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Vehicle-to-Grid: Impacts on the electricity market and consumer cost of electric vehicles

Abstract:
Higher battery storage capacity in electric vehicles (EV) can potentially serve two purposes: First, the larger the capacity, the less need for inconvenient recharging during long trips. Second, the larger the capacity, the larger the potential gains from vehicle-to-grid (V2G) electricity supply during peak prices or during periods of imbalance. We present an analytical model for the intertwinement of the consumers' choice of battery capacity and the potential for supplying power to the electricity market. We show that V2G increases the consumers' choice of battery capacity, and it may reduce the cost of owning an EV vis-à-vis a traditional car. Furthermore, V2G alleviates the capacity pressure on peak hours, and thereby reduces the need for investment in backup power, saving social costs. Based on a future scenario for the Belgian electricity market, we provide a numerical illustration indicating that the savings may be substantial.

Keywords: Electric vehicles; Vehicle-to-grid; V2G; Electricity market

JEL classification: Q41; Q42; Q54; R42

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Sammendrag

Større batterikapasitet i elbiler kan gi flere gevinster for bileierne. For det første betyr større kapasitet at man har mindre behov for å lade i løpet av en lang biltur. For det andre kan økt batterikapasitet gi større inntekter av å stille ledig kapasitet tilgjengelig for tilbakeføring av strøm til nettet. Tilbakeføring av strøm kan gjennomføres ved bruk av såkalte toveis ladere (Vehicle-to-grid, V2G). V2G innebærer at elbileieren kan fylle opp batteriet med strøm når prisene er lave og selge overskuddsstrøm tilbake til nettet når prisene er høye. Elbilbatteriet kan også brukes til å dekke strømbehovet i eget hjem deler av døgnet. I denne artikkelen presenterer vi en analytisk modell for sammenhengen mellom valg av størrelsen på bilbatteriet og mulighetene for å selge strøm tilbake til nettet. Vi viser at V2G vil øke størrelsen på batterikapasiteten, og det kan redusere kostnadene ved å eie elbil. V2G vil også redusere presset på el-systemet i høylasttimer, og bidrar dermed til mindre behov for investeringer i kraftforsyningen, noe som igjen vil innebære lavere samfunnsøkonomiske kostnader. En numerisk illustrasjon, basert på et fremtidig scenario for det belgiske elektrisitetsmarkedet, viser at innsparingen kan bli betydelig.

1 Introduction

Electric vehicles (EV) are dependent on electricity supply from the grid, and they will increase total electricity demand. On the other hand, since an EV with a large battery only needs charging occasionally, the EV may also store electricity in order to smooth out daily variability in demand and supply of electricity. Moreover, the EV can also help to balance the grid by supplying quick power when there is a local imbalance.

To our knowledge, this is the first economics paper to model the interlinkages between the consumers' choice of battery capacities and the power market, where the optimal battery capacities and power production capacities are endogenously determined.

First, we derive a model for the EV users' choice of battery capacity. The battery capacity serves two purposes. The larger the capacity, the smaller the need for inconvenient recharging during long trips. Furthermore, the larger the storage capacity, the larger the potential gains from vehicle-to-grid (V2G) flows during peak prices. We show that consumers' optimal choice of battery capacity increases with the price difference between peak and off-peak prices and in the number of long trips per year.

Second, we integrate our model of EV users' choice of battery capacity with a simple model of the electricity market. We show how consumers' optimal battery capacity choices affect the equilibrium electricity prices during peak and off-peak hours, and the optimal investments in power plants. One finding is that viable V2G solutions increase welfare as the need for investment in backup power capacity decreases.

We consider two qualitatively different equilibriums in the electricity market; one where the peak and off-peak prices will remain constant, and one where the prices are affected by the introduction of EVs and the V2G option. In the first case, the car owners receive all the benefits from the V2G option. In the latter case, the V2G option benefits both the car owners and the other consumers of electricity. The peak price decreases and the off-peak price increases, which means exchanging lower-value KWh by higher-value KWh in the peak period.

Finally, given that a V2G system exists, a sufficient condition to decentralize the social optimum is to have peak load pricing in the electricity sector, and marginal cost pricing of fast charging along roads.

In the following, we first discuss some of the relevant literature (Section 2). Thereafter we derive the private demand function for battery capacities (Section 3). We find the consumers' optimal choice of capacity and show how the V2G technology decreases the consumer cost of EVs, and thereby may contribute to a larger rollout of EVs. In the next section, we introduce a simple model of the electricity market (Section 4), which we use to discuss the social value of V2G. Next, we discuss social versus private optimum (Section 5) and discuss how V2G affects the equilibrium prices depending on the characteristics of the electricity market (Section 6). We present a simple numerical illustration of our model in Section 0, and give some remarks on the EVs potential balancing services in Section 8. Concluding remarks are given in the last section.

2 Literature review

Electrification of the car fleet contributes to reduced greenhouse gas emissions, as well as local pollution, and is promoted by a variety of policy instruments in several countries (Holtsmark and Skonhoft, 2014; Lieven, 2015; Kemfert, 2016, Zhang et al., 2016; Li et al., 2017). In the present paper, we focus on the EV's capacities for storing electricity and the implications for the power market, and do not explicitly model the rollout of EVs. There are several studies pointing to the need for energy storage and flexible demand in a future energy system with a large share of intermittent energy sources, like wind and solar, see Newbery (2018). EV batteries, through V2G options, can thus play an important role in achieving a low-carbon electricity market (ELIA, 2017; Erbach, 2016). In the present paper, we focus on variations in demand between peak and off-peak hours (see Williamson, 1966), and do not explicitly address the price effects of fluctuations in intermittent power supply. We, however, expect that our main results carry over to the case with a large share of intermittent power supply.

There is an extensive technical literature investigating the impact on the electricity system of the bidirectional power flows from EVs. EVs can contribute to the power market by supplying peak power as well as quick-response services required to maintain grid stability and security (ancillary services), like spinning reserves and regulation services (see, inter alia, Kempton and Tomić, 2005 and Mwasilu et al., 2014). Clement-Nyns et al. (2011) investigate how EVs can support the distribution grid in terms of voltage control and congestion management. Lund and Kempton (2008) and Loisel et al. (2014) analyze how EVs can improve the ability to integrate intermittent wind power.

Our view of the technical literature is that there is significant potential for financial return when the vehicle-to-grid (V2G) technology is used for frequency regulations, but less so for peak shavings (Freeman et al., 2017 and White and Zhang, 2011). The latter results follow from the battery degradation under frequent charging and discharging.

However, battery degradation following from V2G operations has been subject to debate. Dubarry et al. (2017) argued that the additional cycling to discharge EV batteries to the power grid is detrimental to battery performance, whereas Uddin et al. (2017) presented a simulation study claiming that V2G can extend the life of Lithium-ion batteries in EVs. In Uddin et al. (2018) authors of both papers discussed these contradictory results. A main conclusion in Uddin et al. (2018) is that V2G can be economically viable, but demands a smart control algorithm, which takes into consideration the impact of V2G cycling on the battery life. Although not yet commercially available to private households, there are several ongoing innovation and trial projects aimed at making V2G chargers commercially available.[1] Nissan LEAF has recently been approved to feed into the grid in Germany.[2]

The EVs contribution to V2G services will depend on the driving patterns as the vehicles have to be connected to the grid to charge and discharge the batteries. Sioshansi and Denholm (2010) analyzed the value of plug-in hybrid electric vehicles (PHEVs) as grid resources for the Texas electric power system and found that the EVs could benefit the electricity system by providing ancillary services such as frequency control. They based the driving patterns upon a household travel survey, but all the PHEVs were assumed to have the same storage capacity. PHEVs have a far smaller battery capacity than EVs. In the present paper, we make the consumers' choice of battery capacity endogenous, and take into account that EVs can have a larger impact on electricity markets through their larger battery capacity.

3 Demand for EV Battery Capacity

In order to concentrate on the essentials, we opt for a simple model in which both the stock of EVs and the driving pattern of consumers are given. We assume that there is a peak and an off-peak period in the electricity market each day. The EV battery can help to store power for the grid by buying off-peak and using or selling it in the peak period.

There are N EV owners, and they are denoted by $i = 1,\ldots,N$. Every day of the year, an EV owner makes either a short trip with distance d_s or a long trip with distance d_l. Furthermore, we let x_s represent the number of short trip days per year, and x_l the number of long trip days per year. The owners differ with respect to how many long trips they make per year. For each owner, we assume that the number of long

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[1] See, e.g., http://parker-project.com/ and https://www.ovoenergy.com/electric-cars/vehicle-to-grid-charger

[2] . https://www.reuters.com/article/autos-electricity-germany/nissan-leaf-approved-for-vehicle-to-grid-use-in-germany-idUSL8N1X32XW. 17.11.2018

trips is uniformly distributed on the interval $[\underline{x}, \overline{x}]$.[3] Since we must have $x_s + x_l = 365$, we can use the number of long trips as the central variable to differentiate the EV owners. Moreover, we express the distances d_s and d_l in KWh needed for a trip in a standardized EV.

The EV batteries can be charged either at home, or at the destination of the trip, or on the route during a long trip. On short trip days, EV owners reserve their charging to hours when the price of electricity is low, for instance, at home at times when there is plenty of capacity available (say coal power or nuclear at night). We assume that for short trips, the battery is always fully loaded and the power not needed for the small trip is available to supply the grid. Thus, with a battery size B_i, $(B_i - d_s)$ is available to supply to the grid on short trip days.

For the days when the consumer makes a long trip, the consumer has to fast charge along the route. The EV owners do not prefer this kind of charging as fast charging is more costly due to the need for separate stations with high KW effects. Moreover, fast charging takes time, which could have been spent driving. Consider now the private costs for EV owner i of operating her vehicle during a year when she makes x_l^i trips per year. The total cost equals:

$$c(B_i) = 365 p_e^o B_i - p_e^p (B_i - d_s)(365 - x_l^i) + [p_{ch}(d_l - B_i) + v(d_l - B_i)^2] x_l^i + a B_i \qquad (1)$$

The first term represents the cost of charging the battery fully in the off-peak period (at price p_e^o). The EV-owner will charge the battery fully every day. The next term is the income from supplying the grid: On all short trip days, she will sell the surplus to the grid at peak prices p_e^p.[4]

The third component is the cost associated to charging en route, which consists of a monetary part and a disutility part. The monetary part is the cost of buying the additional power en route $(d_l - B_i)$ at the price p_{ch}. The disutility part is a quadratic function of the power needed en route where the parameter v scales this time cost.

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[3] Even if the number of long trip days per year is an integer variable, we employ, as an approximation, the continuous uniform distribution since this is more convenient from an analytical point of view as it allows derivation.

[4] We disregard any conversion loss in the theoretical part of the paper as it has no impact on our results. In the numerical model we let $\eta(B_i - d_s)$ be the electricity available for supplying the grid (where $\eta < 1$ is the conversion factor). We also assume that the cost of the charging/de-charging facility at home or at the office can be neglected.

The fourth component represents the annualized cost of battery capacity. We use a cost that is proportional to the size of the battery, which is in line with current supply options offered for EVs.

The total cost of operating an EV with access to V2G technology can be compared to the cost of operating an EV if V2G solutions are not available, $c^0(B_i)$. When comparing situations with and without V2G solutions, we consider the case where the peak and off-peak prices are not affected by whether V2G is available or not.[5]

If V2G solutions are not available, the consumer will only fully charge her vehicle if she is going on a long trip. The total cost per year of an EV is then given by:

$$c^0(B_i) = (365 - x_l^i)p_e^o d_s + [p_e^o B_i + p_{ch}(d_l - B_i) + v(d_l - B_i)^2]x_l^i + aB_i \tag{2}$$

where the first term is the charging for the short trip days, the second term is the total cost of a long trip, and the third term is the battery related costs.

3.1 Choice of battery capacity by the EV owner

The EV owner chooses a battery capacity B_i that minimizes her costs. For both the case with V2G and the case without V2G we assume $d_s < B_i < d_l$. Indeed, a battery size that is smaller than what is needed for a short trip is rather unlikely as it implies a large disutility from charging en route on every trip the consumer makes. On the other hand, a battery size larger than what is needed for a long trip is also rather unlikely as its only purpose would be to store electricity. For this storage function, there likely exist cheaper stationary options.

When we rule out corner solutions, we can use the first order conditions to find the battery capacities that minimize (1) and (2):

$$B_i^* = d_l - \frac{a-(p_{ch}-p_e^o)x_l^i}{2vx_l^i} + \frac{(p_e^p-p_e^o)x_s^i}{2vx_l^i} \tag{3}$$

$$B_i^0 = d_l - \frac{a-(p_{ch}-p_e^o)x_l^i}{2vx_l^i} \tag{4}$$

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[5] This will be the case if the electricity market is characterized by constrained capacity only in peak hours and constant marginal cost of backup power; see Section 0.

It is easy to see that $B_i^* > B_i^0$. Thus, not surprisingly, independently of the number of long trips per year, the EV owners will choose a higher battery capacity if V2G is available, as it is profitable to use the storage capacity during days with short trips.

Finally, due to the disutility from waiting while the car is charged on long trips we find that both battery capacities are increasing in the number of long trips x_l^i:[6]

$$\frac{\delta B_i^*}{\delta x_l^i} = \frac{a}{2v(x_l^i)^2} - \frac{365(p_e^p - p_e^o)}{2v(x_l^i)^2} > 0 \tag{5}$$

$$\frac{\delta B_i^0}{\delta x_l^i} = \frac{av}{2(vx_l^i)^2} > 0 \tag{6}$$

From (5) and (6) we note that the difference in battery size between the "No-V2G" and "V2G" is smaller, the higher the number of long trips per year, e.g., $\frac{\delta B_i^*}{\delta x_l^i} < \frac{\delta B_i^0}{\delta x_l^i}$.

Consumers with a low number of long trips per year have more to earn on V2G. Furthermore, consumers with a high number of long trips per year invest in battery capacity to save en route charging costs and on time spent charging en route independently of whether V2G is an option or not.

The expressions for the optimal battery sizes (3) and (4) can be inserted into the cost functions (1) and (2), respectively, to yield minimized EV costs with and without V2G. We have that the cost of an EV is always lower with V2G than without V2G:[7]

$$c^0(B_i^0)\text{-}\ c(B_i^*) = \left[d_l - d_s + \frac{p_{ch} - p_o}{2v} - \frac{a}{2vx_l^i}\right]\left(p_p - p_o\right)x_s^i > 0 \tag{7}$$

Since $d_l - d_s + \frac{p_{ch} - p_o}{2v} - \frac{a}{2vx_l^i} > 0$ when we assume $d_s < B_i < d_l$.

Assuming that consumers compare the cost of an EV with the cost of a gasoline car and will choose the car type with the lowest cost, we can conjecture that V2G will lead to a higher number of EVs.

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[6] Note that we have assumed that the battery capacity is smaller than the capacity needed for long trips, (d_l), which implies that $a > (p_e^p - p_e^o)365$.

[7] See the Appendix A1 for the derivation.

3.2. Total battery capacity

In order to investigate how V2G may affect the electricity market, we need to derive the total battery capacity in the EV fleet. The total capacity is given by:

$$\Gamma = \frac{N}{\overline{x}-\underline{x}} \int_{\underline{x}}^{\overline{x}} B(s)ds \tag{8}$$

Remember that consumers' preferences for long trips are uniformly distributed on $(\overline{x} - \underline{x})$. The term outside the integral sign is thus the density of consumers at each point along this interval, while the integral adds the choice of each type of consumer. Inserting for B(s), the consumer optimal battery capacities (3) and (4) in the cases with and without V2G, we obtain:

$$\Gamma^{*} = N\left[d_l - \frac{a-365(p_e^p - p_e^o)}{2v(\overline{x}-\underline{x})} ln\left(\frac{\overline{x}}{\underline{x}}\right) + \frac{p_{ch}-p_e^p}{2v}\right] \tag{9}$$

$$\Gamma^{0} = N\left[d_l - \frac{a}{2v(\overline{x}-\underline{x})} ln\left(\frac{\overline{x}}{\underline{x}}\right) + \frac{p_{ch}-p_e^o}{2v}\right] \tag{10}$$

For the difference in battery capacities, we have:

$$\Gamma^{*} - \Gamma^{0} = \frac{N(p_e^p - p_e^o)}{2v}\left[\frac{365}{(\overline{x}-\underline{x})} ln\left(\frac{\overline{x}}{\underline{x}}\right) - 1\right] > 0 \tag{11}$$

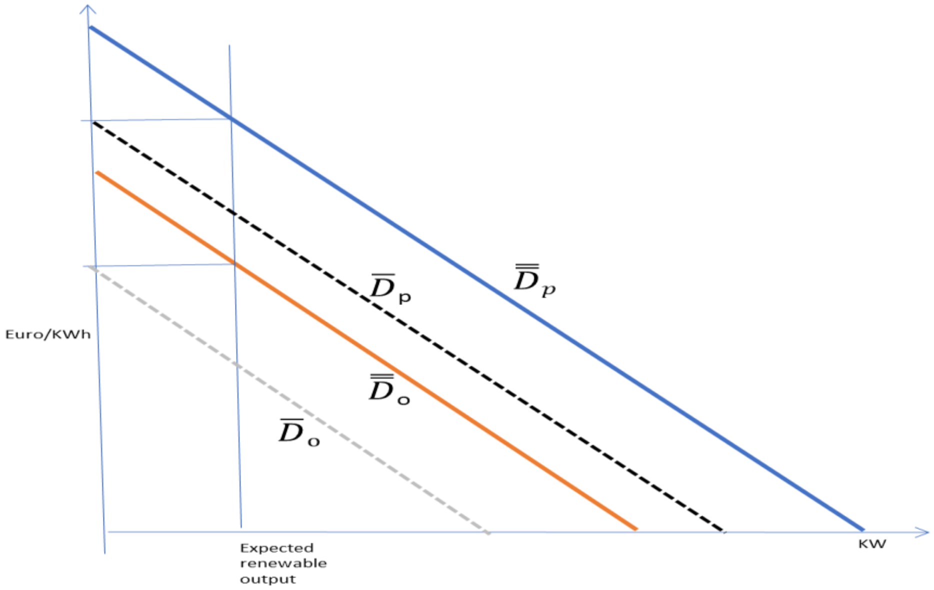
We note that the difference in total available EV battery capacity in the cases with and without V2G is increasing in the price difference between peak and off-peak periods, and is decreasing in the level of disutility from en route charging.

4 V2G and the electricity market

We assume that there are two demand periods within a day. Peak demand periods could be evening hours cooking, heating and washing at home, while off-peak hours could be the night hours.

Electricity supply consists of renewables and conventional power like natural gas, nuclear and coal. As the output of renewable electricity is intermittent and as storage of electricity remains costly, there is a need for additional backup power, see e.g. Ambec and Crampes (2012). We assume that natural gas functions as the backup power technology. To simplify our model, we further assume that the level of renewable output is uncorrelated with peak and off-peak demand.[8] We can then derive the expected residual demand functions, $\overline{D_p}$ and $\overline{D_o}$, for a representative day from the expected renewable supply as shown in Figure 1 below.

Figure 1: Electricity market representation



The solid lines, $\overline{\overline{D_p}}$ and $\overline{\overline{D_o}}$, represent electricity demand from all other uses than charging of EVs in the peak and off-peak period, respectively. We then subtract expected renewable output from $\overline{\overline{D_p}}$ and $\overline{\overline{D_o}}$ to derive the two residual demand functions $\overline{D_p}(p_e^p)$ and $\overline{D_o}(p_e^o)$. These will be the bases for the rest of our analysis.

We also assume perfectly competitive producers of electricity, and that the V2G option does not shift the peak demand period to become the off-peak period. Note that one can imagine that the peak period for electricity demand is very short. This can be the case when there is a local balancing problem. With a very short peak period, one needs to invest in extra feed-in capacity (kW) to make the battery potential (kWh) useful. We return to this issue later.

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[8] In a country with steady sunny weather and a lot of sun power, this assumption is unrealistic, see Luz et al. (2018). However, for Europe, which has highly variable weather, we find the assumption more realistic. See for instance Gaure and Golombek (2019).

4.1. The production cost of electricity

We assume one type of backup power technology with operating cost c_0 per MWh and a daily capacity cost I per MW. Moreover, we denote the number of off-peak hours per day by h_o, and the number of peak hours per day by h_p. Finally, we let γ denote the probability that fast charging en route will happen in the peak demand period.

4.2. The en route charging cost

We assume that all houses, offices and parking lots are already equipped to charge or discharge EVs during off-peak hours, and we do not model the investment in such equipment.[9] However, due to the extra cost of fast charging equipment, the cost of charging en route (p_{ch}), will typically exceed the average electricity price $\left[\gamma p_e^p + (1-\gamma)p_e^o\right]$. See further discussion in section 5.

4.3. Consumption

In the V2G case the charging demand function (D_p) in a peak hour is given by:

$$D_p = \frac{N}{365(\overline{x}-\underline{x})h_p}\int_{\underline{x}}^{\overline{x}}\left[\gamma\left(d_l - B(s)\right)s\right]ds \tag{12}$$

Consumers only charge during peak hours on long trip days by a fast charger, and the amount of electricity they demand depends on their battery size, as shown by the expression inside the integral. The integral must be multiplied by the factor $\frac{N}{(\overline{x}-\underline{x})}$ to get the density of consumers along the interval $(\underline{x}, \overline{x})$. Remember that consumers are uniformly distributed on this interval with respect to how many long trip days they have per year.

The charging demand function with V2G in an off-peak hour (D_o) is given by:

$$D_o = \frac{N}{365(\overline{x}-\underline{x})h_o}\int_{\underline{x}}^{\overline{x}}\left[365B(s) + (1-\gamma)\left(d_l - B(s)\right)s\right]ds \tag{13}$$

Inside the integral is the charging demand from an arbitrary consumer. Every day during off-peak hours, the consumer charges her battery fully. In addition, there is need for charging when out driving on long trip days. Again, the integral must be multiplied by the density factor.

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[9] Alternatively, the cost could be included in the parameter a from the cost function as a larger battery will in general require a home charger with higher capacity.

Without V2G the charging demand function in a peak hour is given by:

$$D_p^0 = \frac{N}{365(\overline{x}-\underline{x})h_p} \int_{\underline{x}}^{\overline{x}} \left[\gamma\left(d_l - B(s)\right)s\right] ds \tag{14}$$

Equation (14) is identical to (12) with the only difference that the battery size is smaller in (14) than in (12). Consequently, peak hour demand from EVs will be higher without V2G than with V2G, that is, $D_p^0 > D_p$. The result follows from the fact that the need for en route charging is smaller due to the larger battery sizes with V2G.

The charging demand function without V2G in an off-peak hour is given by:

$$D_o^0 = \frac{N}{365(\overline{x}-\underline{x})h_o} \int_{\underline{x}}^{\overline{x}} [(365 - s)d_s + (B(s) + (1-\gamma)(d_l - B(s)))s] ds \tag{15}$$

The EV charging demand in off-peak hours without V2G differs from the corresponding expression with V2G. First, without V2G, EV owners only charge d_s on short trip days. On long trip days, they charge their battery fully in the off-peak hours, and in addition, they charge en route in the off-peak hours with probability $(1-\gamma)$.

It is easy to show that $D_o^0 < D_o$.[10] The reason is that without V2G, the EV owners will only charge their battery fully before they are going on a long trip, while with V2G the EV owners charge it fully every day.

4.4. The supply of electricity with V2G

EVs will only supply electricity in peak hours, and we denote this supply by X^{V2G}. This supply is given by:

$$X^{V2G} = \frac{N}{365(\overline{x}-\underline{x})h_p} \int_{\underline{x}}^{\overline{x}} [(B(s) - d_s)(365 - s)] ds \tag{16}$$

By assumption, EV owners only supply electricity to the grid on short trip days.

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[10] See Appendix A2

It is easy to show that we may have $X^{V2G} - D_p > 0$.[11] This must hold if γ is small. Thus, V2G could decrease net demand for electricity in peak hours. This turns out to be the case in our application based on numerical model (Section 0).

We are now in the position to illustrate the effect on the electricity market of V2G. Our main focus will be on the case where capacity is constrained only in peak hours (Section 0). An electricity market where capacity is constrained in both periods in equilibrium is discussed in Section 6.

4.5. Electricity market equilibrium

As already discussed, due to fluctuations in demand, there is need for traditional backup power. Let *K* denote the amount of backup power capacity in the market. For the capacity investments to be profitable, we must have for the peak price:

$$p_e^p = c_0 + \frac{I}{h_p}, \tag{17}$$

where *I* denote the capacity investment cost (per day).

In the off-peak hours, there is spare capacity, and hence the only equilibrium price is:

$$p_e^o = c_0 \tag{18}$$

Thus, the prices are equal to the short run and long run marginal costs of the backup power production technology. We can then solve for $X^{V2G}, D_o, D_p, D_o^0, D_p^0, \overline{D_o}\ and\ \overline{D_p}$. Note that V2G cannot influence electricity prices as long as net demand in the peak period is higher than total demand in off-peak periods, that is:

$$D_p(p_e^p, p_e^o, p_{ch}) + \overline{D_p}(p_e^p) - X^{VTG}(p_e^p, p_e^o, p_{ch}) > D_o(p_e^p, p_e^o, p_{ch}) + \overline{D_o}(p_e^o) \tag{19}$$

where p_e^p, p_e^o are given from (17) and (18) above.

Compared to the situation without V2G, investments in capacity are lower according to:

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[11] See Appendix A2

$$-\Delta K = \left(D_p^0 + \overline{D_p}\right) - \left(D_p + \overline{D_p} - X^{VTG}\right) = D_p^0 - D_p + X^{VTG} \tag{20}$$

Thus, the savings in capacity are not only equal to V2G supply, but there is also an effect from the reduced need of charging en route in peak hours, due to higher battery capacities in general e.g. $D_p^0 - D_p > 0$.

From (16) we see that the saving of generating capacity is particularly high when the number of peak hours is low: Then the battery capacity that has a volume dimension (KWh) can be used to reduce generating capacity (KW) a lot. Of course, when the peak period becomes very thin one needs a high transfer capacity (KW) from the battery to the grid and this can be costly in itself. Below we illustrate the electricity market effects in Figure 2:

Figure 2: The electricity market effects of V2G with fixed prices

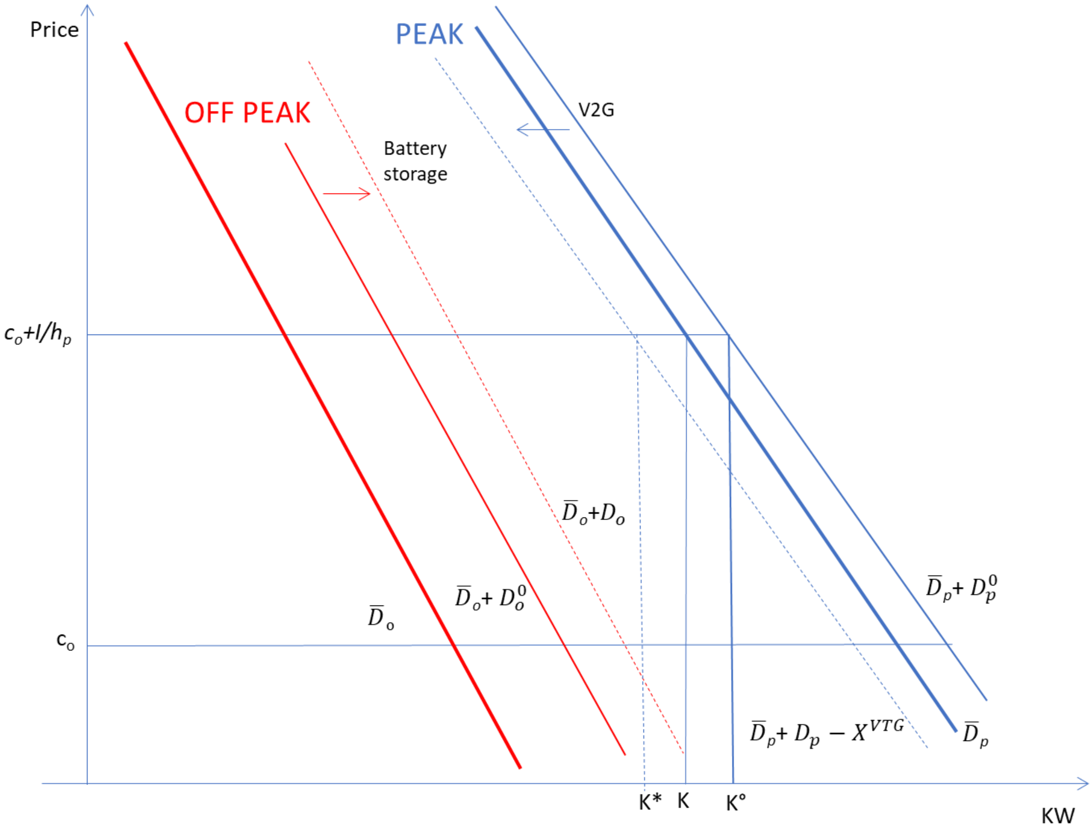


Figure 2 illustrates the role of V2G on the overall electricity market with fixed electricity prices given by (17) and (18). The solid, bold lines are electricity demand in peak and off-peak periods without demand from EVs (see Figure 1). Then, starting with the case where the EV owners cannot sell to the grid, they select a battery capacity that allows them to limit recharging en route during long trips. This gives the solid (not bold) demand functions in the off-peak ($D_o^0 + \overline{D_o}$) and peak period ($D_p^0 + \overline{D_p}$). We note that demand increases in both periods, but more in the off-peak period since we assume that EV owners do most of their charging off-peak. However, because there is always some en route charging, demand also increases in the peak period. The optimal level of backup generating capacity investment in this case equals K°, which is greater than K – the original level before EVs were introduced.

When we introduce the V2G option, EV owners start to sell electricity to the grid in the peak period on short trip days. This shifts the residual peak period demand function to the left e.g. the stippled line denoted $\left(\overline{D_p} + D_p - X^{VTG}\right)$. In the figure we assume $X^{V2G} - D_p > 0$, and consequently, the backup power capacity can be reduced to K* < K. Note that we must have K* < K^0. In the off-peak, power demand will be shifted further to the right as EV owners always want to charge the battery fully in order to sell electricity to the grid at peak times. This is illustrated by the stippled line denoted $\left(\overline{D_o} + D_o\right)$.

As prices of electricity in peak and off-peak periods remain constant, the EV owner will receive all the benefits of the option to sell to the grid: a profit from the sales to the grid and the reduced disutility from a larger battery implying less recharging en route. The backup power generators make no profit before allowing sales to the grid, and no profit after allowing sales to the grid. However, V2G still increases social welfare, as the need for backup power capacity is reduced. Clearly, the higher the available total battery capacity in EVs, the larger the reduction in backup power capacity.

5 Social versus private optimum

The choice of battery capacity by car consumers, derived above, can be compared with the social optimum value of battery power. The social optimum is reached when the total electricity system costs plus the costs of EVs, the EV users' disutility of charging, and the charging services en route are minimized. Given optimal peak and off-peak pricing, the social cost can be written:

$$\Omega{=}365\left[h_p \cdot p^p \cdot \left(\overline{D_p} + D_p - X^{V2G}\right) + h_o \cdot p^o \cdot \left(\overline{D_o} + D_o\right)\right]$$
$$+ \frac{N}{(\overline{x}-\underline{x})} \int_{\underline{x}}^{\overline{x}} \left[\left[v\left(d_l - B(s)\right)^2 \cdot s\right] + aB(s) + K\left(d_l - B(s)\right) \cdot s\right] ds \tag{21}$$

where $D_p,\ D_o, X^{V2G}$ are given by (12), (13) and (16), and $K\left(d_l - B(s)\right)$ is the social cost of providing charging services (fast charging) en route (exclusive the cost of electricity).

By minimizing Ω with respect to $B(x^i)$ we find the following first order conditions for the social optimal battery choice (B_i^{**}) for the EV owner *i*:

$$B_i^{**} = d_l - \frac{a-(K'^{**}+(\gamma p_e^p+(1-\gamma)p_e^o)-p_e^o)x_l^i}{2vx_l^i} + \frac{(p_e^p-p_e^o)x_s^i}{2vx_l^i} \tag{22}$$

where K'^{**} is the social marginal cost of providing charging services en route ($\frac{\delta K}{\delta B(x^i)}$).

By comparing (3) with (22) , we see that if the driver pays the full social marginal cost of charging (charging services plus electricity cost; $p_{ch} = K'^{**} + (\gamma p_e^p + (1-\gamma)p_e^o)$, the private optimal battery capacity equals the social optimal capacity. Hence, a sufficient condition to decentralize the social optimum is to have peak load pricing in the electricity sector, and marginal cost pricing of fast charging along roads.

6 V2G and the price of electricity

In the previous sections we have considered an electricity market with excess capacity in off-peak hours, and where the V2G technology did not alter the equilibrium electricity prices. However, the introduction of V2G may alter the equilibrium electricity prices. If full charging of EVs at off-peak hours becomes sufficiently widespread, it may be optimal with full capacity utilization of backup power also in off-peak periods. In this case, we must find prices (p_e^o, p_e^p) that equalize net residual demand in the two periods. Moreover, we must ensure that the average price over the peak and off-peak hours is equal to the long run marginal cost of electricity. This implies solving the following two equations where p_e^o, p_e^p are the two unknowns:

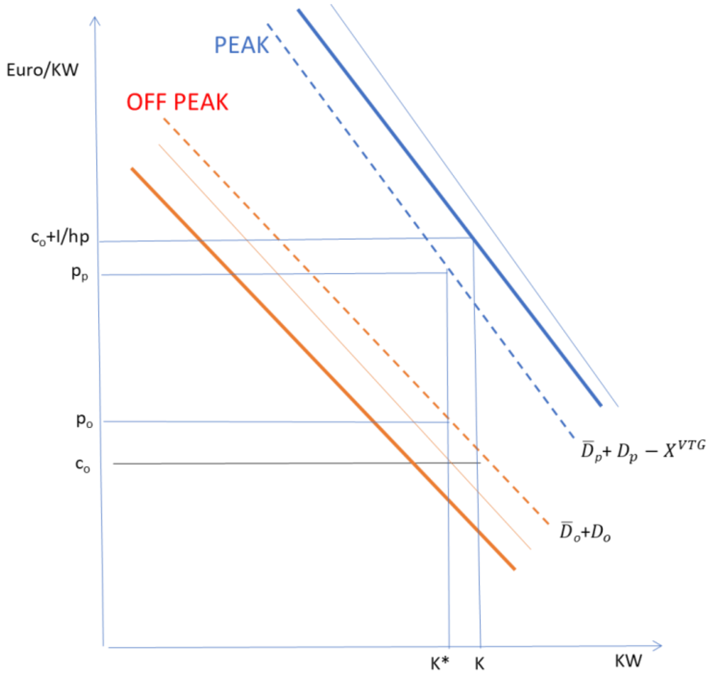
$$D_p(p_e^p, p_e^o, p_{ch}) + \overline{D_p}(p_e^p) - X^{VTG}(p_e^p, p_e^o, p_{ch}) = D_o(p_e^p, p_e^o, p_{ch}) + \overline{D_o}(p_e^o) \tag{23}$$

$$\frac{h_o}{24}p_e^o + \frac{h_p}{24}p_e^p = c_0 + I \tag{24}$$

The solution must satisfy $c_0 \leq p_e^o < p_e^p$ in order for our model to hold.

Allowing EV owners to sell to the grid will benefit them, but not as much as in the model with spare capacity in off-peak hours. The reason is that the price difference between peak and off-peak periods will be smaller. All other electricity consumers will however benefit from having smaller differences between the off-peak price and the peak price as it implies exchanging lower valued KWh by higher valued KWh. In Figure 3 below, we illustrate the electricity market equilibrium effects of V2G, in the case with full utilization of the backup capacity at all times:

Figure 3: The electricity market effects of V2G with endogenous prices



As above the solid, bold lines are electricity demand in peak and off-peak periods without demand from EVs, the solid (not bold) lines are demand functions in the off-peak and peak period without V2G, and the stippled lines are demand functions in the off-peak and peak period with V2G.

When we introduce the V2G option, the electricity prices change, that is, the price is lower in peak periods and higher in off-peak periods. Note that if they had not changed, off-peak demand would have been higher than peak demand, violating condition (23). Still, backup power capacity can be reduced from K to K*. Thus, social welfare increases both for this reason and because lower valued KWh are exchanged with higher valued KWh.

7 Numerical illustration

Our point of departure will be the Belgian electricity market in 2040. The Belgian network operator ELIA (ELIA, 2017) has made a scenario study for Belgium in 2040, and we will use some of the figures from the scenario results (see Appendix A3).

Assuming no EVs, we calibrate the following demand functions to fit with prices and quantities given by ELIA (2017):

$$\overline{D_j} - \overline{S} = M_j\left(p_e^j\right)^{-\epsilon} \tag{25}$$

where j=o,p, $\overline{S}$ is the expected supply of renewables, M_j is a market size parameter and ϵ the elasticity of demand in the two periods, which we have set to -0.8. Since we assume that renewable production is uncorrelated with peak and off-peak demand, we simply subtract expected renewable production from the electricity demand functions for the peak and off-peak periods.

Even if V2G is possible, we assume that only a fraction of the EV owners use their EV in this way. The other fraction is assumed to behave as if V2G were not possible (and invest in battery capacity accordingly). We then get the following results for the Belgian electricity market with and without V2G:

Table 1: Numerical illustration results

	Base line No EVs	**2 mill. EVs No V2G**	**2 mill. EVs V2G available (10%)**	**2 mill. EVs V2G available (20 %)**
V2G fraction	0.0	0.0	10%	20%
Off-peak el price €/MWh	95.2	95.2	95.2	101.5
Peak el price €/MWh	149.9	149.9	149.9	124.9
Charge peak MW	14,000.0	14,028.0	12,805.0	12,731.0
Charge off-peak MW	10,183.0	12,084.0	12,540.0	12,731.0
Net EV charge peak MW	0.0	28.0	-1,195.0	-2,124.0
EV charge off-peak MW	0.0	1,872.0	2,328.0	2,924.0
Backup cap. invest. MW	8,429.0	8,457.0	7,234.0	7,131.0

There are four scenarios. The baseline scenario is without EVs at all (second column). The calibration of the electricity demand functions is based on this scenario. Then we introduce EVs, but allow for no V2G. We note from comparing the second and the third column that in particular, demand in off-peak hours increases due to EV charging. Since there also is some peak charging, backup capacity increases slightly.

In the third scenario we allow 10 percent of the EV owners to participate in V2G. This does not change the electricity market prices; we still have marginal cost pricing in off-peak hours, and the price in peak hours also covers capital costs. However, due to the EV supply, the need for backup capacity investment is reduced by almost 15 percent. Note from the seventh row that $X^{V2G} - D_p = 1{,}195$. Thus, net demand in peak periods is significantly reduced.

Finally, if we increase EV owner participation to about 12 percent, the electricity market no longer runs with spare capacity in the off-peak hours. This is due to the increased demand from EV owners wanting to charge their batteries in order to sell electricity in peak hours. Consequently, peak and off-peak prices change and move closer to each other. In Table 1 we show numbers for a participation rate of 20%. Note that due to the high EV electricity supply in peak hours, backup capacity investment is further reduced. Note also from the third and fourth row that the price difference between peak and off-peak hours has been reduced.

8 Balancing services

So far, we have considered the case where the battery can serve as electricity storage in order to smooth out daily variability in demand and supply of electricity. However, from time to time, the electricity market may experience balancing problems. In this case, the system operator needs to employ reserve capacity for electricity production at very short notice. This normally comes from idle power plants that receive some payment from the system operator in order to be available; see for instance Joskow and Tirole (2007). The network operator can alternatively pay EV owners for available capacity during balancing days. An EV connected to the grid can help to solve the balancing problem.

Let the probability of a normal day and of a “balancing problem” day be p_n and $(1 - p_n)$, respectively. Since demand is higher in the peak period, we assume that the balancing problem occurs sometime within the peak period. Moreover, we assume that the system operator knows, just before entering the peak period, whether there will be a balancing problem this particular day.

On a long trip day, the battery is fully loaded at the end of the off-peak period, but it will be needed for the long trip. The opportunity cost of using the available battery capacity for balancing is then much higher: postponing the trip and/or stopping more frequently to charge en route. Hence, we neglect this option.

On a short trip day, we have assumed that the EV owner provides a capacity of $B_i - d_s$ during peak hours, and hence that she can supply $(B_i - d_s)/h_p$ per hour to the grid. We have also assumed that the vehicle will be available for discharging all of its capacity during the peak hours. The short trip can still happen within peak hours, but we then require a larger effect for discharging than $(B_i - d_s)/h_p$. In order to simplify, we assume that the vehicle is always connected during peak hours.

Let the market price of making balancing capacity (kW) available per hour be denoted by I^B. We can then reformulate the optimal battery size problem for the EV owner:

$$\begin{aligned} c(B_i) &= 365p_e^oB_i - p_n\, p_e^p(B_i - d_s)\big(365 - x_l^i\big) \\ &+(1-p_n)I^B\,\frac{(B_i-d_s)}{h_p}\,(365 - x_l^i) \qquad (26) \\ &+[p_{ch}(d_l - B_i) + v(d_l - B_i)^2]x_l^i + \mathrm{a}B_i \end{aligned}$$

The first and second line differ from the original expression in (1). If EVs are available for balancing services, each day with short trips, the EV owner will either gain from pure supply of electricity to the grid, or she will gain from helping the system operator solving the balancing problem. We see that the larger the compensation for balancing services $(\frac{I^B}{h_p})$ relative to the peak price (p_e^p), the larger the optimal battery capacity is, compared to the situation in which the EV owner only supplies electricity to the grid.

9 Concluding remarks

We have used a highly stylized model to analyze the impact of V2G on the consumers' choice of battery capacities and the impact on the electricity market. From the model, we find that, compared to no V2G charging technology, V2G:

- Increases the EV owners' choice of battery capacity
- Decreases the cost to the consumer of having an EV
- Reduces the need for backup power capacity
- Increases social welfare.

Obviously, a stylized model cannot capture all characteristics of the electricity market, consumers' charging and driving behavior, and technological development. However, it is reasonable to expect that the electricity market will face periods with higher net demand (peak hours) and periods with lower net demand (off-peak hours). It is thus a robust result that V2G will give EV owners the possibility to earn money on peak shaving. The consumers' profitability of peak shaving will inter alia depend on the fluctuations in net demand, the battery degradation under frequent charging, the cost of V2G chargers, and driving and charging behavior.

A policy recommendation from our analysis is that regulators planning to support the introduction of V2G should signal that early, in order to ensure that EV owners can adjust their choice of optimal battery capacity accordingly.

Another policy implication of our result is that an optimal planning of investments in power plants must consider not only the number of electric cars in the market, but also the consumers' choice of battery capacities, and the development of V2G technology. Our numerical illustration of the Belgian electricity market in 2040 indicates that V2G can significantly reduce the need for power production capacity, even if only a small fraction of the EV owners sell electricity to the grid.

The social value of V2G will differ across countries depending on their electricity system. Countries with a large proportion of base load plants (nuclear and/or coal plants), will tend to have a larger difference between the electricity prices in peak and off-peak periods. Such countries will also have more to gain from utilizing the electricity storage capacities in EVs, and the EV owners will have a larger incentive to buy batteries with large storage capacity. On the other side of the spectrum, we have hydro-energy countries with large reservoir storage. Supply from hydro energy plants is easily increased to meet demand, and there is less price fluctuation between peak and off-peak periods. However, the EV batteries can still help local capacity and balancing problems. Furthermore, as the European electricity market becomes more integrated, through more interconnectors, and the share of wind and solar increases, hydro-energy countries will also face larger fluctuations in the value of electricity during 24-hour periods. The price difference on electricity between peak and off-peak will then provide additional incentives for EV battery storage and V2G options in these countries.

We showed in our model that the private optimal battery capacity equals the social optimal capacity if we have peak load pricing in the electricity sector and marginal cost pricing of fast charging along roads. However, fast charging stations will typically be selling differentiated products, due to the variety of geographical locations. Hence, the market structure can be characterized as monopolistic competition. It is a well-known result that this structure leads to prices that exceed the marginal cost (socially optimal charging cost), and there will be excess capacity, although the supplier receives no super-normal profit. We see from (3) and (4) that when the price of charging en route is higher than the socially optimal price, the battery capacities will exceed the socially optimal capacity.

Another potential source for inefficiency is subsidies on battery capacity. EVs are heavily subsidized in many countries (Kemfert, 2016). Due to network externalities, subsidizing EVs to increase the stock can be a socially optimal policy (Greaker and Midttømme, 2016). In our model, the number of EVs are given, but the model can point to situations where subsidizing EVs can lead to inefficiencies. The price of an EV is typically increasing as the battery size increases. If subsidies are linked to the price of the car (as for instance purchase subsidies), the consumers in our model will buy a larger battery (more expensive car) than without the subsidies, and the total battery capacities will exceed the socially optimal level.

In our model, we assumed that the agents faced the socially optimal electricity prices. There are, however, many distortionary taxes in the electricity market. For instance, the electricity bills for consumers in Europe typically contain renewable energy levies, fees, surcharges etc. (Grave et al., 2016). The larger consumer taxes on electricity, the lower the consumers' income from charging is in off-peak hours and unloading in peak hours. Such taxes will therefore lead to insufficient investment in batteries, compared to the social optimum. Note, however, that the distortions following from the price difference between consumer and producer prices can be avoided if the EV owners use the EV battery capacity to meet their own residential demand during peak hours.

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Appendix

A1 The derivation of equation (7)

By rearranging the expressions for the EV costs and using the first order condition for the optimal battery size, we obtain:

$$c(B_i^*) = v((d_l)^2 - (B_i^*)^2)x_l^i + p_e^p d_s x_s^i + p_{ch} d_l x_l^i$$

$$c\left(B_i^0\right) = v\left((d_l)^2 - \left(B_i^0\right)^2\right)x_l^i + p_e^o d_s x_s^i + p_{ch} d_l x_l^i$$

By inserting for the optimal battery size in the two cases and subtracting $c(B_i^*)$ from $c\left(B_i^0\right)$ we obtain (7).

A2 Proofs of $D_o^0 < D_o$, and that $(X^{V2G} - D_{p)})$ can be positive

By rearranging the integrals, it is easy to see that $D_o^0 < D_o$:

$$D_o = \frac{N}{365(\overline{x} - \underline{x})h_o} \int_{\underline{x}}^{\overline{x}} [(365 - s)B^*(s) + (1 - \gamma)d_l s + \gamma B^*(s)s]ds$$

$$D_o^0 = \frac{N}{365(\overline{x} - \underline{x})h_o} \int_{\underline{x}}^{\overline{x}} [(365 - s)d_s + (1 - \gamma)d_l s + \gamma B^0(s)s]ds$$

since $B^*(s) > B^0(s) > d_l$.

For the net supply from EVs with V2G in peak hours, we have:

$$X^{V2G} - D_p = \frac{N}{365(\overline{x} - \underline{x})h_o} \int_{\underline{x}}^{\overline{x}} [(B^*(s) - d_s)365 + ((1 - \gamma)B^*(s) + \gamma d_l - d_s)s]ds$$

This is clearly positive for $\gamma = 0$. By continuity it will be positive for $\gamma > 0$.

A3 The numerical model

According to ELIA (2017), yearly total demand in 2040 is expected to amount to 97.6 TWh, and the peak load is expected to be 14 GW. On a yearly basis, we assume 2200 peak hours and 6560 off-peak hours. Thus, we have $h_p = 6$ and $h_o = 18$, and thus, the off-peak hour load must be 10.18 GW. The variable fuel cost of gas power is €50 per MWh. Adding the cost of ETS permits, we get to €95.2 per MWh which we assume is the price in off-peak hours. Assuming an investment cost of €850,000 per MW, a 20 year lifespan, and a real interest rate of 12 percent, we obtain for the capital cost per year €113,797. In order to cover this in 6 peak hours per day, the capital cost charge must be €51.7 per MWh. Thus, our peak hour price of electricity is €146.9 per MWh.

Moreover, we assume that the expected production from renewables is 5.6 GW per hour. This implies an average renewable share of 50 percent. Moreover, we set the number of EVs to 2 million (ELIA, 2017). Note that only a fraction of the EV owners will use their EV for V2G. We also introduce a conversion factor (η), that is, we assume that only 90 percent of the available battery capacity can be sold to the grid. Furthermore, the KWh use for a long trip is 60; for a short trip it is set to 10. The number of long trips per year varies between 10 and 100, which we think encompasses the bulk of the real distribution.

In order to simulate the numerical model, we solve the integrals (12) - (16):

$$D_p = \frac{\gamma N}{365 h_p}\left[\frac{a-365(\eta p_e^p - p_e^o)}{2v} - \frac{p_{ch}-\eta p_e^p}{4v}\left(\overline{x}+\underline{x}\right)\right]$$

$$D_o = \frac{\Gamma^*}{h_o} + \frac{(1-\gamma)N}{365 h_o}\left[\frac{a-365(\eta p_e^p - p_e^o)}{2v} - \frac{p_{ch}-\eta p_e^p}{4v}\left(\overline{x}+\underline{x}\right)\right]$$

$$D_p^0 = \frac{\gamma N}{365 h_p}\left[\frac{a}{2v} - \frac{p_{ch}-p_e^o}{4v}\left(\overline{x}+\underline{x}\right)\right]$$

$$D_o^0 = \frac{N d_s}{h_o} + \frac{N}{365 h_o}\left[\frac{2v(d_l-d_s)+(p_{ch}-p_e^o)\gamma}{2v}\frac{(\overline{x}+\underline{x})}{2} - \frac{\gamma a}{2v}\right]$$

$$X^{V2G} = \frac{\eta(\Gamma - N d_s)}{h_p} + \frac{\eta N}{365 h_p}\left[\frac{a-365(\eta p_e^p - p_e^o) - v(d_l-d_s)(\overline{x}+\underline{x}) - (p_{ch}-\eta p_e^p)\frac{(\overline{x}+\underline{x})}{2}}{2v}\right]$$

The model is programmed in Excel Solver.