# Dynamic Responses of Padma Multipurpose Bridge Truss due to Moving Train Load

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**ABSTRACT**

Padma Multipurpose Bridge (PMB) is one of the most important projects in the history of Bangladesh due to its regional importance, economic benefit, and primary connectivity of one-third population of the country. The bridge is 6.15km long, connecting the ends of Mawa and Janjira in Bangladesh. The entire project is challenging to construct and complex in design as it contains both four-lane highways and train tracks supported by a double-deck composite warren truss. In this study, the dynamic response of the truss due to the moving train has been analyzed using the Moving Element Method (MEM). In this process, a separate finite element model has been developed using Finite Element (FE) program to convert the double deck truss into an equivalent beam. Analysis has been conducted for a series of different load cases, converging to the most realistic case where the actual train parameters are considered. Parametric studies have been carried out to determine the dynamic responses of the bridge with varying pier spacing and speed of the train. The most optimal solution has been discussed with the effect of the vibration of the train acting on the multi-purpose Padma bridge. The bridge's dynamic amplification factor (DAF) at a design speed of below 100km/hr is found 1.05. The parametric study shows that the critical train speed for the PMB is 1400km/hr resulting in the bridge resonance with a DAF of 18. It is also evident that with the increase of pier spacing the resonance of the bridge is expected to occur at a relatively lower speed.

## 1. INTRODUCTION

Ever since the 18th century, the use of the railway has been commonly adopted in many countries. This mode of transport has many advantages over conventional car usage and can be used for inter-city, inter-country, and even inter-continent traveling and may be regarded as a viable alternative to air travel. In developed countries like Europe and Japan (Loubinoux, 2012), HSR has been a popular form of transportation between states and countries. HSR not only shortens the time for inter-city traveling, but it also greatly reduces the use of conventional car usage, leading to a decrease in carbon footprint. This in turn, translates into significant energy savings and reduced demand for oil (Institute, 2018). The International Union of Railways (UIC) has stated that energy efficiency for high-speed rail is about four times compared to car and nearly nine times compared to air travel (Institute, 2018).

With a speed of more than 200 kilometers per hour, HSR may not be feasible for intra-city traveling in a city like Dhaka, though it could be a potential mode of transport to inter-cities (Dhaka and other Cities) and neighboring countries. The average speed of a Bangladeshi train is 60km/h, and the maximum speed is well below 100 km/h, which is much lower than the speed of high-speed rail. However, Bangladesh has been in collaboration with China for a high–speed rail to connect Southeast Asia. It is essential to carry out a detailed investigation, including the dynamic response of HSR to understand the impact of adopting such an expensive investment.

Fast-moving loads like high-speed train loads could cause excessive vibration on the supporting structural system during movement, which yields substantial dynamic stresses to the supporting columns (Cao *et al.*, 2018). The dynamic responses caused by the moving train may

amplify the structural responses of the support structures, which could cause discomfort to the passengers. Furthermore, the railway in many developed countries is discretely supported such as pier supported, where the deflections of the rail track between the supports are a point of interest. Therefore, train–track dynamics research should be evaluated for the different structural and loading conditions. Several studies are carried out for train-track dynamics assuming tracks to be continuously supported by subgrades. These include the work by C.G. Koh (Koh *et al.*, 2003) and Dai *et al.* (2018b), which presented an analytical solution for moving loads resting on foundations with different stiffness. Dai et al (2018b) evaluated that the motion of the discrete supports due to the moving train load may experience more numerical complications in the analysis procedure.

Bridge pier and superstructures are considered critical structural components of bridges and are always a point of interest to engineers, researchers, and other stakeholders. Previously, bridge piers were investigated by many researchers, particularly under seismic loading or innovative reinforcing materials (Farzana and Ahmed, 2020). Evaluating Performance-based damage states of the pier using a displacement-based design approach may illustrate the accurate pier responses and hence potential repair and retrofitting (Mahmud and Ahmed, 2020, Ahmed *et al.*, 2021) strategy after the seismic event (Farzana and Ahmed, 2022). Recently, a study on the seismic evaluation of the Padma Multi-purpose bridge pier shows that the seismic demand of the bridge pier is only 36% of its capacity for an earthquake return period of 2475 yrs (Ahmed & Moniruzzaman, 2022). However, the dynamic response of the padma bridge pier or bridge superstructures due to the seismic event or moving train has not been investigated yet.

The focus of this study is to investigate the dynamic response of the Padma Multipurpose Bridge, a mega lifeline project of Bangladesh. The bridge structure consists of a series of steel warren trusses supported by piers at every 150m. The main bridge has a total length of 6.15 km with two different levels, servicing the vehicles and cars on concrete decking at the top and railway system travel across the middle level (Bangladesh Bridge Authority, 2022). This structure can be considered rather complex for a composite truss bridge with an effective length of 150m between discrete supports with different level vehicle movements. The construction work of the bridge has already been completed and is already operational for vehicles. The design speed of the train is set to 80-100 Km/hr for the bridge, where the train in developed countries is operated in the range of 200-300 km/hr. At this stage, assessing the dynamic response of the Padma multi-purpose bridge at higher speeds is crucial.

This paper presents the results of investigating the dynamic response of the Multipurpose Padma bridge of Bangladesh using the moving element method. Here, a conventional railway track is incorporated into the bridge where the truss is simplified into an equivalent beam. The analysis focuses on the vertical displacement of the bridge despite being subjected to bending in both vertical and lateral directions. Parametric studies are conducted to understand the effect of spacing between discrete supports and the train speed on the dynamic displacement of the bridge. Although the truss system is designed for both motor vehicles and trains at two different bridge levels, MEM analyses are performed for the bridge solely for the moving train in view of the fact that the magnitudes of train loads are much higher than road vehicles.

## 2. METHODOLOGIES, ASSUMPTIONS, AND FORMULATIONS

The Moving Element Method (MEM) has been adopted to compute the dynamic response of the bridge under different parameters. For a more simplified analysis, the Euler–Bernoulli beam theory is adopted in view of the slenderness of the bridge girder. Furthermore, train–track interaction and wheel-rail contact models are not included. The computer program, SAP2000 has been utilized to carry out Finite Element Analysis (FEA). Composite warren trusses are simplified to an equivalent beam with Finite Element Method (FEM) by assuming coherent beam properties for the whole bridge. A MEM software code written in MATLAB is developed and employed to carry out the numerical analysis. The dynamic response of the bridge under different parameters has been further studied with the MEM formulation.

### A. Computational Method and Assumptions
A moving train with derived parameters is primarily modeled to travel at a constant speed of 100km/h. Track properties have been incorporated into the bridge without dampers and springs on the Moving Load. This is justifiable since the study's main purpose is to evaluate the dynamic response of the bridge truss under different situations.

In this study, the composite superstructure of the Warren Type Steel – Truss Girder is transformed into an equivalent beam with coherent properties. As there are two levels of service lines, namely, the motor vehicles and the railway viaduct, there may be challenges to incorporating the moving cars and trains simultaneously into the numerical solution. Therefore, only the moving train traveling on the railway viaduct is considered for analysis, as the speed and weight of the moving train govern the majority of the vehicle live load of the Multipurpose Padma Bridge. The train is idealized into four concentrated moving loads, simulating the wheel loads of the locomotive. The parameters for the train and structural conditions are insufficient to induce resonance effect to the bridge, hence having a nominal vibration effect for analysis. Nonetheless, non – operational parameters have been employed to investigate the dynamic response of the Multipurpose Padma Bridge.

### B. Truss as an Equivalent Beam
The truss is modeled as an equivalent beam with parameters derived from the existing truss geometry and subjecting the beam to moving loads. Figure 1 shows a truncated section of the Padma Bridge that has been analyzed to fit with the Moving Element Method. Two scenarios will be investigated; the case of numerical validation, where the adopted method is verified, and

parametric studies carried out with actual loading conditions. In the parametric analysis, results with different parameters will be discussed, converging into the critical contributing factor for the dynamic response of the bridge caused by the moving train. Excitation due to rail corrugation, wheel-rail contact model, and track irregularity factor are ignored in this study since it may create outliers for the research and increase complexity for the formulation. Track interactions and wheel-rail contact model are adopted from the previous research conducted by C.H. Lee et al (Lee *et al.*, 2006) and Sun et al. (Sun *et al.*, 2014).
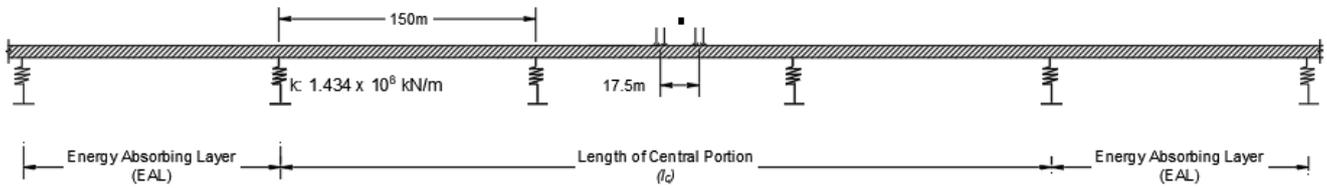


**Figure 1**: Equivalent beam model

### C. Moving Element Method

The moving Element Method has been widely adopted and applied to analyze highway bridges, and railroad structures with high-speed moving loads (Lin and Trethewey, 1989). The MEM is employed in this case study due to the following reasons:

i. Force and displacement vectors at every contact point can be avoided.

ii. It allows finite elements with unequal lengths to be formulated. This could be useful when the distances between the wheels have different dimensions.

iii. Infinite boundary conditions can be assumed as the boundary constantly moving at every step – simulate realistic train conditions where boundary conditions are ignored.

The MEM method considers the mass-spring model with two displacement degrees of freedom (DOF) – $u_1$ and $u_2$,

where the DOF of lumped ballast mass is ignored. In the formulation, each nodal point has two degrees of freedom (DOFs), and nodal points are equally spaced on the track model. Spacing between the nodal points depends on the element size and is based on the parameters of the moving train load. Nonetheless, a smaller element size requires more time steps which relate to a more accurate analysis. This study adopts a constant time step of 80 between two discrete supports. According to the Euler – Bernoulli Beam Theory (Erochko, 2020), displacement along ($x$ - the axis) is ignored. Therefore, $u_1$ refers to the translational deformation along the $z$-axis, and $u_2$ refers to the deformation along the $y$ – axis. In MEM, the $x$ coordinate of the train model is fixed in the longitudinal direction of the beam with an arbitrary origin. Therefore, the default input for the start of the analysis will be at $x = 0$ and $t = 0$ (Koh *et al.*, 2003). While the beam is moving towards the left, the point mass P will move from Node 1 (N1) to Node 80 (N80).
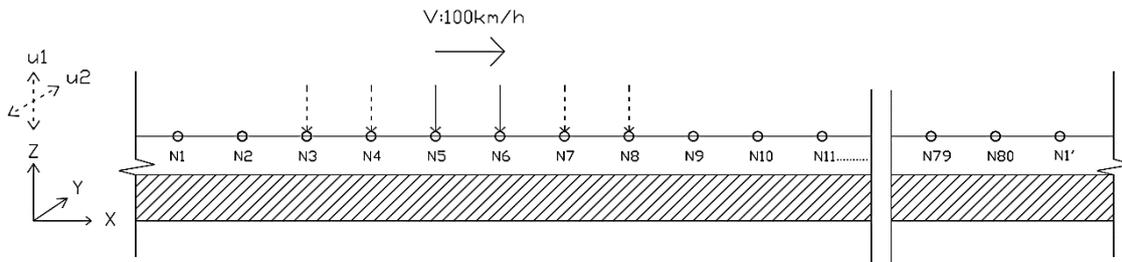


**Figure 2**: Moving Element Model Idealization

In this MEM formulation, parameters must be ascertained for train, tracking, supporting, and moving element parameters for the MEM domain. For the moving element parameters, these are the specifications that need to be inserted; domain and element size ($l_{domain}$ and $l$), the duration for each time step ($t$), number of time steps ($N_{ped}$) to travel from n1 to n1' and distance traveled ($l_{travel}$).

Number of time steps to travel between two same points, $N_{ped}$, is assumed. A sufficiently large value should be adopted to ensure more steps between two repetitive points. This confirms that most nodes' response is captured with accurate results. However, it should not be exceedingly large as the movement of the model is still controlled by the element size. Uniform element size, $l$, is assumed to be 2.5m as it corresponds with the spacing

between two wheels for the selected train. The ratio for the length of $l_{domain}$ must be kept constant throughout the whole study to ensure consistent dynamic responses of the moving load.

With the spacing between two piers ($l_s$), and the speed of the train (V), the duration of each time step ($t$) can be derived with Equation (1)

$$t = \frac{l_s}{N_{ped} \times V} \qquad (1)$$

Note that $l_s$ is different from $l_{travel}$, where $l_s$ is the spacing between two piers and $l_{travel}$ is the total distance set for P to travel (includes a series of $l_s$). The governing Equation for the vertical displacement of the rail beam can be idealized as shown in Equation (2).

$$EI_{zz}\left(\frac{\partial^4 z}{\partial x^4}\right) + \sum m\left(\frac{\partial^2 z}{\partial t^2}\right) + k(z) = P(t)\delta(x - Vt) \quad (2)$$

Here P is the magnitude of the moving load on the track, m is the mass of the self–weight of the beam, k represents the stiffness of the piers, and δ represents the Dirac Delta function of the train. Dirac Delta function is a function that denotes a point mass where the difference in integral is mapped to zero(Eftekhari). Note that z denotes the rail's displacement while u indicates the displacement of the point mass, which is the vehicle model (Koh *et al.*, 2003).

A moving ordinate, r, can be translated from the global ordinate system by generalizing the initial node and the subsequent node with the speed and time taken for the moving load to travel. Therefore, the Equation can be written as

$$r = x_0 - x_1 - Vt \quad (3)$$

where x is a global ordinate system. Incorporating equations (2) and (3), the vertical displacement of the beam can be converted into the moving ordinate system in the following Equation (Koh *et al.*, 2003),

$$EI_{zz}\left(\frac{\partial^4 z}{\partial r^4}\right) + \sum m\left[V^2\left(\frac{\partial^2 z}{\partial r^2}\right) - 2V\left(\frac{\partial^2 z}{\partial r \partial t}\right) + \left(\frac{\partial^2 z}{\partial t^2}\right)\right] + k(z)$$
$$= P(t)\delta(r + x_1) \quad (4)$$

where the Delta function (δ) is employed for tracking the locations of the vertical wheel loads with respect to the speed of train (V), bridge ($EI_{zz}$, M) and pier (k) properties. The function varies with P(t), which is the mass of the point load, and the speed of the moving particle, which can be referred to ($r + x_1$).

Therefore, the slope can be calculated based on Equation (2)

$$\emptyset = \frac{y}{\epsilon}, \emptyset = \frac{M}{EI} \quad (5)$$

Based on all the abovementioned assumptions, the Bernoulli – Euler Beam equation can be derived and used to find the design parameters as

$$\emptyset \approx \frac{d^2\Delta}{dx^2} = \frac{d\theta}{dx}; \frac{d^2\Delta}{dx^2} = \frac{M}{EI} \quad (6)$$

### D. Train Parameters

Train parameters are given and based on AASHTO LFRD specifications as shown in Figure 3 (Kim 2017, Dai *et al.*, 2018a). Since the primary consideration is to examine the dynamic response on the bridge under train loading, the stiffness and damping coefficient is assumed as 1. The train parameters and their values are presented in Table 1.
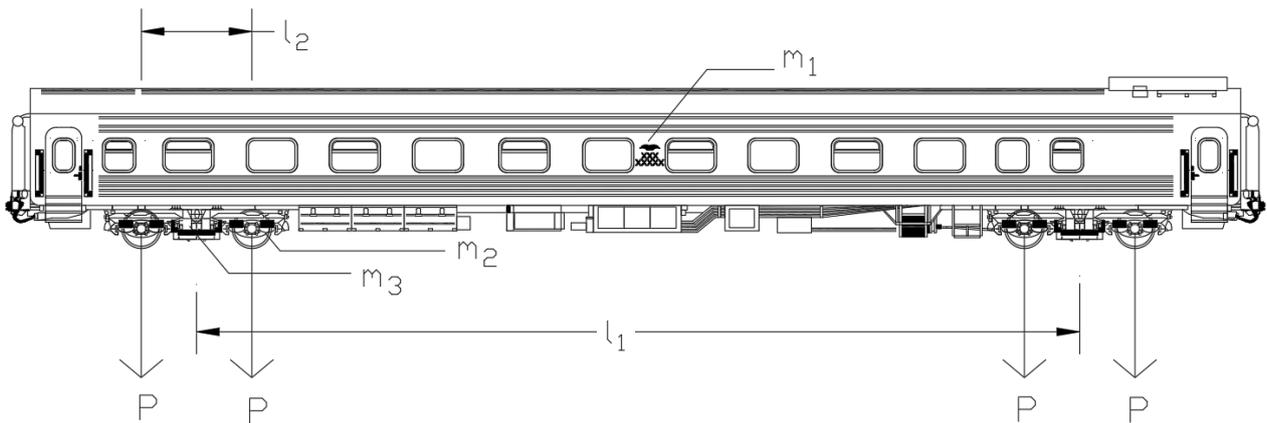


**Figure 3**: Train model of one locomotive

**Table 1**
Train Parameters (Dai *et al.*, 2018a)

| Description | Notation | Value | Unit |
|---|---|---|---|
| Mass of Carriage | $m_1$ | 26000 | kg |
| Mass of Wheelset | $m_2$ | 750 | kg |
| Mass of Bogie | $m_3$ | 2309 | kg |
| Distance between Bogie to Bogie | $L_1$ | 17.5 | m |
| Distance from Wheelset to Wheelset | $L_2$ | 2.5 | m |
| Wheel Load | P | 78.74 | kN |
| Primary Suspension Stiffness Coefficient* | $k_p$ | 1.87 | MN/m |
| Primary Suspension Damping Coefficient* | $c_p$ | 500 | kNs/m |
| Secondary Suspension Stiffness Coefficient* | $k_s$ | 1.78 | MN/m |
| Secondary Suspension Damping Coefficient* | $c_s$ | 196 | kNs/m |

### E. Conversion of Truss to Equivalent Beam for MEM

In order to accommodate the moving element method for the series of warren trusses of PMB, equivalent beam properties are determined from the actual truss properties. The entire bridge contains 6 identical modules, and each module includes six equally sized spans of 150m each. One similar module is modeled in SAP200 to understand their dynamic modal shapes. Subsequently, a single 150m span is also modeled to determine the stiffnesses in all three directions. Structural properties from the chords of the composite truss system have been extracted from the actual drawings to derive the equivalent beam properties. Figure 4a depicts an illustration of the composite truss superstructure, annotating the location of the chords. The element is pinned supported at the sides and roller supported at the other ends. Similarly, the finite element model for one module comprising six spans has been developed as shown in Figure 4(b). Modal analysis has

been conducted for the module to find the mode shapes that match the loading direction. Twelve different modes are analyzed to determine the critical mode for moving the train. Among them, mode eight is found vital to the train

loading as shown in Figure 4(c). In this mode, the vertical displacement Uz is more critical as most loads mainly act in the z – direction. Mode 8 shows the largest deflection about the z-axis out of the 12 different modal analyses.
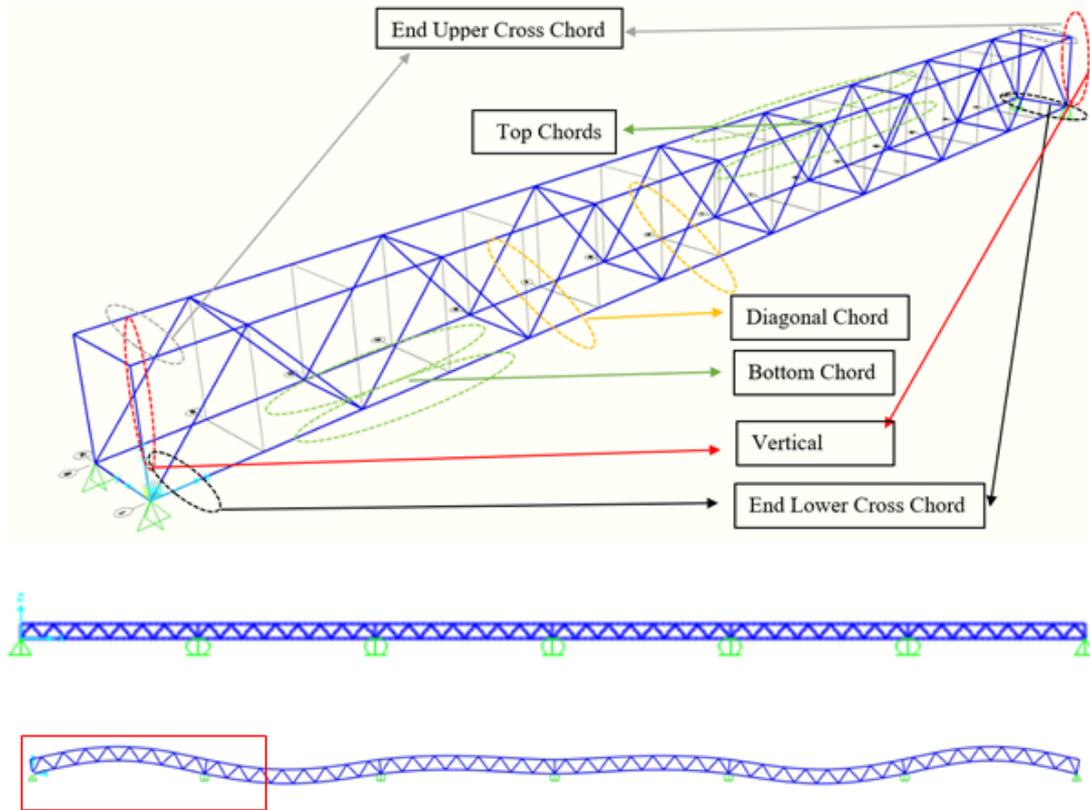


**Figure 4**: (a) Components of each module of the bridge, (b) One Repetitive Module of the whole bridge, and (c) Typical deflected shape of the module

Based on this, mode 8 is further used to elaborate the mass participation of the structure. After modeling in SAP2000, it was deduced that the total weight of a truncated element is 27776.5kN, which is 185.2kN/m for the whole bridge. Using the developed model axial stiffness has been determined and hence equivalent solid cross-sectional area has been evaluated. Similarly, stiffness in longitudinal and transverse directions is also determined through the required forces to yield unit deformation in the corresponding directions. The important stiffness parameter in the Z direction (along the loading direction) has been determined using the self-weight of the structure. The area and stiffness parameters of the equivalent beams are presented in the table.

**Table 2**
Equivalent beam parameters

| Description | Notation | Value | Unit |
|---|---|---|---|
| Length of Bridge | L | 6,150 | m |
| Length of Truncated Beam | $l_s$ | 150 | m |
| Mass | M | 185.2 | kN/m |
| Cross-Sectional Area | A | 1.22 | m² |
| Second Moment of Area About Minor (Y-Y) Axis | $I_{yy}$ | 750 | m⁴ |
| Second Moment of Area about Major (Z-Z) Axis | $I_{zz}$ | 37 | m⁴ |

The track has been incorporated into the equivalent beam properties with the rail properties in place. The damping of the rail is considered as 10% of the critical damping associated with the circular eigen frequencies of the equivalent beam with respect to the rigid-body mode. The formulation of MEM code assumes the wavelength of track irregularity is assumed as 1m; the amplitude of track irregularity is 1mm for the whole study. However, the train stiffness and spring are excluded in this study which may lead to a slight difference in the wheel-rail contact model.

In this analysis, the loading conditions of the moving train and the equivalent parameters from Table 2 are used in the simulation. For numerical validation, only two wheels are idealized, which is a representation of half of the locomotive. After a series of validation, these parameters are determined for the formulation. $N_{ped}$, the number of steps to travel the length of $l_s$, is assumed to be 80. Therefore, the duration of each time step (0.0675) can be determined from the predefined values for *V* and $l_s$. To prevent P from traveling incomplete domain, the size must be multiple of $l_s$. For accurate response prediction, at least five spans should be modeled; where each *EAL* and $l_c$ length should be at least 1:3. Domain size ($l_{domain}$) must be sufficiently large to ensure convergence of results. Therefore, these parameters are assumed for the domain; *EAL* as 450m, and $L_c$ as 1500m. With two *EALs* on each

end of $l_c$, the domain size will be 2400m. Two domains will be modeled to ensure stability for a transient response for a more accurate result. Therefore, the total distance for P to travel is 4800m. Each element size, $l$, will be modeled as 2.5m, as the distance from wheel to wheel is 2.5m; $l = l_1$. Element size refers to the length of travel for each time step. In conclusion, in each time step (0.0675), P will travel 2.5m.

## 3.   RESULTS & DISCUSSIONS

Using the Matlab program for the moving element method, results are extracted to observe the dynamic response of the Padma Multipurpose Bridge. The extracted results determine the vertical dynamic responses with different speeds as pier spacings and their mutual dependencies. Before presenting the dynamic response, MEM has been validated with the Finite element solution for the static response of the bridge. In the verification, maximum static deflections at the mid-span are also checked through the FEA of SAP2000.

### A.   Midspan Deflection using FEA

In the MEM formulation, it encompasses a series of FEM equations. Finite Element Analysis is adopted with the same model and loading conditions and is carried out using SAP2000. Five spans were modeled and P is loaded on the third span from the left, spaced 2.5m apart. Although it was indicated that *EAL* and $l_s$ are 450Nm and 1500m respectively, a much smaller domain is used for comparison analysis. Therefore, a domain size of 750m is simulated; where *EAL* is 150m and $l_c$ is 450m with an exact proportion of 1:3. The displacement curve over five spans is displayed in Figure 5, where the crests of the curve are observed at 240m, which is the second span, and 505m, which is the fourth span of the whole domain. This indicates that there is a slight hogging effect of 0.2mm at these two points. Comparatively, a significant increment of vertical displacement is observed at 375m, which matches the position of P.
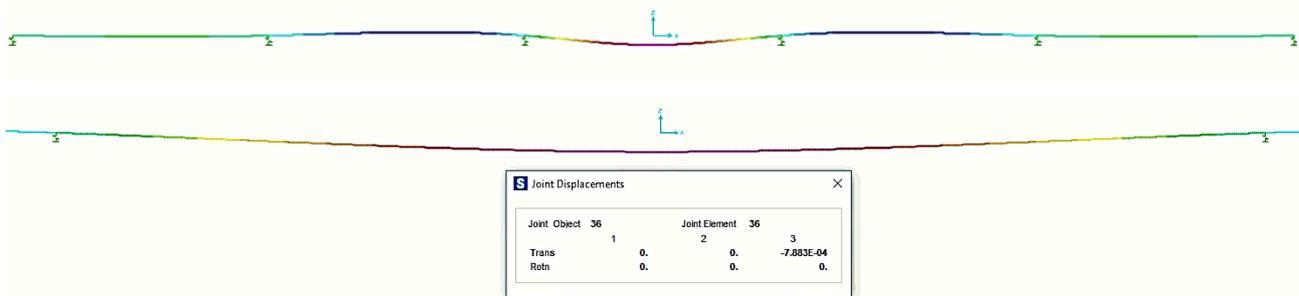


**Figure 5**: Deflection of the whole domain and Max Displacement at Mid-span using SAP2000

As the steady-state response of the rail beam is of interest, static loading is simulated in this analysis. Using the equivalent parameters, five spans are modeled where only the displacement under the loading at the middle span is considered. The observed maximum vertical displacement at the mid-span is 0.789mm.

The analytical governing Equation for vertical displacement can be written as

$$\delta_{zz} = \frac{Pb(3l_s^2 - 4a^2)}{48EI_{zz}} \qquad (7)$$

Since, P, two identical loads are placed symmetrically, the Equation for total displacement can be idealized as

$$\delta_{zz} = 2\left(\frac{Pb(3l_s^2 - 4a^2)}{48EI_{zz}}\right) \qquad (8)$$

The Equation evaluating the deflection for self–weight as the uniformly distributed on the equivalent beam is

$$\delta_{zz} = \frac{5wl_s^4}{384EI_{zz}} \qquad (9)$$

where E is the young modulus for steel, 200 GPa. As observed in Figure 5, the maximum vertical displacement for the moving load is 0.786mm at mid-span. This can be also directly calculated using Equation (7). Using Equation (8), the static displacement due to the self–weight of the equivalent beam can be calculated. The maximum vertical displacement due to dead load is 165mm. According to

Eurocode 7, the deflection limit of the given span is L/360 – the most conservative assumption for deflection limits.

$$\frac{L}{360} \rightarrow 420mm > 165mm \qquad (10)$$

Therefore, a maximum displacement of 165.78mm fulfills the serviceability of the given structure. However, the serviceability design of the Padma bridge may be over-designed as most of the parameters utilized are conservative.

### B.   Comparison between MEM and FEA

Comparatively, the MEM formulation gave a slightly larger displacement of 0.798mm while the FEM analysis led to a value of 0.784mm. As computed, the difference in results is only 1.2%. Moreover, the MEM formulation may produce more reliable results as the element model size and domain may be larger compared to the one defined in FEM.

### C.   Dynamic Response of the PMB using MEM

Since the study's main purpose is to investigate the dynamic response of the bridge, only the finite element method may not be efficient in producing the required results. It may be cumbersome to carry out parametric studies using FEM as the whole model needs to be remodeled in a larger domain. For example, the change of $l_s$ requires remodeling of the whole domain in SAP2000 as the spacing must be edited individually. Non-uniform discretization of track elements can be adopted in MEM.

Therefore, the adoption of the Moving Element Method results in a more efficient and accurate way of analysis.

As a basic outcome of the MEM, a repeated sinusoidal curve is observed throughout the response of the composite truss, where the peak of the curve relates to the position of P (directly on the supports); the crest of the curve is related to the position when P is at the mid-span of the equivalent beam. Realistically, the vertical displacement of the bridge girder above the piers where the axial rigidity is the largest as such this direction of the bridge should experience the least displacement, and the mid-span has the largest displacement. Therefore, this results in a sinusoidal curve with smooth connecting points. The profile has been compared to verify the accuracy of the dynamic response of a moving train as conducted by (Dai *et al.*, 2018b, Dai *et al.*, 2018a). The dynamic response patterns are quite close and similar to those studies.
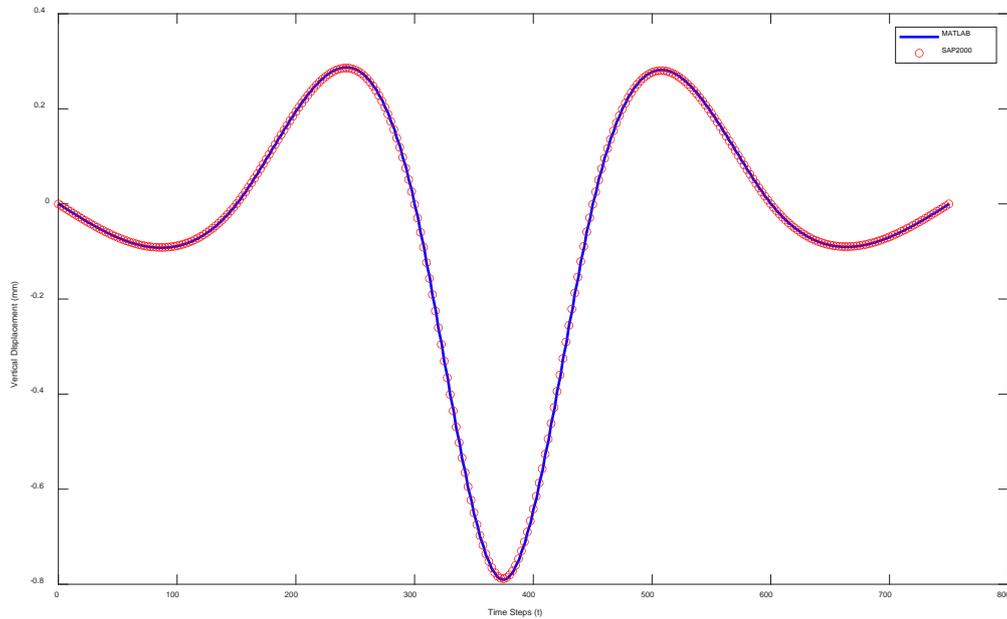


**Figure 6:** Static deflection curve using MEM and FEM
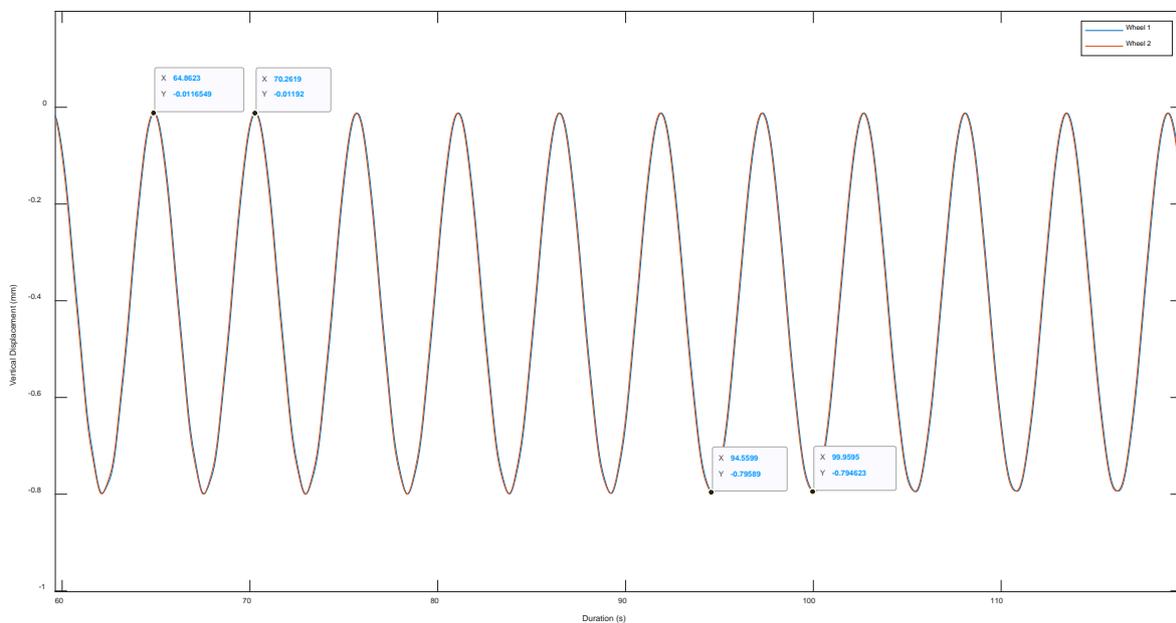


**Figure 7:** Dynamic response of the PMB (Time Steps)

Figure 7 demonstrates the dynamic response of the bridge with time steps. As the size of the time step is too insignificant compared to the domain size, it leads to the cramped illustration of the graph. In this figure, we observed that the transient response is in the first half of the analysis where the values fluctuate from 0.81mm to 0.793mm. It is important to model a larger domain to notice an accurate displacement due to the vibration of the bridge. With a larger domain modeled, the values began to converge to 0.795mm towards the end of the analysis where $l_{travel} = 2l_{domain}$. With a speed of 100km/h, the rail used 172.8 seconds to travel 4800m. Since the point of

interest of the study is the transient response due to moving load, subsequent analysis will utilize the results between 60 to 120 seconds, the period where the transient response of the sinusoidal curve has stabilized.

In order to compare the static and dynamic effects, static and dynamic loads have been plotted to do the comparison as shown in Figure 8. The maximum dynamic amplification factor (DAF) is ≈ 1 (1.05), where DAF denotes the ratio of maximum dynamic to static response on the Padma bridge(Dai *et al.*, 2018a). Therefore, it can be concluded that the vibration effect caused by the

moving load is not that dominant for the design rail speed of the Multipurpose Padma Bridge. In this plot, it can be observed that the difference between static and dynamic loading is about 0.1mm, where the DAF ≈ 1 (1.033). Despite increasing the effective length between pier supports, the static and dynamic response of the bridge is similar. An exaggerated effective length of 400m between supports is investigated and resulted in a DAF of 1. Therefore, the effective length of pier spacing is independent of the dynamic response at an effective length at $l_s = 150m$.
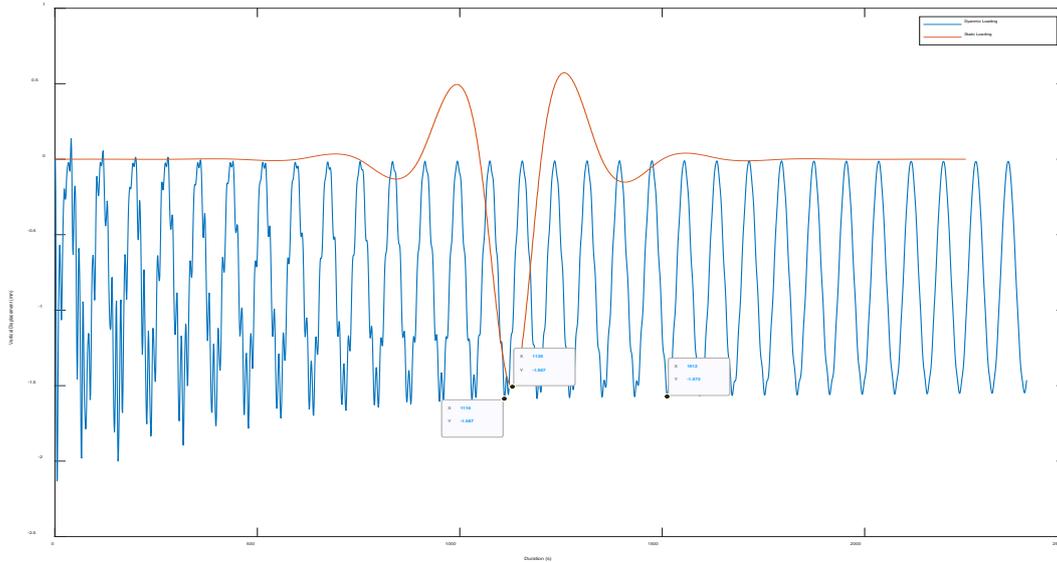


**Figure 8:** Comparison between dynamic and static loading
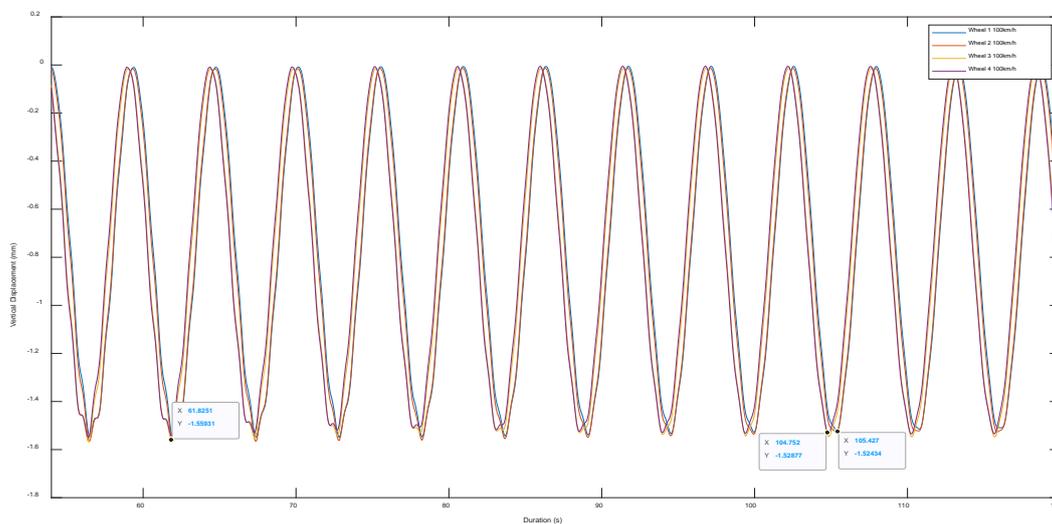


**Figure 9:** Dynamic displacement response of different wheels

Figure 9 depicts the dynamic response of the Padma Bridge due to loading conditions where the four moving loads are considered in one locomotive of the whole train model. It was assumed that the magnitude of the moving load is identical. It was clear that the dynamic response is quite stable. It is also observed that the vertical dynamic displacements are very close for all-wheel loads and their dynamic response overlaps each other at different time steps.

## 4. PARAMETRIC STUDIES

Parametric studies are conducted to investigate the speed and spacing dependencies of the dynamic response of the PMB. According to Dai et al(Dai *et al.*, 2018b), with the increase in speed, dynamic loading will result in a larger amplitude in the vertical displacement. The speed of the moving train also affects the frequency of the sinusoidal curve of the dynamic response. As the second moment of

area for the equivalent beam contributes to vertical displacement, the effective length between discrete supports will be investigated.

### A. Speed Dependencies

As the train is taken as a moving load, the effect of different speeds on the dynamic response of the bridge has been investigated. At Padma bridge, the train is expected to travel at a top speed of 80km/h. To be more conservative, the train is assumed to travel at a constant speed of 100km/h. Six different speeds; 100km/h, 150km/h, 200km/h, 250km/h and 300km/h and 350km/h are considered in this parametric study. Speed is investigated up to 350km/h, since the fastest high-speed rail (excluding maglev trains) in Asia has a record high of 317km/h-during normal operations.

In Figure 10, the dynamic response of moving load with different speeds has been plotted for comparison. Different regions of 40 to 100s are employed as the moving load with a faster speed does not have sufficient travel length to see a good comparison. A different domain size ratio may affect the displacement value. The amplitude of vertical displacement is consistent for three different speeds; the difference comes only as a fraction of a millimeter. Considering a massive structure like the Padma bridge, the difference can be neglected where $\left(\frac{\partial z}{\partial t}\right)_{V=100} = \left(\frac{\partial z}{\partial t}\right)_{V=300}$.
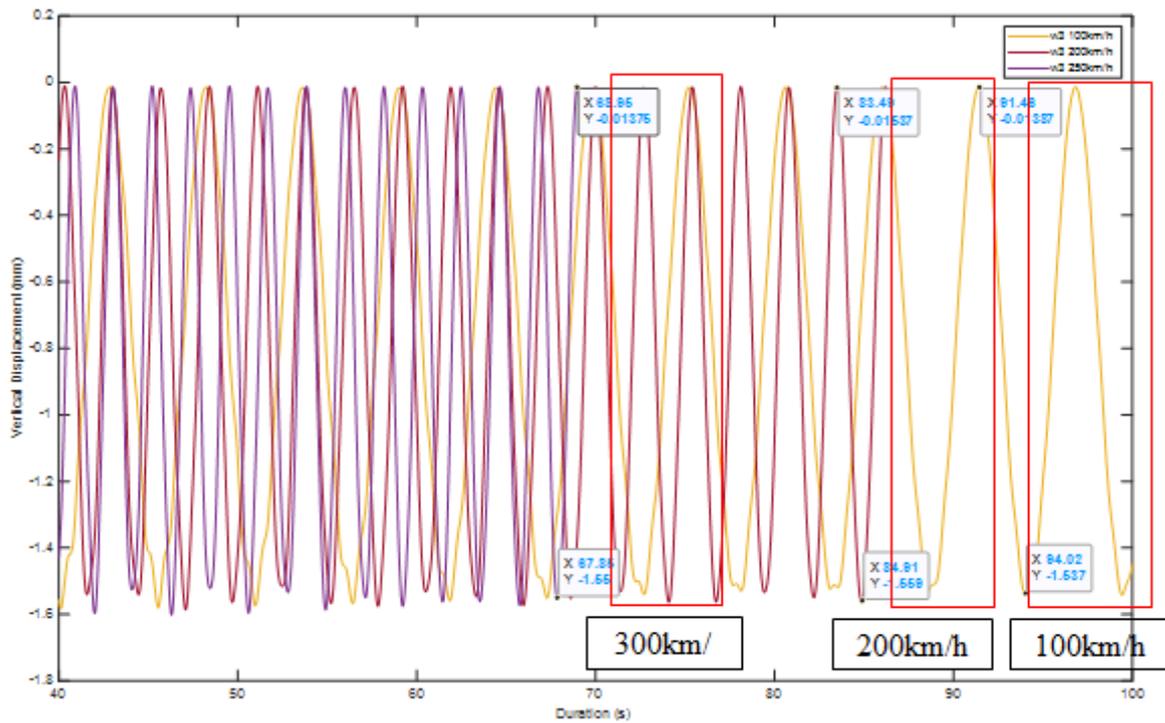


**Figure 10:** Dynamic response of moving load with varying speed ($l_s$ = 150m)
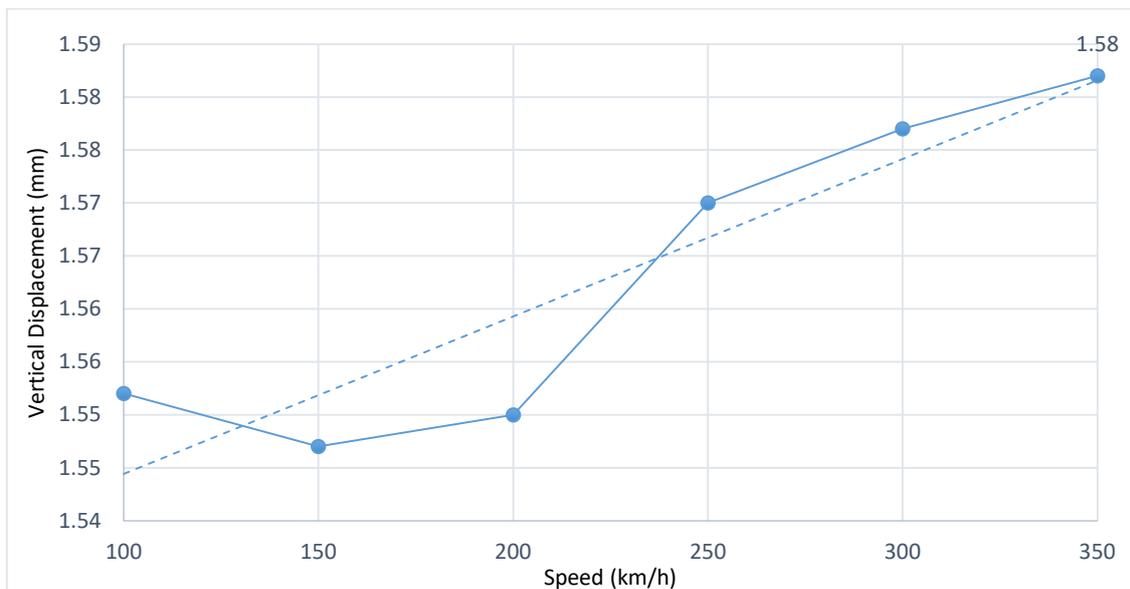


**Figure 11:** Relationship between speed and vertical displacement

The relationship between the speed of the moving load and the bridge's vertical displacement can be observed in Figure 11. In order to minimize observation error, an average of three different data points from the graph of every analysis has been used for comparison. An inconsistent plot is observed with a range of vertical displacement from 1.547mm to 1.583mm. The difference is about 0.04mm (2%), which can be interpreted as an observation error. According to Dai et al(Dai *et al.*, 2018b), moving loads can result in a higher amplitude of dynamic response where the amplitude of vertical displacement is amplified compared to static loading conditions. Hence, the speed of the moving load is insufficient to result in an exaggeration of the bridge's vertical displacement. Therefore, there are no dynamic amplifications of the displacement response of the bridge due to the increased speed of up to 350 km/hr.

Comparatively, the quasi-static analysis has also been carried out with the same loading conditions ($P_o$) to observe the difference between the dynamic and static load. Quasi – static analysis refers to a numerical solution that is based on constant speed and zero damping, simulating the same concentrated loading. The solution resulted in a vertical displacement of 1.513mm, which represents a slightly dynamic response of 0.07mm difference compared to the vertical displacement of 1.58mm when the moving load is at a speed of 350km/h. The response is nominal because the resonance of the movement is not excited enough due to the low speed of the moving load, leading to a small dynamic response. At the critical speed, the dynamic response of the bridge will be much amplified due to the source of excitation from resonant speeds(Dai *et al.*, 2018b). Nonetheless, the structural capacity of the equivalent beam is competent to resonate with the frequency of the moving load, where the dynamic response of the bridge is independent of the speed of the train.

The investigation has been extended to evaluate the critical speed of a moving load to stimulate the dynamic response where much amplified vertical displacements were observed. Pier spacings are kept constant at 150m. In Figure 12, the amplitude of vertical displacement is examined at different intervals of speed which are much faster. Although the speed employed for the load is not practical, the basis of adoption is to examine the resonant speed to excite the Padma bridge. The analysis depicts a clear increment in vertical displacement from 1200km/h, where the amplitude starts to increase from 3.43mm to the peak of 27.41mm at 1400km/h. The vertical displacement increases almost 18 times, while the initial vertical displacement is 1.55mm. The range of critical speed can be noticed between the range of 1400km/h to 1450km/h where the amplitude of vertical displacement declined from 27.41mm to 0.063mm, a much smaller displacement value compared to the initial speed of the moving load. This also translates to a DAF of 18, where the amplitude of the vertical displacement is amplified. Since the speed employed is not practical to be operational at any stage, still we can conclude that the dynamic response of the bridge is independent of the moving load's speed.

### B. Pier Spacing Dependencies
The dynamic response of a bridge with a different effective length between supports is also investigated using MEM. The moving load is imposed to travel at 100km/h where the pier spacing will be analyzed from 100m to 200m, at an interval of 10m. A range of 100m to 200m is utilized to keep the analysis within the structural limits of the Padma Bridge.
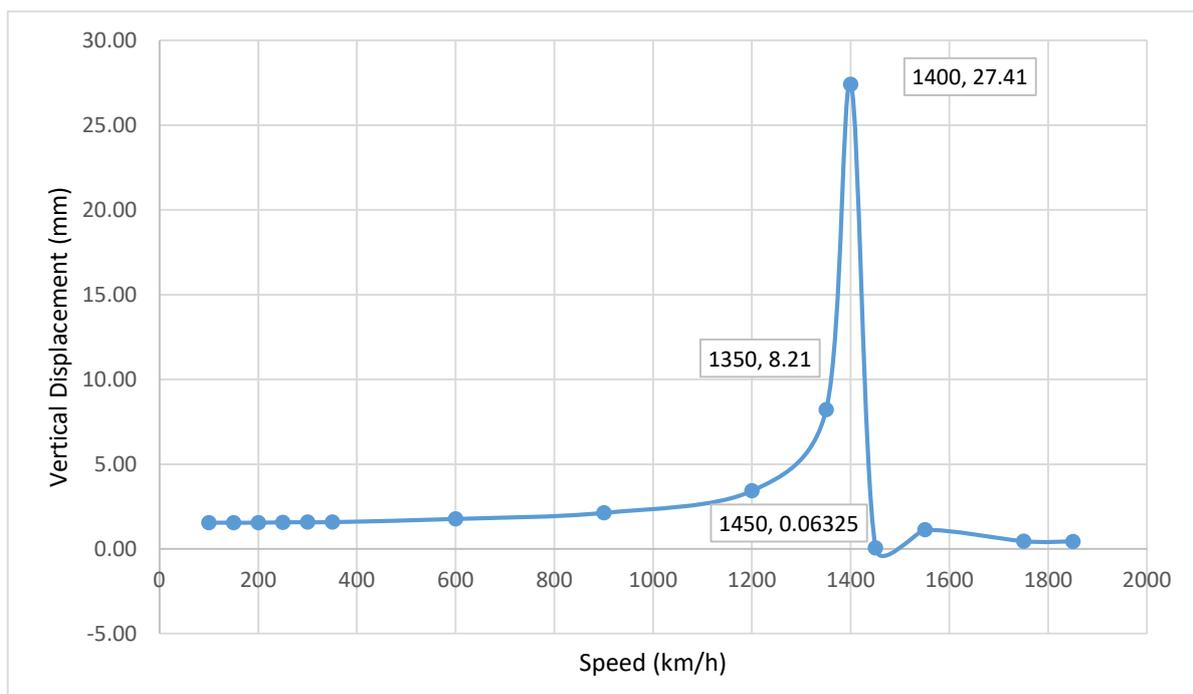


**Figure 12:** Relationship between speed (non-operational) and vertical displacement

**Figure 13:** Relationship between pier spacings and vertical displacement

Since it was discussed that the speed of the moving load is independent of the vibration effect, it was distinctively clear that the amplitude of vertical displacement increases linearly when the spacing of the pier supports the increase. However, the reason behind the increase is mainly due to the structural stiffness of the equivalent beam in the longitudinal direction. A longer effective length between pier supports allows for a higher vertical displacement, where the effective length between supports is denoted as $l_s$. Furthermore, there is four imposed loading on the bridge, which will contribute to a larger difference in deflection for the calculation. However, we are interested in the difference between dynamic and static displacement for the longest effective length of 200m as it shows the highest displacement value for dynamic loading. Despite having the same ratio for *EAL* and $l_c$ *of 1:3 and* $l_{travel} =$ $2l_{domain}$, the transient response of 200m may need a longer time to stabilize. It is observed from figure 14 that the vertical displacement of the bridge increases significantly when the effective length of the pier spacing increase. Accompanied by dynamic loading, the increment for the vertical displacement can be distinctively observed. Therefore, it can be concluded that the amplitude for vertical displacement increase with the pier spacing infinitely, where the type of loading (dynamic or static) remains independent to the Padma Bridge's vertical displacement.



**Figure 14:** Comparison between dynamic response with $l_s$ = 100m, 150m, 200m

### C. Correlation Between Pier Spacings and the Speed of Train

The correlation between different pier spacings and the speed of moving load has also been investigated. As observed earlier, the speed of the moving load is insufficient to induce resonance to the beam. Therefore, the speed of the moving load is independent of the dynamic response of the bridge. The independence of pier spacing is observed previously, where the effective length between piers increases with the amplitude of vertical displacement. In Figure 15, the speed of the moving load is kept constant at 100km/h, 200km/h, and 300km/h, where the vertical displacement is taken for different pier spacings.

It is observed from the relation that the moving load with different speeds has similar vertical displacement for up to 200m. The difference between the maximum and minimum vertical displacement can be neglected where it is less than 0.5mm. However, it is noticeable that the amplitude starts to diverge from 170m, where a higher displacement is observed for the moving load at 300km/h. Due to technical difficulties and excess exhaustion of software, the relationship is only plotted up to 200m where only a minor differentiation can be observed. However, it is believed that the model will continue to diverge and displacement for the moving load with a faster speed will subsequently govern. Since the speed of the train is kept constant, the model will not reach a resonant speed and the vertical displacement will increase infinitely if the pier spacings continue to increase without changing the flexural stiffness.

In Figure 16, the pier spacings are kept constant at 100m, 150, and 200m with varying speeds. It was discussed that the increment of pier spacings increases the vertical displacements infinitely until the structural capacity of the beam is challenged. Hence, results for three different spacings have varying displacement values. As the Equation of the line is computed for each series, it is observed that the gradient of the line increases when the pier spacings increase. A steeper gradient for the Equation translates to a higher increment of displacement values when the speed of the moving load increases. Thus, the resonant speed of the moving load should be faster for shorter pier spacings, as the increment of vertical displacement with a slower moving load is smaller. As the increment of vertical displacement is larger, it will take a shorter time to reach the resonant speed. Therefore, the dynamic response for an effective length of 200m is investigated with different speeds.

In Figure 17, the difference between the resonant speed of two different pier spacings can be clearly observed. The source of excitation from the moving load will induce sufficient resonance to the bridge for the displacement to decrease significantly after reaching the resonant speed. The analysis depicts a clear increment in vertical displacement from 1000km/h, where the amplitude starts to increase from 7mm to the peak of 23.5mm at 1200km/h. The vertical displacement increases almost six times, while the initial vertical displacement is 3.78mm. The range of critical speed can be noticed between the range of 1200km/h to 1300km/h where the amplitude of vertical displacement declined from 23.5mm to 0.44mm, a much smaller displacement value compared to the initial speed of the moving load. This also translates to a DAF of 6.2, where the vertical displacement amplitude is amplified.

The resonant speed for pier spacing of 200m is 1200km/h, which is comparatively slower than the resonant speed of 1400km/h for spacing of 150m. As discussed, longer pier spacing has a steeper gradient of increment for vertical displacement. Therefore, the maximum vertical displacement for a longer spacing will be employed at a slower speed. The study also evaluates that the resonant speed decreases when the effective length between discrete supports increases.
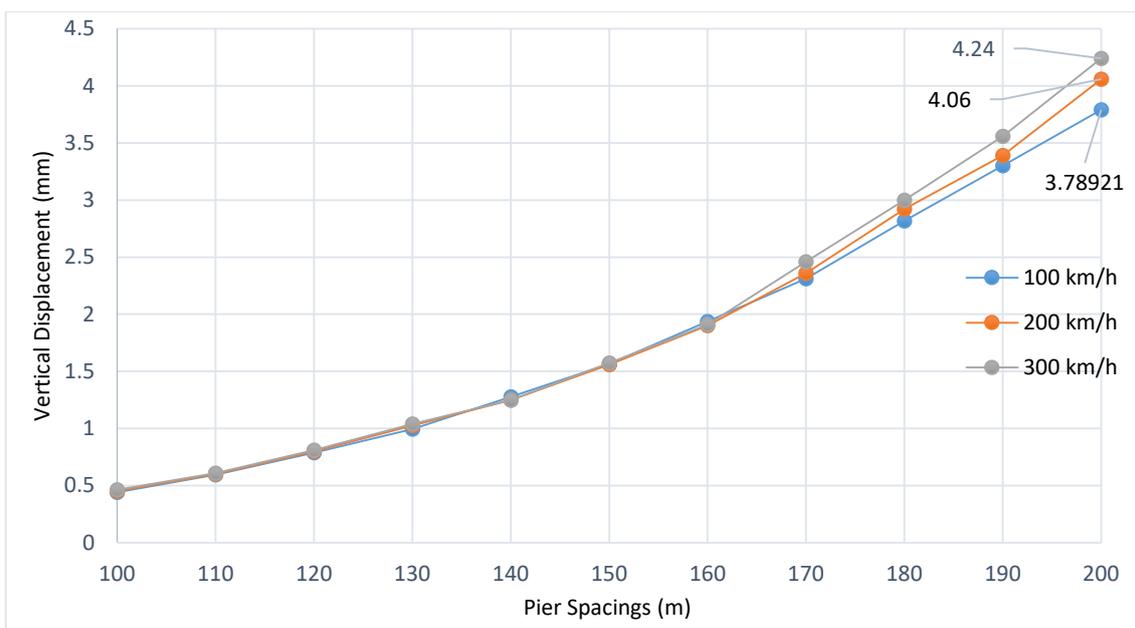


**Figure 15:** Relationship between pier spacings and vertical displacement for different speed
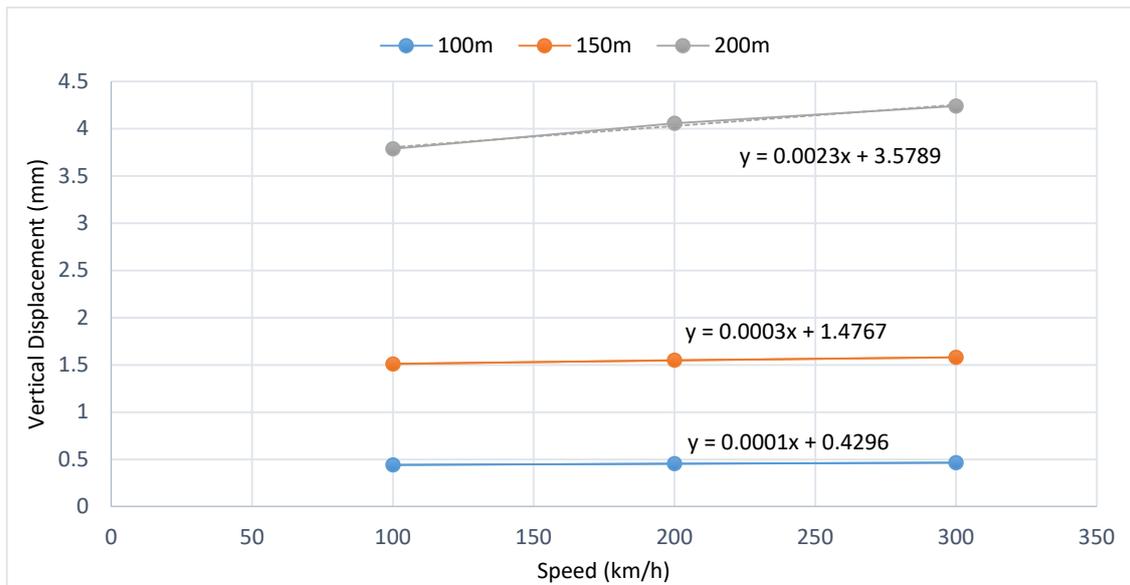
**Figure 16:** Relationship between speed of moving load and vertical displacement with different pier spacings
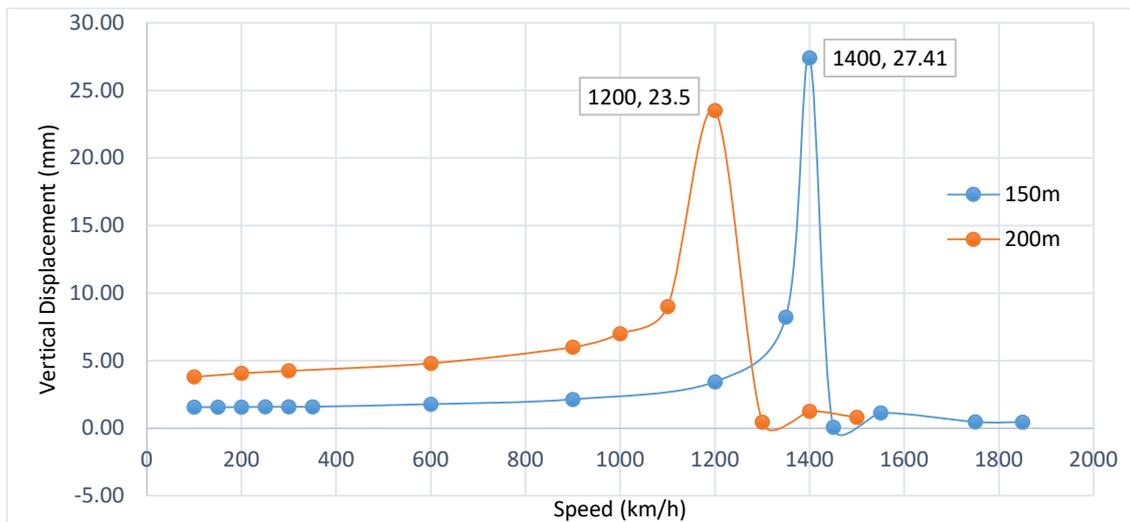


**Figure 17:** Comparison between the dynamic response of 150m and 200m

## 5.   CONCLUSIONS

In this study, the dynamic response of the Padma Multipurpose Bridge due to the loadings of a moving train is investigated using the moving element method. The accuracy of the computational method has been verified and compared with the Finite Element Method. The complexity of the structural bridge has been simplified into an equivalent beam, where the deformation at vertical and lateral directions are used to derive the  properties of the equivalent beam. The basis of simplification is mainly for the research of the vibration responses of the bridge under four concentrated moving loads, which is the representation of one moving locomotive. The effects of various factors such as train speed and pier spacings were investigated and the conclusions can be drawn as follows:

The dynamic amplification factor (DAF) of the bridge for the design train speed of 100km/hr is found to be 1.05. The speed of the moving train load has a very nominal effect on the DAF where the induction of resonance from the load is insufficient to trigger excitation for a substantial difference for dynamic response to be observed. The critical speed of the Moving Load that causes resonance in PMB is found to be approximately 1400km/h. The dynamic amplification factor is found to be 28 at this critical speed.

The effective length between piers is nearly independent of the DAF, where the vertical displacement for dynamic and static loading is nominal. However, the increase in the effective length of the piers contributes to a higher vertical displacement in the bridge. This is mainly due to the bridge's structural capacity or reduced stiffness (at higher spacings).

Although the critical speed for the moving load is not mobilized, the relationship between the pier spacings and the speed of the moving load is examined. The resonant

speed for the bridge decrease when the pier spacings increase. For example, the the pier spacing is increased from 150m to 200m, the critical result that causes resonance reduces to a relatively lower speed of 1200km/hr. The parametric study also shows that larger pier spacings lead to an increase in the vertical displacement with a non-linear pattern. In conclusion, a longer effective span length yields higher vertical displacement as well as a reduction in the critical speed that causes resonance to the dynamic response of the bridge.

In this numerical investigation, the study is limited to only moving train load for accounting the dynamic response of the PMB though the double deck truss is subjected to both vehicles and trains. Only standard and specific types of trains are considered for the analysis. The response may change with train type, load intensity, and the spacings. Overall, the bridge is expected to experience negligible dynamic response for any practical design speed of future trains.

## ACKNOWLEDGEMENTS

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