

Furthermore, individuals with ASD exhibit lower peak saccade velocities, and longer durations result in slower saccade. Patients were observed to spend more time accelerating saccades to reach peak velocity but not

decelerating them. Attentional deficits are also believed to contribute to saccade initiation abnormalities in individuals with ASD. Increased latency variability in individuals with ASD is indicative of increased variability in attention control [54].

Eye-tracking technology has been used to record these atypicalities in gaze dynamics. Moreover, the combination of eye-tracking technology and machine learning could ultimately improve our understanding of visual processing differences in individuals with ASD and might lead to earlier diagnosis and more effective intervention. The next chapter will explore how machine-learning techniques have been applied to eye-tracking data in autism research.

2.5 Basic concepts in machine learning

Before proceeding further with our discussion, it is crucial to have a clear grasp of the essential terminology and concepts associated with machine learning approaches, which are frequently utilized to model gaze dynamics in ASD participants. In this section, we will provide definitions and explanations [55] for the key terms used throughout the next section. These terms are as follows:

- **Object annotation**: The process of object annotation involves identifying and labeling objects within an image, allowing us to describe important characteristics such as the object's location, size, and category.
- **Image pre-processing**: At times, it is necessary to perform certain adjustments on our images before they can be used in a machine learning algorithm. Some examples of these adjustments include noise reduction, resizing the image, and normalization, which are done to make the image suitable for use as input data. This process is commonly known as image pre-processing.
- **Image augmentation**: Image augmentation is the process of applying transformations to images, such as rotations or zooms, in order to generate additional variations of the original images. This is done to increase the size of the image dataset, which can lead to better performance and accuracy in machine learning models.
- **Minimum Redundancy Maximum Relevance (MRMR) feature selection method**: MRMR is a process used to select the most important and relevant features for a specific task. This helps to minimize the number of features used as input. One important aspect of MRMR is that it avoids selecting similar or redundant features that may not add much new information.
- **Logistic Regression:** In statistics, logistic regression is a widely used statistical model that is used to estimate the probability of an event occurring, based on one or more independent variables. In binary

logistic regression, the dependent variable takes on only two values, typically labeled "0" and "1". The independent variables can either be binary variables (with two classes) or continuous variables (with any real value).

Unlike linear regression, which cannot be used to estimate the probability of a dependent variable due to the possibility of negative probabilities, logistic regression uses a function that produces outputs between 0 and 1 for all values of the independent variable. This function is commonly referred to as the sigmoid function, and its output value is always between zero and one.

The sigmoid function results in an S-shaped curve, which ensures that regardless of the value of the independent variable, the predicted probability will always be sensible and not negative. Therefore, logistic regression is a powerful tool for predicting the probability of an event occurring, particularly in cases where the dependent variable is binary.

- **Support Vector Machine (SVM)**: SVM is a supervised learning algorithm that can be used for tasks such as classification and regression. An SVM algorithm is designed to find a hyperplane in P-dimensional space (P corresponds to the number of features) that distinguishes data points from one another. Several possible hyperplanes may be selected to separate the two classes of data points. Hyperplanes are decision boundaries that categorize data points. There are different classes of data points that fall on either side of the hyperplane. Additionally, the hyperplane's dimension depends on how many features there are. Data points near the hyperplane are called support vectors, and they influence the hyperplane's position and orientation.

- **Deep Support Vector Machine (DSVM)**: DSVM is a type of machine learning algorithm that combines the power of deep learning and support vector machines (SVM). The architecture of DSVM is similar to a deep neural network, which has multiple hidden layers of neurons. In a regular deep neural network, the neurons in the hidden layers are connected by weights, but in DSVM the weights are initialized using SVM functions that have been modified with a special parameter called the SVM regularization parameter. To get the best output for a given input, DSVM updates all the SVM functions in the hidden layers to find the most optimal set of weights. This approach allows DSVM to benefit from the strengths of both deep learning and SVM, resulting in a powerful algorithm that can be used for a variety of tasks.

- **Decision Trees (DT)**: Supervised learning techniques, such as DT, can be applied to both classification and regression problems, but are most commonly used to solve classification problems. Essentially, it is a tree-structured classifier with internal nodes representing features,

branches representing decision rules, and leaf nodes representing outcomes

- **Random Forests (RF)**: For making decisions, Random Forest, which is an ensemble method, utilizes the power of multiple decision trees. Every tree in the random forest predicts a class and the class with the most votes becomes our model's prediction.

- **Boosted Decision Tree (BDT)**: In the Boosting method for classification, misclassified events are given a higher weight so that future learners can focus on these examples and improve their accuracy. BDT is a type of machine learning algorithm that combines many decision trees into one model. The algorithm trains the decision trees iteratively, using re-weighted versions of the training data in each iteration. The weights are adjusted to give more importance to the examples that were misclassified in the previous iteration. This approach helps BDT to create a more accurate model by focusing on the areas where it made mistakes and improving upon them. In the end, the algorithm combines all the decision trees to create a powerful model that is better at classifying data and making predictions than any individual decision tree could be on its own.

- **Convolutional Neural Network (CNN)**: CNNs are a type of deep learning algorithm that have been designed to perform image classification, object detection, and other related tasks. They use a series of convolutional filters to analyze the input image, and then apply non-linear transformations and pooling operations to extract important features.

- **Linear Discriminant Analysis (LDA)**: LDA is a statistical method that helps us to identify the features that best separate different groups of objects or events. It tries to find a linear combination of these features that highlights the differences between the groups. This is useful for tasks like classification, where we want to assign new objects or events to the correct group based on their features. LDA is a commonly used method for classification in machine learning.

- **Principal Component Analysis (PCA)**: PCA is a method that is commonly used to make sense of complex data sets with a lot of features. It helps to reduce the number of dimensions, making it easier to understand the information contained within the data. This method helps to maintain as much information as possible while enabling the data to be visualized in a way that is easier to comprehend.

- **Evaluation metrics:** The assessment of a classifier or predictor's performance can be accomplished through several metrics, with varying preferences based on the specific objectives of different fields. In the medical domain, the use of specificity and sensitivity

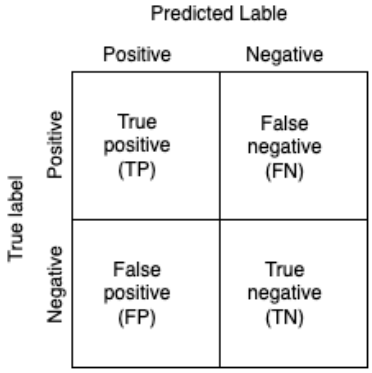


Figure 2.7: Comparison between actual and predicted conditions.

is frequent, while the computer science field favors precision and recall. The evaluation metrics are calculated based on the comparison between the predicted and real data, as illustrated in Figure 2.7.

Accuracy is a widely used metric in classification tasks, which evaluates the algorithm's ability to correctly classify a given data point. It is calculated as the proportion of correctly predicted data points out of all the data points. The calculation of accuracy based on Figure 2.7 can be expressed mathematically as follows:

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN}. \tag{2.1}$$

Recall, also known as **sensitivity**, is an essential evaluation metric, especially in medical applications. It refers to a test's ability to correctly identify individuals with a particular disease as positive. A highly sensitive test has fewer false negative results, reducing the likelihood of missing cases of the disease. The recall based on Figure 2.7 can be calculated using the following formula:

$$Recall = \frac{TP}{TP + FN}. \tag{2.2}$$

Precision is another important metric that is often used to assess the performance of a classification model. Precision is a measure of the accuracy of the positive predictions made by the model. It answers the question: "How many of the positive identifications made by the model were correct out of all the positive identifications?". In other words, precision tells us how precise the positive predictions made by the model are. Based on Figure 2.7 it can be calculated using the following formula:

$$Precision = \frac{TP}{TP + FP}. \tag{2.3}$$

The **F1 score** is an evaluation metric that combines the precision and recall measures to provide a single value that summarizes the overall

performance of a classifier. It is calculated as the harmonic mean of precision and recall, giving equal weight to both measures. The F1 score can be calculated using the following formula:

$$F1 = 2 * \frac{\cdot precision \cdot recall}{precision + recall}. \tag{2.4}$$

- **Receiver Operating Characteristic (ROC)**:A ROC curve is a graphical representation of the trade-offs between different classification thresholds. The plot displays the relationship between the true positive rate (sensitivity) and the false positive rate (specificity) for various threshold values. A well-performing classification model should exhibit more true positives than false positives at all thresholds.

  The ROC curve is used to determine the optimal position, which is towards the top left corner, where the sensitivity and specificity are at their maximum values. The area under the ROC curve (AUC) is a commonly used metric to assess the classification model's accuracy. The AUC value represents the degree of separation between true and false positives. A higher AUC indicates that the model can more accurately predict the training dataset's members.

  A model with an AUC of 0.5 indicates random classification performance, and a good classifier should strive to stay as far away from this value as possible. An AUC value of 1 is ideal, indicating perfect classification performance. In general, a higher AUC value implies a better classifier. Therefore, it is desirable to have an AUC value that is as close to 1 as possible.

2.6 Predictive modeling of gaze dynamics in autism spectrum disorder using machine learning techniques

As we discussed, atypical visual attention patterns reflect higher-order differences in information processing, as the focus of attention directs the input of information from the environment. Visual attention is related to concentration, interest, perception, learning, the ability to form joint attention, cognitive effort, and other indicators, the combination of which can be used to detect autism. For example, many people with ASD tend to avoid the eye region when looking at faces.

As a cost-effective method of measuring gaze behavior, eye tracking has become increasingly popular for diagnosing and predicting ASD. Researchers have applied various machine learning models to eye-tracking data, in addition to choosing different stimuli, in order to understand the difference between autistic and typically developing individuals (of varying ages). As part of this literature review, we will explore some of these machine learning models and their applications in understanding autism through eye-tracking data.

The study by Wang et al. [56] provided further insights into the visual processing differences in individuals with ASD. They used a novel three-layered computational saliency model to examine how people with ASD and controls allocate visual attention and eye movements while viewing natural scene images. The study was conducted with 20 people with ASD and 19 controls who were matched on age, IQ, gender, race, and education. The model took into account different factors such as pixel level (e.g., contrast), object level (e.g., shape), and semantic-level attributes (e.g., faces) to 5551 annotated objects with a SVM classifier to predict fixation allocation. These findings suggest that individuals with ASD may have different strategies for processing and allocating visual attention. The results showed that people with ASD exhibited a stronger image center bias, regardless of object distribution, compared to controls. They also showed reduced saliency for faces and locations indicated by social gaze, as well as increased pixel-level (such as color, intensity, and orientation) saliency at the expense of semantic (such as face) saliency. Strong center bias in ASD was associated with a slower saccade velocity but not with fewer fixations or a different distribution of objects. This finding could have implications for our understanding of how individuals with ASD process and respond to visual information and may inform the development of interventions and treatments for ASD [56].

Various stimuli and approaches have been used to study visual processing in ASD individuals. For example, the study by Zhao et al. (2021) [57] builds on this by examining whether eye-tracking data from face-to-face conversations could be used to differentiate between children with ASD and typical development children. They utilized four areas of interest (AOIs) in four conversation sessions, including the eyes, mouth, whole face, and whole body, and calculated the percentage of visual fixation time on each AOI as a feature. They also included features related to session length. The study investigated whether combining features related to visual fixation and length of conversation would result in better classification performance. Four machine learning classifiers (such as SVM, decision trees, and random forests) were utilized to determine maximum classification accuracy and their corresponding features. The results showed that eye-tracking data collected from face-to-face conversations could be used to accurately classify children with and without ASD, with a maximum classification accuracy of 92.31 percent for the SVM classifier when combining features on both visual fixation and session length. This finding may provide further evidence of the potential usefulness of eye-tracking technology as a diagnostic tool for ASD, and may also have implications for understanding the social and communication differences between children with and without ASD during face-to-face interactions.

The SVM is one of the most commonly used classifiers among researchers. For example, Kang et al. (2020) [58] conducted a study to explore machine-learning techniques for identifying children with ASD. This was done based on data from EEG and eye-tracking measurements. In order to achieve this, they extracted features from two modalities - EEG and eye-tracking - and used them as inputs to an SVM algorithm. In the

study, 97 children between the ages of 3 and 6 performed eye-tracking tests while their resting-state EEG data was recorded. These tests used either own-race or other-race stranger faces as stimuli. Analyzing EEG data involved power spectrum analysis while analyzing eye tracking data involved selecting AOIs to analyze face gaze.

A minimum redundancy maximum relevance (MRMR) feature selection method was used with SVM classifiers to identify autistic children. According to their results, the classification accuracy from combining the two types of data reached 85.44 percent. This was when 32 features were selected. This finding may have significant implications for ASD early detection and treatment. Moreover, it could be used to gain a better understanding of neural and behavioral differences between children with and without ASD.

Yaneva et al. [59] conducted a study in which they used a 60 Hz Gazepoint GP3 video-based eye-tracker to observe the eye movements of 31 adults with high-functioning autism and 40 adults without autism while viewing web pages. Their goal was to determine how the visual processing of these two groups differed. According to this study, autism could be detected automatically with an accuracy rate of around 74%. The authors also examine the effects of a variety of factors on the accuracy of classification, such as gaze-based and page-related features, the number of tasks, and different approaches and levels of granularity to defining AOIs. With and without specific instructions regarding location and at different times, people with autism process information contained on web pages differently. Aside from the content and granularity level of the AOI, they also found that the visual complexity of the pages and the gender of the participants did not affect the classification accuracy [59].

Instead of adults Kanhirakadavath et al. [60] also tested the applicability of eye-tracking data in children to aid in the early detection of autism using machine-learning techniques. Their study examined the effectiveness of various machine learning techniques to identify the most accurate model for predicting autism based on images of Eye-Tracking Scan Paths (ETSP). This data set consists of 547 graphical eye-tracking scan paths taken from 328 typically developing children and 219 children with autism. In order to avoid model overfitting, they used image augmentation. On the populated dataset neural network model outperformed typical machine learning approaches such as BDT, and DSVM (see Figure 2.8 [60]).

Even though machine learning techniques have been successful in some cases for analyzing gaze data, they may not be the most appropriate method for studying the temporal dynamics of gaze behavior in individuals with autism. Most of these techniques require training models on large datasets of gaze data and then using those models to make predictions about new data. The literature currently lacks investigation into the mathematical properties of eye-tracking data collected from individuals with and without autism. To address this gap, I have chosen to use models that can capture the dynamic nature of gaze behavior and account for the stochastic (random) nature of the data [61]. This approach will allow me to gain insight into my dataset and potentially uncover new findings.

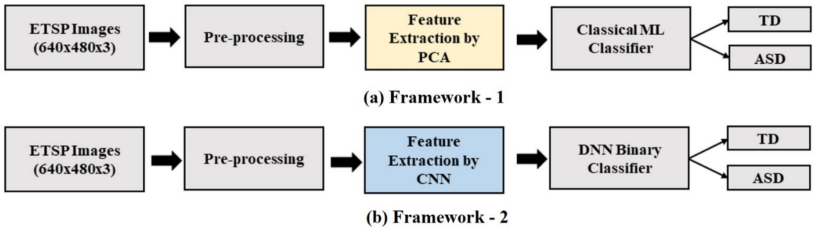


Figure 2.8: Proposed models for image pre-processing and feature extraction in the classification of ASD and typically developing children, taken from Ref. [60]. The framework includes two main components: (1) image pre-processing, which involves RGB to grayscale conversion and image resizing, and (2) feature extraction, which is performed by either a machine learning algorithm using PCA or a CNN with four layers. PCA reduces the dimensionality of the data while preserving important information.

Hidden Markov Model (HMM) is a mathematical model that is useful for analyzing time-series data, including gaze patterns. By employing HMMs, researchers can capture the intricate temporal dynamics of gaze and compare them between different groups, such as individuals with and without autism. This approach can help gain insights into the underlying gaze behavior processes and identify differences between groups that may not be evident using simpler machine-learning methods. Using HMMs may lead to a better understanding of gaze behavior in this population and potentially uncover new insights that were previously unknown.

2.7 Markov processes and hidden Markov models

Markov processes, also known as Markov chains (shown in Figure 2.9 [62]), are mathematical models used to study systems that undergo a series of probabilistic transitions from one state to another over time.

A Markov process is a stochastic process, which means that it involves random variables. It is called "Markov" because it satisfies the Markov property, which states that the probability of transitioning to a new state depends only on the current state, and not on any of the previous states. In other words, the future behavior of the system is independent of its past behavior given the present state.

HMM is a mathematical model for representing sequences of observable events that arise from unobserved (hidden) processes. There is no knowledge of the underlying processes that generated the events [62]. There are many applications of HMMs, including speech recognition, handwriting recognition, bioinformatics, natural language processing, and many other areas where the underlying process generating a sequence of observations is unknown.

A full probabilistic description of a system such as the Figure 2.9 [62])requires knowledge of the current state and all previous states (in addition to knowledge of the probability of a transition between two

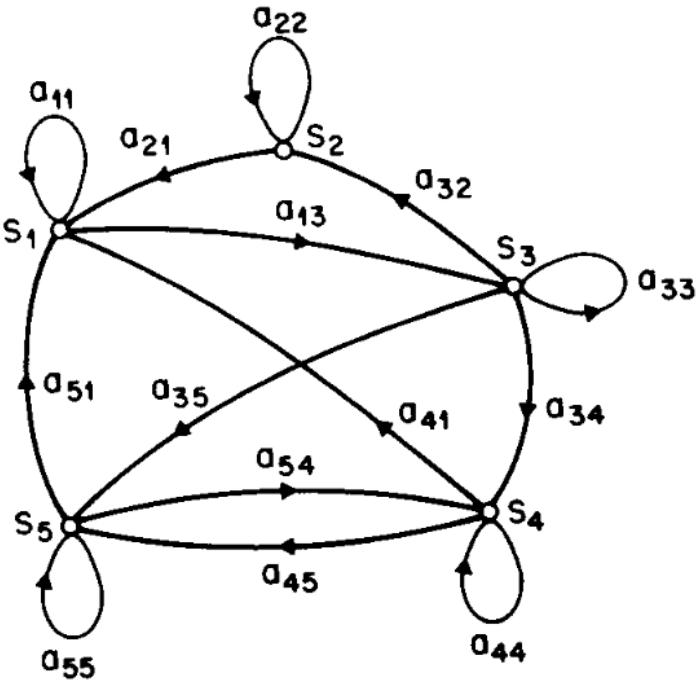


Figure 2.9: A visual representation of a Markov chain, taken from Ref. [62], consisting of 5 distinct states (S_1 to S_5) and pre-defined transition probabilities. For example the transition probability from state S_1 to S_5, labeled as a_{12}.

states). In HMM, there is an assumption based on the Markov property stating that our decision in each state is conditionally independent of our history given our immediate predecessor (the current hidden state only depends on the previous hidden state).

It is possible to characterize a HMM with a set of parameters $\{N, M, A, B, \pi\}$ which are define as follow:

- $N =$ number of distinct states in the model (a set of N distinct states can be written as $S = \{S_1, S_2, \ldots, S_N\}$). Although these states are hidden, they generally correspond to something we know about the world. These states are **ergodic**, which means all states are connected. If we consider $t =$ as a time instance associated with the state. Then q_t is actual state at time t, $\forall i$: $q_i \in S, 1 \leq i \leq t$. And $Q = \{q_0, q_1, \ldots, q_T\}$ is a sequence of states of the Markov process.

- $M =$ number of observable symbols. They are what can be observed, and we can identify the set of possible observations in each state as $V = \{v_1, v_2, \ldots, v_M\}$. An observation sequence $O = (O_1, O_2, \ldots, O_T)$ is a sequence of observed symbols, but we do not know which specific values in the sequence correspond to which state in the HMM.

- $A = \{a_{ij}\} =$ state transition matrix in which $a_{ij} = P(q_t = S_j | q_{t-1} =$

$S_i)$, $1 \leq i, j \leq N$ and $\sum_{j=1}^{N} a_{ij} = 1$.

If $a_{ij} = 0$, then a transition between two states S_i and S_j is impossible.

Rows in A correspond to hidden states, and columns correspond to probabilities of transitioning from one hidden state to another.

- $B = \{b_j(k)\}$ = Observation probabilities matrix (probability distribution of seeing one of the observable symbols given that you are in a particular state, each state has different distribution and we represent it by B) in which $b_j(k) = P(v_k \text{ at } t | q_t = S_j)$, $1 \leq j \leq N$ and $1 \leq k \leq M$.
- $\pi = \{\pi_i\}$ = initial hidden state probabilities (probability of beginning in the particular state) in which $\pi_i = P(q_1 = S_i)$, $1 \leq i \leq N$.

The following are three typical types of problems that can be solved using hidden Markov models:

- Given a sequence of observations O and $\lambda = (\pi, A, B)$, calculate the probability that a model generated O. Or for a given set of model probabilities $\lambda = (\pi, A, B)$) and a sequence of observations O, calculate $P(O|\lambda)$. By using the **forward backward algorithm**, we can solve this problem.
- Given a sequence of observations O and $\lambda = (\pi, A, B)$, determine the best sequence of hidden states Q that result in O. Or for a given set of model probabilities $\lambda = (\pi, A, B)$ and a sequence of observations O, calculate the maximum a posteriori probability estimate of the most likely Q. By using the **Viterbi algorithm**, we can solve this problem.
- Given only a sequence of observations O, determine the optimal set of model probabilities π, A, and B. Or for a sequence of observations O, guess an initial set of model probabilities $\lambda = (\pi, A, B)$ and use the forward and **Viterbi algorithm** iteratively to recompute $P(O|\lambda)$ as well as to readjust λ. Whenever $P(O|\lambda)$ stops increasing, or after a set number of iterations, the calculations will stop.

The first problem is known as the evaluation problem. This involves calculating the probability that a model produced a given sequence of observations. Alternatively, we can view this problem as a way to score how well a given model matches a given observation sequence. This view is particularly useful when choosing between multiple models, as it helps us to select the model that best matches the observations. The second problem is the decoding problem, which involves uncovering the hidden part of the model - specifically, finding the correct state sequence. However, for most models, there is no single correct state sequence to be found. Therefore, we typically use an optimality criterion to find the best possible solution. The choice of criterion depends on the intended use of the uncovered state sequence. For example, it could be used to learn about the model's structure or to find optimal state sequences for continuous

speech recognition. The third problem is the training problem, which involves optimizing the model parameters so that it can best describe a given observation sequence. To do this, we use a training sequence to adjust the model parameters. This problem is crucial for most HMM applications, as it allows us to create the best models for real phenomena by optimally adapting model parameters to observed training data [62].

We are working on the third problem of HMMs, which is finding the optimal set of model probabilities given a sequence of observations. Specifically, we are using eye-tracking data to estimate the parameters of the HMM, such as the transition probabilities between states and the mean and standard deviation of velocity observations for each state.

After defining our parameters, we will conduct a power analysis to determine if there are significant differences between the two groups regarding these parameters.

2.8 Power analysis

When conducting a statistical hypothesis test, there are two main hypotheses to consider: the null hypothesis and the alternative hypothesis. The null hypothesis assumes that there is no difference between the two groups, while the alternative hypothesis assumes the opposite. If there is no difference between the populations of the two groups being compared, it is unlikely that the samples from these populations will show differences in the test. However, there is still a chance of making an error, which is called a type I error. To minimize the risk of this error, we need to decide how much risk we are willing to take. In medical situations, for example, a type I error could be quite serious, and we may want to have a smaller risk [63].

The level of significance, or α, is defined as the probability of rejecting the null hypothesis when it is true. Another type of error is a type II error, which is the probability of accepting the null hypothesis when it is not correct. We want to balance these two types of errors, and the statistical power of a hypothesis test is the probability of correctly rejecting the null hypothesis or accepting the alternative hypothesis if it is true. The higher the statistical power for a given test, the lower the probability of making a type II error [63].

To calculate the statistical power, we can use the formula:

$$Power = 1 - \beta = P(\text{reject}H_0|H_1\text{is true}), \tag{2.5}$$

where β is the probability of a type II error, or failing to reject the null hypothesis when it is false. The power of a statistical test is influenced by several factors, including the alpha level (when the alpha level decreases, power also decreases), sample size, and effect size.

The effect size is a measure of the magnitude of the difference between groups or the strength of the relationship between variables in a study. Common effect size measures include Cohen's d, Pearson's r, and odds ratios. Cohen's d is a commonly used effect size measure for comparing the means of two groups. It is calculated as:

$$d = \frac{\mu_1 - \mu_2}{sp}, \tag{2.6}$$

where μ_1 and μ_2 are the means of the two groups being compared, sp is pooled standard deviation and is calculated as:

$$sp = \sqrt{\frac{[(n_1 - 1) \times s_1^2 + (n_2 - 1) \times s_2^2]}{n_1 + n_2 - 2}}, \tag{2.7}$$

where s_1 and s_2 are the standard deviations of the two groups, and n_1 and n_2 are the sample sizes of the two groups.

So, power analysis is an essential tool in experimental design, consisting of four variables: Effect Size, Significance Level, Power, and Sample Size. These variables are interconnected in such a way that a change in one will affect the other three. Power analysis enables us to determine the fourth variable when the other three variables are known. A power analysis can be used to determine the minimum sample size needed to achieve a given power level or to estimate the power of a study based on the sample size.

There are several software packages available that can perform power analyses, including Python, R, and SAS. These packages typically require the user to specify the effect size, the significance level, and the desired power level and can provide sample size estimates or power estimates for a given sample size.

Chapter 3

Data and methods

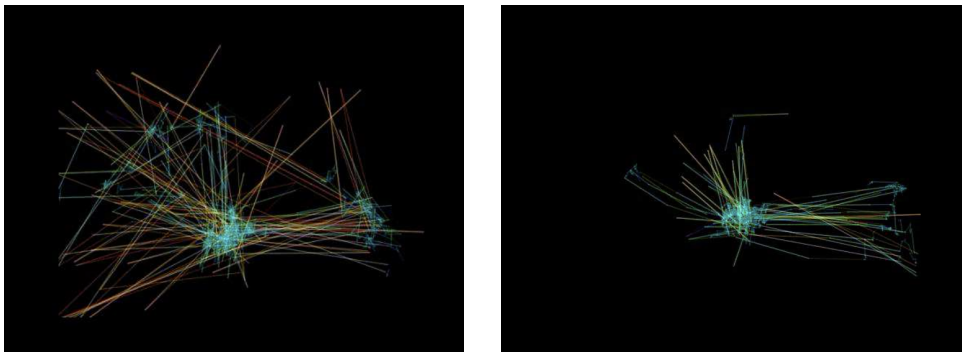
3.1 Data description

Carette et al. [15] developed a raw eye-tracking data set to study and analyze gaze behavior in children with and without autism. The aim was to generate insights and develop useful ASD applications, such as machine learning models for ASD diagnosis. The study involved collaboration between psychologists from the CRP-CPO lab and AI researchers from the MIS lab at the University of Picardie Jules Verne in France.

The eye-tracking data were collected from 59 children aged 3 to 12 years, including 29 with ASD and 30 TD children from schools in the Hauts de France region, using an SMI Red-M eye tracker with a 60 Hz sampling rate.

In this study, eye gaze was stimulated by presenting photos and videos on a screen. The ocular activity was analyzed using a variety of videos from different perspectives. Visual items, such as colorful balloons and cartoons, were used in the photos and videos to capture children's interest. As depicted in Figure 3.1 [15], human actors were included in the videos to

Figure 3.1: The figure, taken from Ref. [15], illustrates a screenshot taken from one of the videos used to record eye-tracking data. The video features an actress holding a colorful balloon, designed to attract the attention of child participants.



(a) (b)

Figure 3.2: Figure, taken from Ref. [15], (a) shows the eye-tracking paths of one autistic participant, while Figure (b) depicts the eye-tracking paths of typically developing participants. The comparison highlights potential differences in gaze patterns between these groups, providing insights into the mechanisms underlying visual attention in autism. The color shift along the line denoted the underlying pattern of eye movements. To achieve this, the RGB values were adjusted in accordance with the velocity, acceleration, and jerk at each instance. To prevent color values from exceeding the limit, all values were restricted to a quarter of the diagonal length of the screen. As a result, Saccadic eye movements were represented by yellow or white, with the latter indicating extremely fast eye movements that exceeded the limit. Fixations were represented by cyan, providing a visual representation of eye motion.

attract children's attention to the visual items. The study employed various types of stimuli, such as static and dynamic naturalistic scenes, static faces or objects, cartoons, and joint attention stimuli. The average duration of an eye-tracking experiment was around five minutes.

The eye-tracking device captured three types of eye movements, including fixations, saccades, and blinks. Additionally, the points of gaze which refer to the location on a visual display where a person is looking at a particular moment in time coordinates were recorded. This can be used to track eye movements and analyze gaze behavior.

3.2 Data processing and visualization

Figures 3.2 depict the eye-tracking paths of participants with and without ASD in a study conducted by Carette et al. [15]. We are using the same data set. Analysis of the data revealed that participants with ASD tended to look more frequently toward the bottom of the screen, where the eye-tracking device was positioned.

In our data exploration, we focused on two participants initially: one with ASD (participant 15) and one TD participant (participant 46) for the first experiment. The main variables of interest were the x and y positions of the eye at different time points, along with the classification of the

eye movements (fixations, saccades, blinks, and null data) as recorded by the eye tracker. However, we excluded blink and null data in the initial analysis as they were of little interest and had a small number of occurrences. We then separately analyzed the data for each participant to gain a better understanding of the data statistics.

3.2.1 Introduction to eye-tracking metrics and terminology

Eye tracking is a powerful tool for understanding visual attention and cognitive processes. However, interpreting the data can be challenging without a foundational understanding of key metrics. In this section, we will introduce and define several relevant eye-tracking metrics that will be used throughout the following section. By establishing a clear understanding of these concepts, we can better interpret and analyze the eye-tracking data presented in the following section.

x and y-position of the point of gaze: The x-position of the point of gaze represents the horizontal location on the screen where the participant is looking, while the y-position of the point of gaze represents the vertical location. These coordinates are denoted by x and y, respectively, and are typically measured in terms of pixel coordinates on the screen.

Distance between two eye coordinates: The Euclidean distance between two eye coordinates (x_1, y_1) and (x_2, y_2) is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \tag{3.1}$$

where d is distance.

Velocity: Velocity is defined as the rate of change of position over time which can be calculated as:

$$v = \frac{d}{t_2 - t_1}, \tag{3.2}$$

where d is the distance from one point to another point and $t_2 - t_1$ is the time duration between those two points.

In the context of eye-tracking, we can calculate the velocity for the horizontal position of the gaze (v_x), the vertical position of the gaze (v_y), velocity during fixation ($v_{fixation}$), and velocity during the saccade ($v_{saccade}$) using the following formulas:

$$v_x = \frac{x_{t_2} - x_{t_1}}{t_2 - t_1}, \tag{3.3}$$

where (x_{t_1}) and (x_{t_2}) are two consecutive eye coordinates in the horizontal direction at time t_1 and t_2 respectively.

$$v_y = \frac{y_{t_2} - y_{t_1}}{t_2 - t_1}, \tag{3.4}$$

where (y_{t_2}, t_1) and (y_{t_1}, t_2) are two consecutive eye coordinates in the vertical direction at time t_1 and t_2 respectively.

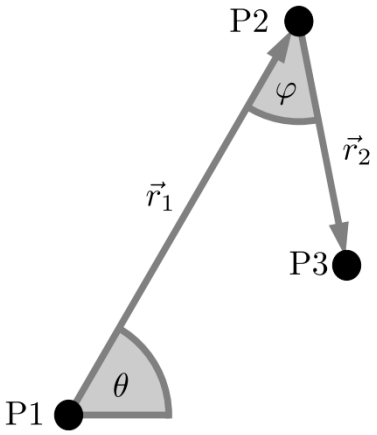


Figure 3.3: Illustration of the position (P_i) of three points, taken from Ref. [64]: P_1, P_2, and P_3. $\vec{r}_1$ shows the relocation from P_1 to P_2. ϕ is the angle between consecutive relocation vectors, and θ is the angle between the first relocation vector and the horizontal axis.

We can calculate $v_{fixation}$ and $v_{saccade}$ using the same formula as velocity, but only considering data points classified as fixations and saccades, respectively.

$$v_{fixation} = \frac{d_{fixation}}{t_{fixation}}, \tag{3.5}$$

where $d_{fixation} = \sqrt{(x_{f_2} - x_{f_1})^2 + (y_{f_2} - y_{f_1})^2}$ represents the total distance traveled during two consecutive fixations, and $t_{fixation}$ represents the duration of the fixation.

$$v_{saccade} = \frac{d_{saccade}}{t_{saccade}}, \tag{3.6}$$

where $d_{saccade} = \sqrt{(x_{s_2} - x_{s_1})^2 + (y_{s_2} - y_{s_1})^2}$ represents the total distance traveled during two consecutive saccades, and $t_{saccade}$ represents the duration of the saccade.

Overall speed (v_s): Overal speed or scalar velocity refers to the speed of eye movement without considering direction. Usually expressed as pixels or degrees per second, it is the magnitude of the velocity vector. It measures how fast the eyes are moving without regard to whether they are moving up, down, left, or right. For example, If the participant's gaze is moving quickly to the right (with a high x-position velocity (v_x)), but not moving up or down (with a low y position velocity v_y), the overall velocity (v_s) would still be high. Using the scalar value of velocity, you can measure or compare the overall speed of eye movements during a task. Accordingly, it calculates as follows:

$$\sqrt{v_x^2 + v_y^2}. \tag{3.7}$$

Angle between two consecutive measurements: To find this angle, we need to measure the position of the gaze at different points in time. Let's say we have three measurements of the position (P) of the gaze on the screen: one at time 1, one at time 2, and one at time 3 same as what we have in Figure 3.3 taken from [64]. Using these three measurements and considering relocation ($\vec{r}$) between the positions, we can calculate the velocities of the gaze at the first two points and at the second two points. The velocity is just how fast your gaze is moving on the screen. We can calculate the velocity magnitude by dividing the distance the gaze moves between two points or relocation by the time it takes to move that distance.

Once we have the velocities at each pair of points, we can use a formula to find the angle between those velocities. This angle tells you how much the gaze direction changes between those two points and can be calculated as:

$$\phi = cos^{-1}\left(\frac{v_1 \cdot v_1}{\|v_1\| \cdot \|v_2\|}\right), \tag{3.8}$$

where $\mathbf{v}_1$ and $\mathbf{v}_2$ are the velocities at two consecutive points. The symbol $\|\cdot\|$ denotes the magnitude (or absolute value) of the vector and $\cdot$ denotes the dot product between the two vectors. The angle ϕ is measured in radians. If we need the result in degrees, you can convert it by multiplying it by $\frac{180}{\pi}$.

By repeating this process for every pair of consecutive points, we can get an idea of how the gaze direction changes over time while the participant is watching the screen.

Angle of given velocity with horizontal axis: We can calculate the angle of each velocity component v_x and v_y with respect to the horizontal axis as shown in Figure 3.3, denoted as θ.

Autocorrelation (ρ_k): The autocorrelation shows how a signal correlates with itself over time. x-position velocity autocorrelation in eye-tracking datasets is the correlation between the x-position velocity at different time lags. Over time, it can identify trends or patterns. The autocorrelation of a time series X_t at lag k can be computed as:

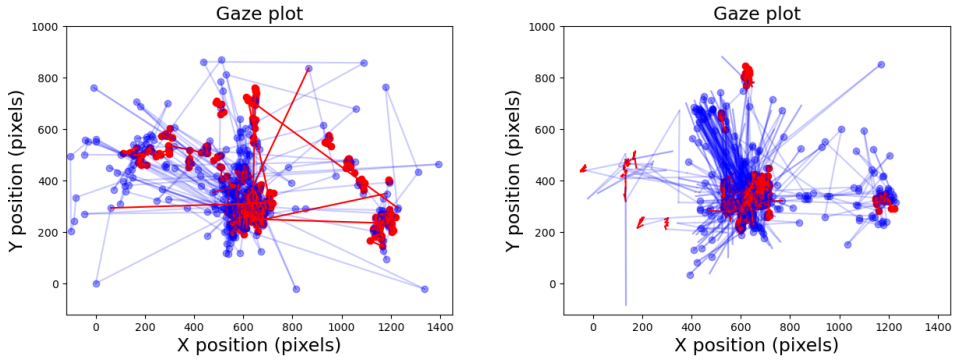
$$\rho_k = \frac{\sum_{t=k+1}^{n}(X_t - \bar{X})(X_{t-k} - \bar{X})}{\sum_{t=1}^{n}(X_t - \bar{X})^2}, \tag{3.9}$$

where $\bar{X}$ is the mean of the time series and n is the number of observations in the time series.

A high autocorrelation means the x velocity at a given moment is highly correlated with the x velocity one second later. A repetitive gaze pattern may indicate this.

A low autocorrelation means the x velocity at a given moment is not strongly correlated with the x velocity one second later.

Moving average: A moving average is a common method of smoothing time-series data. Essentially, it involves calculating the mean of a specified number of consecutive data points, known as the "window size," and then moving the window forward one point at a time until the end of the



(a) (b)

Figure 3.4: Visual representation of eye movements during an eye-tracking task for an autistic individual (a) and a typically developing participant (b). The red parts of the figure depict the recorded fixations, indicating the times at which the individual's attention was attracted to different areas. The blue parts represent the recorded saccades, showing the times at which the individual made rapid eye movements between different areas.

data series is reached. Using a window size of n, the moving average is calculated as follows:

$$MAV_i = \frac{x_i + x_{i-1} + ... + x_{i-n+1}}{n}, \tag{3.10}$$

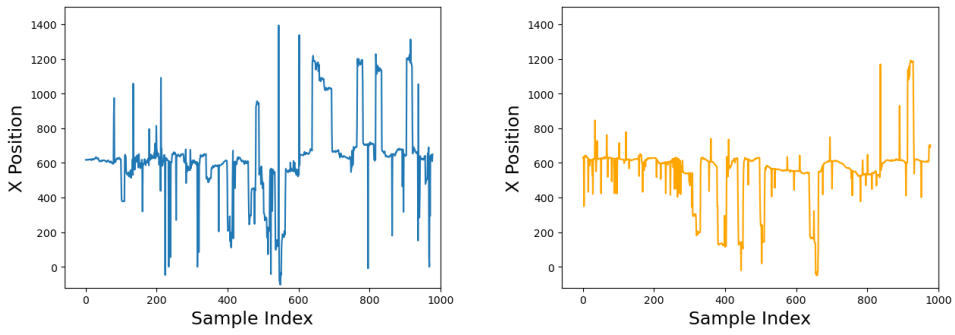
where MAV_i is the moving average at time point i, and x_i to x_{i-n+1} are the data points included in the window.

When a moving average index is calculated between consecutive eye tracking data points for participants with and without autism and plotted over a time window, it can be used to estimate the distance covered by the participants' gaze over the given time window. A higher moving average index indicates a wider area of coverage over the time window, while a lower index indicates a narrower area.

3.2.2 Interpreting eye-tracking data from autistic and typically developing participants

The eye-tracking data presented in this study suggests that individuals with autism may exhibit different scanning patterns compared to typically developing individuals. Figure 3.4 depicts the gaze paths of the two participants, showing that the participant with ASD covered a wider area horizontally compared to the TD participant, who might have more attention directed towards the center of the screen. Additionally, the ASD participant had longer fixation durations on specific areas except for the center, resulting in a more random or fragmented scan path.

Figure 3.5 corroborates the findings from the gaze pattern visualization. The variability of the x-position values, as shown in this figure with peaks and valleys, is larger for the autistic participant, suggesting that their eye



(a) (b)

Figure 3.5: Horizontal gaze patterns of an autistic individual (Figure a, blue) and a typically developing individual (Figure b, orange) during the same eye-tracking task.

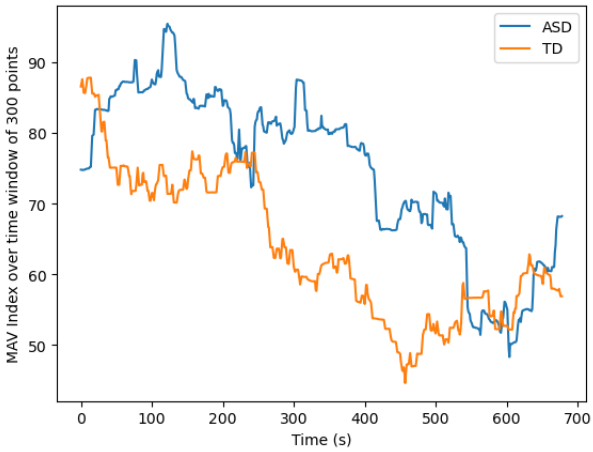
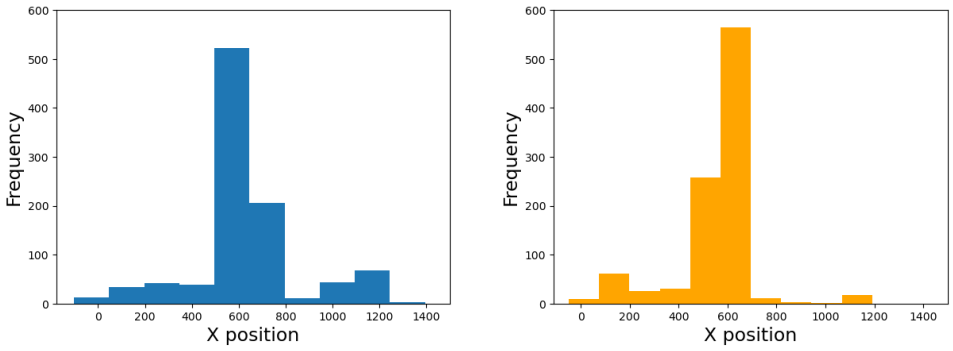


Figure 3.6: The figure demonstrates estimating distance covered by participants with and without autism through moving average analysis of eye tracking data. This analysis technique involves using a moving average index to estimate the distance covered by participants' gaze over a specific time window (in this case, 300 points).

movements might be more erratic. The moving average index shown in Figure 3.6 also suggests that the ASD participant's gaze covered more ground than the TD participant.

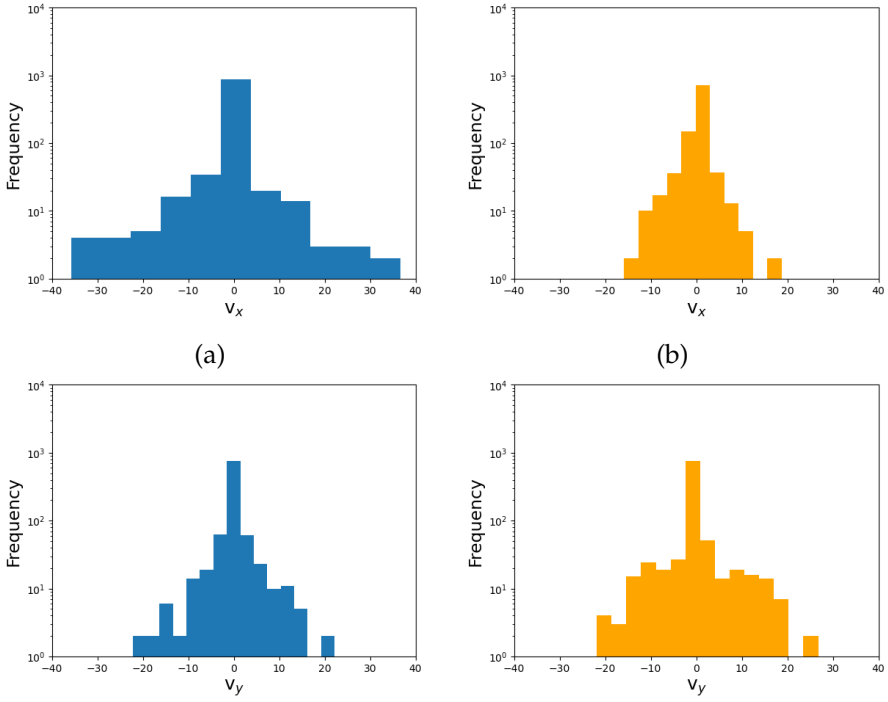
Figure 3.7 shows during the eye-tracking task, both participants attempted to return to the x-position of 600 pixel on the screen, with a few data points falling outside this amount. This suggests that their gaze is primarily focused on the center of the screen, with occasional deviations to the left or right. However, the autistic participant exhibited more variability in their gaze patterns, as evidenced by the larger spread of data points around the central region.

We can analyze the horizontal and vertical movements of gaze by examining the values of v_x and v_y, which respectively indicate the speed



(a) (b)

Figure 3.7: The figures compare horizontal eye movement histograms of a participant with autism (Figure (a)) and a typically developing participant (Figure (b))



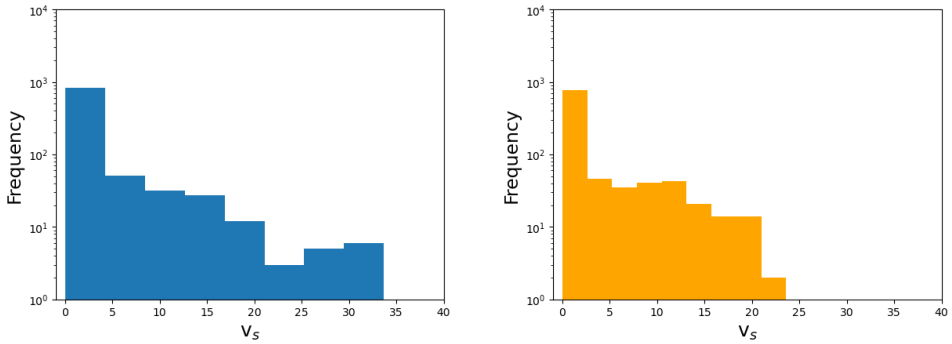
(a) (b)

(c) (d)

Figure 3.8: Figure displays four histograms that illustrate the distribution of eye movement velocities for participants with and without autism. The first two histograms, Figures (a) and (b), depict the horizontal velocity distribution for autistic and TD participants, respectively. The next two histograms, Figures (c) and (d), show the vertical velocity distribution for autistic and TD participants, respectively.

and direction of gaze movement in the horizontal and vertical directions on the screen.

Based on the histograms of horizontal velocity shown in Figure 3.8,



(a) (b)

Figure 3.9: This plot compares the gaze velocity scalar (v_s) values between individuals with and without autism, which reflects the overall speed of their eye movement on the screen. The results reveal that the participant with ASD exhibited a broader range of gaze velocity values than the TD participant, suggesting that the former made more rapid gaze shifts.

both ASD (Figure 3.8a) and TD (Figure 3.8b) participants had a similar frequency of eye movements with low horizontal velocity values around zero. However, there were some differences in velocity ranges between the two groups. While most horizontal velocities for TD participants fell between -10 and 10, those with ASD had wider horizontal velocity ranges, ranging from -20 to 20. This suggests that the ASD participant may make rapid gaze shifts at a higher rate than the TD participant.

However, when it comes to horizontal eye movements, both individuals with autism (Figure 3.8c) and those without (Figure 3.8d) exhibit almost similar speeds and directions. There is a slightly greater concentration of data points in the -20 to 20 range for the TD participant, indicating slightly higher speed in that range, but overall the two groups have almost similar horizontal velocity distributions.

In addition to examining the horizontal and vertical velocity of eye movements, we can also gain insights into the overall speed of eye movements, which is represented by the velocity scalar (v_s) values. The velocity scalar plots in Figures 3.9a and 3.9b demonstrate that the ASD participant had a higher range of high-velocity values reaching to almost 35, indicating more sudden and rapid changes in gaze direction. In contrast, the TD participant had a higher frequency of lower velocity values, suggesting more consistent and smoother eye movements.

Since the histograms for v_x and v_s showed differences between participants, we decided to investigate whether data points in v_x and v_s are correlated with each other. To do this, we examined the autocorrelation plots.

Figure 3.10 illustrates that there is no significant correlation between data points for either v_x or v_s for either participant, as seen by the small fluctuations around zero in the autocorrelation plots. This lack of correlation indicates that the velocity values are essentially random, with

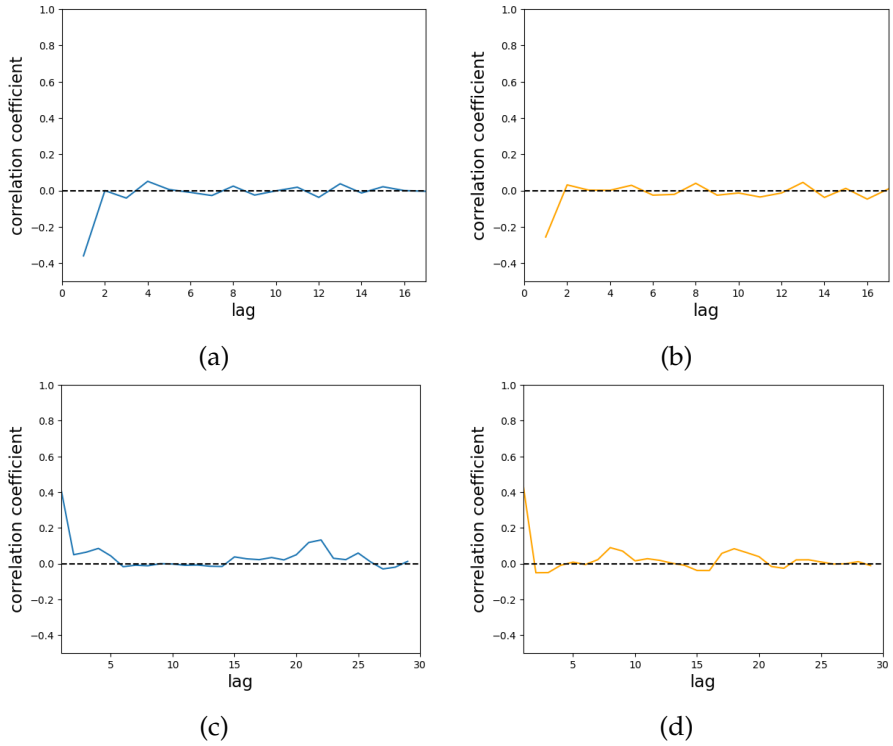


Figure 3.10: Autocorrelation plots for eye movement variables. (a) autocorrelation of v_x for the ASD participant. (b) autocorrelation of v_x for the TD participant. (c) autocorrelation of v_s for the ASD participant. (d) autocorrelation of v_s for the TD participant.

no discernible pattern.

However, there is a positive autocorrelation in the first lag for the v_x for both participants as shown in Figure 3.10a and 3.10b, and a negative autocorrelation in v_s as shown in Figure 3.10c and 3.10d. This suggests that the gaze direction from one moment to another is positively correlated, while the gaze's overall speed is negatively correlated.

The positive autocorrelation in the first lag of the v_x suggests that the gaze direction from one moment to another is positively correlated, meaning that if the gaze moves quickly to the right at a given moment, it's more likely to keep moving that way at the next moment. The negative autocorrelation in the v_s suggests that the gaze overall speed is negatively correlated, meaning that it's more likely to slow down or move at a slower speed at the next moment. This kind of pattern could be indicative of specific gaze behavior, such as fixations or saccades.

The angle histograms depicted in Figure 3.11 provide valuable insights into the distribution of eye movement directions and the stability and consistency of gaze movements.

Figure 3.11 reveals that there is no qualitative difference between the two participant groups.

From the single angle histograms shown in Figure 3.11c and Figure 3.11d, it is evident that the angles are concentrated around 90 or 270 de-

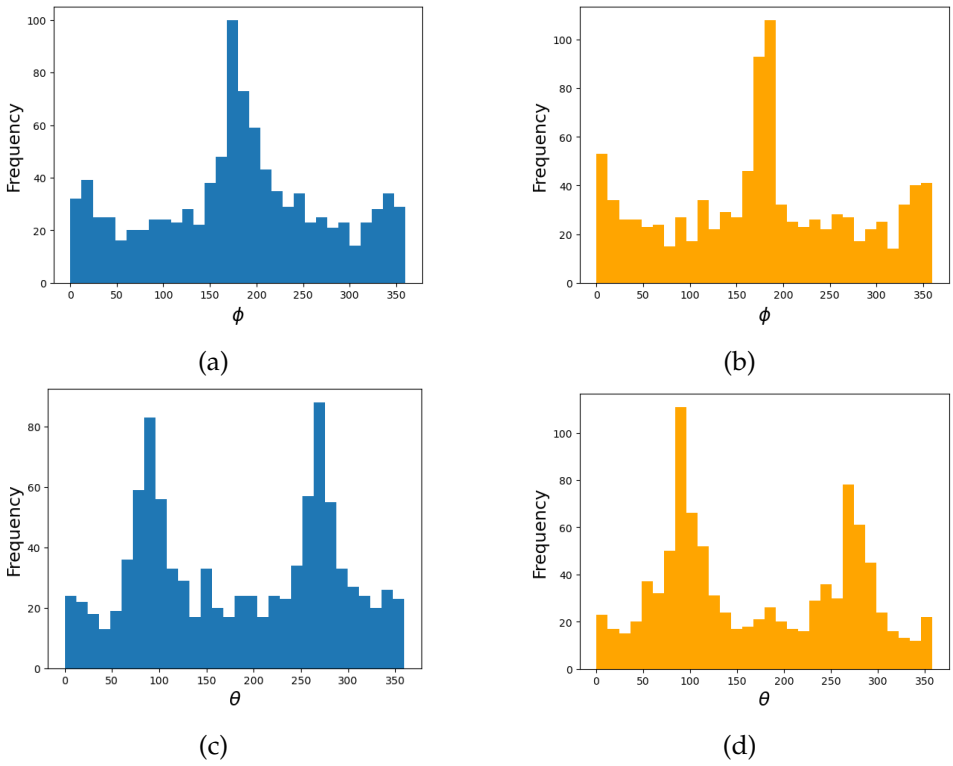


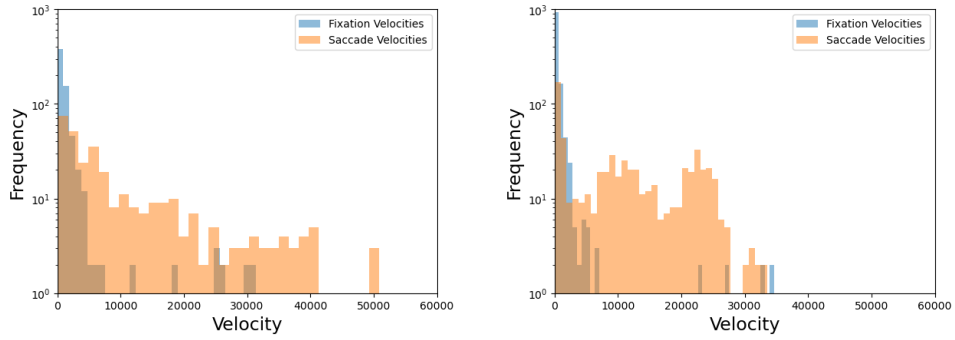
Figure 3.11: Histograms of the angle between v_x and v_y for ASD and TD participants are shown in figures (a) and (b), respectively. Histograms of the angle between v_x and v_y with respect to the x-axis for ASD and TD participants are shown in figures (c) and (d), respectively

grees, indicating a greater frequency of vertical eye movements compared to horizontal ones. Conversely, if the angles were concentrated around 0 or 180 degrees, it would indicate a higher frequency of horizontal eye movements.

Moreover, both Figures 3.11a for ASD participants and 3.11b for TD participants indicate periodic eye movements in both groups.

Having examined the gaze direction and speed patterns in our eye-tracking data set, we now turn our attention to analyzing fixation and saccade velocities in both ASD and TD participants. Fixations and saccades are essential components of eye movements that reflect different cognitive and perceptual processes, and their velocity profiles can provide insights into the underlying mechanisms that drive gaze behavior. By examining the histograms of fixation and saccade velocities for both groups, we aim to investigate potential group differences in these fundamental eye movement parameters.

The comparison of fixation and saccade velocities for ASD and TD participants in Figure 3.12 shows that the fixation velocities of the ASD participant are more widely spread out compared to the TD participant. This could suggest that the individual with ASD has difficulty maintaining fixation on stimuli. On the other hand, the distribution of saccade velocities



(a) (b)

Figure 3.12: Comparison of fixation and saccade velocities for ASD and TD participants. Figure (a) displays the histogram of fixation and saccade velocities for an autistic individual, while Figure (b) displays the same histogram for a typically developing individual. The x-axis represents the velocity values in degrees per second, and the y-axis represents the frequency of occurrence. The blue bars represent the fixation velocities and the orange bars represent the saccade velocities.

for both participants is quite similar, making it difficult to differentiate between them based on this figure alone.

As a result of the information above, it seems that the data set may be of use to researchers interested in studying the differences between individuals with ASD and those with TD, as well as shedding light on potential underlying mechanisms associated with ASD.

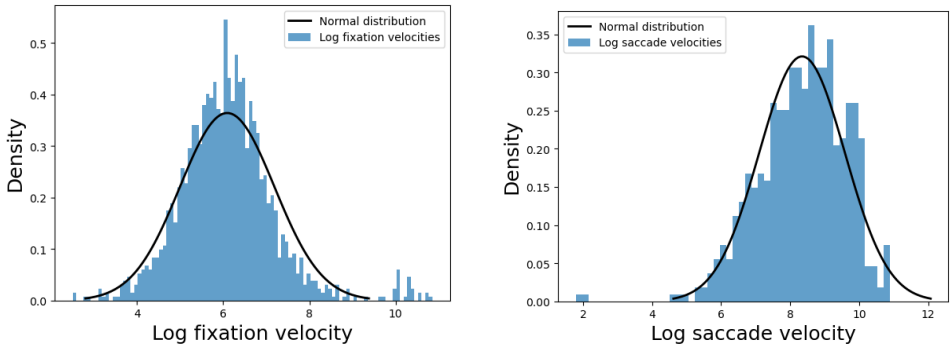
3.3 Methodology plan

In Section 2.7, we discussed the characterization of a Hidden Markov Model (HMM) using a set of parameters N, M, A, B, π. Our hidden states are defined as $N = \text{Fixation}, \text{Saccade}$, and M represents the velocity data, with two observable velocities: fixation velocity (v_{fixation}) and saccade velocity (v_{saccade}). The state transition matrix A represents the transition rates (expected number of events) between saccade and fixation states, which can be calculated using the formula:

$$T = -log(n/m * 60), \tag{3.11}$$

where T is the transition rate, n represents the count of transitions between two distinct states, whereas m denotes the total number of states, which could either be saccadic or fixation. As the data were sampled at a rate of 60 Hz, we normalized our calculations by using a denominator of 60. In fact, the eye tracker samples the position of the eyes 60 times per second (number of frames per second (fps)). The duration of each frame is $1/60$ seconds, which is approximately 16.7 milliseconds.

The probability distribution of observing velocities, given a particular state of fixation or saccade, is denoted by the variable B, which assumes a



(a) (b)

Figure 3.13: Figure shows the normal distribution fitted on the log fixation velocities (a) and log saccade velocities (b)

lognormal distribution [65].

As an example, as shown in Figure 3.13 for one participant in one experiment, both fixation and saccade velocities exhibit a log-normal distribution, and the data has been fitted to the log-normal distribution.

Additionally, the initial probability of starting from each state, represented by the variable π, is determined by dividing the number of observable states saccade or fixation by the total number of states.

The analysis of the data indicated potential differences between the two groups of participants, those with and without autism. To further investigate and identify any significant differences, we plan to follow the steps outlined in Figure 3.14. Our first step will involve data preprocessing to separate the participants.

The dataset comprises 25 CSV files, each representing an eye-tracking experiment. It is important to note that different experiments involved different participants from both groups.

Upon analyzing the complete dataset and separating the participants, we discovered that data was unavailable for participants 12 and 16, who were both diagnosed with autism. Consequently, the final number of participants comprised 27 children with autism and 30 children without autism. The experiments involved 167 samples from autistic participants and 226 samples from typically developing children.

Our initial attempt is to assume that the current classification of saccade and fixation with the algorithm used for eye-tracking is correct. We will investigate whether there are any differences between the two groups using this classification method.

Our next step involves extracting hidden Markov parameters from each participant in both groups. The following are the key parameters used in our analysis:

Transition rates between saccade and fixation: Two transition rates are considered: one from saccade to fixation (T_{s2f}) and the other from fixation to the saccade (T_{f2s}).

Mean and standard deviation of fixation velocity: These parameters

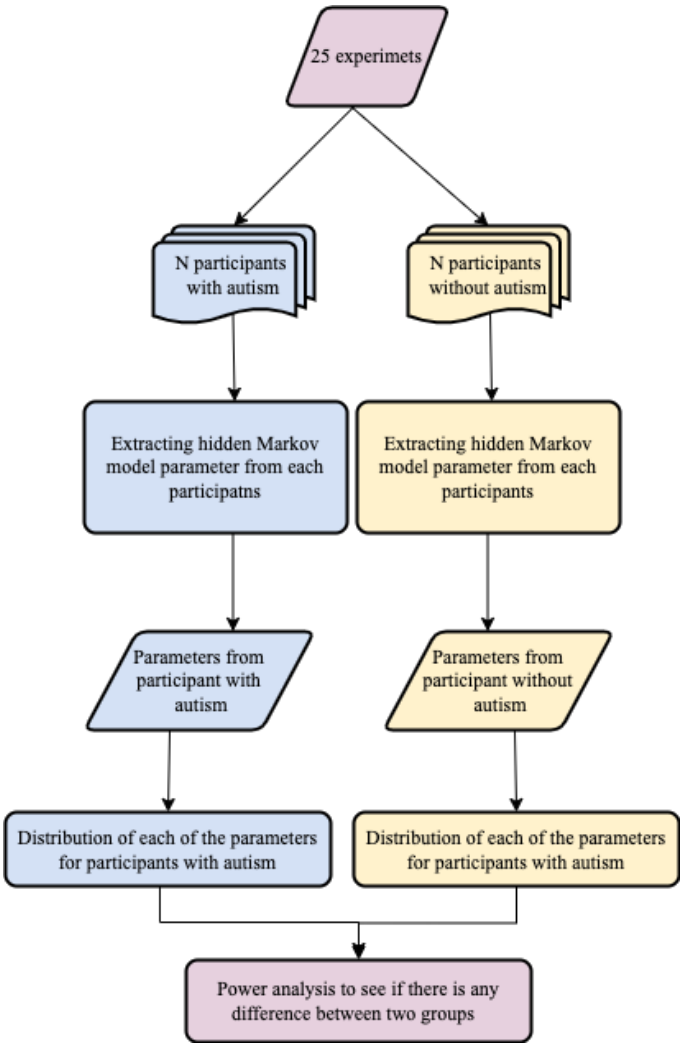


Figure 3.14: Plan for data analysis.

refer to the mean ($\bar{X}_{fixation}$) and standard deviation ($s_{fixation}$) of the velocity observed during the fixation state, which is modeled by a log-normal distribution.

Mean and standard deviation of saccade velocity: These parameters refer to the mean ($\bar{X}_{saccade}$) and standard deviation ($s_{saccade}$) of the velocity observed during the saccade state, also modeled by a log-normal distribution.

Following this step, we will draw the distribution of each parameter for each group to visually inspect for any differences. Finally, we will perform a power analysis between the two groups for each parameter to determine if the observed differences are statistically significant.

Chapter 4

Results and Discussion

In this chapter, we present the results of our analysis of eye-tracking data for both autistic and non-autistic participants. Table 4.1 presents a summary of the dataset that will be analyzed in this study.

Specifically, we analyzed the transition rates from saccade to fixation and fixation to saccade, as well as the mean and standard deviation of fixation and saccade velocities for both groups. An example of the extracted parameters for participant 1 in different experiments is provided in Table 4.2.

After the process of extracting parameters from the sample of size 393, we calculated the distribution of each parameter separately for each group and compared them.

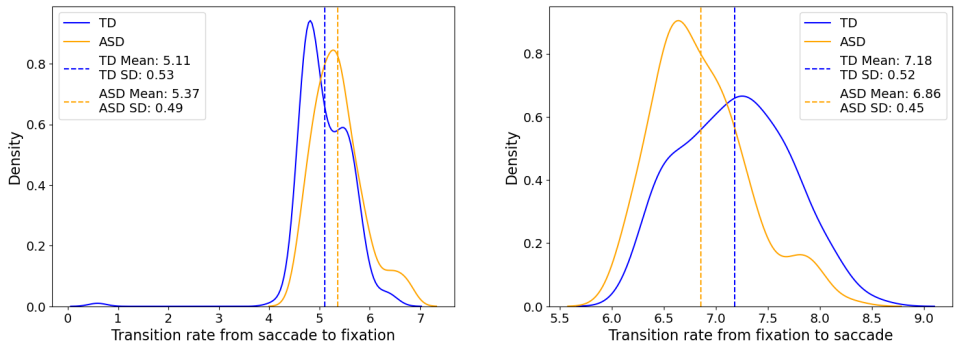
The distributions of the transition rates from saccade to fixation and from fixation to saccade for the TD and ASD groups are displayed in Figures 4.1a and 4.1b, respectively. The TD group has a distribution of the transition rate from saccade to the fixation that is skewed towards lower values, with a higher peak compared to the ASD group. This suggests that individuals in the TD group tend to have longer saccade durations and less frequent fixation compared to the ASD group. On the other hand, the ASD group has a distribution of the transition rate from fixation to saccade that is also skewed towards lower values but with a higher peak compared to the TD group.

Total number of participants	57
Number of autistic participants	27
Number of non-autistic participants	30
Age (Mean / Median)	7.88 years / 8.1 years
Number of experiments	25
Number of samples from the autistic group	167
Number of samples from the non-autistic group	226

Table 4.1: Data statistics overview.

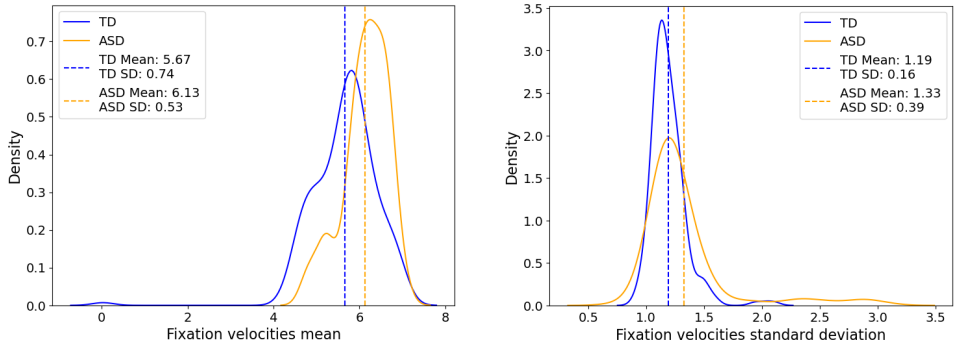
Experiment	T_{s2f}	T_{f2s}	$\bar{X}_{fixation}$	$s_{fixation}$	$\bar{X}_{saccade}$	$s_{saccade}$
9	4.96	6.90	6.70	1.02	8.29	1.15
12	4.94	7.14	6.58	0.94	8.33	1.14
18	5.33	6.94	6.29	1.16	8.49	1.38
19	4.96	6.87	6.71	1.07	8.54	1.10

Table 4.2: Table shows transition rates and velocity distribution parameters. T_{s2f} is the transition rate from saccade to fixation, and T_{f2s} is the transition rate from fixation to saccade. $\bar{X}_{fixation}$ and $s_{fixation}$ represent the mean and standard deviation of the fixation velocities, respectively. Similarly, $\bar{X}_{saccade}$ and $s_{saccade}$ represent the mean and standard deviation of the saccade velocities, respectively.



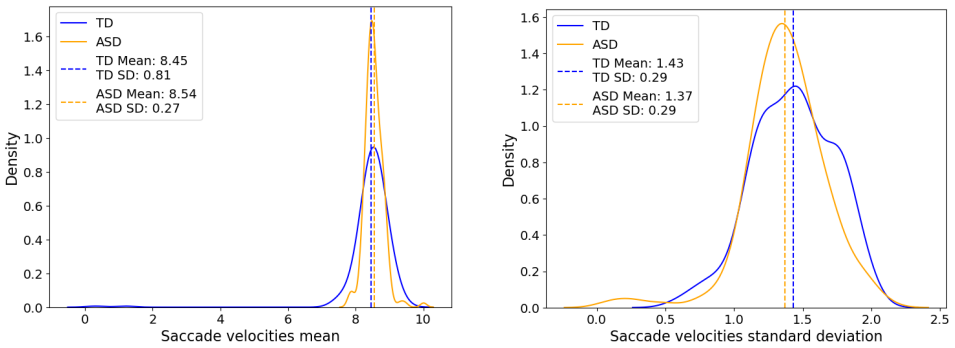
(a) (b)

Figure 4.1: Figure (a) displays the distribution of the transition rate from saccade to fixation, and figure (b) illustrates the distribution of the transition rate from fixation to saccade for the autistic and non-autistic groups. The distribution of the autistic group is depicted by the orange line, while the blue line represents the distribution of the non-autistic group



(a) (b)

Figure 4.2: Figure (a) shows the distribution of the fixation velocity means and figure (b) displays the distribution of the fixation velocity standard deviation for both participants with and without autism. The distribution of the autistic group is depicted by the orange line, while the blue line represents the distribution of the non-autistic group.



(a) (b)

Figure 4.3: Figure (a) shows the distribution of the saccade velocity means and figure (b) displays the distribution of the saccade velocity standard deviation for both participants with and without autism. The distribution of the autistic group is depicted by the orange line, while the blue line represents the distribution of the non-autistic group.

This indicates that individuals in the ASD group tend to have longer fixation durations and less frequent saccades compared to the TD group. Therefore, there might be a significant difference between the two groups in terms of their oculomotor behavior.

The results from the fixation velocities mean plot, which is more about the average speed of eye movements during fixations, in Figure 4.2a suggest that the ASD group exhibits a higher peak and a more positively skewed distribution towards higher values, indicating that individuals with ASD might have longer fixation durations compared to TD individuals since on average they tend to have faster fixation velocity. It is what exactly we saw in the transition rate in Figure 4.1b. Conversely, the fixation velocity standard deviation plot in Figure 4.2b suggests that the TD group has a higher peak and less symmetrical distribution compared to the ASD group, indicating that TD individuals have more consistent and stable fixation durations. There seems to be a difference in the fixation velocity characteristics between the TD and ASD groups.

The saccade velocities standard deviation plots in Figures 4.3b shows that the ASD group has a higher peak compared to the TD group. It means that there is a higher density of individuals in the ASD group with similar or higher saccade velocity standard deviations compared to the TD group. Based on the fact that the standard deviation is the same in both distributions and the mean values are almost identical, it is possible that there is no significant difference between the two groups.

The saccade velocities mean plot in Figures 4.3a, same as saccade velocities standard deviation plots, does not provide information on whether this difference is significant.

Next, we will investigate whether there is a significant difference between the two groups in terms of various parameters by conducting a power analysis. By performing power analysis, we can determine the

	T_{s2f}	T_{f2s}	$\bar{X}_{fixation}$	$s_{fixation}$	$\bar{X}_{saccade}$	$s_{saccade}$
Effect size	0.51	0.66	0.71	0.48	0.14	0.21
Sample size	62	37	33	71	749	365

Table 4.3: Table showing the effect size and required sample size for each parameter using a significance level of 0.05 and power of 0.8 for conducting a two-sample t-test to compare the groups in our study. These sample sizes will enable us to accurately and reliably test our null hypothesis for each parameter.

required sample size to detect significant differences between the two groups with a desired level of statistical power.

In our study, each observation is independent, meaning that the measurements for one observation do not affect measurements for any other observation. We have set our null hypothesis to test whether the mean of the first group is equal to the mean of the second group for each parameter. To test this hypothesis, we will use the t-test, which is a commonly used statistical test for two independent samples.

Before conducting the t-test, we need to determine the sample size required for the test. To do this, we have set the significance level at 0.05 and the power at 0.8. Additionally, we have calculated the effect size using Cohen's d. Table 4.3 presents the required sample size for each parameter along with its corresponding effect size.

Overall, our study has ensured that each observation is independent, and we have carefully calculated the required sample size to ensure accurate and reliable results when conducting the t-test.

Since the effect size is very small, with values of 0.14 and 0.21 for the mean and standard deviation of saccade velocities, respectively, this suggests that there is only a small difference between the mean saccade velocities of the ASD and TD groups and that the variability within each group is similar. As a consequence, the statistical test may not demonstrate a significant difference between the two groups, which would lead to the acceptance of the null hypothesis that states that the two groups are similar. This finding differs from what we found in the literature review, which suggests that saccade velocity is slower in children with autism [54].

In order to proceed with the remaining parameters, we have calculated the power values using the necessary sample size and effect size that were previously calculated, along with a significance level of 0.05 for each parameter. The results of these calculations are presented in Table 4.4.

Based on the table, the calculated power values for each parameter are all above 0.97, which indicates that there is a high probability of correctly rejecting the null hypothesis if it is indeed false. This means that with the given sample size and effect size, we have sufficient statistical power to detect significant differences between the groups for each parameter. These parameters, namely the transition rate from saccade to fixation, transition rate from fixation to saccade, fixation velocity mean, and standard deviation, exhibit informative characteristics and indicate a

	T_{s2f}	T_{f2s}	$\bar{X}_{fixation}$	$s_{fixation}$
EEfect size	0.51	0.66	0.71	0.48
Sample size	62	37	33	71
Power	0.977	0.974	0.977	0.979

Table 4.4: The table presents the calculated power values, using the effect size and required sample size calculated in the previous step, for each parameter with a significance level of 0.05.

	precision	recall	f1-score
TD	0.63	0.97	0.77
ASD	0.96	0.58	0.72

Table 4.5: The table presents evaluation metrics values based on the confusion matrix.

statistically significant difference between the two groups. These findings suggest that these parameters may hold promise for future research in this area.

In this study, we utilized logistic regression as a classification model to distinguish between individuals with ASD and TD individuals. To accomplish this, we derived four parameters from HMM and used them as inputs to our model. The dataset was split into training and testing sets with a ratio of 80:20, respectively.

Our findings demonstrate that, based on 5-fold cross-validation, the logistic regression classifier achieved an Area Under the Curve (AUC) of 0.85, as illustrated in Figure 4.4. Additionally, the accuracy of the model was found to be 75 percent. This result surpasses the AUC values reported in previous studies, such as Carette et al.'s research [15] where logistic regression was also utilized for classification. To gain further insight into the classifier's performance, we analyzed the confusion matrix depicted in Figure 4.5. Our study's analysis of the logistic regression classifier using test data revealed that the classifier correctly predicted 26 children with ASD (True Positives, TP) and 33 TD children (True Negatives, TN). However, it also falsely predicted one child as an ASD participant (False Positive, FP) and 19 children as TD when they actually have ASD (False Negatives, FN).

After computing various evaluation metrics based on the confusion matrix, we presented them in Table 4.5. The F1-score, which is a weighted average of the precision and recall, where a value of 1 represents perfect precision and recall, was found to be 0.72 for ASD participants. This score is lower than the F1-score for TD participants (0.77), indicating that the model's performance for identifying ASD participants is not as good as its performance for identifying TD participants.

Such misclassification is undesirable in our study, as it may result in incorrect diagnosis and treatment decisions for the affected children.

However, these findings suggest that our approach, which incorporates

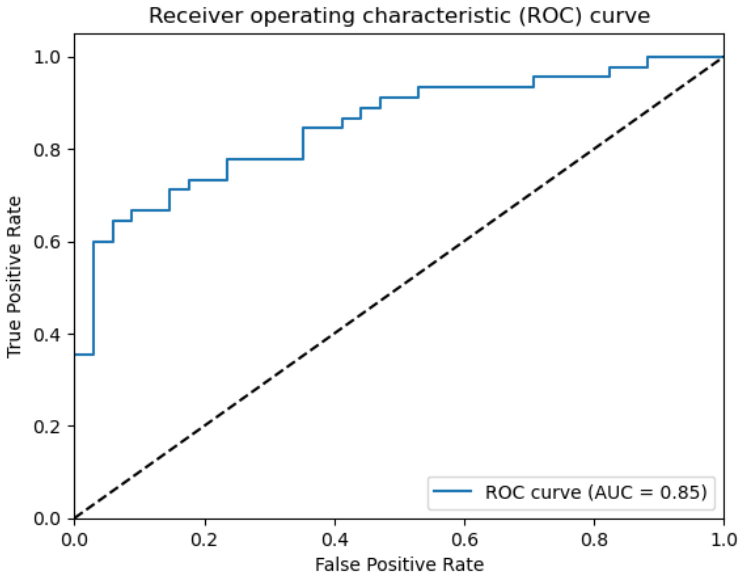


Figure 4.4: The ROC curve of the classification by logistic regression.

HMM-derived parameters and logistic regression classification, may offer an improved and more accurate method for classifying individuals with ASD. Although we achieved a higher AUC in our study, there is still room for improvement.

After exploring the eye-tracking dataset and drawing a series of plots depicting the standard deviation of the x and y positions, we observed that there might be differences between the two participant groups. Specifically, the y position standard deviation showed greater differences, with the mean y position standard deviation for ASD participants appearing to be larger than that for TD participants, as shown in Figure 4.6. This could indicate that there is more variability in the y position of ASD participants compared to TD participants, providing a measure of how much the eye gaze location varies in the up and down direction across a series of time points or trials.

We investigated this feature further to determine whether it could distinguish between the two groups. However, after conducting a power analysis for both x and y position standard deviation for both groups, we found that the statistical power was low, even with sufficient sample size. The power for the x position standard deviation was 0.331, and for the y position standard deviation, it was 0.41. This suggests that our study may not have had sufficient statistical power to detect a significant difference between the two groups in terms of the standard deviation of eye position.

The low power for the x position may be due to the low effect size of 0.36, while the high degree of variability in the data may explain the low power for both x and y positions. Using these two features in the classification resulted in a negative outcome, with AUC dropping from 0.85 to 0.76. This suggests that these features were not effective in distinguishing between the two groups.

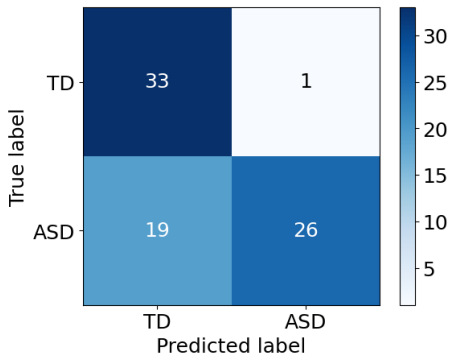


Figure 4.5: The figure displays a confusion matrix for the classification model, providing an overview of the predicted and actual classification results for each class, ASD and TD participants. This matrix can be utilized to determine multiple evaluation metrics, including accuracy, precision, recall, and F-score.

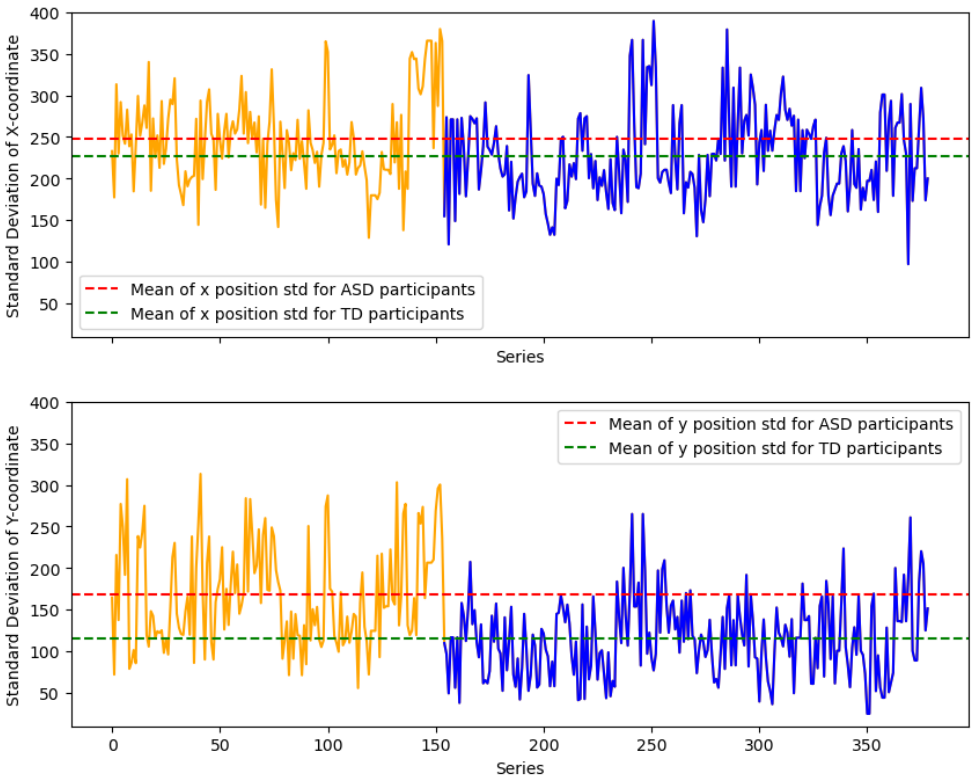


Figure 4.6: The figure shows the standard deviation of x and y eye gaze positions for ASD and TD participants. Orange lines represent ASD participants, and blue lines represent TD participants. The plot indicates that ASD participants have higher variability in both x and y positions, but the difference is more noticeable for y position.

Chapter 5

Conclusion

In this study, we investigated the statistics of fixation and saccade in eye movements using a hidden Markov model on an eye-tracking dataset from autistic and non-autistic participants who had a free viewing task in different experiments. We extracted six parameters from the dataset and found that four of them, namely transition rate from saccade to fixation, transition rate from fixation to saccade, mean, and standard deviation of fixation velocities were informative enough to distinguish between the two groups. Our objective was to determine whether the distribution parameters of fixations and saccades can effectively differentiate individuals with autism from those who are neurotypical. We have discovered that the distribution parameters of fixations, along with the transition rate from saccade to fixation and fixation to saccade, could potentially aid in achieving this objective. However, we could not find the difference between the saccade velocities mean and standard deviation between participants with and without autism. This finding contradicts the existing literature, which suggests that saccade velocity is slower in children with autism. However, it is crucial to note that discrepancies in study design, methodology, and participant characteristics may contribute to divergent findings across studies, as is the case with the present study that utilized a free viewing task for autistic participants rather than saccade guidance, as seen in the existing literature. Moreover, it is important to acknowledge that the current study may have limitations regarding sample size or statistical power that could influence the ability to detect significant differences. Further research is necessary to reconcile these discrepancies and to gain a more comprehensive understanding of the relationship between autism and saccade velocity.

In terms of the classification of individuals with ASD versus TD individuals using logistic regression, our study was able to achieve superior results compared to those reported in the literature. Specifically, our approach, which incorporated four parameters derived from HMM as inputs to the logistic regression classifier, yielded an improved classification performance in terms of accuracy and/or AUC metrics.

One limitation of this study is that we had to include participants performing different tasks to increase the sample size. We assume that

the eyes behave consistently regardless of the task being performed when analyzing the features such as transitions, fixation, and saccade mean and velocities. However, it is possible that this assumption is not entirely accurate. It may be necessary to conduct an experiment that focuses on a single task to obtain equal sample sizes and improve the validity of the results. Unfortunately, due to time constraints, it was not feasible to conduct such an experiment in the scope of this thesis

As a future direction, it may be useful to investigate the gaze dynamics of individuals with autism in the absence of social stimuli. One way to achieve this is to design an experiment in which participants are presented with non-social stimuli, such as objects, and their eye movements are recorded using eye-tracking technology. By comparing the gaze dynamics of individuals with and without autism in this context, we may gain a deeper understanding of how autism affects visual attention and perception.

In conclusion, this study provides some insight into the differences in eye movements between individuals with and without autism. The results suggest that differences in fixation velocity may be a more informative metric than saccade velocity when distinguishing between the two groups. The lack of significant differences in saccade velocity between the two groups is a novel finding that contradicts the existing literature.

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Appendix A

Code repository

The codes used in the project are available on GitHub at the following address:

https://github.com/MaryamLG/Eyetracking.

The code files are written in Python format, and the extracted parameters for each group are provided in Excel format.