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The certifier for the long run[☆]

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A R T I C L E I N F O

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A B S T R A C T

We build a workhorse model to study the optimal and the equilibrium certifier from a long-run perspective. Firms enter the market, and invest in their capacity to provide quality, before the certification threshold is determined. With a certifier that cares about quality and externalities (such as an NGO), the threshold is demanding and the firms' profits are small. Anticipating this, only a few firms enter the market, and they invest heavily. With a certifier mostly concerned with the firms' profits (such as an industry association), the results are reversed. The relative importance of externalities, investments, and entry determines the socially optimal certifier identity as well as the type of certifier that is most likely to operate in equilibrium. The theory's predictions are empirically testable and shed light on the variety of certifiers across markets and over time.

1. Introduction

Incomplete information can destroy markets (Akerlof, 1970). At the time of purchase, consumers are often unaware of the durability of products because of noise and lags before the products break down. Health risks or benefits associated with products may be unobserved through one's lifetime. Many consumers care about social and environmental aspects, but these aspects may not be observed directly or immediately. For these and other types of "credence goods", consumers do not observe the exact quality of products they purchase (Dulleck et al., 2011). Consequently, investments in quality might not be profitable for the producers.

Given the complexity of new goods, there is an increasing demand for third parties that can verify or certify the quality of products. On the demand side, empirical evidence suggests that consumers are willing to pay more for products with

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Table 1
Examples of certifiers.

Commercial certifiers	NGOs
Online platforms	**Sustainable agriculture**
Amazon, eBay, Airbnb	Fairtrade, Rainforest Alliance
Credit ratings	**Sustainable forestry and fisheries**
Moody's,	Forest Stewardship Council,
Standard & Poor's,	Marine Stewardship Council
Fitch Ratings	**Energy efficiency**
Safety certification	U.S. Green Building Council
Underwriters Laboratories	**Consumer products**
	Green Seal

green or energy-efficiency labels.[1] On the supply side, firms are evidently making costly investments to satisfy certification requirements.[2]

But who should certify? Certifiers come in many variants, but most certifiers are one of three types. Activist or non-governmental organization (NGO) certifiers seek to reward good products by labeling those and only those that satisfy certain criteria. The Fairtrade Labelling Organizations International eV (or "Fairtrade" for short), the Forest Stewarship Council®, the U.S. Green Building Council (the organization managing the LEED rating system), and the Green Seal are examples of such certifiers. These certifiers play a prominent role when it comes to energy efficiency and sustainable production (ranging from agriculture and forestry to household goods).

In other markets, corporate or industry associations often define "best practice" or standards that they can verify. Table 1 shows that online selling platforms, such as eBay, Airbnb, or Amazon,[3] are themselves providing certification for their most reliable sellers. Interestingly, this certification is provided in-house even though the platforms could, in principle, delegate certification to some consumer rights association. In still other markets, for-profit certifiers provide certification services. Arguably the most prominent example is credit rating agencies. As we will show, industry and for-profit certifiers follow similar criteria when it comes to standard setting. For this reason, we place them in the same category. Table 1 lists the examples mentioned so far, together with a few others.

A third certifier type is bureaucratic or government agencies. Well-known examples are the Energy Star, the U.S. Department of Agriculture Organic Label, and the EU Ecolabel. In practice, there is a continuum of certifier types, since the decision-making boards in government programs are often composed of stakeholders from industry as well as from NGOs.[4]

What explains the variation, and what are the consequences of it? When do governments prefer to let representatives of industries or NGOs define the standards of publicly run certification programs? Which certifiers are socially optimal, and which ones establish themselves in equilibrium?

The purpose of this paper is to address all these questions head-on. We develop a model that is intentionally stylized, so that it easily can be generalized in various directions. Even the simplest model must include consumers, firms, and a certifier. The certifier maximizes a weighted sum of firms' profits and quality. The weights depend on the certifier's type, known by everyone.

In the basic short-term version of the model, the certifier specifies a quality requirement before firms decide whether to improve the quality of their products in order to satisfy the requirement. Firms have heterogeneous costs and the more demanding the requirement is, the smaller is the mass of firms finding it worthwhile to satisfy. This prediction is in line with empirical evidence. Hui et al. (2018) show that when eBay replaced its "Powerseller" badge for virtuous sellers with the more demanding "Top Rated Seller" badge, the share of badged sellers dropped. Elfenbein et al. (2015) compare different product markets on eBay and find that in markets with fewer certified products, certification commands a higher price premium. Our basic model is further predicting that an NGO prefers a more demanding requirement than does the government agency, who in turn prefers a tougher requirement than does an industry association. This prediction is also in line with the evidence.[5]

[1] Eichholtz et al. (2010), Hainmueller et al. (2015), de Janvry et al. (2015) provide evidence of price premia associated with green labels. Bjørner et al. (2004) and Hallstein and Villas-Boas (2013) measure the positive effect of green labels on consumers' willingness to pay. For evidence of consumers' responsiveness to energy-efficiency labels, see Newell and Siikamäki (2014) and Davis and Metcalf (2016).

[2] de Janvry et al. (2015) provide evidence on costly investments by coffee farmers to obtain fair-trade labels; Eichholtz et al. (2013) find large differences in energy efficiency between labeled and non-labeled buildings.

[3] For Airbnb, see the program Airbnb Plus ; for eBay, see the Top Rated Seller Program ; for Amazon, see a forum discussion on the Amazon's Choice badge.

[4] To the best of our knowledge, all certifiers run by government agencies work with third parties. The Energy Star, for example, is a program run by the U.S. Environmental Protection Agency and U.S. Department of Energy in consultation with industry representatives (see the Environmental Protection Agency web page). The U.S. Department of Agriculture Organic Food Label and the EU Ecolabel are more inclusive: They consult with representatives of consumers, firms, and environmental associations (see National Organic Standards Board and EU Ecolabelling Board).

[5] Supporting anecdotal evidence can be found in sectors in which firms can get certificates from industry-backed certifiers as well as from NGOs. Construction and forest products are two examples. In both sectors, the most commonly used NGO standard is recognized to be more stringent than the industry one (see Portland Tribune, The Washington Post and GreenBiz). Additional evidence comes from certifiers that have changed their main

In the basic model, the government prefers to certify itself rather than to delegate certification to somebody else. This preference changes in our long-term model. There, firms take actions such as investing in technology or entering a new market after they know the identity of the certifier but before the certification specifies the exact requirement. This timing is reasonable as new certifiers usually take years to obtain recognition from consumers and firms, although established certifiers can change their standards on short notice.[6]

The long-term actions are introduced sequentially. First, we let firms invest in capacity to provide quality before the certification requirement is known. In the construction industry, for example, energy efficiency improvements require the development of new materials and construction techniques. In the healthcare sector, improvements in medical technology require long-term investments. We show that such investments are especially rewarding if the certification requirement is expected to be strict. When it is socially efficient to motivate investments, a certifier that emphasizes quality over profits is likely to be better. In such a case, we show that governments may benefit from delegating certification to NGOs.

Next, we endogenize the market structure. Profits are lower when the certification requirement is very demanding. Thus, when it is costly for firms to enter the market, fewer firms find it optimal to enter if the certifier is an organization expected to emphasize quality. A larger number of firms enter if the certifier is, instead, a profit-maximizing industry association. When entry is welfare-enhancing and sensitive to expected profit, everyone benefits if certification is delegated to the industry itself.

Our characterization of the optimal certifier has normative policy implications. After all, a government often dictates the composition of a public certifier's standard-setting boards. Governments can also influence private certifiers by requiring public procurement to prioritize products with certain labels, or by subsidizing NGOs involved in the certification process.

We also endogenize the certifier type. If it is very costly to establish oneself as a certifier (due to expensive monitoring technologies, for example), then neither industry associations nor NGOs will volunteer to certify. If this expense declines, then, sooner or later, a single stakeholder will enter as a certifier. The first certifier to enter is more likely to be an NGO if investments and externalities are important, we show. In contrast, an industry association is more likely to enter first if the potential for further firm entry is important, while externalities are small. The comparative statics of the equilibrium identity are qualitatively similar to the comparative statics of the results we derived when the government could dictate the identity of the certifier.

Our predictions are testable and align well with the examples listed in Table 1. As we explain in the concluding section, both externalities and firms' investments in their capacity to provide quality are important in agriculture and industries relying on the use of natural resources. By comparison, online selling platforms are recent phenomena, and the potential for sellers to enter these markets is still significant. Our theory thus predicts that the industry certifies for online selling platforms, while NGOs certify when it comes to agricultural and natural resource products, exactly as Table 1 illustrates.

Literature. This paper is the first to analyze the consequences of certifiers' inability to commit ahead of time to their future requirements, and the implications that this has for the optimal certifier identity. Therefore, we bridge the certifier literature and the literature on strategic delegation.

Several papers have studied the effect of certification on firms' choice of product quality. Albano and Lizzeri (2001), for example, extend the model of Lizzeri (1999) and characterize the profit-maximizing strategy of a certifier in an environment where product quality is endogenous. Miklós-Thal and Schumacher (2013) tackle a similar question in a dynamic setting with short-lived sellers, while Siegel (2020) shows that the socially optimal standard is higher when quality becomes endogenous. Relatedly, Bizzotto and Vigier (2022), Boleslavsky and Kim (2018), Boleslavsky and Cotton (2015), Dubey and Geanakoplos (2010), and Costrell (1994) consider the way grading schemes influence students' efforts.

The effect of certification on the market structure has also been investigated. Harbaugh and Rasmusen (2018) show that a non-profit certifier can provide more information to consumers by assigning coarse grades, if one just takes into account that the equilibrium distribution of firm types will depend on the grading. Choi and Mukherjee (2020) endogenize both product quality and market structure.

These papers, however, assume that certifiers are first-movers and they focus on firms' actions *after* the certification standard is set. Thus, they ignore the possibility that firms often need to make long-term entry and investment decisions long before they know the certification threshold. When firms make such long-term decisions, we show how it is beneficial to delegate certification authority, just like Rogoff (1985) showed that governments may want to delegate monetary policy to a central bank. The benefits of strategic delegation has been discussed for bargaining situations by Schelling (1956), for principal-agent settings by Aghion and Tirole (1997), for cheap-talk games by Dessein (2002), and for organizational design by Alonso et al. (2008). Harstad (2010) showed that the concern for bargaining power, and the concern for coalition membership, go in opposite directions when it comes to delegation in political contexts. In the present paper, the concern for entry, and the concern for investments, go in opposite directions.

stakeholders. The main credit rating agencies switched in the1970′s from an "investor pays" to an "issuer pays" business model. The former model, by focusing on investor profits, made the rating agencies' objectives comparable to those of NGOs, while the latter, by focusing on issuers' profits, made the rating agencies more similar to industry-backed certifiers. This shift has been criticized as a source of rating inflation (see Rivlin and Soroushian (2017)).

[6] For example, the standards of the LEED rating system are revised approximately every five years. The first paragraph of the LEED v4.1 web page describes the latest version of these standards as follows: "Today's version of LEED, LEED v4.1, raises the bar on building standards to address energy efficiency, water conservation, site selection, material selection, day lighting and waste reduction."

With this, we add to the relatively small literature on the optimal certifier identity. Stahl and Strausz (2017) compare seller-paid certification and buyer-paid certification and show that the first variant results in ratings that are more informative. While they consider various business models, they only consider profit-maximizing certifiers. We, in contrast, consider certifiers with different objective functions. The conflict between buyers' and sellers' preferences for information disclosure is emphasized by Hopenhayn and Saeedi (2022), who show that even when it is possible to separate products into multiple categories, a certifier has little to lose from sticking to the two-tiered policy considered also in our paper.[7]

Our analysis also aims at predicting which type of certifier is more likely to arise in different markets. Bonroy and Constantatos (2014) and Baron (2011) take a similar positive approach; yet the dynamic aspect of our analysis is absent in their models.[8]

Outline. In Section 2, we present the baseline model. In Section 3, we allow firms to invest in their capacities to provide quality and, in Section 4, firm entry is endogenous. Section 5 makes the certifier type endogenous, and discusses certification fees. Section 6 concludes by returning to the anecdotal evidence. All proofs are contained in the Appendix and a number of extensions are discussed in the Online Appendix.[9]

2. The benchmark analysis

2.1. The benchmark model

This section presents a simple benchmark model. The model consists of consumers, firms, and a certifier.

Consumers: There is a mass one of consumers benefiting from quality. Each of them can buy a unit of a good to get $u(x) = v(x) - p(x)$, where $v(x) = v_0 + x$ is the value of a good with quality x and $p(x)$ is the price of such a good. The consumer gets a utility of 0 from not buying.

The following analysis is unchanged if we allow for additional consumers that do not benefit from quality.[10]

Firms: There is a mass $m > 1$ of heterogeneous firms. Each firm produces up to one unit of a good. To produce a good of quality x_i, firm i incurs the cost $\tilde{q}_i x_i^\alpha$, where $\alpha \in (1, 2)$.[11] The distribution of each $\tilde{q}_i$ is uniform and i.i.d. over the interval $[q, q + \Delta]$. Consequently, if we order firms according to the $\tilde{q}_i$'s, we can write $\tilde{q}_i = q + i\Delta/m$ for $i \in [0, m]$. We let firm i's profit be denoted by $\pi_i(x_i) = \mathbb{1}_i\big(p(x_i) - \tilde{q}_i x_i^\alpha\big)$, where $\mathbb{1}_i \in \{0, 1\}$ measures the production level of firm i.

The certifier: At the time of purchase, consumers do not observe the product quality, but observe whether a product has a certificate. Without the possibility to certify, the equilibrium quality would be zero in our model. The certifier determines and publicly announces a requirement, $\underline{x}$, and certifies all goods with this or higher quality.[12] Certification is offered at no cost to the firms.[13] Certification fees are analyzed in Section 5.3.

After learning $\underline{x}$, each firm i decides on x_i. Since quality is costly, in equilibrium, $x_i \in \{0, \underline{x}\}$. We let $n(\underline{x}) \in [0, m]$ measure the equilibrium mass of firms that satisfy the requirement $\underline{x}$.

The certifier's objective function may take several aspects into account. One extreme variant is that the certifier chooses $\underline{x}$ to maximize aggregate profit, as given by

$$\Pi(\underline{x}) \equiv \int_0^m \pi_i(x_i)di.$$

This objective function is reasonable if the certifier is a trade association. Another special case arises if the certifier maximizes the aggregate quality:

$$U(\underline{x}) \equiv \int_0^{n(\underline{x})} \underline{x}di = n(\underline{x})\underline{x}.$$

This objective function is reasonable if the certifier is an NGO.

In many cases, the quality may be a public good.[14] Then, if an individual buyer's marginal value of $\underline{x}$ stays normalized at one, the *social* value of quality can be $sU(\underline{x}) = sn(\underline{x})\underline{x}$ for some large value, $s > 1$. Clearly, the quality-maximizing $\underline{x}$ is independent of s. Furthermore, as long as each consumer is infinitely small, the level of s will not influence one consumer's

[7] The buyers' optimal information disclosure is rarely studied. Exceptions are Roesler and Szentes (2017) and Harbaugh and Rasmusen (2018).

[8] Our positive analysis is somewhat related to Biglaiser (1993), who looks at incentives to operate as a middleman between sellers and buyers and to provide buyers with information about the product.

[9] The Online Appendix is available on Harvard Dataverse, at https://doi.org/10.7910/DVN/HTFPAW.

[10] Thus, it would be straightforward to permit the consumers to be heterogeneous in this way: Consumers that do not benefit from quality would simply buy the least expensive good. The following analysis would be unchanged with this type of heterogeneity, as long as the total number of firms were larger than the total number of consumers.

[11] Our analysis stays essentially unchanged if the cost took the form $q_0 + \tilde{q}_i x^\alpha$, where $q_0 \in [0, v_0]$. Also, we focus on $\alpha \in (1, 2)$ since the outcome will be trivial otherwise: For $\alpha \geq 2$, all certifiers set a standard such that a mass 1 of firms get a certificate.

[12] Pass/fail standards are common: The directory Ecolabel Index listed 291 labels adopting a pass/fail standard and 97 providing multi-tiered standards. The data were obtained from the Ecolabel Index web page in August 2022.

[13] A minority of labels operate for-profits: In August 2022, the Ecolabel Index listed 349 labels that defined themselves as NGOs, government labels, or as run by an industry association. Only 97 defined themselves as for-profit organizations.

[14] The aggregate quality may represent the industry's total reduction in emissions, pesticides, the use of child labor, or cruel animal treatment.

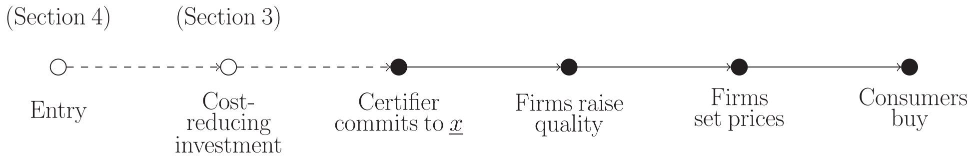

Fig. 1. The timing of the game.

decision, since s captures the externality on the other consumers. It is therefore inconsequential to exclude externalities associated with quality from the consumers' objective function above.[15]

That said, the externality does matter for a government or bureaucracy, B, which may take social welfare into account when setting the requirement. If B is the certifier, B maximizes a weighted sum of aggregate profits and aggregate quality:

$$W_b(\underline{x}) \equiv bU(\underline{x}) + (1-b)\Pi(\underline{x}),$$

for some $b \in (0, 1)$. It seems reasonable to let b increase in the externality s.[16] It is inconsequential to let the certifier ignore consumer surplus.[17]

To describe our results, we also refer to an NGO, or "activist certifier", A. A places weight $a = 1$ on quality and weight 0 on profit. A trade association, or commercial certifier, C, places weight $c = 0$ on quality and 1 on profit. (For the results below, it is sufficient to assume $1 \geq a > b > c \geq 0$.)

We will also refer to a general certifier identity, D, which can be any of the above. D maximizes $dU(\underline{x}) + (1-d)\Pi(\underline{x})$, $d \in [0, 1]$. When the government delegates to a board, it might be able to select any d. Such delegation can be important when certifiers are unable to build reputations for setting particular requirements before firms enter or invest.[18]

Timing: The four black dots in Fig. 1 illustrate the timing of the benchmark game. The white dots refer to the more interesting versions analyzed in later sections. The firms know the identity of the certifier from the very beginning. (For the model and the results, it is irrelevant whether the consumers also know this identity.) After the certifier specifies $\underline{x}$, the firms simultaneously and independently decide whether to set the quality high enough so as to be certified. Every consumer observes $\underline{x}$, the products that are certified and those that are not, and buys at most one product. The prices clear the market. The investment stage is included in Section 3, and the entry stage in Section 4. Section 5 extends the game by deriving the equilibrium identity of the certifier.

2.2. The market equilibrium

The timing just described leads to a unique subgame-perfect equilibrium that we can find when we solve the game by backward induction. We start by determining the market prices as a function of $\underline{x}$ and of n. Suppose, first, that $n \in (0, 1)$. In this case, some consumers will purchase certified products, while others will not. As $m > 1$, competition among the firms drives profits to zero for the firms producing non-certified products. Therefore, $p(0) = 0$, so every consumer is guaranteed at least a surplus equal to $u(0) = v_0$.

Scarcity of certified products implies that consumers are willing to pay up to $p(\underline{x})$ given by $v(\underline{x}) - p(\underline{x}) = u(0)$. It follows that $p(\underline{x}) = \underline{x}$.

Given $p(\underline{x})$, the profit of firm i producing a certified product is:

$$\pi_i(\underline{x}) = \underline{x} - \left(q + \frac{i\Delta}{m}\right)\underline{x}^{\alpha}.$$

Now, consider the stage at which firms decide on whether to seek certification. Firm i benefits from meeting the quality requirement if:

$$\pi_i(\underline{x}) \geq 0 \Leftrightarrow i \leq \frac{m\left(\underline{x}^{1-\alpha} - q\right)}{\Delta}.$$

With this, the equilibrium mass of certified firms is:

$$n(\underline{x}) = \frac{m\left(\underline{x}^{1-\alpha} - q\right)}{\Delta},$$

[15] The analysis is also unchanged if there are negative externalities associated with consuming low-quality products, as we explain in the Online Appendix. If the negative externality is, for example, $s(1 - v(\underline{x}))$, decreasing in the quality $\underline{x}$, the constant s can be ignored and we can re-define this externality as a positive externality when a buyer consumes a product with large $\underline{x}$.

[16] Note that $sU(\underline{x}) + \Pi(\underline{x}) = [bU(\underline{x}) + (1-b)\Pi(\underline{x})]/(1-b)$ if $b = s/(1+s) \in (0, 1)$.

[17] The next section proves that, in our model, both individual and aggregate consumer surplus are constant (and independent of $\underline{x}$) whenever $n(\underline{x}) \in [0, 1]$. In the Online Appendix, we discuss how the model can be modified so that the consumer surplus becomes relevant.

[18] Reputation is analyzed by Hui et al. (2018) and Elfenbein et al. (2015), but note that governments often delegate the certification process to committees consisting of all kinds of representatives. The committees place various weights on profits vs. quality, depending on their composition. With this, the choice of certifier, as measured by d, may be fine-tuned by B, as a function of b.

under the condition $m\left(\underline{x}^{1-\alpha}-q\right)/\Delta \in (0,1)$. As we prove below, this condition will require that:

$$\Delta > \frac{2mq(\alpha-1)}{(2-\alpha)}.$$

We assume this inequality to hold, i.e., firm heterogeneity is sufficiently large so that some but not all firms seek certification. We explain the consequences of relaxing this assumption below: see the *Remark on Heterogeneity*.

In line with the evidence mentioned in the Introduction (see the discussion of Elfenbein et al. (2015)), $n(\underline{x})$ decreases with the requirement $\underline{x}$. It is also intuitive that $n(\underline{x})$ decreases in the cost, q, while $n(\underline{x})$ increases in the mass of firms, m. The aggregate profit and the aggregate quality can be written, respectively, as:

$$\Pi(\underline{x}) = \frac{\underline{x}^{\alpha} m}{2\Delta}\left(\frac{1}{\underline{x}^{\alpha-1}} - q\right)^2 \text{ and } U(\underline{x}) = \frac{m}{\Delta \underline{x}^{\alpha-1}}\left(\underline{x} - q\underline{x}^{\alpha}\right).$$

2.3. The equilibrium requirement

At the certification stage, any certifier D prefers the requirement

$$\underline{x}_d = \arg\max_{\underline{x}} dU(\underline{x}) + (1-d)\Pi(\underline{x}).$$

Given the explicit formulae for $U(\underline{x})$ and $\Pi(\underline{x})$, it is straightforward to derive our first benchmark result.

Proposition 1. *The equilibrium requirement is:*

$$\underline{x}_d = \left(\frac{(2-\alpha)(1+d)}{q\left(1+\sqrt{1-\alpha(2-\alpha)\left(1-d^2\right)}\right)}\right)^{\frac{1}{\alpha-1}},$$

implying that $\underline{x}_d$ is increasing in d, and that

$$n(\underline{x}_d) = \frac{mq}{\Delta}\left(\frac{1+\sqrt{1-\alpha(2-\alpha)\left(1-d^2\right)}}{(2-\alpha)(1+d)} - 1\right) \in (0,1).$$

Consequently,

$$\underline{x}_a > \underline{x}_b > \underline{x}_c > 0 \text{ and } 0 < n(\underline{x}_a) < n(\underline{x}_b) < n(\underline{x}_c) < 1.$$

In other words, A prefers a more demanding certification requirement than does B, and B prefers a more demanding requirement than does C. These differences are in line with the evidence discussed (see footnote 5).

Remark on Heterogeneity. Note that n is large when d is small, as when $d = c$. Nevertheless, we always have $n(\underline{x}_c) < 1$ if $\Delta > 2mq(\alpha-1)/(2-\alpha)$. If this inequality fails, it is still true that $\underline{x}_a \geq \underline{x}_b \geq \underline{x}_c$, but these inequalities can bind. In particular, if $\Delta < 2mq(\alpha-1)/(2-\alpha)$, then $n(\underline{x}_c) = 1$.[19] If Δ is reduced further, then, eventually, $n(\underline{x}_b) = 1$, and, therefore, $\underline{x}_b = \underline{x}_c$. Finally, if $\Delta < mq(\alpha-1)/(2-\alpha)$, then $\underline{x}_a = \underline{x}_b = \underline{x}_c$.

2.4. The optimal certifier

If B represents a benevolent planner, the socially optimal certifier is a certifier D, associated with weight $d(b)$, so that B's objective function is maximized. In the basic model, $d(b) = b$, that is, B prefers to certify itself.

Proposition 2. *The optimal certifier's identity is given by the function $d(b) = b$. Consequently, $B \succeq A$ and $B \succeq C$.*

Here, B's (strict) preference ordering over certifiers' identities is represented by the binary relation $\succeq$ ($\succ$). So, we have $B \succeq A$ if and only if $W_b(\underline{x}_b) \geq W_b(\underline{x}_a)$, for example. Proposition 2 is trivial, but stated as a benchmark result for later comparisons.

[19] A further reduction in $\underline{x}$ would reduce aggregate profit, however, since then a larger number of certified products would eliminate the scarcity of certified products.

3. Endogenous cost of quality

This section endogenizes the firms' cost of improving the product quality. As before, the identity of the certifier is known at the start of the game. Each firm i can invest any amount $y^i \in [0, q]$ at a cost $k\left(y^i\right)^2/2$. The investment reduces the cost parameter q to $q - y^i$. Thereafter, the firm-specific shock is realized, so that $\tilde{q}_i$ is drawn from a distribution uniform over the interval $\left[q - y^i, q - y^i + \Delta\right]$. This timing ensures that firms are heterogeneous, in equilibrium, even if they invest the same. After the average investment level is observed, the certifier determines $\underline{x}$ and the rest of the game is as in Section 2. To ensure that each firm's objective function is concave, we assume:

$$k > \frac{1}{\Delta q^{\frac{\alpha}{\alpha-1}}}.$$

We assume that this situation, in which all firms can invest, arises with probability $\psi \in [0, 1]$. The model in the previous section is a special case with $\psi = 0$. One may interpret ψ as the development stage of the industry, so that ψ is large for emerging markets with plenty of unexploited investment opportunities.

3.1. Equilibrium investments

In this subsection, we consider the equilibrium level of investment in the presence of an arbitrary certifier, D. Each firm has rational expectations regarding the future quality requirement and how it will depend on all the firms' investments. However, because there is a continuum of firms, each of them finds it impossible to influence the future decision on $\underline{x}_d$.

Given the aggregate profit, $\Pi(\underline{x})$, derived in Section 2.2., and the expected $\underline{x}_d$, the expected profit for a firm is:

$$\frac{\underline{x}_d^{\alpha}}{2\Delta}\left(\underline{x}_d^{1-\alpha} - (q - y^i)\right)^2 - k\frac{\left(y^i\right)^2}{2}.$$

The first-order condition for y^i is satisfied when y^i equals

$$y_d \equiv \frac{\underline{x}_d - q\underline{x}_d^{\alpha}}{\Delta k - \underline{x}_d^{\alpha}},$$

The second-order condition is $\Delta > \underline{x}_d^{\alpha}/k$. When this inequality holds, all firms invest the same quantity, y_d, and this quantity depends on d.

In equilibrium, a larger investment by every firm leads to a higher standard for any given d. Nevertheless, the proof of the next proposition confirms that the equilibrium is unique and that a larger d leads to a larger y_d, and thus to a larger $\underline{x}_d$.

Proposition 3. *There exists a unique equilibrium outcome. In equilibrium, all firms invest the same amount, and the expected investment, $\mathbb{E}y_d$, is an increasing function of the expected $\underline{x}$, and of d. Consequently,*

$$\mathbb{E}y_a > \mathbb{E}y_b > \mathbb{E}y_c.$$

The proposition states that firms invest the most in their capacities to provide quality when the certifier is A, and the least when the certifier is C.

3.2. The optimal certifier with investments

For any given level of $\underline{x}$, the equilibrium investment is always smaller than the socially optimal investment level. After all, each firm invests to maximize profits, at thus a marginally larger investment level has a second-order effect on profits. However, a larger investment increases the mass of firms that will satisfy the quality requirement, and thus the aggregate quality increases. This is a first-order benefit, explaining why social welfare increases with investment.

To motivate firms to invest, B prefers to delegate certification to a certifier that places a larger weight on quality, since the equilibrium $\underline{x}$ will then be larger. The larger $\underline{x}$ encourages the firms to invest. Intuitively, the benefit of increasing d is larger when it is more likely that the firms can invest. When the benefit of quality (b) is large, it is especially important to motivate firms to invest in their capacities. Thus, a larger b makes it more attractive to delegate to a certifier with a large d.

Proposition 4.

(i) *The optimal certifier is given by a function $d(\psi, b) > b$, increasing in both arguments.*
(ii) *Consider any pair $\underline{D}$ and $\overline{D}$ associated with $\overline{d} > \underline{d} \geq b$. If $\overline{D} \succeq \underline{D}$ for some (ψ, b), then $\overline{D} \succ \underline{D}$ for all $\left(\psi', b'\right) > (\psi, b)$.*[20]

The proposition implies that B never benefits from delegating the certifying responsibility to C, but B benefits from delegating to A if investments are important and externalities are large. Fig. 2 illustrates the parameter space under which B benefits from delegating to A.[21]

[20] We follow the convention that the two vectors be ranked as $\left(\psi', b'\right) > (\psi, b)$ if and only if $\psi' \geq \psi$ and $b' \geq b$ with at least one strict inequality.

[21] For Fig. 2, we set: $\Delta = 3.5$, $\alpha = 1.3$, $k = 0.3$, and $q = 1$.

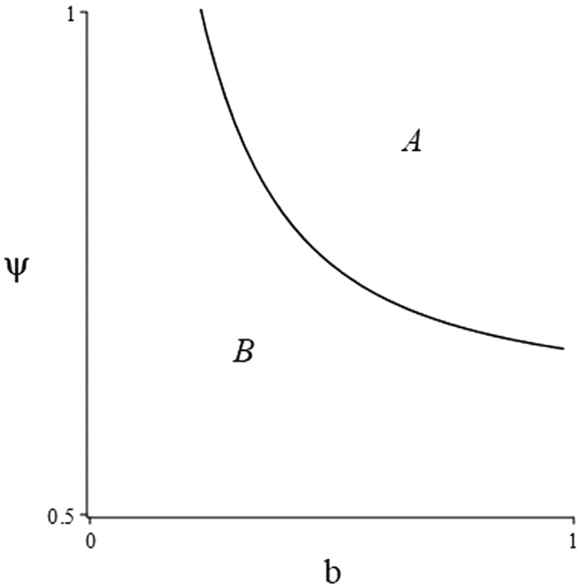

Fig. 2. Optimal certifier with endogenous cost of quality.

4. Endogenous market structure

We now permit entry at the beginning of the game, before the rest of the game proceeds exactly as in Section 3. (As before, the identity of the certifier is known before the game starts.) We suppose that a mass $\underline{m}$ of firms is already (or exogenously) in the market, while another mass $\Delta_E = \overline{m} - \underline{m}$ might enter.[22] Each firm i faces an entry cost e_i, distributed uniformly and i.i.d. with density σ over the interval $[\underline{e} - 1/(2\sigma), \underline{e} + 1/(2\sigma)]$. Firm i prefers to enter if and only if the expected profit is weakly larger than e_i.

This situation, in which additional firms can enter, arises with probability $\psi_E \in [0, 1]$. The model in Section 3 is a special case with $\psi_E = 0$. One may interpret ψ_E as the development stage of the industry, so that ψ_E is large for recent industries where many firms are yet to enter.

4.1. Equilibrium entry

We have already proven that the level of m does not influence any investment decision or the certifier's choice of a quality threshold. The expected profit for each firm takes into account the option of investing later. This profit will be independent of m as long as $n(\underline{x}) < 1$. Consequently, depending on the expected quality requirement, which, in turn, depends on the certifier's characteristic d, there will be a threshold $e(d)$ such that firm i enters if and only if $e_i \leq e(d)$. The expected mass of firms will be given by

$$\mathbb{E}m_d = \underline{m} + \Delta_E \psi_E Pr(e_i \leq e(d)).$$

It is intuitive that entry is larger if $d \in [0, 1]$ is smaller.

Proposition 5. *The expected equilibrium mass of firms, $\mathbb{E}m_d$, is a decreasing function of the expected $\underline{x}$, and thus of d. Consequently,*

$$\mathbb{E}m_a \leq \mathbb{E}m_b \leq \mathbb{E}m_c,$$

with strict inequalities as long as $\underline{m} < \mathbb{E}m_b < \overline{m}$.

In order to focus on the most interesting parameter region, we assume in the following that:

$$\underline{m} < \mathbb{E}m_a < \mathbb{E}m_b < \mathbb{E}m_c < \overline{m}. \tag{1}$$

This assumption holds if the firm facing the smallest possible entry cost always prefers to enter, while the firm facing the highest possible entry cost will never enter, regardless of the identity of the certifier.

[22] In line with our previous assumption on heterogeneity, we assume $\Delta > 2\overline{m}q(\alpha - 1)/(2 - \alpha)$.

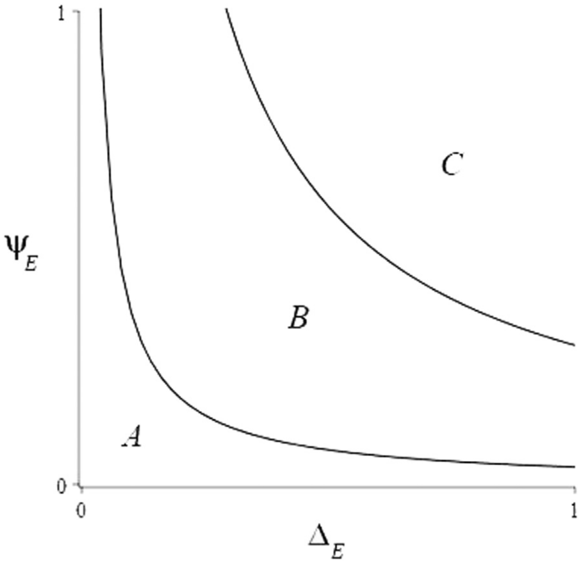

Fig. 3. Optimal certifier with endogenous market structure.

4.2. The optimal certifier with entry and investments

Entry is always valuable to B in this model, regardless of the weight b. If more firms have entered the market, there is a larger mass of firms that may find it affordable to satisfy any requirement. When more goods are certified, the total quality and the total profit both increase.[23]

For potential entrants, entry is attractive when the expected profit is large, and profits will be large when the certifier places a small weight d on quality. Thus, it can be beneficial to delegate certification authority to C if entry is very important. Encouraging entry is important when σ is large, since then a small change in $\underline{x}$ (or, equivalently, in d) dramatically influences the expected mass of firms. Similarly, a larger Δ_E makes it desirable to reduce d. However, if b is large, it is optimal for B to delegate strategically by increasing d, since a larger d raises the equilibrium $\underline{x}$ and thus firms' quality investments. Proposition 6 formalizes these insights.[24]

Proposition 6.

(i) *The optimal certifier is given by a function* $d_E(\sigma, \Delta_E, \psi_E, -b)$*, decreasing in all its arguments.*
(ii) *Consider any pair* $\underline{D}$ *and* $\overline{D}$ *characterized by* $\underline{d}$ *and* $\overline{d} > \underline{d}$*. If* $\underline{D} \succeq \overline{D}$ *for some* $(\sigma, \Delta_E, \psi_E, -b)$*, then* $\underline{D} \succ \overline{D}$ *for all* $\left(\sigma', \Delta_E', \psi_E', -b'\right) > (\sigma, \Delta_E, \psi_E, -b)$.

Part (ii) implies that B is less likely to prefer that A certifies, and more likely to prefer that C certifies, if entry is of significant importance, while the benefit of quality is limited. In line with this result, Fig. 3 illustrates the parameter spaces under which it is better to delegate certification authority to A, B, and C, respectively.[25]

The proposition explains when B benefits from delegating certification authority to private certifiers emphasizing profits or to NGOs that instead focus on quality. Although the results can be interpreted normatively, they also provide testable empirical predictions in cases when governments can influence the identity of the certifier.

5. Extensions

5.1. The equilibrium certifier

In reality, a government may be unable to dictate the identity of the certifier, and establishing oneself as a certifier is costly. We investigate in this section how the various stakeholders' incentives to enter as a certifier vary across industries and with the industry's stage of development. Interestingly, we will show that the certifier that enters first in equilibrium

[23] This preference may change if entry itself is associated with negative externalities, for example because firms that enter but fail to sell in this market can sell their low-quality products to other markets. We discuss this possibility in the Online Appendix.

[24] The proof of the following result assumes that B does not internalize the firms' entry costs when ranking certifiers. The result would continue to hold (although the proof would have to be modified) also if B internalized these costs.

[25] For Fig. 3, we set: $b = 0.5$, $\Delta = 3.5$, $\alpha = 1.3$, $k = 0.3$, $q = 1$, $\psi = 1$, $m = 1.1$, $\underline{e} = 0.0055$, and $\sigma = 434$.

can be the same as the certifier that is socially optimal: Qualitatively, our predictions are the same regardless of whether B appoints the certifier (as above) or whether we consider the equilibrium with entry (as below).

To investigate the incentive to certify, note that if there is no certifier, then firms provide zero quality and profits are zero. Therefore, any certifier D receives some positive payoff, denoted V_d, for being present in the market. The cost of establishing oneself as a certifier is, say, $\kappa + \kappa_d$. We permit this cost to consist of a common component, κ, and a component that varies with d, κ_d.

Consider, for instance, two potential certifiers, $\underline{D}$ and $\overline{D}$, characterized by parameters $\underline{d}$ and $\underline{d} < \overline{d}$, respectively. If the common cost κ is very high, then no one enters as a certifier. If κ is gradually reduced, then, sooner or later, one of the stakeholders will find it optimal to enter. For example, if $V_{\underline{d}} - \kappa_{\underline{d}} < \kappa < V_{\overline{d}} - \kappa_{\overline{d}}$, then $\overline{D}$ is willing to enter but $\underline{D}$ is not. We will say that $\overline{D}$ "can certify first" if $V_{\overline{d}} - \kappa_{\overline{d}} \geq V_{\underline{d}} - \kappa_{\underline{d}}$ and that $\overline{D}$ "certifies first" if $V_{\overline{d}} - \kappa_{\overline{d}} > V_{\underline{d}} - \kappa_{\underline{d}}$.

Note that the benefit for one stakeholder to enter and out-compete an existing certifier is smaller than the benefit from entering if there is no other certifier in the market. This discouragement effect motivates us to limit attention to the situation with a single certifier.

The value V_d shifts if the other parameters of the model are adjusted, and this shift in V_d varies systematically with d. For example, and in contrast to the analysis we have provided so far, the spillover s can now be important, because a larger s increases the importance of quality. It is thus intuitive that a larger s increases A's willingness to establish itself as the certifier more than C's willingness.

Proposition 7. *Consider any pair of potential certifiers, $\underline{D}$ and $\overline{D}$, characterized by quality weights $\underline{d}$ and $\overline{d} > \underline{d}$, respectively.*

(i) *Consider the situation with a fixed market structure. If $\overline{D}$ can certify first for some (ψ, s), then $\overline{D}$ certifies first for all $\left(\psi', s'\right) > (\psi, s)$.*
(ii) *Consider the situation in which additional firms can enter. If $\underline{D}$ can certify first for some $(\sigma, \Delta_E, \psi_E, -s)$, then $\underline{D}$ certifies first for all $\left(\sigma', \Delta'_E, \psi'_E, -s'\right) > (\sigma, \Delta_E, \psi_E, -s)$.*[26]

Let A and C be the (only) potential certifiers. Part (i) implies that, if the market structure (m) is fixed, then A is more likely to certify first if firms' are more likely to invest, or if the spillover associated with quality is large. This implies, in turn, that if A can certify first in the basic model without investments, then A is definitely certifying first in the model with investments (but without entry).

Part (ii) implies that when the market structure is endogenous, then C is more likely to certify first if it is likely that several new firms can enter, or if the spillover is small. This implies, in turn, that if C can certify first in the model without entry, then C is definitely certifying first if entry is possible.

These predictions are qualitatively consistent with the predictions we derived when the certifier was directly appointed by B. With this consistency, our analysis provides robust testable predictions for how the certifier's identity varies across markets and over time, and for how this identity influences the entry of new firms, the firms' investments in quality, the certification requirement, and the number of firms that end up meeting this requirement.

5.2. The choice of certifier over time

Our analysis suggest the following development of optimal and equilibrium certification. In a new market, when it is still possible and likely that new firms can enter (in that ψ_E is large), it is socially desirable that certification be delegated to a stakeholder that places a large weight on profits. At this stage, an industry association may itself be the best provider of certifications. The industry association is also most willing to pay any fixed operational cost, according to Proposition 7. At a later stage, the market structure might be developed (and the probability for further entry, ψ_E, might be small), but firms may still take actions that enhance their capacities to provide quality (in that ψ is large). At this stage, it would be socially optimal that the certifier placed a large weight on quality and, therefore, it would be optimal that an NGO will be responsible for certifications. (This will also be the equilibrium if the "fixed cost", discussed in the previous subsection, can be reclaimed once a certifier ends its business.) Finally, when further investments seem less likely (in that ψ is small), the government prefers to take control of the certification requirement.[27]

[26] To be consistent with Section 4, the proof of this proposition assumes that when a potential certifier considers whether to enter, it does not internalize the producing firms' subsequent entry cost. This assumption describes private certifiers such as industry certifiers, NGOs, and hybrid ones. To the extent that private certifiers care about firms' costs, they should not care about costs incurred by firm that are still outside the industry.

[27] If the governments learn about the externality over time, however, the dynamics can be quite different: Then, the government may prefer to delegate to an NGO as soon as one learns about the externality (f.ex, an environmental problem) and the possibilities to reduce the harm by making additional investments. Analogously, the government may want to switch to industry certifications if one learns about the possibilities (or benefits) of additional entry.

5.3. Private certifiers and certification fees

We have abstracted from certification fees to keep the analysis tractable, but fees should be incorporated in future research. Interestingly, a fee-maximizing certifier would set the same standard as the profit-maximizing certifier C in our model.

To see this, note that firm i is willing to pay at most $\underline{x} - (q + i\Delta/m)\underline{x}^{\alpha}$ to be certified. Hence, the total number of firms willing to pay some fee f to be certified is

$$m\frac{\underline{x} - q\underline{x}^{\alpha} - f}{\underline{x}^{\alpha}\Delta},$$

and the total revenue from collecting this fee is

$$fm\frac{\underline{x} - q\underline{x}^{\alpha} - f}{\underline{x}^{\alpha}\Delta}.$$

With this, it is easy to take first-order conditions and derive the revenue-maximizing f and $\underline{x}$. For a given f, the revenue-maximizing $\underline{x}$ is[28]

$$(1-\alpha)\underline{x}^{-\alpha} + \alpha f\underline{x}^{-\alpha-1} = 0 \Rightarrow \underline{x} = \frac{f\alpha}{\alpha - 1},$$

For a given $\underline{x}$, the revenue-maximizing f is

$$f = \frac{\underline{x} - q\underline{x}^{\alpha}}{2}.$$

When we combine the two conditions, we find that the revenue-maximizing $\underline{x}$ is equal to $\underline{x}_C$, derived above (see Proposition 1). This is intuitive, as a revenue-maximizing certifier, who captures the firms' profits, prefers to set the requirement so as to maximize these profits.

Suppose that also other certifiers (such as A and B) collect fees and place some weight on these revenues. When this weight is larger, A and B will be more similar to C and the preferred quality requirement will be smaller. If B, but not A, places a larger weight on collecting the fees, B should be more likely to prefer C to be the certifier instead of A. If A, but not B, places a larger weight on collecting fees, A becomes more similar to C (and thus to B) when setting the quality requirement, and B should be more likely to prefer A to be the certifier instead of C, everything else equal.

6. Conclusion

To sustain something else than markets for lemons, it is essential that the buyer is able to trust that the product attributes are valuable. This paper provides an analysis of the role of the certifier. To focus on the most important long-term consequences, we endogenize the firms' ability to provide quality, the market structure, and also the certifier's identity. The assumptions of the model are motivated by empirical observations, as discussed in the Introduction. Thus, the resulting normative recommendations and positive predictions deserve careful scrutiny as well.

As discussed in the Introduction, the certifier's identity varies systematically across industries. Table 1 shows that NGOs are active as certifiers of food (organic food and fair-trade products), natural resources (forests and fisheries), and energy efficiency. The qualities of such products are associated with important environmental externalities. Moreover, it is important that firms make long-term investments to be able to provide high-quality products. When externalities and investments are important, the optimal certifier, as well as the equilibrium certifier, is an NGO, according to our predictions: See Propositions 4, 6, and 7. These predictions match well with the evidence in Table 1.

For the more recent online-selling-platform industry, the certifier is instead associated with the industry itself, as summarized in Table 1. Since this industry is new and, to a large extent, still emerging, the market structure is still evolving and entry remains important. According to our theory, the optimal certifier, as well as the equilibrium certifier, is then an industry association: See Propositions 6 and 7, and the illustration in Fig. 3.

A serious empirical investigation is naturally beyond the scope of the present paper. However, our theoretical predictions, combined with the anecdotal evidence, suggest that such empirical research will be important for our understanding of the certifier.

The Online Appendix shows that the model can be extended to include multiple dimensions of quality, negative externalities of entry, and nontrivial consumer surplus. Other generalizations should be explored in future research.[29] Our model is simple and tractable, and it should thus be a useful starting point for several such extensions.

[28] The relevant second-order condition is: $\underline{x} < (1+\alpha)f/(\alpha-1)$. This condition holds locally when $\underline{x} = f\alpha/(\alpha-1)$. The first-order condition is then sufficient.

[29] While we have limited attention to binary licenses, motivated by the empirical regularity of this type of licensing, future research should allow for a ladder of quality thresholds. For a discussion of multi-tiered certification see, for example, Farhi et al. (2013). While we have limited attention to a single certifier, future research should explore the interaction between multiple and heterogeneous certifiers. The presence of multiple certifiers, in turn, raises novel issues. For instance, a proliferation of certificates can generate confusion among consumers (see Heyes et al. (2020)).

CRediT authorship contribution statement

Jacopo Bizzotto: Conceptualization, Methodology, Formal analysis, Formal analysis, Writing – original draft. **Bård Harstad:** Conceptualization, Methodology, Formal analysis, Formal analysis, Writing – original draft.

Data Availability

Data will be made available on request.

Appendix of Section 2

Proof of Proposition 1. We first derive $\underline{x}_d$. As discussed in Section 2.2,

$$\underline{x}_d \in \arg\max W_d(\underline{x}), \text{ where } W_d(\underline{x}) \equiv dU(\underline{x}) + (1-d)\Pi(\underline{x}).$$

For $\underline{x} \geq q^{\frac{1}{1-\alpha}}$, no firm gets a certificate (see Section 2.2), hence $W_d(\underline{x}) = 0$. For $\underline{x} \leq (q + \Delta/m)^{\frac{1}{1-\alpha}}$, a mass 1 of firms gets a certificate. In this case, $W_d(\underline{x}) \leq \overline{W}_d(\underline{x}) \equiv \underline{x} - (q + \Delta/(2m))\underline{x}^{\alpha}$.[30] Assumption $\Delta > 2(\alpha-1)mq/(2-\alpha)$ implies that $\overline{W}_d$ is an increasing function of $\underline{x}$, as long as $\underline{x} \leq (q+\Delta/m)^{\frac{1}{1-\alpha}}$.

If $\underline{x} \in ((q+\Delta/m)^{\frac{1}{1-\alpha}}, q^{\frac{1}{1-\alpha}})$, then $n(\underline{x}) \in (0,1)$, and

$$W_d(\underline{x}) = \tilde{W}_d(\underline{x}) \equiv (1+d)\underline{x}^{2-\alpha} - 2q\underline{x} + q^2(1-d)\underline{x}^{\alpha}.$$

The first-order condition with respect to $\underline{x}$ for this function corresponds to

$$(2-\alpha)(1+d)\underline{x}^{2(1-\alpha)} - 2q\underline{x}^{1-\alpha} + (1-d)\alpha q^2 = 0.$$

This condition holds if and only if $\underline{x} \in \{(\chi_1(d)/q)^{\frac{1}{\alpha-1}}, (\chi_2(d)/q)^{\frac{1}{\alpha-1}}\}$, where

$$\chi_1 \equiv \frac{(2-\alpha)(1+d)}{1+\sqrt{1-\alpha(2-\alpha)(1-d^2)}}, \text{ and } \chi_2 \equiv \frac{(2-\alpha)(1+d)}{1-\sqrt{1-\alpha(2-\alpha)(1-d^2)}}.$$

The second-order condition corresponds to

$$\underline{x} < q^{\frac{1}{1-\alpha}}((2-\alpha)(1+d))^{\frac{1}{\alpha-1}}.$$

This condition is satisfied if $\underline{x} = (\chi_1(d)/q)^{\frac{1}{\alpha-1}}$, and violated if $\underline{x} = (\chi_2(d)/q)^{\frac{1}{\alpha-1}}$. Assumption $\Delta > 2(\alpha-1)mq/(2-\alpha)$ ensures that $(\chi_1(d)/q)^{\frac{1}{\alpha-1}} > (q+\Delta/m)^{\frac{1}{1-\alpha}}$. Straightforward algebra ensures that $(\chi_1(d)/q)^{\frac{1}{\alpha-1}} < q^{\frac{1}{1-\alpha}}$. We are left with two candidate for corner solutions: $(q+\Delta/m)^{\frac{1}{1-\alpha}}$, and $q^{\frac{1}{1-\alpha}}$. As

$$\overline{W}_d\left(\left(\frac{\Delta}{m}+q\right)^{\frac{1}{1-\alpha}}\right) = \tilde{W}_d\left(\left(\frac{\Delta}{m}+q\right)^{\frac{1}{1-\alpha}}\right),$$

and $\tilde{W}_d$ is continuous, we rule out the first candidate. As $W_d(\chi_1(d)/q) > 0$, we rule out the second candidate. We conclude that $\underline{x}_d = (\chi_1(d)/q)^{\frac{1}{\alpha-1}}$.

We show next that $\underline{x}_d$ is strictly increasing in d. Note that

$$\frac{d\underline{x}_d}{d(d)} > 0 \Leftrightarrow \sqrt{1-\alpha(2-\alpha)(1-d^2)} > \alpha(2-\alpha)(1+d) - 1. \tag{2}$$

If $\alpha(2-\alpha)(1+d) - 1 < 0$, then (2) holds. Suppose instead that $\alpha(2-\alpha)(1+d) - 1 \geq 0$. Then (2) holds if $1-\alpha(2-\alpha)(1-d^2) > (\alpha(2-\alpha)(1+d)-1)^2$. A few steps of algebra are sufficient to verify that this inequality holds. □

Appendix of Section 3

Lemma 1. *In the subgame in which investment is possible, there exists an equilibrium in which all firms invest the same amount.*

Proof. We construct an equilibrium in which all firms invest the same amount. Suppose firm i anticipates standard $\underline{x}$. If, following investment y^i, the firm anticipates raising quality to $\underline{x}$ with some probability in (0,1) (and anticipates a mass of firms not larger than 1 to be doing the same, so that $p(\underline{x}) = \underline{x}$), then firm i's expected profits are

$$\pi^{\dagger}(\underline{x}, y^i) \equiv \frac{\underline{x}^{\alpha}}{2\Delta}\left(\underline{x}^{1-\alpha} - q + y^i\right)^2 - \frac{k(y^i)^2}{2}.$$

[30] If every firm with cost not larger than $q + \Delta/m$ sets quality at $\underline{x}$, and every other firm sets quality at 0, then $W_d(\underline{x}) = \overline{W}_d(\underline{x})$.

Let $y^*(\underline{x})$ be defined, implicitly, by $\frac{\partial \pi^\dagger(\underline{x},y)}{\partial y}|_{y=y^*(\underline{x})} = 0$. Hence, for $\underline{x} \neq (\Delta k)^{\frac{1}{\alpha}}$:

$$y^*(\underline{x}) = \frac{\underline{x} - q\underline{x}^\alpha}{\Delta k - \underline{x}^\alpha}. \tag{3}$$

We denote with $\underline{x}_d^\dagger(y)$ the standard set in equilibrium by certifier D, as long as all firms invest y. Note that $\Delta > 2mq(\alpha-1)/(2-\alpha)$ implies $\Delta > 2m(q-y)(\alpha-1)/(2-\alpha)$, for any $y \in [0,q]$. Hence, Proposition 1 implies that, for any $y \in [0,q)$:

$$\underline{x}_d^\dagger(y) = \left(\frac{\chi_1(d)}{q-y}\right)^{\frac{1}{\alpha-1}}. \tag{4}$$

Equation (4) is equivalent to: $y = q - \chi_1(d)(\underline{x}_d^\dagger(y))^{1-\alpha}$. Replacing y with $y^*(\underline{x}_d^\dagger(y))$, this last equation becomes:

$$t(d, \underline{x}_d^\dagger(y)) \equiv \Delta k(q - \chi_1(d)(\underline{x}_d^\dagger(y))^{1-\alpha}) - (1-\chi_1(d))\underline{x}_d^\dagger(y) = 0. \tag{5}$$

Next, we show that there exists a unique $\underline{x}_d^* \in (0, q^{\frac{1}{1-\alpha}})$ such that $t(d, \underline{x}_d^*) = 0$. The following remarks will prove useful:

(i) for any any $\underline{x} > 0$, $d^2 t(d,\underline{x})/d\underline{x}^2 < 0$;

(ii) $\lim_{\underline{x}\to 0^+} t(d,\underline{x}) = -\infty$; and

(iii) $t(d, q^{\frac{1}{1-\alpha}}) > 0$.

Remarks (i) and (ii) are immediate. Remark (iii) follows from assumption $k > \Delta^{-1} q^{\frac{\alpha}{1-\alpha}}$, as shown:

$$k > \frac{1}{\Delta q^{\frac{\alpha}{\alpha-1}}} \Leftrightarrow (1-\chi_1(d))\left(k - \frac{1}{\Delta q^{\frac{\alpha}{\alpha-1}}}\right) > 0 \Leftrightarrow t\left(d, q^{\frac{1}{1-\alpha}}\right) > 0,$$

where the first equivalence holds as $\chi_1(d) < 1$, and the second equivalence amounts to rearranging terms.

Remarks (i)-(iii) together imply that, for any $d \in [0,1]$, there exists a unique $\underline{x}_d^* \in (0, q^{\frac{1}{1-\alpha}})$ such that $t(d, \underline{x}_d^*) = 0$.

We show that there exists an equilibrium in which every firm invests $y^*\left(\underline{x}_d^*\right)$.

Note first that $y^*(\underline{x}_d^*) \in (0,q)$ (call this Remark (iv)).[31] Furthermore, Proposition 1 ensures that if every firm, except possibly firm i, invests $y^*\left(\underline{x}_d^*\right)$, then $n(\underline{x}_d^*) \in (0,1)$ (call this Remark (v)).

Next, we show that if firm i anticipates $\underline{x} \in (0, q^{\frac{1}{1-\alpha}})$, and a mass of firms not larger than 1 satisfying the requirement, then firm i's optimal choice of investment is $y^*(\underline{x})$ (call this Remark (vi)). Let

$$h(z, \underline{x}, y^i) \equiv z\left(\underline{x} - \left(q - y^i + \frac{\Delta}{2}z\right)\underline{x}^\alpha\right) - \frac{k(y^i)^2}{2}.$$

If firm i anticipates $\underline{x} \in (0, q^{\frac{1}{1-\alpha}})$, a mass of firms not larger than 1 satisfying the requirement, then firm i's profits equal:

$$\begin{cases} h(0,\underline{x},y^i) & \text{if } y^i \le q - \underline{x}^{1-\alpha}; \\ \pi^\dagger(\underline{x},y^i) & \text{if } y^i \in (q-\underline{x}^{1-\alpha}, q-\underline{x}^{1-\alpha}+\Delta); \\ h(1,\underline{x},y^i) & \text{if } y^i \ge q - \underline{x}^{1-\alpha} + \Delta. \end{cases}$$

Note that $\partial^2 h(1,\underline{x},y^i)/\partial(y^i)^2 = \partial^2 h(0,\underline{x},y^i)/\partial(y^i)^2 = -k$. Furthermore, $h(0,\underline{x},y^i) = \pi^\dagger(\underline{x},y^i)$ for $y^i = q - \underline{x}^{1-\alpha}$, and $h(1,\underline{x},y^i) = \pi^\dagger(\underline{x},y^i)$ for $y^i = q - \underline{x}^{1-\alpha} + \Delta$. Hence, continuity of $\pi^\dagger(\underline{x},y^i)$ with respect to y^i implies that firm i's profits are a continuous function of y^i, for any $y^i \in [0,q]$.

Note that $(0, q^{\frac{1}{1-\alpha}}) \subset (0, (k\Delta)^{\frac{1}{\alpha}})$. Hence, $\partial^2\pi^\dagger(\underline{x},y^i)/\partial(y^i)^2 = \underline{x}^\alpha/\Delta - k < 0$. As $\pi^\dagger(\underline{x},y^i) \ge \max\left\{h(1,\underline{x},y^i), h(0,\underline{x},y^i)\right\}$, for any $\underline{x} > 0$, and any $y^i \in [0,q]$, firm i's profits are a strictly concave function of y^i, for any $y^i \in [0,q]$. Remark (vi) follows. Remarks (iv)-(vi) establish the existence of an equilibrium in which all firms invest the same amount. □

Lemma 2. *In the subgame in which investment is possible, there exists a unique equilibrium outcome.*

Proof. In equilibrium, if the mass of firms expected to get a certificate is larger than 1, then all firms invest 0. Proposition 1 ensures that, if all firms invest 0, then there is no equilibrium in which the mass of firms expected to get a certificate is larger than 1. We conclude that in any equilibrium the mass of firms expected to get a certificate is not larger than 1.

Let $\underline{x} \in (0, (k\Delta)^{\frac{1}{\alpha}})$. Remark (vi) in the proof of Lemma 1 implies that the equilibrium outcome we have found is the unique equilibrium outcome in which $\underline{x} \in (0, q^{\frac{1}{1-\alpha}})$. Furthermore, if $\underline{x} \in [q^{\frac{1}{1-\alpha}}, (k\Delta)^{\frac{1}{\alpha}})$, then $y^*(\underline{x}) < 0$. Hence, the optimal investment level is 0. As, for any $d \in [0,1]$, $\underline{x}_d^\dagger(0) = (\chi_1(d)/q)^{\frac{1}{\alpha-1}} < q^{\frac{1}{1-\alpha}}$, we conclude that there exists no equilibrium in which $\underline{x} \in [q^{\frac{1}{1-\alpha}}, (k\Delta)^{\frac{1}{\alpha}})$.

[31] Remark (iv) holds as $\underline{x}_d^* \in \left(0, q^{\frac{1}{1-\alpha}}\right)$, and $k > \frac{1}{\Delta q^{\frac{\alpha}{\alpha-1}}}$.

Consider now $\underline{x} = (k\Delta)^{\frac{1}{\alpha}}$. In this case, $\partial\pi^{\dagger}(\underline{x}, y^i)/\partial y^i = k\big(x^{1-\alpha} - q\big) < 0$, for any $y^i \in [0, q]$. Hence the optimal investment level is 0. By an argument analogous to the one in the last paragraph, we conclude that there exists no equilibrium in which $\underline{x} = (k\Delta)^{\frac{1}{\alpha}}$.

Let $\underline{x} > (k\Delta)^{\frac{1}{\alpha}}$. Then, $\partial^2\pi^{\dagger}(\underline{x}, y^i)/\partial(y^i)^2 = \underline{x}^{\alpha}/\Delta - k > 0$. Hence, in equilibrium, either all firms invest 0, or a positive mass of firms make an investment sufficiently large to ensure that they get a certificate with probability 1. The first case can be ruled out as $\underline{x}_d^{\dagger}(0) < (k\Delta)^{\frac{1}{\alpha}}$. The second case can be ruled out as follows. As $\partial^2 h(1, \underline{x}, y^i)/\partial(y^i)^2 = -k < 0$, in equilibrium, all firms that make a positive investment must invest the same amount. In equilibrium, $\underline{x}$ should maximize the certifier's objective function, conditional on the firms that invested 0 not getting the certificate (we know that for $\underline{x} > (k\Delta)^{\frac{1}{\alpha}} > q^{\frac{1}{1-\alpha}}$, a firm that invests 0 never gets a certificate). Proposition 1, ensures that, for any $d \in [0, 1]$, the optimal standard is such that firms that make a positive investment get a certificate with probability smaller than 1, thus yielding a contradiction. This observation implies that the equilibrium we have characterized in the proof of Lemma 1 is unique. □

Lemma 3. *For any $d \in [0, 1]$, $\underline{x}_d^*$ is increasing in d.*

Proof. From the proof of Lemma 1, $\partial t(d, \underline{x})/\partial\underline{x}|_{\underline{x}=\underline{x}_d^*} > 0$, for any $d \in [0, 1]$. Hence, it is sufficient to show that $\partial t(d, \underline{x})/\partial d < 0$, for any $\underline{x} \in (0, q^{\frac{1}{1-\alpha}})$, and any $d \in [0, 1]$. This is the case as, for any $d \in [0, 1]$,

$$\frac{\partial t(d, \underline{x})}{\partial d} < 0 \Leftrightarrow \left(\frac{\underline{x}}{\Delta k} - \underline{x}^{1-\alpha}\right)\chi_1'(d) < 0,$$

and the latter inequality holds as $\chi_1'(d) > 0$, and

$$\underline{x} < q^{\frac{1}{1-\alpha}} \Rightarrow \underline{x} < (\Delta k)^{1/\alpha} \Leftrightarrow \frac{\underline{x}}{\Delta k} - \underline{x}^{1-\alpha} < 0.$$

□

Lemma 4. *For any $d \in [0, 1]$, $dy^*(\underline{x})/d\underline{x}|_{\underline{x}=\underline{x}_d^*} > 0$.*

Proof. Note that (3) implies:

$$\begin{aligned} \frac{dy^*(\underline{x})}{d\underline{x}} > 0 \Leftrightarrow \frac{\big(1 - q\alpha\underline{x}^{\alpha-1}\big)(\Delta k - \underline{x}^{\alpha}) + \alpha\underline{x}^{\alpha-1}(\underline{x} - q\underline{x}^{\alpha})}{(\Delta k - \underline{x}^{\alpha})^2} > 0 \Leftrightarrow \\ \big(1 - q\alpha\underline{x}^{\alpha-1}\big)\Delta k - \big(1 - q\alpha\underline{x}^{\alpha-1}\big)\underline{x}^{\alpha} + \alpha q^{\alpha}\big(1 - q\underline{x}^{\alpha-1}\big) > 0 \Leftrightarrow \\ f(\underline{x}) \equiv \big(\underline{x}^{1-\alpha} - q\alpha\big)\Delta k + (\alpha - 1)\underline{x} > 0. \end{aligned}$$

As $(\Delta k)^{\frac{1}{\alpha}} > q^{\frac{1}{1-\alpha}}$, then $f'(\underline{x}) = (\alpha - 1)(1 - \underline{x}^{-\alpha}\Delta k) < 0$, as long as $\underline{x} \in (0, q^{\frac{1}{1-\alpha}})$. Thus, in light of Lemma 3, it suffices to show that $f(\underline{x}_1^*) > 0$. If $\underline{x}_1^* \le (\alpha q)^{\frac{1}{1-\alpha}}$, then $f(\underline{x}_1^*) > 0$. Suppose instead that $\underline{x}_1^* > (\alpha q)^{\frac{1}{1-\alpha}}$. Then:

$$f(\underline{x}_1^*) > 0 \Leftrightarrow \Delta k < \frac{(\alpha - 1)\underline{x}_1^*}{q\alpha - \big(\underline{x}_1^*\big)^{1-\alpha}}.$$

Note also that $t(1, \underline{x}_1^*) = 0$ (see the proof of Lemma 1 for the definition of $t(\cdot, \cdot)$). Condition $t(1, \underline{x}_1^*) = 0$ is equivalent to:

$$\Delta k = \frac{(\alpha - 1)\underline{x}_1^*}{q - (2 - \alpha)\big(\underline{x}_1^*\big)^{1-\alpha}}. \tag{6}$$

We now show that:

$$\frac{(\alpha - 1)\underline{x}_1^*}{q - (2 - \alpha)\big(\underline{x}_1^*\big)^{1-\alpha}} < \frac{(\alpha - 1)\underline{x}_1^*}{q\alpha - \big(\underline{x}_1^*\big)^{1-\alpha}}.$$

This inequality is equivalent to:

$$\frac{q\alpha - \big(\underline{x}_1^*\big)^{1-\alpha}}{q - (2 - \alpha)\big(\underline{x}_1^*\big)^{1-\alpha}} < 1.$$

As (6) implies $q - (2 - \alpha)\big(\underline{x}_1^*\big)^{1-\alpha} > 0$, then

$$\begin{aligned} \frac{q\alpha - \big(\underline{x}_1^*\big)^{1-\alpha}}{q - (2 - \alpha)\big(\underline{x}_1^*\big)^{1-\alpha}} < 1 \Leftrightarrow \\ q\alpha - (\underline{x}_1^*)^{1-\alpha} < q - (2 - \alpha)(\underline{x}_1^*)^{1-\alpha} \Leftrightarrow \end{aligned}$$

$$q^{\frac{1}{1-\alpha}} > \underline{x}_1^*.$$

In the proof of Lemma 1 we showed that $q^{\frac{1}{1-\alpha}} > \underline{x}_d^*$, for any $d \in [0, 1]$. □

Proof of Proposition 3. The proposition follows from Lemmata 1–4, and the remark that there exists a unique equilibrium outcome of the subgame in which investment is not possible (see Proposition 1). □

Proof of Proposition 4. Let

$$U^\dagger(\underline{x}, y) \equiv \frac{m\underline{x}^{1-\alpha}}{\Delta}(\underline{x} - (q-y)\underline{x}^\alpha),$$
$$\Pi^\dagger(\underline{x}, y) \equiv m\pi^\dagger(\underline{x}, y), \text{ and}$$
$$W_b^\dagger(\underline{x}, y) \equiv bU^\dagger(\underline{x}, y) + (1-b)\left(\Pi^\dagger(\underline{x}, y) - \frac{ky^2}{2}\right),$$

where $\pi^\dagger(\cdot,\cdot)$ is defined in the proof of Lemma 1. If D is the certifier, then Certifier B's payoff equals:

$$\psi W_b^\dagger(\underline{x}_d^*, y_d) + (1-\psi)W_b(\underline{x}_d). \tag{7}$$

It is easy to check that (7) is a continuous function of d. Hence, compactness of the interval [0,1] ensures that a weight d that maximizes (7) exists. It is also easy to check that it is (generically) the case that

$$W_b^\dagger(\underline{x}_d^*, y_d) \neq (1-\psi)W_b(\underline{x}_d).$$

Hence, it is generically true that the maximand is unique. This maximand is a function $d(\psi, b)$.

We show next that $d(\psi, b) > b$. First, we show that $\partial^2\Pi^\dagger(\underline{x}, y)/\partial\underline{x}^2|_{\underline{x}=\underline{x}_d^*} < 0$, for any $y \in [0, q)$, and any $d \in [0, 1]$:

$$\begin{aligned}
&\frac{\partial^2\Pi^\dagger(\underline{x}, y)}{\partial\underline{x}^2}|_{\underline{x}=\underline{x}_d^*} < 0 \Leftrightarrow\\
&\frac{\partial^2\pi^\dagger(\underline{x}, y)}{\partial\underline{x}^2}|_{\underline{x}=\underline{x}_d^*} < 0 \Leftrightarrow\\
&\left(\frac{\sqrt{2-\alpha}}{\sqrt{\alpha}(q-y)}\right)^{\frac{1}{\alpha-1}} > \underline{x}_d^* \Leftarrow\\
&\left(\frac{\sqrt{2-\alpha}}{\sqrt{\alpha}(q-y)}\right)^{\frac{1}{\alpha-1}} > \underline{x}_1^* \Leftrightarrow\\
&(\alpha-1)^2 > 0.
\end{aligned}$$

Note that the left arrow follows from Lemma 3.

Next, we show that $\partial^2U^\dagger(\underline{x}, y)/\partial\underline{x}^2|_{\underline{x}=\underline{x}_d^*} < 0$, for any $y \in [0, q)$, and any $d \in [0, 1]$:

$$\frac{\partial^2U^\dagger(\underline{x}, y)}{\partial\underline{x}^2}|_{\underline{x}=\underline{x}_d^*} < 0 \Leftrightarrow (2-\alpha)(1-\alpha)(\underline{x}_d^*)^{-\alpha} < 0 \Leftrightarrow \alpha \in (1, 2).$$

We conclude that $\partial^2W_b^\dagger(\underline{x}, y)/\partial\underline{x}^2|_{\underline{x}=\underline{x}_d^*} < 0$, for any $d \in [0, 1]$, and any $y \in [0, q)$. A marginal increase in d increases marginally both $\underline{x}_d^*$ (see Lemma 3), and y_d (see Lemma 4). As $d\underline{x}_d^*/d(d) > 0$, while, for any $y \in [0, q)$, $\partial^2W_b^\dagger(\underline{x}, y)/\partial\underline{x}^2|_{\underline{x}=\underline{x}_d^*} < 0$, and $\partial W_b^\dagger(\underline{x}, y)/\partial\underline{x}|_{\underline{x}=\underline{x}_b^*} = 0$, then fixing $y \in [0, q)$, a marginal increase in $\underline{x}_d^*$ weakly increases $W_b^\dagger(\underline{x}_d^*, y)$, as long as $d \leq b$. For any $d \in [0, 1]$, holding $\underline{x}_d^*$ fixed, a marginal increase in y_d strictly increases $W_b^\dagger(\underline{x}_d^*, y_d)$, as it leaves profits unchanged, while increasing the mass of firms that gets a certificate, thus increasing $U^\dagger(\underline{x}_d^*, y_d)$. Setting $y = 0$, the arguments just presented imply that also $W_b(\underline{x}_d)$ is an increasing function of d, as long as $d \in [0, b)$ (and a decreasing function of d, as long as $d \in (b, 1]$). Hence, $d(\psi, b) > b$.

Next, we prove part (ii) of the proposition. Standard arguments ensure that the rest of part (i) follows from part (ii).

Consider a pair $\underline{D}$ and $\overline{D}$ characterized by $\underline{d} \geq b$, and $\overline{d} > \underline{d}$, respectively. Assume $\overline{D} \succeq \underline{D}$, for some (ψ, b). As we just established that $W_b(\underline{x}_d)$ is an increasing function of d, as long as $d \in [0, b)$ (and a decreasing function of d, as long as $d \in (b, 1]$), then $W_b(\underline{x}_{\underline{d}}) > W_b(\underline{x}_{\overline{d}})$. Hence, $\overline{D} \succeq \underline{D}$ implies $W_b^\dagger(\underline{x}_{\underline{d}}, y_{\underline{d}}) > W_b^\dagger(\underline{x}_{\overline{d}}, y_{\overline{d}})$. Thus, $\overline{D} \succ \underline{D}$ for all $(\psi', b) > (\psi, b)$.

Next, we establish that $\overline{D} \succ \underline{D}$ for all $(\psi, b') > (\psi, b)$. First, we show that, for any $d \in [0, 1]$,

$$\frac{dU^\dagger(\underline{x}_d^*, y_d)}{d(d)} = \frac{\partial U^\dagger(\underline{x}_d^*, y)}{\partial y}|_{y=y_d}\frac{dy_d}{d(d)} + \frac{\partial U^\dagger(\underline{x}, y_d)}{\partial\underline{x}}|_{\underline{x}=\underline{x}_d^*}\frac{d\underline{x}_d^*}{d(d)} > 0.$$

For any $d \in [0,1]$, it is immediate that $\frac{\partial U^{\dagger}(\underline{x}_d^*, y)}{\partial y}|_{y=y_d} > 0$, while Lemma 4 establishes that $dy_d/d(d) > 0$, and Lemma 3 establishes that $d\underline{x}_d^*/d(d) > 0$. Furthermore,

$$\frac{\partial U^{\dagger}(\underline{x}, y_d)}{\partial \underline{x}}|_{\underline{x}=\underline{x}_d^*} > 0 \Leftrightarrow \underline{x}_d^* < \left(\frac{2-\alpha}{q-y_d}\right)^{\frac{1}{\alpha-1}},$$

for any $d \in [0,1]$, thus $U^{\dagger}(\underline{x}_{\overline{d}}^*, y_{\overline{d}}) > U^{\dagger}(\underline{x}_{\underline{d}}^*, y_{\underline{d}})$. It is also easy to check that $U(\underline{x}_{\overline{d}}) > U(\underline{x}_{\underline{d}})$. Hence, either

$$\psi\left(\Pi^{\dagger}(\underline{x}_{\overline{d}}^*, y_{\overline{d}}) - \frac{k\left(y_{\overline{d}}\right)^2}{2}\right) + (1-\psi)\Pi(\underline{x}_{\overline{d}}) \geq$$
$$\psi\left(\Pi^{\dagger}(\underline{x}_{\underline{d}}^*, y_{\underline{d}}) - \frac{k\left(y_{\underline{d}}\right)^2}{2}\right) + (1-\psi)\Pi(\underline{x}_{\underline{d}}),$$

and therefore $\overline{D} \succ \underline{D}$ for all $(\psi, b') > (\psi, b)$, or, else, the last highlighted inequality is violated, in which case $\overline{D} \succ \underline{D}$ for all values of ψ. □

Appendix of Section 4

Proof of Proposition 5. A firm's expected profits, net of the entry cost, equal

$$\mathbb{E}\pi^{\dagger}(d) \equiv \psi\pi^{\dagger}(\underline{x}_d^{\dagger}(y_d), y_d) + (1-\psi)\pi^{\dagger}(\underline{x}_d^{\dagger}(0), 0),$$

where $\pi^{\dagger}(\cdot,\cdot)$ is defined in the proof of Lemma 1. We show next that, for any $d \in (0,1]$, and any $y \in [0, y_1]$, $\pi^{\dagger}(\underline{x}_d^{\dagger}(y), y)$ is a decreasing function of d. Note, first, that:

$$\frac{d\pi^{\dagger}(\underline{x}_d^{\dagger}(y), y)}{d(d)} = \frac{\partial \pi^{\dagger}(\underline{x}, y)}{\partial \underline{x}}|_{\underline{x}=\underline{x}_d^{\dagger}(y)} \frac{d\underline{x}_d^{\dagger}(y)}{d(d)}.$$

Equation (4) ensures that, for any $d \in [0,1]$, and any $y \in [0,q)$, $d\underline{x}_d^{\dagger}(y)/d(d) > 0 \Leftrightarrow \chi_1'(d) > 0$, where $\chi_1(d)$ is defined in the proof of Proposition 1. We showed in the proof of Proposition 1 that, indeed, $\chi_1'(d) > 0$ for any $d \in [0,1]$. Hence, we conclude that $d\underline{x}_d^{\dagger}(y)/d(d) > 0$ for any $d \in [0,1]$, and any $y \in [0,q)$. Furthermore,

$$\frac{\partial \pi^{\dagger}(\underline{x}, y)}{\partial \underline{x}}|_{\underline{x}=\underline{x}_d^{\dagger}(y)} < 0 \Leftrightarrow$$
$$\alpha(\underline{x}_d^{\dagger}(y))^{\alpha-1}((\underline{x}_d^{\dagger}(y))^{1-\alpha} - q + y)^2 + 2(1-\alpha)((\underline{x}_d^{\dagger}(y))^{1-\alpha} - q + y) < 0.$$

For any $y \in [0, y_1]$, we showed in the proof of Lemma 1 that $(\underline{x}_d^{\dagger}(y))^{1-\alpha} - q > 0$, and therefore $(\underline{x}_d^{\dagger}(y))^{1-\alpha} - q + y > 0$. Hence the last highlighted inequality is equivalent to:

$$\alpha(\underline{x}_d^{\dagger}(y))^{\alpha-1}((\underline{x}_d^{\dagger}(y))^{1-\alpha} - q + y) + 2(1-\alpha) < 0 \Leftrightarrow$$
$$\underline{x}_d^{\dagger}(y) > \left(\frac{2-\alpha}{\alpha(q-y)}\right)^{\frac{1}{\alpha-1}}.$$

As $\underline{x}_0^{\dagger}(y) = ((2-\alpha)/(\alpha(q-y)))^{\frac{1}{\alpha-1}}$ (see the proof of Lemma 1), and $\underline{x}_d^{\dagger}(y) > \underline{x}_0^{\dagger}(y)$ for any $d \in (0,1]$ (see the proof of Lemma 3), then the last highlighted inequality holds, for any $d \in (0,1]$.

By the envelope theorem, $d\pi^{\dagger}(\underline{x}_d^{\dagger}(y), y)/d(d)|_{y=y_d} = d\pi^{\dagger}(\underline{x}_d^{\dagger}(y), y_d)/d(d)$. We have thus established that both $\pi^{\dagger}(\underline{x}_d^{\dagger}(y_d), y_d)$, and $\pi^{\dagger}(\underline{x}_d^{\dagger}(0), 0)$ are decreasing functions of d, hence $\mathbb{E}\pi^{\dagger}$ is decreasing in d. The proposition follows. □

Proof of Proposition 6. Let:

$$\mathbb{E}u^{\dagger}(d) \equiv \frac{1}{m}\left(\psi U^{\dagger}(\underline{x}_d^*, y_d) + (1-\psi)U(\underline{x}_d)\right),$$

where $U^{\dagger}(\cdot,\cdot)$ is defined in the proof of Proposition 4, while $\underline{x}_d^*$ is defined in the proof of Lemma 1. In equilibrium, for any $d \in [0,1]$, $e(d) = \mathbb{E}\pi^{\dagger}(d)$ (see the proof of Proposition 5 for a definition of $\mathbb{E}\pi^{\dagger}(\cdot)$). Let:

$$w_b^{\dagger}(d) \equiv b\mathbb{E}u^{\dagger}(d) + (1-b)\mathbb{E}\pi^{\dagger}(d).$$

Certifier B's objective function is thus:

$$W_b^E(d) \equiv \underline{m}w_b^{\dagger}(d) + \psi_E\Delta_E\sigma\int_{\underline{e}-\frac{1}{2\sigma}}^{\mathbb{E}\pi^{\dagger}(d)}\left(w_b^{\dagger}(d) - (1-b)e\right)de =$$

$$\underline{m}w_b^\dagger(d)+\psi_E\Delta_E\sigma\left(\mathbb{E}\pi^\dagger(d)-\underline{e}+\frac{1}{2\sigma}\right)\left(w_b^\dagger(d)-(1-b)\frac{\mathbb{E}\pi^\dagger(d)+\underline{e}-\frac{1}{2\sigma}}{2}\right).$$

It is easy to check that W_b^E is a continuous function. Hence, a weight $d\in[0,1]$ that maximizes (7) in the interval [0,1] exists. Moreover it is (generically) the case that

$$\psi_E\sigma\left(\mathbb{E}\pi^\dagger(d)-\underline{e}+\frac{1}{2\sigma}\right)\left(w_b^\dagger(d)-(1-b)\frac{\mathbb{E}\pi^\dagger(d)+\underline{e}-\frac{1}{2\sigma}}{2}\right)\neq w_b^\dagger(d).$$

Hence, it is generically true that the maximand is unique. This maximand is a function $d_E(\sigma,\Delta_E,\psi_E,-b)$. We prove here part (ii) of the proposition (the rest of part (i) is an immediate consequence of part (ii)).

Consider a pair $\underline{D}$ and $\overline{D}$ characterized by $\underline{d}$, and $\overline{d}>\underline{d}$, respectively. Suppose $\underline{D}\succeq\overline{D}$ for some $(\sigma,\Delta_E,\psi_E,-b)$. Note that

$$\begin{aligned}W_b^E(\underline{d})=\underline{m}w_b^\dagger(\underline{d})+&\\ \psi_E\Delta_E\sigma\Bigg[\left(\mathbb{E}\pi^\dagger(\overline{d})-\underline{e}+\frac{1}{2\sigma}\right)\left(w_b^\dagger(\underline{d})-(1-b)\frac{\mathbb{E}\pi^\dagger(\overline{d})+(\underline{e}-\frac{1}{2\sigma})}{2}\right)+&\\ \left(\mathbb{E}\pi^\dagger(\underline{d})-\mathbb{E}\pi^\dagger(\overline{d})\right)\left(w_b^\dagger(\underline{d})-(1-b)\frac{\mathbb{E}\pi^\dagger(\underline{d})+\mathbb{E}\pi^\dagger(\overline{d})}{2}\right)\Bigg].&\end{aligned}$$

Hence, if $w_b^\dagger(\underline{d})\geq w_b^\dagger(\overline{d})$, then

$$\begin{aligned}W_b^E(\underline{d})\geq\underline{m}w_b^\dagger(\overline{d})+&\\ \psi_E\Delta_E\sigma\Bigg[\left(\left(\mathbb{E}\pi^\dagger(\overline{d})-\underline{e}\right)+\frac{1}{2\sigma}\right)\left(w_b^\dagger(\overline{d})-(1-b)\frac{\mathbb{E}\pi^\dagger(\overline{d})+(\underline{e}-\frac{1}{2\sigma})}{2}\right)+&\\ \left(\mathbb{E}\pi^\dagger(\underline{d})-\mathbb{E}\pi^\dagger(\overline{d})\right)\left(w_b^\dagger(\overline{d})-(1-b)\frac{\mathbb{E}\pi^\dagger(\underline{d})+\mathbb{E}\pi^\dagger(\overline{d})}{2}\right)\Bigg]=&\\ W_b^E(\overline{d})+\psi_E\Delta_E\sigma\left(\mathbb{E}\pi^\dagger(\underline{d})-\mathbb{E}\pi^\dagger(\overline{d})\right)\left(w_b^\dagger(\overline{d})-(1-b)\frac{\mathbb{E}\pi^\dagger(\underline{d})+\mathbb{E}\pi^\dagger(\overline{d})}{2}\right)>&\\ W_b^E(\overline{d}).&\end{aligned}$$

Thus, if $w_b^\dagger(\underline{d})\geq w_b^\dagger(\overline{d})$, then $\underline{D}\succ\overline{D}$ for all values of σ, Δ_E, and ψ_E.

Suppose, instead, that $w_b^\dagger(\underline{d})<w_b^\dagger(\overline{d})$. It is immediate that, in this case, if $\underline{D}\succeq\overline{D}$ for some Δ_E and ψ_E, then $\underline{D}\succ\overline{D}$ for any $\Delta_E'>\Delta_E$, and for any $\psi_E'>\psi_E$. Note also that

$$\frac{dW_b^E(d)}{d\sigma}=\frac{1}{\sigma}\left(W_b^E(d)-\underline{m}w_b^\dagger(d)-\frac{\psi_E\Delta_E b}{2}\mathbb{E}u^\dagger(d)\right).$$

As $W_b^E(\underline{d})\geq W_b^E(\overline{d})$, $w_b^\dagger(\underline{d})<w_b^\dagger(\overline{d})$, and $\mathbb{E}u^\dagger(\underline{d})<\mathbb{E}u^\dagger(\overline{d})$, we conclude that $dW_b^E(\overline{d})/d\sigma<dW_b^E(\underline{d})/d\sigma$. This, in turn, implies $\underline{D}\succ\overline{D}$, for any $\sigma'>\sigma$.

Next, we show that $\underline{D}\succ\overline{D}$ for any $b'<b$. First, note that:

$$\begin{aligned}W_b^E(d)=b\left(\underline{m}+\psi_E\Delta_E\left(\sigma\left(\mathbb{E}\pi^\dagger(d)-\underline{e}\right)+\frac{1}{2}\right)\right)\mathbb{E}u^\dagger(d)+&\\ +(1-b)\left(\underline{m}\mathbb{E}\pi^\dagger(d)+\psi_E\Delta_E\sigma\left(\mathbb{E}\pi^\dagger(d)-\underline{e}+\frac{1}{2\sigma}\right)^2\right).&\end{aligned}$$

Note that the term multiplying $(1-b)$ is larger for $d=\underline{d}$ than for $d=\overline{d}$, that is:

$$\begin{aligned}&\underline{m}\mathbb{E}\pi^\dagger(\underline{d})+\psi_E\Delta_E\sigma\left(\mathbb{E}\pi^\dagger(\underline{d})-\underline{e}+\frac{1}{2\sigma}\right)^2>\\ &\quad\underline{m}\mathbb{E}\pi^\dagger(\overline{d})+\psi_E\Delta_E\sigma\left(\mathbb{E}\pi^\dagger(\overline{d})-\underline{e}+\frac{1}{2\sigma}\right)^2.\end{aligned}$$

Hence, either $\underline{D}\succ\overline{D}$ for any b, or else $\underline{D}\succeq\overline{D}$ for some b implies $\underline{D}\succ\overline{D}$, for any $b'<b$. This concludes the proof of part (ii) of the proposition. □

Appendix of Section 5

Proof of Proposition 7. (i) Consider the model without firm entry. Let:

$$v_d^\dagger \equiv ds\mathbb{E}u^\dagger(d) + (1-d)\mathbb{E}\pi^\dagger(d),$$

where $\mathbb{E}u^\dagger(\cdot)$ and $\mathbb{E}\pi^\dagger(\cdot)$ are defined in the proofs of Proposition 6 and Proposition 5. The payoff of certifier D from being in the market is $mv_d^\dagger$.

Suppose $v_{\bar{d}}^\dagger \geq v_{\underline{d}}^\dagger$ for some (ψ, s). The proof of Proposition 4 showed that $U^\dagger(\underline{x}_d^*, y_d)$ is increasing in both its arguments (we defined $U^\dagger(\cdot,\cdot)$ in the same proof, and $\underline{x}_d^*$ in the proof of Lemma 1), and $U(\underline{x}_d)$ is increasing in $\underline{x}_d$. Lemma 4 ensures that y_d is increasing in d. Hence, $v_{\bar{d}}^\dagger > v_{\underline{d}}^\dagger$, for any $(\psi, s') > (\psi, s)$. Next, we prove that $d^2\mathbb{E}v_d^\dagger/(d(d)d\psi) > 0$, or, equivalently,

$$\frac{d}{d(d)}\left(dsU^\dagger(\underline{x}_d^*, y_d) + (1-d)\left(\Pi^\dagger(\underline{x}_d^*, y_d) - \frac{mk(y_d)^2}{2}\right)\right) > \frac{d}{d(d)}(dsU(\underline{x}_d) + (1-d)\Pi(\underline{x}_d)), \tag{8}$$

where $\Pi^\dagger(\cdot,\cdot)$ is defined in the proof of Proposition 4. By the envelope theorem, $\frac{d}{d(d)}(dsU(\underline{x}_d) + (1-d)\Pi(\underline{x}_d)) = sU(\underline{x}_d) - \Pi(\underline{x}_d)$. As y_d maximizes firms' profits given $\underline{x}_d^*$, the envelope theorem also ensures:

$$\frac{d}{d(d)}\left(dsU^\dagger(\underline{x}_d^*, y_d) + (1-d)\left(\Pi^\dagger(\underline{x}_d^*, y_d) - \frac{mk(y_d)^2}{2}\right)\right) = sU^\dagger(\underline{x}_d^*, y_d) - \Pi^\dagger(\underline{x}_d^*, y_d) + \frac{mk(y_d)^2}{2} + ds\frac{\partial U^\dagger(\underline{x}_d^*, y_d)}{\partial(y_d)}.$$

Hence, (8) is equivalent to:

$$sU^\dagger(\underline{x}_d^*, y_d) - \Pi^\dagger(\underline{x}_d^*, y_d) + \frac{mk(y_d)^2}{2} + ds\frac{\partial U^\dagger(\underline{x}_d^*, y_d)}{\partial y_d} > sU(\underline{x}_d) - \pi(\underline{x}_d).$$

As we discussed above, $\partial U^\dagger(\underline{x}_d^*, y_d)/\partial y_d > 0$. Moreover, $mk(y_d)^2/2 > 0$. Thus, the last highlighted inequality holds if:

$$sU^\dagger(\underline{x}_d^*, y_d) - \Pi^\dagger(\underline{x}_d^*, y_d) > sU(\underline{x}_d) - \Pi(\underline{x}_d).$$

As, for any $d \in [0,1]$, $U^\dagger(\underline{x}_d^*, y_d) > U(\underline{x}_d)$, then the last highlighted inequality holds for any $s \geq 1$, as long as:

$$U^\dagger(\underline{x}_d^*, y_d) - \Pi^\dagger(\underline{x}_d^*, y_d) > U(\underline{x}_d) - \Pi(\underline{x}_d). \tag{9}$$

In order to show that (9) holds, we show that $U^\dagger(\underline{x}_d^*, y_d) - \Pi^\dagger(\underline{x}_d^*, y_d)$ is increasing in y, for any $y \in [0, q)$. In the proof of Lemma 1, we showed that $\underline{x}_d^* = (\chi_1(d)/(q-y_d))^{\frac{1}{\alpha-1}}$, where $\chi_1(d)$ is defined in the proof of Proposition 1. A few steps of algebra ensure:

$$U^\dagger(\underline{x}_d^*, y_d) - \Pi^\dagger(\underline{x}_d^*, y_d) = \frac{2m}{\Delta}(\chi_1(d))^{\frac{2-\alpha}{\alpha-1}}(1-\chi_1(d))^2(q-y_d)^{\frac{\alpha-2}{\alpha-1}}.$$

Note that $(q-y)^{\frac{\alpha-2}{\alpha-1}}$ is increasing in y, for any $y \in [0,q)$, and $(\chi_1(d))^{\frac{2-\alpha}{\alpha-1}}(1-\chi_1(d))^2 > 0$. We have thus proved that (9) holds. Thus, for any $d \in [0,1]$, $d^2v_d^\dagger/(d(d)d\psi) > 0$. It follows that $v_{\bar{d}}^\dagger \geq v_{\underline{d}}^\dagger$ for any $(\psi', s) > (\psi, s)$.

(ii) Consider the model with firm entry. The payoff of certifier D from being in the market equals $\mathbb{E}m_d v_d^\dagger$.

Suppose $\mathbb{E}m_{\underline{d}} v_{\underline{d}}^\dagger = \mathbb{E}m_{\bar{d}} v_{\bar{d}}^\dagger$ for some $(\sigma, \Delta_E, \psi_E, -s)$. The proofs of Propositions 4 and 5 show that $d\mathbb{E}u^\dagger(d)/d(d) > 0 > d\mathbb{E}\pi^\dagger(d)/d(d)$. Hence, $\mathbb{E}m_{\underline{d}}\mathbb{E}u^\dagger(\underline{d}) < \mathbb{E}m_{\bar{d}}\mathbb{E}u^\dagger(\bar{d})$. It follows that $\mathbb{E}m_{\underline{d}}v_{\underline{d}} > \mathbb{E}m_{\bar{d}}v_{\bar{d}}$, for any $(\sigma, \Delta_E, \psi_E, -s')$ such that $s' < s$. Furthermore,

$$\mathbb{E}m_d = \underline{m} + \psi_E\Delta_E\sigma\left(\mathbb{E}\pi^\dagger(d) - \underline{e} + \frac{1}{2\sigma}\right). \tag{10}$$

As $\mathbb{E}m_{\underline{d}} > \mathbb{E}m_{\bar{d}}$, then $\mathbb{E}m_{\underline{d}}v_{\underline{d}}^\dagger = \mathbb{E}m_{\bar{d}}v_{\bar{d}}^\dagger$ implies $v_{\underline{d}}^\dagger < v_{\bar{d}}^\dagger$. Hence $\mathbb{E}m_{\underline{d}}v_{\underline{d}}^\dagger > \mathbb{E}m_{\bar{d}}v_{\bar{d}}^\dagger$ for any $(\sigma, \Delta_E', \psi_E', -s) > (\sigma, \Delta_E, \psi_E, -s)$.

Fix $(\sigma, \Delta_E, \psi_E, -s)$ such that $\mathbb{E}m_{\underline{d}}v_{\underline{d}}^\dagger = \mathbb{E}m_{\bar{d}}v_{\bar{d}}^\dagger$; By (10), this corresponds to:

$$\left(\underline{m} + \psi_E\Delta_E\sigma\left(\mathbb{E}\pi^\dagger(\underline{d}) - \underline{e} + \frac{1}{2\sigma}\right)\right)v_{\underline{d}}^\dagger = \left(\overline{m} + \psi_E\Delta_E\sigma\left(\mathbb{E}\pi^\dagger(\bar{d}) - \underline{e} + \frac{1}{2\sigma}\right)\right)v_{\bar{d}}^\dagger.$$

Suppose

$$(\mathbb{E}\pi^{\dagger}(\underline{d}) - \underline{e})v_{\underline{d}}^{\dagger} \leq (\mathbb{E}\pi^{\dagger}(\overline{d}) - \underline{e})v_{\overline{d}}^{\dagger}.$$

As $\mathbb{E}\pi^{\dagger}(\underline{d}) > \mathbb{E}\pi^{\dagger}(\overline{d})$, then $v_{\underline{d}}^{\dagger} < v_{\overline{d}}^{\dagger}$, and, furthermore, $\mathbb{E}m_{\underline{d}}v_{\underline{d}}^{\dagger} < \mathbb{E}m_{\overline{d}}v_{\overline{d}}^{\dagger}$, which contradicts our assumption. We conclude that $(\mathbb{E}\pi^{\dagger}(\underline{d}) - \underline{e})v_{\underline{d}}^{\dagger} > (\mathbb{E}\pi^{\dagger}(\overline{d}) - \underline{e})v_{\overline{d}}^{\dagger}$. It follows that $\mathbb{E}m_{\underline{d}}v_{\underline{d}} > \mathbb{E}m_{\overline{d}}v_{\overline{d}}$, for any $(\sigma', \Delta_E, \psi_E, -s) > (\sigma, \Delta_E, \psi_E, -s)$. This observation concludes the proof. □

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