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PRESERVICE ELEMENTARY TEACHERS' MENTAL COMPUTATION STRATEGY USE IN SUBTRACTION ON TWO-DIGIT NATURAL NUMBERS

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Abstract: In this paper it is investigated if the mental computation strategies in the research literature are enough to satisfactorily categorize the mental computation strategy use by preservice elementary teachers (PETs) in subtraction on two-digit natural numbers. The PETs' use of mental computation strategies is measured operationally with a written questionnaire. The paper indicates that the strategies used by PETs are generally seen contained in the research literature on elementary school pupils' mental computation strategy use, but that there are additional strategies used by PETs, which are presented and discussed in the paper.

Key words: mental computation strategy, preservice elementary teacher, subtraction, written questionnaire.

1. Introduction

Mental computation is part of the elementary school curricular content globally, and in recent years there have in many countries been an increased focus on mental computation in elementary school teacher education (Csíkos, 2016; Hartnett, 2007; Lemonidis et al, 2014). There are many advantages of becoming better at mental computation, for example that it improves number sense (Hajra & Kofman, 2017; Heirdsfield et al, 2002, 2011), it gives a better understanding of the place value system and elementary calculation rules (Gürbüz & Erdem, 2016; Maclellan, 2001; Reys, 1984, 1992; Sowder, 1990, 1992, 1994), and it is often involved in everyday use of mathematics (Baranyai et al, 2019a; Thompson, 2010). Mental computation strategies in connection to elementary school pupils have been well researched, but less so in connection to preservice elementary teachers (PETs). In this article the latter group's strategy use will be investigated. A brief background to mental computation is given in section 2.1–2.4 before stating the research question in section 3.1.

2. Mental computation

2. 1. What is mental computation?

In the research literature one can find different definitions of mental computation, or synonyms such as mental calculation, mental math, and mental arithmetic (Lemonidis, 2016; QCA, 1999; Sowder, 1988; Thompson, 1999). An overarching trait among the definitions, and the one that will be used in this paper, is calculating without use of any equipment (Baranyai et al, 2019b; Lopez, 2014; McIntosh & Dole, 2000; Reys et al, 1995).

2. 2. What is a mental computation strategy?

Mental computation strategies are different ways that arithmetic problems are solved mentally (Hartnett, 2007; Threlfall, 2000, 2002). For example, to calculate $45 - 32$ one can subtract the tens and ones separately ($40 - 30 = 10$ and $5 - 2 = 3$) and then add the results ($10 + 3 = 13$), which is a mental computation strategy called 1010 ("ten-ten"). Some strategies are more general like this, while others are more dependent on coincidences in the calculation such that 36 is the double of 18 in the calculation $36 - 18$. Mental computation strategies which elementary school pupils use to calculate with natural numbers up to 100 have been well researched and documented for addition and subtraction (Beishuizen, 1993; Blöte et al, 2000; Heirdsfield, 1997, 2001; Klein & Beishuizen, 1998). Some of these strategies

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have official names, such as SA (standard algorithm done mentally), 1010 (separately adding tens and ones), N10 (stringing), N10C (stringing with compensation), A10 (bridging through multiples of ten), and B (balancing). There exist variants of these definitions and their names, but generally there is consensus on what they should mean (Heirdsfield, 2004; Varol & Farran, 2007). Elementary school pupils' use of mental computation strategies for multiplication and division has been researched to a lesser degree (Callingham, 2005; Heirdsfield et al, 1999; Murray et al, 1994; Oliver et al, 1991). There are fewer studies on preservice elementary teachers' (PETs') mental computation strategy use in addition and subtraction (Baranyai et al, 2019a; Hajra & Kofman, 2017; Whitacre, 2007), multiplication (Baranyai et al, 2019a; Lemonidis et al, 2014; Whitacre, 2007), and division (Mutawah, 2016).

2. 3. Why learn mental computation strategies?

To do mental computation efficiently, one need to learn several different strategies and know when to use which strategy (Hajra & Kofman, 2017; McIntosh, 2003; QCA, 1999). Many mental computation strategies are possible for pupils to discover on their own, but one cannot presume that all pupils will be able to do so (Murphy, 2004). There is evidence to suggest that pupils are often not directly exposed to mental computation strategies in school but are rather left to themselves to devise more or less efficient strategies. Some pupils then get stuck in unwieldly mental computation strategies, such as doing the standard algorithm mentally, and therefore need to learn more efficient strategies in an organized and systematic way (Askew, 1997; Baranyai et al, 2019b; Hajra & Kofman, 2017; Joung, 2018; McIntosh et al, 1995). Hope and Sherrill (1987) highlight that pupils with less developed skills mostly use standard written methods for mental computation, while pupils with higher developed skills use a variety of mental strategies. Thompson (2009) stresses the importance of teaching and using mental calculation strategies, since the traditional methods are not effective enough to improve pupils' numeracy proficiency.

2. 4. Why should preservice elementary teachers study mental computation strategies?

Even though mental strategies are a desired focus for computational instruction in schools, Hartnett (2007) suggests that teachers have been slow to adopt such a focus in their classroom, and that a possible barrier to adopting a mental strategies approach is the teachers' own lack of knowledge about possible mental computation strategies. Since PETs are the next generation of teachers it is important that they know and master mental computation strategies. They need a strong foundation of the mathematics of mental computation and the ability to apply this important calculation method as well as use efficient strategies of their own (Heirdsfield & Cooper, 2004; Lemonidis et al, 2014; Threlfall, 2002). Mental mathematics ability is considered a hallmark of number sense (Hajra & Kofman, 2017; Sowder, 1992), and good number sense is especially essential for elementary school teachers. Without it they are ill-equipped to make sense and take advantage of children's often unorthodox but very number sensible solution strategies (Whitacre, 2007).

3. Method

3. 1. Purpose of article and research question

In the light of section 2.1−2.4, it is proposed that it is important to know the current knowledge and proficiency base of PETs on mental computation. Knowing which strategies PETs are aware of and use provides valuable information for continuing professional development and improving teacher content knowledge on mental computation, and for use in related research (Heirdsfield & Lamb, 2005; Valenta & Enge, 2013). In this article a written questionnaire is utilized to conduct research into PETs' mental computation strategy use. A written questionnaire has an advantage over interviews when it comes to gathering a large amount of data. One disadvantage is if some PETs calculate the exercises using pen and paper even though they are instructed not to do so. In Månsson (2022) it was demonstrated that using written questionnaires to survey and categorize PETs' strategy use can be a valuable and reliable method. This paper is based on the premise that to conduct research into PETs' strategy use it is important to first determine if the existing research literature in fact covers the strategies used by PETs. For instance, when conducting research on PETs' strategy use it is important that different research papers use the same list of strategies, and that it covers the strategies used by PETs. A search through the research literature reveals that there is no article containing an exhaustive and complete list of all strategies occurring in the research literature. The categories vary, and they do not always account for all possible strategies. Also, since most research on mental computation have been done with elementary school pupils it is not obvious that the mental strategies considered in research in connection with pupils are the same as those used by PETs. Since PETs have more schooling in mathematics than elementary school pupils it is possible that PETs use a different set of strategies than the pupils do. Further, the availability and ability of PETs to participate in written questionnaires on mental computation also makes them suitable research participants. A relevant research question not emphasized in the research literature is therefore:

When preservice elementary teachers do mental computation in subtraction on two-digit natural numbers, what strategies, beside those that can be found in the literature, do they draw on?

Depending on what the research reveals for the research question there could be a need to improve and extend the list of strategies in the research literature (presented in section 3.4). The answer to the research question is presented in section 3.6. Note that for practical reasons the investigation here is limited to two-digit numbers, which are also more interesting than one-digit numbers when it comes to stimulating the development of number sense and insightful flexible number operations (Beishuizen et al, 1997; McIntosh et al, 1992).

3. 2. Research participants

In 2021-2022, written mental computation strategy questionnaires were given to 148 first- and second-year PETs at two different mid-sized universities in Norway. The PETs were chosen by availability, and they were not provided with any prior training on mental computation strategies prior to administering the questionnaire.

3. 3. Measures

The PETs' use of mental computation strategies was measured with a written questionnaire with 23 exercises (Figure 1):

1. $17 - 13$	**2.** $70 - 34$	**3.** $65 - 59$	**4.** $92 - 68$	**5.** $29 - 15$
6. $54 - 30$	**7.** $37 - 24$	**8.** $63 - 47$	**9.** $92 - 45$	**10.** $80 - 30$
11. $94 - 49$	**12.** $51 - 25$	**13.** $82 - 79$	**14.** $64 - 24$	**15.** $80 - 41$
16. $99 - 33$	**17.** $59 - 27$	**18.** $70 - 35$	**19.** $98 - 39$	**20.** $26 - 13$
21. $98 - 49$	**22.** $47 - 43$	**23.** $61 - 39$		

Figure 1. *Questionnaire exercises*

The exercises were chosen so that many different strategies would be induced and used by the PETs. There is no general theory on how to find such exercises, and it is in the nature of things that is hard to construct exercises inducing strategies that one is not aware of. However, the ambition was to construct exercises that at least could induce the strategies in Table 1. Figure 2 shows the questionnaire instructions and how each exercise was presented to the PETs.

Important instructions! For every exercise do these steps in order:

1. **Calculate the exercise in your head.**
    - The calculation must be done solely in your head. You are **not** allowed to write anything on the paper.
    - If you do not know the answer after you have been thinking for a while, do not write anything and instead go to the next exercise.

2. **Write down the answer.**
3. **Explain mathematically how you were thinking.**

Exercise 1

$17 - 13 =$ ______

Explanation:____

____

Figure 2. *Questionnaire instructions together with first exercise*

The PETs' written explanations of their strategy use were categorized by the author by referring to the list of mental computation strategies in section 3.4. This was demonstrated in Månsson (2022) to be an informative and reliable approach. It is an operational method, basing the categorization solely on the PETs' own written explanations, and not speculating on how they were "really thinking" when they calculated.

3. 4. List of mental computation strategies

In Table 1 is a summary of the mental computation strategies found through the literature review, which was used in the categorization of the PETs' explanations. This list of strategies was the result of an exhaustive search in the research literature. After each strategy a reference to where the definition can be found in the research literature is provided. If several articles agreed on a definition of a strategy, then only one of those articles were referenced. The strategies have been named after the most common and established name occurring in the literature. In the parenthesis after each strategy name or abbreviation are name variations that occur in the research literature. Strategies identified as being similar are placed together in the table, but other than that they are placed in no order.

Table 1. *Mental computation strategies for subtraction on two-digit natural numbers*

Strategy	Definition	Source
Auto [Automatic calculation]	Retrieve the answer automatically or from memory.	(Lucangeli et al, 2003)
Counting	$7-4=3$ because $7 \ldots 6,5,4,3$ $7-4=3$ because $7 \ldots 6,5,4$. It's 3. $7-4=3$ because $\underbrace{5,6,7}_{3}$ $39-24$: $29,19,\underline{15}$ $87-54=33$ because $\underbrace{64,74,84}_{30},\underbrace{85,86,87}_{3}$	(Thompson, 1999) (McIntosh & Dole, 2005)
Convert to addition	$14-8=6$ because 8 and what makes 14? $51-49=2$ because $49+?=51$	(McIntosh & Dole, 2005) (Hajra & Kofman, 2017)
Completion of the subtrahend	$73-36 \to 36+4=40 \to 40+30=70$ $\to 70+3=73 \to 4+30+3=37$ $75-43 \to 43+7=50 \to 50+25=75$ $\to 7+25=32$	(Karantzis, 2010) (Cooper et al, 1995)
Addition of tens until the number is surpassed	$73-46 \to 46+30=76 \to 76-3=73$ $\to 30-3=27$ $73-46 \to 73+3=76 \to 76-46=30 \to 30-3$ $=27$	(Van de Walle, 2007)
Short jump	Bridging the difference in subtraction problems in one or two steps instead of subtracting the second number from the first one. $65-59$: $65 \curvearrowright 59 = 6$	(Blöte et al, 2000)

	$65-59$: $59 \curvearrowright 60 \curvearrowright 65 = 1+5 = 6$ $65-59$: $65 \curvearrowright 60 \curvearrowright 59 = 5+1 = 6$	
Doubles and near doubles	$12-6=6$ because $2\times 6 = 12$. $13-6$: $6+6=12$, so it is 7.	(Thompson, 1999) (McIntosh & Dole, 2005)
Using tens as the unit	$80-50 = 8$ tens – 5 tens = 3 tens $= 30$	(McIntosh & Dole, 2005)
SA [standard algorithm done mentally]	Mental image of pen and paper algorithm, placing numbers under each other, as on paper, and carrying out the operation, right to left.	(Heirdsfield, 2001)
1010 [decomposition, regrouping, splitting, partitioning]	$58-26 \to 50-20 = 30 \to 8-6 = 2 \to 30+2 = 32$ $42-15 = (40-10)+(2-5) = 30+(-3) = 27$ $42-15 \to 40-10 = 30 \to 12-5 = 7 \to 20+7 = 27$ $42-15 \to 30-10 = 20 \to 12-5 = 7 \to 20+7 = 27$ $53-24 \to 50-20 = 30 \to 3-4 =$ "down 1" $\to 29$ $53-24 \to 20+20 = 40 \to 4+9 = 13 \to 29$	(Beishuizen, 1993) (Joung, 2018) (Beishuizen, 1993) (Cooper et al, 1995) (Heirdsfield et al, 1999)
u-1010 [1010 right to left]	$58-26 \to 8-6 = 2 \to 50-20 = 30 \to 30+2 = 32$ $42-15 \to 12-5 = 7 \to 30-10 = 20 \to 20+7 = 27$ $53-24 \to 4+9 = 13 \to 20+20 = 40 \to 29$	(Beishuizen, 1993) (Heirdsfield et al, 1999)
10s [1010 stepwise, cumulative sum]	$65-59 = \left((60-50)+5\right)-9 = (10+5)-9 = 15-9 = 6$ $42-15 \to 40-10 = 30 \to 30+2 = 32 \to 32-2-3 = 27$	(Blöte et al, 2000) (Beishuizen, 1993)
N10 [stringing, jumping, sequencing, cumulative]	$58-26 \to 58-20 = 38 \to 38-6 = 32$ $42-15 \to 42-10 = 32 \to 32-2-3 = 27$ $52-24 \to 24+20 = 44 \to 44+8 = 52 \to 28$	(Beishuizen, 1993) (Heirdsfield et al, 1999)
u-N10 [N10 right to left]	$52-24 \to 52-4 = 48 \to 48-20 = 28$ $52-24 \to 24+8 = 32 \to 32+20 = 52 \to 28$	(Heirdsfield et al, 1999)
N10C [stringing with compensation]	$65-59 = (65-60)+1 = 5+1 = 6$ $68-26 \to 70-26 = 44 \to 44-2 = 42$	(Blöte et al, 2000) (Karantzis, 2001)
Jumping	$87-39 \to 87-30 = 57 \to 57-7 = 50 \to 50-2 = 48$	(Van den Heuvel-Panhuizen & Drijvers, 2020)
A10 [adding-on, bridging through ten]	$65-39 = (65-5)-34 = 60-34 = 26$ $65-49 \to 65-5 = 60 \to 60-40 = 20 \to 20-4 = 16$ $65-49 \to 49+1 = 50 \to 50+10 = 60 \to 60+5 = 65 \to 1+10+5 = 16$	(Blöte et al, 2000) (Klein et al, 1998)
B [balancing, leveling]	$52-24 = (52+6)-(24+6) = 58-30 = 28$ $52-24 \to 22+28 = 50 \to 28$ $65-22 = 63-20 = 43$	(Heirdsfield et al, 1999) (Baranyai et al, 2019b)
Round one or both addends to multiple of ten, then adjust	$79-26 \to 80-30 = 50 \to 50-1+4 = 53$	(Reys et al, 1995)
Round to multiples of five	$79-26 \to 75-25 = 50 \to 50+4-1 = 53$	(Reys et al, 1995)
C10 [Formation of units of 10]	$43-7 = (43-3)-4$	(Lucangeli et al, 2003)

3. 5. Data collection

The questionnaire was administered as part of a mathematics lecture at the university. The PETs were not informed beforehand that they would take a questionnaire, so they had no way of preparing for it.

The PETs' participation in the questionnaire were voluntary, anonymous, and by checking in a box on the test they gave their consent or dissent to that their test results were used anonymously in research purposes. There was no time limit to the test. The PETs were instructed to calculate each exercise mentally, write down the answer, and then write an explanation on how they were thinking when they solved the exercise (see Figure 1). The author categorized their explanations according to which mental computation strategy in Table 1 they used (if any).

3. 6. Results

Instances where the PETs used strategies that deviated from the list of strategies in section 3.4 are here presented and commented on, thereby answering the research question in section 3.1.

3.6.1. Single-digit manipulation, SA, 1010, u-1010, and their overlap. One PET gave the following explanation to exercise 16 ($99-33$): "$9-3=6,\ \ 99-33=66$". A similar example is the following explanation to exercise 14 ($64-24$): "$6-2$ and $4-4$". It is not clear if these explanations should be categorized as SA, 1010, u-1010, or a strategy not included in the list that one could call "Single-digit manipulation". These four strategies are similar and hard to separate from each other. For example, one PET explained his or her calculation of exercise 4 ($92-68$) as "Thinks $12-8=4$ on the one place. $90-60=30$, but have exchanged, therefore it is 20. It becomes 24." In practice the PET does the steps in SA but without explicitly placing numbers vertically under one another. Another PET gave the following explanation to exercise 7 ($37-24$): "First tens, then ones. $3-2$, $7-4$." This is similar to 1010, but it could also be regarded as Single-digit manipulation, or SA done from left to right. A practical approach when categorizing is to group these four strategies and consider them as one strategy.

3.6.2. One of the numbers ends with a zero. When *one* of the numbers ends with a zero some strategy definitions overlap. For example, if using the 1010 strategy in exercise 6 ($54-30$), where the second number ends with a zero, it is natural to write an explanation such as "$50-30+4=24$" instead of the longer "$(50-30)+(4-0)=20+4=24$". One PET gave the following explanation to this exercise: "$50-30=20+4=24$". This could be the strategy 1010, but it could also be 10s, or *Round one or both addends to multiple of ten, then adjust.* Another example is exercise 2 ($70-34$), where instead the first number ends with a zero, that some PETs explained as "$70-30=40,\ 40-4=36$". It is not possible to say with certainty if this is 1010 or N10 since the strategy definitions overlap in this case. Although these explanations do not seem to be examples of new strategies there is a theoretical need to clarify the strategy definitions.

3.6.3. Both numbers ending with zeroes. A common PET strategy in exercise 10 ($80-30$) is to do the single-digit calculation $8-3=5$ and then add on a zero. This strategy is not described in the list but is similar to *Using tens as the unit*. However, the latter strategy there were no explicit instances of among the PETs' explanations. One could therefore make a case for adding a strategy "Single-digit manipulation and adding on zeroes" to the list of strategies.

3.6.4. One of the digits identical. In exercise 1 ($17-13$) and exercise 22 ($47-43$) the first digits are identical. A common PET explanation to these exercises is "$7-3=4$". One PET explained exercise 1 as "7 minus 3 is 4, therefore 17 minus 13 is also 4". These explanations could be categorized as *Auto* (Automatic calculation), a basic fact in the same way as $7-3=4$ can be, but they could also be categorized as a strategy having to do with changing the origin. One PET that explained exercise 1 as "$7-3=4$" also explained exercise 3 ($65-59$) as "$15-9$". It is possible to interpret this as the PET changed the origin to 10 in the first case, and to 50 in the second case. In the first example in section 3.6.7 one PET also changed the origin ("Sees 60 as 0."). "Change of origin" is a new strategy that could be added to Table 1.

In exercises 10 ($80-50$) and exercise 14 ($64-24$) the second digits are identical. The first case has already been considered separately in section 3.6.3. In the second case the PETs seem to observe later that the second digit is identical than when the first digit is identical, so they more often use some normal strategy such as 1010 in this case. However, there are examples of explanations such as the following for exercise 14 ($64-24$): "Disregards the last digit since it will cancel. $60-20=40$". One could consider it as a new strategy, that one could call "Cancelling digits".

3.6.5. Variants of A10 (bridging through ten). Exercise 4 ($92 - 68$) was explained by two PETs as "98 minus 68 is 30, therefore 92 minus 68 is six less" and "$92 - 4 = 88,\ 88 - 68 = 20,\ 20 + 4 = 24$". These two strategies are similar to A10, but instead of changing the second number the first number is changed so that it matches the other number. One could include these variants in the strategy A10 by adding more defining examples, but one could also consider them as strategies separate from A10. That also depends on if one wants A10 to be limited to bridging through the *closest* ten or not.

3.6.6. Jumping on the number line. One PET gave the following explanation to exercise 8 ($63 - 47$): "Jumps upwards on the number line from 47, 3 up to 50 and 13 further to 63 ($3 + 13 = 16$)". It is not clear if this is a case of *Completion of subtrahend*, *Convert to addition*, or *Short jump*, since these strategies are similar. (Note that since the PET gave an explanation involving jumping on the number line it is tempting to categorize this as *Jumping*, but the latter strategy does not involve converting to addition or explicitly jumping on the number line. *Jumping* is a confusing name for that strategy, since it is closer to N10 and A10 than jumping on the number line.) In any case, the explanation given by the PET is covered, although not unambiguously, by the list of strategies.

3.6.7. Negative numbers. Apart from 1010 none of the strategies in Table 1 explicitly involve negative numbers, but some PETs use negative numbers in their explanations. For example, consider the following explanation to exercise 3 ($65 - 59$): "$5 - (-1) = 6$. Sees 60 as 0." Another example is the following explanation to exercise 11 ($94 - 49$): "$4 - 9 = -5 \to 85 - 40 = 45$", which is similar to u-N10. It is possible to define a new strategy "Using negative numbers in the calculations". However, such a strategy would in practice only be used in combination with other strategies, for in the data there were no explanations of the type "$65 - 59 = 65 + (-59) = ..$". It is therefore probably better to include negative numbers only as part of defining examples to other strategies.

3.6.8. Triples. To exercise 16 ($99 - 33$) three different PETs gave the explanations: "$33 + 33 + 33 = 99,\ 33 + 33 = 66$", "Here I thought $\frac{1}{3}$ of 99", and "$33 \times 3 = 99$". One could categorize these as *Convert to addition*, or *Counting*, but they could also be categorized as a new strategy, that one could call "Triples", which would be similar to *Doubles and near doubles*. In that case one should probably also consider the strategies "Quadruples", "Quintuples", etc. However, these would be strategies not often used since they are useful only in a few cases. So, it is probably better not to consider them as new strategies but rather as variants of *Doubles and near doubles*, *Counting*, or *Convert to addition*. As a sidenote, many PETs explain *Doubles and near doubles* by talking in terms of halving rather than doubling. But since noting that $51 - 25$ is "almost half" or "almost double" is basically the same strategy it is natural to consider them both as examples of *Doubles and near doubles*.

3.6.9. Using results from previous calculations. The following explanation was given by a PET to exercise 21 ($98 - 49$): "I remembered exercise 19 and subtracted 10", where exercise 19 is $98 - 39$. It is questionable if the PET actually remembered the answer to exercise 19 or if the he or she looked it up. Nevertheless, remembering results from previous exercises can be categorized as the strategy *Auto* (automatic calculation or recall from memory). However, the definition of *Auto* is ambiguous and open to interpretation. For what is the difference between an automatic calculation and recall from memory? And when it comes to memory there can be both recall of basic facts such as that $7 - 4 = 3$ and recall of non-basic facts. An example of a non-basic fact could be that one knows instantly that $75 - 35 = 40$ since one has seen it many times before. In any event, there does not seemed to be an immediate need for defining a new strategy in addition to *Auto* having to do with remembering short-term results of previous calculations.

3.6.10. Strategy combinations. It is not uncommon for PETs to use combinations of strategies. For example, one PET explained exercise 4 ($92 - 68$) as "thought the ten place first $92 - 60$, thereafter $32 - 8$ ($30 - 6$ via ten)". This can be categorized as *Jumping*, but it could also be categorized as a combination of N10 and A10. Whether or not combinations of strategies should be considered as separate strategies is debatable since that would give many strategies. Then it is perhaps better to group similar strategies (as in Månsson (2022)) that are hard to separate anyway.

3.6.11. Unclear strategy use. In some cases, it is not possible to say with certainty which strategy a PET has used. For instance, in exercise 5 ($29 - 15$) one PET gave the explanation "$20 - 15 = 5,\ 9 +$

$5 = 14$". The explanation has elements of several strategies, such as 10s, N10, N10C, *Round one or both addends to multiple of ten, then adjust*, and *Round to multiples of five*. Other examples are given by the following explanations to exercise 12 ($51 - 25$): "$25 + 25 = 50,\ \ 51 - 25 = 25 + 1 = 26$" and to exercise 2 ($70 - 34$): "$70 - 34 = 36$ since $35 + 35 = 70$, one less gives one more". It is natural to categorize them as *Doubles and near doubles*, but they could also be categorized as *Completion of the subtrahend*, *Convert to addition*, or *Round to multiplies to five*. Other similar examples are given by the following two PETs' explanations to exercise 15 ($80 - 41$): "$80 - 40 = 40,\ \ 39 + 41 = 80$", and "$40 + 40$ is 80, therefore $41 + 39 = 80$". These explanations could be categorized as *Doubles and near doubles*, but also as B (Balancing), *Convert to addition*, or as a new strategy. At any rate, in all these examples the core idea of the strategies is at least partly covered by Table 1, so they are not completely new strategies.

4. Discussion

In section 3.6 were presented several examples of strategies that are not part of Table 1. "Single-digit manipulation", "Single-digit manipulation and adding on zeroes", and "Cancelling digits" are three examples that are commonly used by PETs, but which are not explicitly defined in the research literature. They are hard to distinguish from the strategies SA, 1010, and u-1010. The reason for that is both because they are principally similar, but also that the definitions of the latter three strategies are not precise enough to be able to separate between them and the three just mentioned strategies. It is therefore suggested to either add the three new strategies to Table 1, or to clarify the definitions of SA, 1010, and u-1010 so there is not a need to define them as new strategies.

Then there are the new strategies "Using negative numbers in the calculations" and "Variants of A10". The first one occurs usually in combination with other strategies, but there is a need to clarify the role of negative numbers in the strategy definitions in Table 1. There is also a need to clarify the definition of A10 so that one e.g., knows if it should be limited to the closest ten, if the defining examples of A10 should involve converting to addition, and if one is allowed to change both numbers.

"Change of origin" is a new strategy that could be added to the list of strategies. "Triples, Quadruples, Quintuples, etc." are also new strategies, but since they are not often used, and they are similar to *Doubles and near doubles* it is debatable if they should be considered as new strategies or as variants of *Doubles and near doubles*.

After adding the new strategies to Table 1, clarifying the definitions of some strategies, and possibly removing or merging some of them, one has a list of strategies that is in practicality capable of categorizing all of PETs' mental computation strategies. This constitutes a first step towards having a complete and common list of strategies in the research literature. That would make it is easier to compare results of different research papers, and the strategies would also be more precisely and unambiguously defined.

5. Conclusions

The result of this article is that the mental computation strategies that preservice elementary teachers (PETs) use in mental computation are for the most part covered in the research literature. However, in this paper we have seen several examples of strategies that are not covered or are incompletely covered in the research literature. These are described in detail in section 3.6 and 4, and by name they are "Single-digit manipulation", "Single-digit manipulation and adding on zeroes", "Cancelling digits", and "Change of origin". Then there are also "Using negative numbers in the calculations", "Variants of A10", and "Triples, Quadruples, Quintuples, etc." The later three are probably better to add as defining examples to already existing strategies instead of defining them as completely new strategies.

As discussed in section 4 some of the new strategies could be directly added to the list of strategies, but before doing that it is suggested that one redefine and clarify several of the strategies in the list, to make them precise and clearly distinguishable. However, the aim of this paper was not to do that, but only to find an exhaustive list of strategies. To define the strategies so that they become more precise, and less overlapping, are left for future papers. The problem of having similarly defined strategies can in practical situations be avoided by grouping similar strategies, but theoretically seen it is not a completely satisfactory situation to have overlapping and unclearly defined strategies. There is thus a theoretical need to clarify the definitions in Table 1 to make them more precise and less overlapping.

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