Environmental Policy in General Equilibrium: New Insights from a Canonical Model

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Abstract

This paper derives the incidence of a pollution tax in a stylized general equilibrium framework, building on previous work by Fullerton and Heutel (2007a). Using the CPI as numeraire, we show that tax incidence is a simpler problem than previously thought, and that general insights can be derived without the need to restrict the parameter space. In addition, the counterintuitive possibility that an increase in the tax could increase the pollution level vanishes. The choice of the CPI as numeraire is further justified by the fact that environmental taxes, notably carbon taxes, are typically indexed on inflation.

JEL codes: Q52, H23, H22
Keywords: tax incidence, general equilibrium, relative prices, numeraire

1 Introduction

It has been known since the seminal work of Muth (1964), and later confirmed through the more general results of Heiner (1982) and Braulke (1984), that under fairly general conditions, the law of input demand holds at the industry level even when the prices of outputs or other inputs are allowed to adjust in related markets in response to the input price increase. An interpretation of this law is that an exogenous tax on an industry input reduces the overall use of that input, even if certain firms end up using more of it. That is, derived demand slopes down in the aggregate. If the taxed input causes pollution, pollution will unambiguously be reduced by an increase in the tax.[1] This simple economic logic lies at the core of environmental taxation (Baumol and Oates, 1988).

While the works cited above explicitly allow for the presence of markets besides the one subject to taxation, they rely on partial equilibrium approaches. In seminal contributions to the analysis of the incidence of environmental policy, Fullerton and Heutel (2007a) and Fullerton and Heutel (2010b) extend Harberger (1962)'s model of corporate income taxation and show, among other results, that the desirable effects of environmental taxation on pollution need not apply in a general equilibrium context, even in a closed economy and under the assumption of homogenous spending propensities on the demand side.[2]

[1]The result holds as long as the supply of the dirty input is less than infinitely inelastic. If it is infinitely inelastic, input use remains constant, but it does not increase.

[2]In earlier work, Mieszkowski (1967) relaxes the assumption of identical spending propensities among owners of capital and owners of labor and shows that a series of counterintuitive comparative statics may ensue.

That is, an increase in the tax on a polluting input (e.g., carbon) may in some cases lead to more, not less, pollution. Similarly, in a cap-and-trade system, a decrease in the cap on a polluting input could lead to a decrease in the permit price.

Fullerton and Heutel (2007a) suggest that the ambiguity regarding the applicability of the law of input demand to polluting inputs is due to general equilibrium feedback effects.[3] Here, we argue that whenever the law of input demand is violated for pollution, the equilibrium in their model is unstable in the Marshallian sense.[4] The occurrence of such instability, which may seem tangential to the issue of policy incidence, is in fact closely related to it as all comparative statics, not simply that on the pollution outcome, are reversed in this case, notably those pertaining to relative returns to labor and capital. Thus, the inability to rule out equilibrium instability precludes general conclusions to be drawn regarding policy incidence.

The present paper offers a solution to this problem. Specifically, we show that the existence of unstable equilibria is, in the original model, an artefact of the choice of numeraire good. Perhaps due to the non-market nature of pollution, the tax on the polluting input is modeled as a nominal tax, for instance a \$20/ton CO_2, rather than an *ad valorem* tax, say a 10% tax on energy expenditures. As a result, the effects of this nominal tax on goods provision, resource allocation, and relative prices implicitly depend on which good is chosen as numeraire, as the tax increases the price of the polluting input *relative to that of the numeraire*. Said differently, there are as many *varieties* of environmental taxes—one could say as many environmental policies—as choices of numeraire good. Note that the issue is not about the undeterminacy surrounding the overall magnitude of prices, but, since the model is used to *compare* equilibria, that an *increase* in the nominal tax rate increases the price of pollution relative to the price of the numeraire, while potentially decreasing it relative to other prices.[5] We show that this fundamental dependency of

[3]See, for instance, the discussion in footnote 14 of Fullerton and Heutel (2007a). Note that the numerical example provided in that footnote violates the negative semidefiniteness property of the substitution matrix in the dirty sector. For an example of model parameterization leading to the counterintuitive outcome, see online appendix A.6.

[4]Put simply, Marshallian instability means that if the demand price of a good exceeds its supply price, quantity will move further away from its equilibrium value.

[5]In earlier studies by Bovenberg and Goulder (1997) and De Mooij and Bovenberg (1998) that also consider a pollution tax, capital, the polluting input, and a final consumption good are supplied to/demanded from a small economy in an infinitely elastic fashion, with fixed rates of exchange on the world market. As a result, a tax on the dirty input increases the price of that input relative to that of capital and that of the consumption good. The only price determined endogenously in the model is the wage rate. A government budget constraint imposes that a rise in the pollution tax be matched by a decrease in either the tax on labor or that on capital, giving rise to the possibility of an increase in pollution through an expansion in output. The mechanism behind the increase in pollution in these earlier models is thus distinct from that in

outcomes on numeraire choice, which is admittedly absent in the seminal analysis of Harberger (1962), in fact reappears as soon as one considers an increase in an existing tax, rather than the introduction of a new tax.[6]

The question then arises as to which good should be chosen as numeraire when modeling the effects of a nominal tax. An equivalent way to frame that question is to imagine that instead of being expressed in nominal terms, as in the original model, such tax is expressed in *ad valorem* terms relative to the equilibrium price of some market good. Because the prices of market goods respond to changes in numeraire choice in ways that preserve their relationships to each other, anchoring the pollution tax to the price of a market good removes the dependency of real outcomes on numeraire choice. Note that instead of being anchored to the price of a single good, the tax could also be anchored to a price index reflecting the average level of prices across a set of goods.[7] In that sense, the choice of anchoring price, that is, the *variety* of environmental taxation ultimately represented, may seem overwhelming.[8]

For the purpose of environmental taxation, we argue that a reasonable anchoring price index should be one for which the law of input demand is satisfied, that is, an increase in the price of the dirty input relative to the equilibrium value of the index unambiguously leads to a decrease in input use—and attendant pollution. Such requirement amounts to ruling out equilibria that are unstable in the Marshallian sense and thus ensures that meaningful comparative statics can be derived (Samuelson, 1941). With this constraint in mind, we consider two classes of price indices: those defined over the prices of consumption goods (uses side) and those defined over factor prices (sources side). Using the model of Fullerton and Heutel (2007a), we show that within each class, there exists a unique index that satisfies the stability constraint. We express each index using underlying parameters of the economy. On the uses side, the price index is identical to an inflation

Fullerton and Heutel (2007a) and Fullerton and Heutel (2010b).

[6]Specifically, in online appendix C we show that numeraire choice is irrelevant for the incidence of the marginal corporate income tax, as originally claimed by Harberger (1962). We then show that if one considers a marginal increase in the pre-existing corporate income tax (modeled as a tax on capital in the corporate sector), the choice of numeraire matters and can even lead to perverse comparative statics, e.g., an increase in the use of capital in the corporate sector. This point is conceptually different from McLure Jr (1974)'s critique regarding the use of Harberger (1962)'s displacement model to analyze non-marginal taxes.

[7]A similar point is made in Baylis et al. (2014, pg. 63).

[8]A distinction needs to be made between numeraire and anchoring price. A numeraire is a good, or basket of goods, the price of which is assumed to be invariant to the policy change. An anchoring price for the pollution tax is one to which the value of the tax is attached. For a nominal pollution tax, the *de facto* anchoring price is the numeraire. But for an *ad valorem* tax expressed relative to an explicit anchoring price, the numeraire plays no role besides determining the overall level of prices after the policy change.

index, namely the product of the prices of consumption goods raised to their respective expenditure shares. On the sources side, the price index is equal to the product of factor prices raised to their respective income shares. Importantly, none of these indices depend on behavioral parameters such as substitution elasticities, either in production or consumption.[9] Therefore, a government could anchor the pollution tax to either index without unrealistic informational requirements. For instance, the pollution tax could be tied to the Consumer Price Index (CPI).

With a pollution tax so designed, not only is the effect of a rise in the tax in accordance with the law of input demand and the intent of environmental taxation, but the tax incidence results discussed by Fullerton and Heutel (2007a) in the context of specific parameter values are also shown to hold generally.[10] Namely, if labor and capital are equally substitutable for pollution, then the pollution tax places a disproportionate burden on capital when the polluting sector is capital intensive, as long as goods are sufficiently substitutable in consumption. If not, then the input substitution effect dominates and the demand for capital in the capital-intensive polluting sector rises so that the return to capital increases relative to the wage rate. If both sectors are equally capital-intensive, then capital bears disproportionately more of the tax burden if and only if it is less substitutable for the polluting input than is labor.

Furthermore, because our analysis does not require focussing on special cases, we are able to provide general results regarding incidence on the uses side that were left unexplored in the original study. Specifically, we make explicit the conditions under which the pollution tax may lead to a reduction in the price of the dirty good relative to that of the clean good, a possibility that has been recognized, but not elucidated, in Fullerton and Heutel (2010b) and overlooked in much of the literature. Indeed, incidence studies focussing on the uses side generally take it as a premise that the price of the dirty good will rise relative to that of the clean good, and then compare impacts across groups purchasing these goods in varying proportions (Grainger and Kolstad, 2010; Cronin et al., 2019; Fullerton and Muehlegger, 2019). But if the price of the dirty good decreases relative to that of the clean good, the burden will fall disproportionately on those who purchase relatively less, not more, of the dirty good. We show that this counterintuitive result

[9]Our model assumes homothetic, but not necessarily Cobb-Douglas, consumer preferences.

[10]The special cases analyzed in Fullerton and Heutel (2007a) and Fullerton and Heutel (2010b) are quite restrictive. Fullerton and Heutel (2007a) focus on the case where the clean and dirty industries have equal factor intensities and the case where labor and capital are equally substitutable for pollution in the dirty industry. In their study of a pollution cap, Fullerton and Heutel (2010b) analyze the special case of equal factor intensities, along with the case of zero substitution across inputs in the dirty sector (see their appendix).

can arise if the factor used more intensively by the clean industry (say capital) is a better substitute for pollution than the other factor (say labor), and the price of capital rises relative to that of labor as a result of the pollution tax.

Although fairly recent, the contribution of Fullerton and Heutel (2007a) has had an undeniable influence in the environmental economics literature. Their model has been used in Fullerton and Heutel (2007b), Fullerton and Heutel (2010b), Fullerton and Heutel (2010a), and Fullerton et al. (2011). It has been modified or extended in further contributions by Fullerton and Monti (2013), Baylis et al. (2013), Rivers (2013), Baylis et al. (2014), Rausch and Schwarz (2016), and Goulder et al. (2016). Their analysis has also provided needed clarity on important drivers of the incidence of environmental policy in studies that involve more complex and realistic models of the economy, e.g., Rausch et al. (2011), Rausch and Mowers (2014), or Marten et al. (2019), and in policy discussions, e.g., Morris and Munnings (2013).

Our paper makes four contributions to the literature. First, we show that well-designed pollution taxes need not have ambiguous effects on equilibrium pollution levels in general equilibrium, and that counterintuitive pollution outcomes identified in previous literature imply unstable equilibria. Second, we ask whether there exist anchoring prices that avoid equilibrium instability altogether. We show that the answer is yes, and that, reassuringly perhaps, the set of candidate indices is limited. On the uses side, the only index satisfying the stability constraint for all parameter values is equivalent to the CPI. On the sources side, it is equivalent to a producer price index for primary production factors. Third, using these indices, we demonstrate that tax incidence results previously derived in special cases hold in fact quite generally. We also provide new and general results pertaining to incidence on the uses side. Fourth, we empirically demonstrate how the choice of anchoring price in general equilibrium models influences comparative statics with respect to taxes on non-market goods like pollution. Notably, using a model calibrated to the US economy, we show that for a given relative increase in the price of pollution, predicted pollution impacts may differ by up to 40% depending on the choice of anchoring price, that is, the *variety* of pollution tax considered, and that this choice acts as an essential driver of predicted incidence on the uses side. The fact that our model is extremely stylized suggests that even larger discrepancies could occur in richer models where the number of conceivable anchoring prices is larger.

In terms of policy implications, we note that many recent policy proposals for a U.S. carbon tax already choose to index the tax to inflation, albeit for reasons likely unrelated to the results discussed here (Marron et al., 2015; Aldy, 2016; Vail and Burtraw, 2016; Metcalf, 2018; Climate Leadership Council, 2019; H.R. 763, 2019; H.R. 3966, 2019; H.R. 4058, 2019;

S. 1128, 2019; S. 2368, 2018; S. 2284, 2019).[11] Furthermore, carbon taxes indexed to inflation have already been implemented in a number of countries, including Denmark and Sweden (Withana et al., 2013), Iceland (PMR, 2017), Norway (Haites et al., 2018), Chile, Colombia, and Mexico (UN, 2019), the Netherlands (Andersen et al., 2007), South Africa (Act No. 15, 2019), and the United Kingdom (National Audit Office, 2007).[12] The arguments laid out in the present paper serve to strengthen the case for such indexation. Our simulation results further suggest that accurately modeling the effects of such policies in general equilibrium requires special attention to the choice of anchoring price.

Although our general equilibrium analysis is of direct relevance to environmental policy, our findings on the comparative statics of tax incidence do not rely on any assumption regarding the actual external effects of the dirty input on technology or consumer utility; all that is required for our results is that this input be a non-market good available to the economy in sufficiently large quantity. As such, the framework conceptually applies to other settings, for instance per-period restrictions on the extraction of a natural resource for the purposes of conservation or sustainability.

The question of the impact of numeraire choice in general equilibrium models is not specific to environmental economics. Notably, it has been a topic of discussion in the trade literature, where price uncertainty and imperfect competition have both been shown to render equilibrium outcomes sensitive to the choice of numeraire. As part of a broad literature interested in explaining why and what countries trade, Turnovsky (1974) shows that in the presence of commodity price uncertainty, a small country producing two goods with a Ricardian technology and a single input may choose to specialize in the production of different goods under price uncertainty and the price certainty counterfactual; Flemming et al. (1977) later show that this seemingly striking possibility is contingent upon the choice of numeraire if one uses as the certainty price the arithmetic mean of relative prices, that is, the expected value of the uncertain price ratio. The authors propose the use of the geometric means, which breaks down such dependence, as the relevant certainty price benchmark whenever real variables are homogenous of degree zero in absolute prices. In contrast, Britto (1983) argues for a choice of price counterfactual that reflects the structural conditions that make commodity prices uncertain. Following initial contributions by Gabszewicz and Vial (1972) and Dierker and Grodal (1986), Ginsburgh (1994) shows that the numeraire matters in the Cournot-Walras general equilibrium model, with

[11] Inflation adjustments would be on top of any ramping up of the tax meant to increase pollution reduction incentives over time.

[12] California and Québec operate a cap-and-trade program rather than a carbon tax. Yet, the auction price floor and price ceiling are both indexed to inflation (ICAP, 2019b,a).

welfare effects potentially displaying unexpected signs and magnitudes. The reason is that when firms can influence prices strategically, different choices of numeraire lead to profit functions that are not monotone transformations of each other. Thus, the choice of numeraire affects the objective pursued by non-competitive firms, and thus production plans and real equilibrium outcomes. Kletzer and Srinivasan (1999) discuss plausible implications of such dependency for general equilibrium trade models that feature either monopolistically competitive or oligopolistic firms. Just like the objectives of firms may differ under different price normalizations outside the non-stochastic perfectly competitive case, here we show that different numeraires imply different *varieties* of environmental policy for a nominal tax. One important difference, however, is that we take the nominal tax as given and not the result of any particular optimization. Relatedly, these literatures investigate properties of actual equilibria whereas, in the spirit of Harberger (1962), we focus on comparative statics derived from a linear displacement model.

2 Model and notation

We borrow the assumptions of Fullerton and Heutel (2007a), who build upon the seminal model of Harberger (1962). The model is parsimonious and aims to capture the essential drivers and overall magnitude of environmental tax incidence, rather than provide accurate predictions for a given economy as would a more detailed computable general equilibrium approach. A notable advantage is that it can be solved analytically, providing a "model of the model" (Fullerton and Heutel, 2010a). Relatedly, Fullerton and Ta (2019) demonstrate that a stylized and analytically solvable model of the US economy delivers quantitative predictions on the effects of a carbon tax that are not far from those obtained from the detailed CGE model developed by Goulder and Hafstead (2018), and can help in understanding the drivers of these effects.

There are two economic sectors, X and Y, that use labor (L) and capital (K) as inputs. Sector Y also uses a dirty input, Z. In each sector, production displays constant returns to scale.[13] The economy is endowed with fixed quantities of labor and capital allocated across the two sectors, and with an unspecified quantity of dirty input available for free. The use of the dirty input by sector Y causes pollution, which a tax on input Z aims to reduce. The effect of pollution on utility is left implicit, which means that the model takes the tax as an exogenous policy parameter without attempting to derive its socially optimal level. Instead, the purpose of the model is to predict the effect of a change in the pollution

[13] In sector Y, the constant returns to scale are with respect to all three inputs L, K, and Z.

tax on equilibrium quantities and relative prices.

The model allows for pre-existing *ad valorem* taxes on all goods, but since the focus of our discussion is on environmental taxes (i.e., the tax on input Z), we follow Fullerton and Heutel (2007a) and ignore them in the discussion. That is, the only exogenous change is a rise in the tax on the dirty input, with all other taxes assumed to remain constant. As in Harberger (1962), government is assumed to use additional tax revenue to purchase the two goods in the same proportion as would households under the initial prices. A consequence of this assumption is that the change in the relative aggregate demands for consumption goods only depends on the change in their relative price. An alternative set of assumptions would be that tax revenue is redistributed to households, who have identical homothetic preferences.

We denote by L_X (resp. L_Y) the quantity of labor employed in sector X (resp. sector Y), K_X (resp. K_Y) the quantity of capital employed in sector X (resp.Y), p_X (resp. p_Y) the price of good X (resp. good Y), w (resp. r) the price of labor (resp. capital), and p_Z the price of Z (that is, the tax per unit of Z). Small relative changes in equilibrium variables are denoted with a "hat." For instance, $\hat{p}_Y \equiv \frac{dp_Y}{p_Y}$.

The model is solved by differentiating equilibrium conditions pertaining to production, consumption, and resource availability, yielding the following set of linear equations:[14]

$$\begin{aligned}
\hat{L}_X + \gamma_L \hat{L}_Y &= 0 && (1)\\
\hat{K}_X + \gamma_K \hat{K}_Y &= 0 && (2)\\
\hat{X} - \theta_{XL}\hat{L}_X - \theta_{XK}\hat{K}_X &= 0 && (3)\\
\hat{Y} - \theta_{YL}\hat{L}_Y - \theta_{YK}\hat{K}_Y - \theta_{YZ}\hat{Z} &= 0 && (4)\\
\hat{p}_X - \theta_{XL}\hat{w} - \theta_{XK}\hat{r} &= 0 && (5)\\
\hat{p}_Y - \theta_{YL}\hat{w} - \theta_{YK}\hat{r} &= \theta_{YZ}\hat{p}_Z && (6)\\
\hat{L}_X - \hat{K}_X + \sigma_X\hat{w} - \sigma_X\hat{r} &= 0 && (7)\\
\hat{L}_Y - \hat{Z} - \theta_{YL}(e_{LL} - e_{LZ})\hat{w} - \theta_{YK}(e_{LK} - e_{KZ})\hat{r} &= \theta_{YZ}(e_{LZ} - e_{ZZ})\hat{p}_Z && (8)\\
\hat{K}_Y - \hat{Z} - \theta_{YL}(e_{LK} - e_{LZ})\hat{w} - \theta_{YK}(e_{KK} - e_{KZ})\hat{r} &= \theta_{YZ}(e_{KZ} - e_{ZZ})\hat{p}_Z && (9)\\
\hat{X} - \hat{Y} + \sigma_u\hat{p}_X - \sigma_u\hat{p}_Y &= 0 && (10)
\end{aligned}$$

where $\gamma_L \equiv \frac{L_Y}{L_X} > 0$ and $\gamma_K \equiv \frac{K_Y}{K_X} > 0$ denote the resources allocated to sector Y relative to sector X, $\theta_{XL} \equiv \frac{wL_X}{p_X X}$ denotes the cost share of labor in sector X (and similarly for the parameters θ_{XK}, θ_{YL}, θ_{YK}, and θ_{YZ}), $\sigma_u \geq 0$ is the elasticity of substitution in consumption

[14]The formal derivations are shown in online appendix A.2.

between X and Y, $\sigma_X \geq 0$ is the elasticity of substitution between labor and capital in sector X, and the parameters e_{ij}, for $i, j \in \{L, K, Z\}$, represent Allen elasticities of substitution defined as $e_{ij} = \frac{a_{ij}}{\theta_{Yj}}$, where a_{ij} is the conditional input demand elasticity for input i with respect to the price of input j in sector Y. Microeconomic theory places restrictions on the acceptable values of the θ_{Xj}, θ_{Yj}, and e_{ij} parameters that are described in online appendix A.1.

Equations (1) and (2) pertain to resource use and imply that a change in a resource allocated to one sector must be offset by a change in the resource allocated to the other sector. Equations (3)–(6) are a consequence of profit maximization and constant returns to scale in each sector: effects on output are directly related to effects on inputs through the cost shares, for both quantities and prices. Equation (7) relates the change in the ratio of input demands in sector X to the change in the ratio of input prices using the substitution elasticity. Equations (8) and (9) represent the generalization of this relationship to the three inputs in sector Y, and thus feature the Allen substitution elasticities. Finally, Equation (10) relates the change in the ratio of goods consumed to the change in the ratio of their prices using the elasticity of substitution in consumption.

Given an exogenous change $\hat{p}_Z$, the system describing equilibrium displacement thus has 10 equations for 11 unknowns. Choosing a numeraire good adds the missing relationship, but changing the numeraire also changes the nature of the tax increase and therefore the variety of environmental policy considered. Thus, in the next section, we express the tax on pollution, $\hat{p}_Z$, relative to an explicit price index, thereby eliminating the dependency of comparative static results on numeraire choice.

3 Anchoring the environmental tax to a price index

Consider the following Cobb-Douglas price index:

$$\mathcal{P} = p_X^{\alpha_X} p_Y^{\alpha_Y} w^{\alpha_L} r^{\alpha_K}$$

with positive exponents and $\alpha_X + \alpha_Y + \alpha_L + \alpha_K = 1$ due to homogeneity of degree one. (Since prices are determined up to a multiplicative constant, the price index so defined is also determined up to the same multiplicative constant.) The relative change in $\mathcal{P}$ can then be expressed as

$$\hat{\mathcal{P}} = \alpha_X \hat{p}_X + \alpha_Y \hat{p}_Y + \alpha_L \hat{w} + \alpha_K \hat{r}.$$

If the price index $\mathcal{P}$ is used to anchor the pollution tax, then $p_Z = \mathcal{P} \tau_Z$, where τ_Z is now interpretable as an *ad valorem* tax relative to $\mathcal{P}$, and therefore

$$\hat{p}_Z = \hat{\mathcal{P}} + \hat{\tau}_Z.$$

Note that due to the equilibrium relationship $\hat{p}_X = \theta_{XL}\hat{w} + \theta_{XK}\hat{r}$, including p_X in the price index $\mathcal{P}$ is redundant. That is, any change in the weight on p_X can be exactly offset by changes in the weights on w and r, leaving $\hat{\mathcal{P}}$ unchanged. This is not the case for p_Y because $\hat{p}_Y = \theta_{YL}\hat{w} + \theta_{YK}\hat{r} + \theta_{YZ}\hat{\tau}_Z$, that is, unlike $\hat{p}_X$, $\hat{p}_Y$ implicitly includes $\hat{\tau}_Z$ independently of $\hat{w}$ and $\hat{r}$. Without further loss of generality, we can therefore focus on price indices of the form

$$\mathcal{P} = p_Y^{\beta}\left(w^{\alpha}r^{1-\alpha}\right)^{1-\beta} \tag{11}$$

where $0 \leq \alpha, \beta \leq 1$. If $\beta = 0$, then $\hat{\mathcal{P}} = \alpha\hat{w} + (1-\alpha)\hat{r}$ and the index reflects an average of factor prices, that is, the price of a combined labor-capital input. If $\alpha = \theta_{XL}$, then due to Equation (5) $\hat{\mathcal{P}} = \beta\hat{p}_Y + (1-\beta)\hat{p}_X$ and the index reflects an average of prices of consumption goods. Note that our specification of the price index includes as a special case the normalization made by Fullerton and Heutel (2007a), that is, $\mathcal{P} = p_X$ ($\beta = 0$, $\alpha = \theta_{XL}$). This choice means that the tax on pollution, say SO_2, is expressed as a fraction of the equilibrium price of the clean good, say services.[15] It also includes as special cases the choices of p_Y ($\beta = 1$), w ($\beta = 0$, $\alpha = 1$), or r ($\beta = 0$, $\alpha = 0$) as alternative anchoring prices. Importantly, anchoring the tax on the polluting input to the price of another good is not the same as taxing said good. For instance, consider tying the pollution tax to the price of the dirty good, so that $\hat{p}_Z = \hat{p}_Y + \hat{\tau}_Z$. Taxing good Y instead would create a wedge between the producer and consumer prices of good Y, a wedge that is not captured by Equations (1)–(10) and would lead to different comparative statics.

In what follows, we focus on price indices that reflect prices either on the uses side ($\alpha = \theta_{XL}$) or the sources side ($\beta = 0$). Although we could analyze each case separately, the price index in Equation (11) allows us to handle both cases within a single framework. We also focus on price indices that can be constructed from the observation of an initial equilibrium allocation and do not require knowledge of substitution elasticities, either in production or consumption. The idea is that since the anchoring price index corresponds to a particular policy choice, it is desirable for implementability to restrict the search to indices that can be designed with readily available economic information.

Note that due to the linear nature of our displacement model, the restriction to price

[15]Fullerton and Heutel (2007a) choose the normalization $\hat{p}_X = 0$ while setting $p_Z = \tau_Z$ (that is, τ_Z is a nominal tax), which given the equilibrium condition (5) is equivalent to setting $\beta = 0$ and $\alpha = \theta_{XL}$.

indices of the Cobb-Douglas form is made here without loss of generality. Consider for instance a CES price index on the uses side

$$\mathcal{P} = \left[\beta p_X^{\frac{\sigma-1}{\sigma}} + (1-\beta) p_Y^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

where $0 \leq \beta \leq 1$ and $\sigma \geq 0$. It is easy to show that

$$\hat{\mathcal{P}} = B\hat{p}_X + (1-B)\hat{p}_Y$$

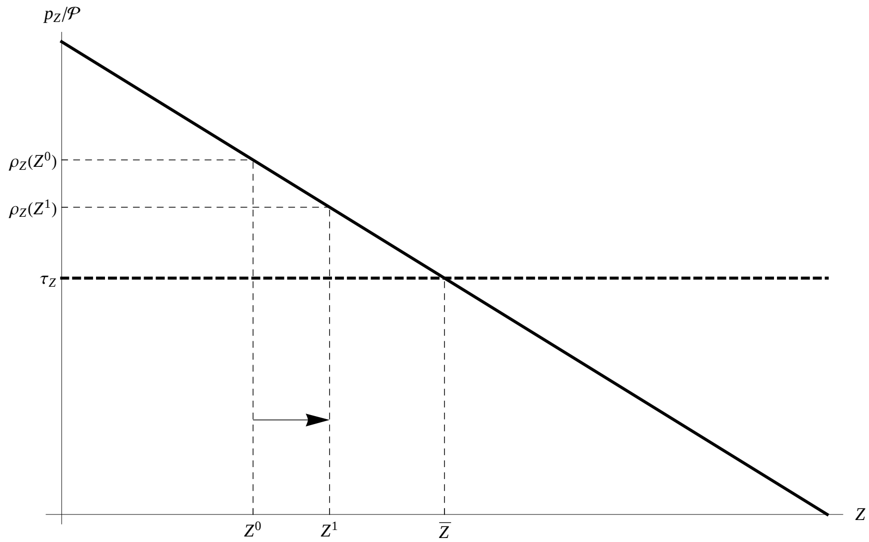
where $B \equiv \frac{\beta p_X^{\frac{\sigma-1}{\sigma}}}{\beta p_X^{\frac{\sigma-1}{\sigma}} + (1-\beta) p_Y^{\frac{\sigma-1}{\sigma}}}$. Therefore, a CES index would not meaningfully expand the set of acceptable indices, as the relative change in the value of the CES index is still reducible to a convex combination of relative price changes. As such, any restriction on the CES parameters β and σ to ensure a downward-sloping demand for the polluting input would necessarily be channelled through the Cobb-Douglas share B.

3.1 Stability of the competitive equilibrium with pollution tax

As explained in Samuelson (1941), "the problem of stability of equilibrium is intimately tied up with the problem of deriving fruitful theorems in comparative statics." Indeed, comparative static exercises consist in predicting how an equilibrium system responds to exogenous changes in external conditions; if equilibria are unstable, it is doubtful that changes in conditions will actually allow the new equilibrium to be reached, and thus the exercise becomes moot. As an analogy, consider the case of a match standing by its tip on a table, an unstable equilibrium in the sense that if the match is slightly tipped it will fall on its side and not return to its initial position. If the table is moved, there exists a new equilibrium where the match still stands by its tip on the now displaced table. But since neither the initial nor the final equilibria are stable, using that comparison to predict what the match would do upon displacement of the table is futile. In contrast, if the match initially lays on its side, the comparison of the two equilibria (before and after the table is moved) provides an informative prediction about what the match would do: it would simply be displaced in space following the table's movement while still laying on its side.

The concept of equilibrium stability requires a definition of the dynamics of the economy in order to infer whether a perturbation of the initial equilibrium would be followed by a return to said equilibrium over time. An equilibrium is said to be stable (in the small) if for sufficiently small displacements, all the variables approach their equilibrium values as time goes to infinity (Samuelson, 1941; Arrow and Hurwicz, 1958). Here we use the

Figure 1: The Marshallian adjustment process in the market for the dirty input



concept of Marshallian stability, as explained for instance in Samuelson (1941) or Plott and George (1992). Marshallian stability mandates that quantity adjusts over time to the difference between the demand and the supply prices of a good.[16]

Specifically, we consider the following Marshallian adjustment process: if at any point in time t, the quantity of polluting input $Z(t)$ is such that the demand price for that quantity (expressed relative to the price index $\mathcal{P}$) exceeds the supply price (that is, the pollution tax τ_Z), then the quantity must increase. The opposite holds in the case when the demand price is less than the supply price. Figure 1 shows a graphical representation of such adjustment process in the normal case of a downward-sloping derived demand, assuming that in the initial condition the quantity (Z^0) is less than the equilibrium value ($\bar{Z}$). Because the derived demand for the input slopes down, the equilibrium is stable: the adjustment process brings quantity closer to its equilibrium value over time. Had the derived demand sloped up, the equilibrium would have been Marshallian unstable.

A simple mathematical representation of the Marshallian process posits the following dynamic relationship:

$$\frac{dZ}{dt} = \rho_Z(Z) - \tau_Z$$

where $\rho_Z(Z)$ represents the inverse demand function for the polluting input, expressed relative to the price index $\mathcal{P}$. Following Samuelson (1941), we can use a first-order

[16] In contrast, Walrasian stability mandates that price adjusts to excess demand, that is, the difference between quantity demanded and quantity supplied. In our model with horizontal supply of polluting input, the quantity of polluting input supplied is not a well-defined function of price, which precludes the use of Walrasian stability.

expansion of the function $\rho_Z(Z)$ around the equilibrium point $\bar{Z}$ to obtain

$$\frac{dZ}{dt} = \rho'_Z(\bar{Z})\left(Z - \bar{Z}\right)$$

where we have made use of $\rho_Z(\bar{Z}) = \tau_Z$. The solution to this differential equation, together with the initial condition $Z = Z^0$, is simply $Z(t) = \bar{Z} + (Z^0 - \bar{Z})e^{\rho'_Z(\bar{Z})t}$, implying that $\lim_{t\to\infty} Z(t) = \bar{Z}$ if and only if $\rho'_Z(\bar{Z}) \leq 0$, that is, the demand for pollution slopes down. This result establishes the fact that whenever the derived demand for pollution slopes up, the equilibrium cannot be stable in the Marshallian sense. In what follows, we characterize price indices $\mathcal{P}$ that ensure that the derived demand for pollution slopes down, that is, the equilibrium is stable in the Marshallian sense.

3.2 Desirable properties of a price index

We are looking for weights α, β on the price index in Equation (11) such that the following three conditions are satisfied:

Condition 1 *The pollution demand elasticity* $\frac{\hat{Z}}{\hat{\tau}_Z}$ *is defined for all parameter values, that is, the determinant of the equilibrium system is never zero and thus does not change sign.*

Condition 2 *The weights* α *and* β *only depend on the observable parameters* γ_K*,* γ_L*,* θ_{XK}*,* θ_{YK}*, and* θ_{YL}*, or combinations thereof.*

Condition 3 *An increase in the pollution tax decreases pollution for all parameter values, that is,* $\frac{\hat{Z}}{\hat{\tau}_Z} \leq 0$.

Condition 1 is a technical condition that ensures that the problem of tax incidence is well-posed. Condition 2 puts limits on the informational requirements needed for the design of the price index. Specifically, static equilibrium shares pertaining to the initial allocation of resources in the economy (γ_L and γ_K) and the industry factor shares (θ_{XK}, θ_{YK}, and θ_{YL}) are allowed, but second-order parameters that are less easily observed (such as substitution elasticities) are excluded. Finally, Condition 3 captures our essential requirement that the law of demand always be satisfied.

3.3 Derivation of the price indices

Our main results regarding the existence and uniqueness of price indices that avoid violations of the law of input demand derive from the following proposition.

Proposition 1 *Condition 1 and Condition 2 imply the following restriction:*

$$[\beta\theta_{YK} + (1-\beta)(1-\alpha)]\,\theta_{XL}(1+\gamma_L) = [\beta\theta_{YL} + (1-\beta)\alpha]\,\theta_{XK}(1+\gamma_K), \tag{12}$$

while Condition 3 is automatically satisfied as long as Condition 1 is.

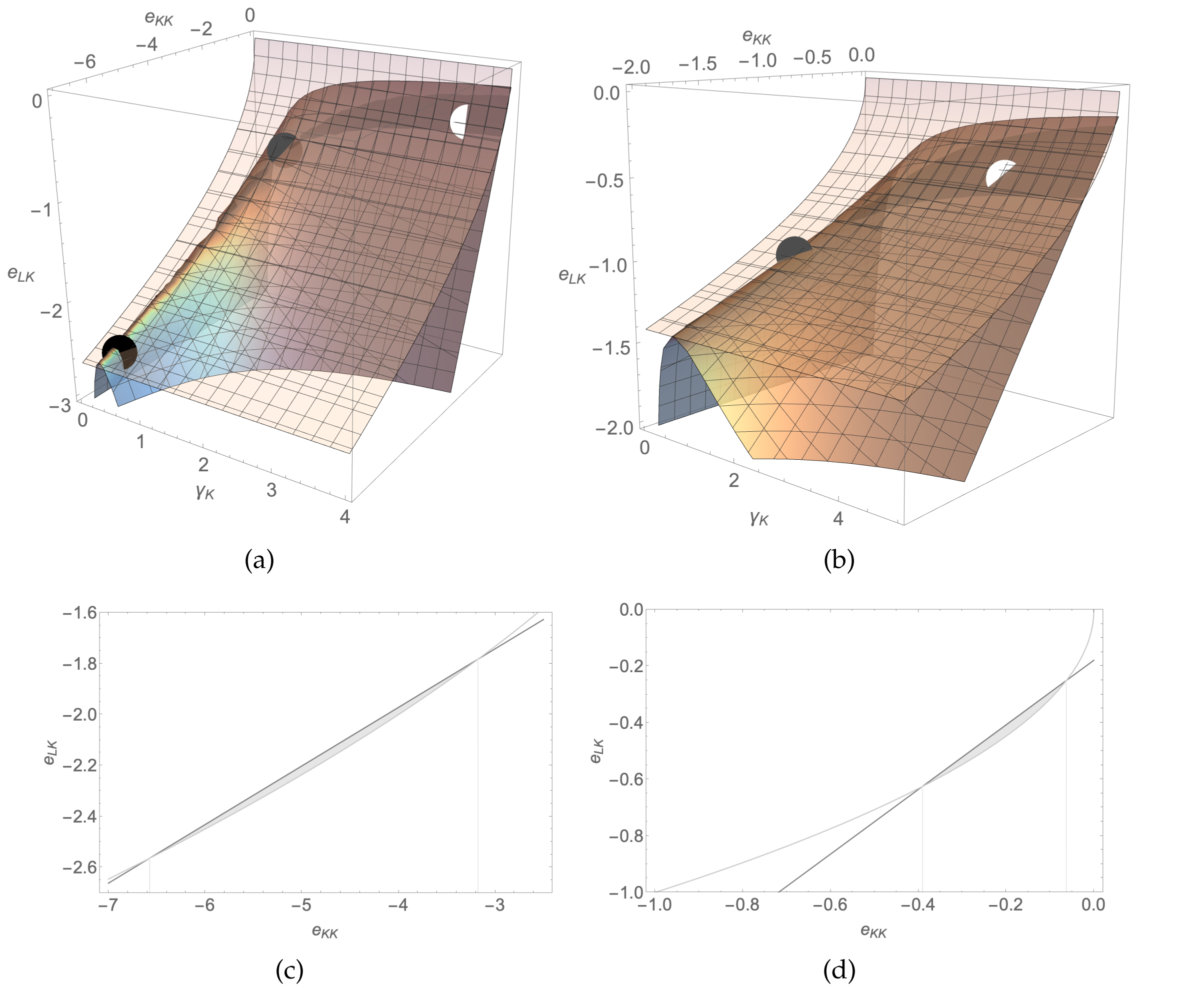
The formal proof can be found in online appendix A.3.[17] There, we show that the choice of (α, β) affects the elasticity $\frac{\hat{Z}}{\hat{\tau}_Z}$ only through its effect on the determinant of the equilibrium system. Equation (12) is shown to be both necessary and sufficient for this determinant to have a constant sign for all possible parameter values. Specifically, we show that if σ_X and σ_u are small enough, values of the Allen elasticities of substitution (e_{LL}, e_{KK}, e_{LK}) may lead to a sign reversal, even when the restrictions from theory are imposed, unless Equation (12) holds. Importantly, the sign reversal requires labor and capital to be complementary in the production of good Y ($e_{LK} < 0$), irrespective of the choice of anchoring price index. Note that when it happens, the reversal in the sign of the determinant of the equilibrium system causes all comparative static results, not just the effect on pollution, to be reversed relative to the normal case. In online appendix B, we further show that there cannot be a sign reversal in a model with a single clean input.

Equation (12) is violated in Fullerton and Heutel (2007a)'s model where $\beta = 0$ and $\alpha = \theta_{XL}$, except in the special case of equal factor intensities in the two sectors ($\gamma_L = \gamma_K$). Consequently, if p_X is chosen as anchoring price there exists a nonempty subset of the parameter space for which all comparative statics are reversed. This subset is depicted in Figure 2 for the special case where $\sigma_X = \sigma_u = 0$, $\gamma_L = 1$, and $e_{LL} = -1$. In panels (a) and (b), the set of acceptable values of $(\gamma_K, e_{KK}, e_{LK})$ supporting the sign reversal for the determinant of the equilibrium system is shown as the region located above the translucent surface and below the gray surface.[18] The choice $e_{LL} = -1$ is a normalization that only affects the scale of e_{KK} and e_{LK}. As such, the values of e_{KK} and e_{LK} can be reinterpreted as those of $\frac{e_{KK}}{|e_{LL}|}$ and $\frac{e_{LK}}{|e_{LL}|}$, respectively. Points located below the translucent surface violate the restrictions from theory and thus do not represent acceptable parameterizations. Specifically, the negative semidefiniteness of the Slutsky matrix implies that $e_{LL}e_{KK} \geq e_{LK}^2$, which in this case imposes that $|e_{LK}| \leq \sqrt{-e_{KK}}$. Within the relevant subspace located above the translucent surface, the parameter region located below the gray surface supports the determinant sign reversal and attendant violation of

[17] Equation (12) is similar to Equation (A-17) in online appendix A.3.

[18] Using the notation of online appendix A.3, this parameter subspace supports $\Delta = C_3 < 0$. Note that when $\beta = 0$ and $\alpha = \theta_{XL}$, $B_L = \theta_{XL}$ and $B_K = \theta_{XK}$. The sign of Δ is then independent of the values of the cost shares, conditional on the choices of γ_L and γ_K.

Figure 2: Violations of the law of input demand with p_X as anchoring price



Note: We set $\sigma_X = \sigma_u = 0$, $\gamma_L = 1$, $e_{LL} = -1$, and assume $e_{LK} \leq 0$. Panels (a) and (b) depict the same region over different parameter ranges. Points located below the translucent surface violate the restrictions from theory. Within the relevant subspace located above the translucent surface, the parameter region located below the gray surface supports violations of the law of input demand. Panel (c) represents the parameter region supporting such violation along the section $\gamma_K = 0.39$ as the shaded area; panel (d) represents it along the section $\gamma_K = 4$.

the law of input demand. The figure shows that this sign reversal is more likely to happen with either small values of the capital intensity γ_K paired with large magnitudes of the Allen elasticities e_{KK} and e_{LK} relative to e_{LL} (black dot), or large values of γ_K paired with small magnitudes of e_{KK} and e_{LK} relative to e_{LL} (white dot). The gray dot corresponds to the numerical example given in online appendix A.6, where γ_K, $|e_{KK}|$, and $|e_{LK}|$ are all close to (but different than) one. Panels (c) and (d) represent sections of the three-dimensional parameter subspace supporting violations along specific values of γ_K. Similarly shaped violation regions exist for alternative values of γ_L. Online appendix A.7 depicts the sign reversal region when p_Y, rather than p_X, is used as the anchoring price.

Although the region depicted in Figure 2 assumes $\sigma_X = \sigma_u = 0$,[19] there also exist parameter values that lead to a sign reversal when $\sigma_X > 0$ and $\sigma_u > 0$. An example is given in online appendix A.6. That being said, the analysis of online appendix A.3 implies that positive values of σ_X (and σ_u) make violations of the law of input demand less likely, everything else equal. This is perhaps reassuring given empirical evidence on labor-capital substitution, which generally points to values strictly greater than zero and sometimes close to one (Thompson and Taylor, 1995; Klump et al., 2012).

We may now specialize the restriction in Equation (12) to the case of indices on the uses side.

Corollary 1 *On the uses side ($\alpha = \theta_{XL}$), Equation* (12) *implies that either $\gamma_L = \gamma_K$ or $\beta = \frac{\gamma_L \gamma_K}{\gamma_L \gamma_K + \gamma_K \theta_{YL} + \gamma_L \theta_{YK}} = \frac{p_Y Y}{p_X X + p_Y Y}$, that is, the desired price index is*

$$\mathcal{P}^{uses} = p_X^{\theta_X} p_Y^{\theta_Y} \tag{13}$$

where $\theta_X \equiv \frac{p_X X}{p_X X + p_Y Y}$ represents the initial expenditure share on good X and $\theta_Y = 1 - \theta_X$.

No other choice of weights will guarantee that the law of input demand holds for all parameter values. In particular, neither p_X nor p_Y as choices of anchoring indices would allow one to rule out upward-sloping demand for the dirty input.[20] Thus, the choice of

[19]If we allow either σ_X or σ_u to be nonzero, we can no longer represent the violation region on a three-dimensional graph, because the parameters θ_{YL}, θ_{YK}, σ_X, and σ_u also determine the sign of the equilibrium system determinant.

[20]Condition 2 is key in ensuring that Equation (12) is not only sufficient, but also necessary for ruling out sign reversals for all parameter values. Imagine for instance that we allowed (α, β) to be indexed on the entire set of parameters. Then, the following uses-side index would trivially avoid violations of the law of input demand:

$$\mathcal{P} = \begin{cases} p_X & \text{if parameters are such that the law of input demand is satisfied} \\ p_X^{\theta_X} p_Y^{\theta_Y} & \text{otherwise} \end{cases} .$$

numeraire would matter when modeling a nominal tax on pollution, as the comparative statics with respect to $\hat{p}_Z$ may have different signs (and different magnitudes) across different normalizations. Of course, this dependency of comparative statics on numeraire choice breaks down if the pollution tax is anchored to an explicit price or price index. But then the choice of anchoring price matters, in the exact same way that the choice of numeraire matters for the nominal pollution tax.

The following corollary addresses the case of price indices on the sources side.

Corollary 2 *On the sources side ($\beta = 0$), Equation* (12) *implies that* $\alpha = \frac{\theta_{XL}(1+\gamma_L)}{\theta_{XL}(1+\gamma_L)+\theta_{XK}(1+\gamma_K)} = \frac{w\bar{L}}{w\bar{L}+r\bar{K}}$*, that is, the price index is*

$$\mathcal{P}^{sources} = w^{\theta_L} r^{\theta_K} \tag{14}$$

where $\theta_L \equiv \frac{w\bar{L}}{w\bar{L}+r\bar{K}}$ *represents the initial contribution of labor to national income and* $\theta_K = 1 - \theta_L$*.*

Again, these weights are uniquely defined.

3.4 Equivalence between our price indices and inflation indices

Inflation is typically measured using a price index that reflects the overall cost of a reference basket of goods under varying prices. Consider that the reference basket is the consumption basket before the change in the pollution tax. Our preferred pollution tax is anchored to the price index $\mathcal{P}^{\text{uses}} = p_X^{\theta_X} p_Y^{\theta_Y}$ where θ_X and θ_Y are the reference budget shares. When prices change in the economy, the change in our price index is:

$$\begin{aligned}
\hat{\mathcal{P}}^{\text{uses}} &= \theta_X \hat{p}_X + \theta_Y \hat{p}_Y \\
&= \frac{p_X X}{p_X X + p_Y Y}\frac{dp_X}{p_X} + \frac{p_Y Y}{p_X X + p_Y Y}\frac{dp_Y}{p_Y} \\
&= \frac{X dp_X + Y dp_Y}{p_X X + p_Y Y} \\
&= \frac{d\text{CPI}}{\text{CPI}} \\
&= \widehat{\text{CPI}}
\end{aligned}$$

where $\text{CPI} \equiv p_X X + p_Y Y$ represents the value of the reference basket. Therefore, expressing the pollution tax as an *ad valorem* tax anchored to $\mathcal{P}^{\text{uses}}$ is equivalent to adjusting a nominal tax on pollution for inflation as measured by the CPI.[21]

[21]In large CGE models, carbon taxes are typically deflated using a GDP deflator called "PGDP," the evolution of which is computed based on reference quantities. Given our assumption that government purchases goods in the same proportion as households and that there is no investment or exports, the

A similar argument can be used to demonstrate that our price index on the sources side represents a producer price index, where the weights on factor prices correspond to the reference shares of each primary factor in national income.

4 Simple tax incidence

4.1 Sources side

The following proposition, proven in online appendix A.4, addresses tax incidence on the sources side.

Proposition 2 *Whenever Equation* (12) *holds,*

$$sign\left\{\frac{\hat{w}-\hat{r}}{\hat{\tau}_Z}\right\} = sign\left\{\sigma_u(\gamma_K-\gamma_L)+\gamma_L(1+\gamma_K)e_{LZ}-\gamma_K(1+\gamma_L)e_{KZ}\right\}.$$

Which sector bears proportionately more of the tax burden therefore depends on the consumption elasticity σ_u, the resource allocation parameters γ_L and γ_K, and the partial substitution elasticities e_{LZ} and e_{KZ}, but not on the cost shares in either sector or the substitution elasticity in the untaxed sector.[22]

These few parameters affect the direction of the change in the price of labor relative to capital in an intuitive yet subtle way. If labor and capital are equally substitutable for pollution ($e_{LZ} = e_{KZ} > 0$), then the pollution tax places a disproportionate burden on capital (i.e., $\hat{w} - \hat{r} > 0$) when the polluting sector is capital intensive ($\gamma_K > \gamma_L$) as long as goods are sufficiently substitutable in consumption (i.e., $\sigma_u > e_{LZ} = e_{KZ}$). If not ($\sigma_u < e_{LZ}$), then the input substitution effect dominates and the demand for capital in the capital-intensive polluting sector rises so that the return to capital increases relative to the wage rate. If both sectors are equally capital-intensive (i.e., $\gamma_L = \gamma_K$), then capital bears disproportionately more of the tax burden if and only if it is less substitutable for the polluting input than is labor.

These economic insights were already discussed, albeit for specific classes of model parameterizations, in the original contribution of Fullerton and Heutel (2007a). Indeed, the authors were prevented from drawing general conclusions by the fact that they could not sign the main equilibrium system denominator except in special cases, which as we

changes in the CPI and PGDP indices are the same, that is, $\hat{\mathcal{P}}^{\text{uses}} = \widehat{\text{CPI}} = \widehat{\text{PGDP}}$. We thank an anonymous referee for pointing this to us.

[22]The size of $\frac{\hat{w}-\hat{r}}{\hat{\tau}_Z}$ itself depends on the full set of model parameters and the choice of anchoring index, see Section 5 and online appendix D.

have shown is a direct consequence of modeling the pollution tax increase relative to the price of good X.

4.2 Uses side

The following proposition, proven in online appendix A.5, addresses tax incidence on the uses side.

Proposition 3 *Whenever Equation* (12) *holds,*

$$
\begin{aligned}
sign\left\{\frac{\hat{p}_Y - \hat{p}_X}{\hat{\tau}_Z}\right\} &= sign\left\{\sigma_X(1+\gamma_L\theta_{XL}+\gamma_K\theta_{XK}) + (\theta_{YL}\gamma_K(1+\gamma_L)+\theta_{YK}\gamma_L(1+\gamma_K))\, e_{LK}\right. \\
&\quad \left. +\gamma_L(1+\gamma_K)(\theta_{XK}-\theta_{YK})e_{LZ} + \gamma_K(1+\gamma_L)(\theta_{XL}-\theta_{YL})e_{KZ}\right\}.
\end{aligned}
$$

As for incidence on the sources side, this expression has an ambiguous sign.[23] However, we show in online appendix A.5 that in the case of equal factor intensities ($\gamma_L = \gamma_K$), $\frac{\hat{p}_Y - \hat{p}_X}{\hat{\tau}_Z} > 0$, that is, users of Y share proportionately more tax burden than users of X irrespective of the values of the substitution elasticities.

Although one may legitimately expect the price of the dirty good to rise relative to that of the clean good when the pollution tax increases (Rausch et al., 2011; Fullerton and Muehlegger, 2019), the opposite may happen depending on the patterns of factor intensity and input substitution in the two industries. For instance, consider the following parameter values, which lead to $\frac{\hat{p}_Y - \hat{p}_X}{\hat{\tau}_Z} < 0$: $\theta_{YL} = 0.5$, $\theta_{YK} = 0.4$, $\gamma_L = 1$, $\gamma_K = 0.25$, $e_{LK} = 1$, $e_{LZ} = -1.9$, $e_{KZ} = 6$, and $\sigma_X = 0.5$. In that case, capital is a better substitute for pollution in the dirty industry ($e_{KZ} > e_{LZ}$). This causes the price of capital to rise relative to that of labor when the pollution tax increases ($\hat{r} - \hat{w} > 0$). Sector X being capital-intensive relative to sector Y ($\frac{\gamma_K}{\gamma_L} = 0.25$), and having more limited substitution possibilities ($\sigma_X = 0.5$ compared to the large magnitudes of e_{LK}, e_{LZ}, and e_{KZ}[24]), the price of good X rises relative to that of good Y ($\hat{p}_X - \hat{p}_Y > 0$). Note that demand conditions (σ_u) play no role in determining the sign of $\hat{p}_Y - \hat{p}_X$, although they play a role in determining

[23]This ambiguity comes in contrast to the effect of a partial factor tax in the standard Harberger model, which unambiguously leads to a relative increase in the price of the good produced by the taxed sector (Mieszkowski, 1967). The replacement of the partial factor tax by a tax on a third, non-market input explains this difference. Fullerton and Heutel (2007a) do not discuss it explicitly, because the special cases they analyze ($\gamma_L = \gamma_K$ and $e_{LK} = e_{LZ} = e_{KZ}$) imply unequivocal increases in the price of good Y relative to that of good X.

[24]The values of e_{LK}, e_{LZ}, and e_{KZ} are not independent due to restrictions pertaining to the negative semidefiniteness of the Slutsky matrix. However, the overall magnitude of theses elasticities is independent of that of σ_X.

the magnitude of the effect.[25] Also note that it is not necessary for one of the cross-price Allen elasticities of substitution to be negative for the price of good X to rise relative to that of good Y; for instance, the following set of parameter values generates comparable effects: $\theta_{YL} = 0.5$, $\theta_{YK} = 0.4$, $\gamma_L = 4$, $\gamma_K = 1$, $e_{LK} = 1$, $e_{LZ} = 1$, $e_{KZ} = 8$, and $\sigma_X = 0.5$. In online appendix A.5, we provide graphical depictions of the parameter region that supports $\frac{\hat{p}_Y - \hat{p}_X}{\hat{\tau}_Z} \leq 0$.

The following proposition, proven in online appendix A.5, generalizes the intuition developed in the examples above, by stating that instances whereby p_Y decreases relative to p_X require (i) the factor used more intensively in sector X to be a better substitute for pollution than the other factor, and (ii) the price of the factor used more intensively in sector X to increase relative to the price of the other factor. Hence, this proposition makes an explicit link between incidence on the sources side and incidence on the uses side.[26]

Proposition 4 *If* $\gamma_L > \gamma_K$, *then* $\frac{\hat{p}_Y - \hat{p}_X}{\hat{\tau}_Z} < 0 \Rightarrow \left\{ e_{KZ} > e_{LZ} \text{ and } \frac{\hat{w} - \hat{r}}{\hat{\tau}_Z} < 0 \right\}$. *Similarly, if* $\gamma_L < \gamma_K$, *then* $\frac{\hat{p}_Y - \hat{p}_X}{\hat{\tau}_Z} < 0 \Rightarrow \left\{ e_{LZ} > e_{KZ} \text{ and } \frac{\hat{w} - \hat{r}}{\hat{\tau}_Z} > 0 \right\}$.

Importantly, Proposition 4 does not imply that incidence on the sources side dictates incidence on the uses sides, as the implications are unidirectional. For instance, if sector Y is labor-intensive, p_Y may only decrease relative to p_X if w decreases relative to r, but this latter condition alone is not sufficient, even if $\sigma_X = 0$. The proposition does imply, however, that whenever the price of Y decreases relative to that of X, incidence on the sources side is entirely determined by the relative factor intensity.

5 Does the choice of anchoring price matter in practice?

In the previous sections, we have formally shown how comparative statics for an *ad valorem* pollution tax depend, from an analytical standpoint, on the choice of anchoring price. Consequently, comparative statics for a nominal tax, as in the original model, depend on the choice of numeraire. Online appendix A.6 also provides an example whereby using p_X as anchoring price leads to the prediction that an increase in the pollution tax increases pollution, whereas the use of our anchoring indices would lead to a decrease in pollution.

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[25]Similarly, substitution possibilities in sector X (σ_X) play no role in determining the sign of the incidence on the sources side.

[26]Although Proposition 4 may seem intuitive, the proof is far from trivial. In online appendix A.5, we actually show a slightly stronger statement than that reported in the proposition, namely that if $\gamma_L > \gamma_K$, then $\frac{\hat{p}_Y - \hat{p}_X}{\hat{\tau}_Z} < 0 \Rightarrow \left\{ e_{KZ} > \frac{\gamma_L(1+\gamma_K)}{\gamma_K(1+\gamma_L)} e_{LZ} \text{ and } \frac{\hat{w} - \hat{r}}{\hat{\tau}_Z} < 0 \right\}$. That latter statement is stronger because $\frac{\gamma_L(1+\gamma_K)}{\gamma_K(1+\gamma_L)} > 1$ for $\gamma_L > \gamma_K$.

Table 1: Model parameters

Parameter	Value
e_{LK}	1
σ_u	1
σ_X	1
γ_L	0.25
γ_K	0.25
θ_{XL}	0.60
θ_{YL}	0.45
θ_{YK}	0.30

Note: Once we have set γ_L, γ_K, θ_{YL}, and θ_{YK}, θ_{XL} is determined by $\theta_{XL} = \frac{\gamma_K \theta_{YL}}{\gamma_K \theta_{YL} + \gamma_L \theta_{YK}}$.

Importantly, the signs of the predicted tax incidence on the sources and uses sides would also be inconsistent for that parameterization between the two choices of anchoring prices (i.e., p_X versus our proposed price indices). Intuitively, cases whereby a rise in the pollution tax relative to p_X (or any other price) leads to reversed comparative statics precisely correspond to cases whereby the value of the pollution tax actually decreases relative to our proposed price indices.

Here, we investigate whether the choice of anchoring price index $\mathcal{P}$ matters in practice, that is, for reasonable model parameterizations reflecting existing economies. This is the same as asking whether, for a nominal pollution tax, the choice of numeraire matters. We use the model parameterization of Fullerton and Heutel (2007a) for the US economy, which is close to that used in Fullerton and Heutel (2010b). In these papers, the polluting sector is defined by selecting polluting industries based on the EPA's Toxic Release Inventory for 2002. We do not limit our analysis to a comparison of our price indices $\mathcal{P}^{\text{uses}}$ and $\mathcal{P}^{\text{sources}}$ to theirs (p_X). Instead, we broaden the scope of the analysis by also considering the following indices: p_Y, w, r. Other indices could be defined, however we believe that the set of chosen indices affords sufficient insights into the empirical question. Baseline parameter values are given in Table 1. All parameter values are fixed, except for the Allen cross-price elasticities e_{KZ} and e_{LZ} which are allowed to take on the values $\{-0.5, 0.0, 0.5, 1.0\}$. We exclude pairs of elasticities that lead to violations of the negative semidefiniteness of the Slutsky matrix in sector Y. Because it is assumed that $\gamma_L = \gamma_K$, $\theta_{XL} = \theta_L$ and therefore the index p_X leads to the exact same results as the index $\mathcal{P}^{\text{sources}}$. This assumption is relaxed in additional simulations reported in online appendix D.2.

Table 2 shows the pollution effects relative to baseline of a 10% increase in the pollution tax relative to various anchoring prices. For instance, when $e_{KZ} = e_{LZ} = 0$, a 10% increase

Table 2: Pollution effect of a 10% increase in the pollution tax relative to various prices

e_{KZ}	e_{LZ}	$\hat{Z}$ (%) anchoring price: p_X ($\mathcal{P}^{\text{sources}}$)	p_Y	$\mathcal{P}^{\text{uses}}$	w	r	st. dev.
0.0	0.0	-2.00	-2.67	-2.13	-2.00	-2.00	0.24
0.5	0.0	-3.58	-4.77	-3.81	-3.54	-3.63	0.43
1.0	0.0	-5.10	-6.80	-5.44	-5.00	-5.26	0.62
-0.5	0.5	-2.70	-3.60	-2.88	-2.76	-2.62	0.33
0.0	0.5	-4.38	-5.83	-4.67	-4.42	-4.31	0.54
0.5	0.5	-6.00	-8.00	-6.40	-6.00	-6.00	0.73
1.0	0.5	-7.58	-10.10	-8.08	-7.50	-7.69	0.92
-0.5	1.0	-4.97	-6.63	-5.31	-5.14	-4.75	0.62
0.0	1.0	-6.70	-8.93	-7.15	-6.84	-6.50	0.83
0.5	1.0	-8.38	-11.17	-8.93	-8.46	-8.25	1.02
1.0	1.0	-10.00	-13.33	-10.67	-10.00	-10.00	1.22

Note: Since $\gamma_L = \gamma_K$, $\theta_{XL} = \theta_L$ and therefore the index p_X leads to the same results as the index $\mathcal{P}^{\text{sources}}$. The standard deviation is calculated accounting for both indices.

in the pollution tax relative to p_X would lead to a 2% decrease in pollution, compared to a 2.67% (resp. 2.13%) decrease in pollution if the anchoring price is chosen to be p_Y (resp. $\mathcal{P}^{\text{uses}}$). More generally, the results in the table show that the choice of anchoring price can matter for the calculation of the predicted effect of a 10% increase in the pollution tax, even if there is no reversal in sign.[27] For a given model parameterization, the standard deviation of predicted pollution impacts across choices of anchoring price lies in excess of 10% of the impact estimates obtained with the CPI ($\mathcal{P}^{\text{uses}}$) or the factor price index ($\mathcal{P}^{\text{sources}}$). The largest discrepancies are found when comparing the use of the wage rate or the rental on capital to that of the dirty good (good Y) as anchoring prices. In such comparisons, predicted pollution effects ($\hat{Z}$) differ by up to about 40%, and the choice of anchoring price leads to variations in predictions that are often comparable to those arising from alternative sets of cross-price elasticities. Predictions obtained using p_X as anchoring price are relatively close to those obtained with the index $\mathcal{P}^{\text{uses}}$. Importantly, alternative choices of anchoring price imply alternative *varieties* of pollution tax: while the increase in the tax rate is held at 10%, the base to which the rate applies is changing. The point here is that a modeler simulating the effect of a nominal tax may assume that the choice of numeraire, which becomes the *de facto* anchoring price, is largely innocuous;

[27] As indicated in Section 3.3, a reversal in sign is precluded by the assumption that $e_{LK} > 0$, that is, labor and capital are substitutes in the production of good Y.

Table 3: Incidence effects of a 10% increase in the pollution tax relative to various prices

e_{KZ}	e_{LZ}	Sources side: $\hat{w} - \hat{r}$ (%) anchoring price: p_X ($\mathcal{P}^{\text{sources}}$)	p_Y	$\mathcal{P}^{\text{uses}}$	w	r	st. dev.	Uses side: $\hat{p}_Y - \hat{p}_X$ (%) anchoring price: p_X ($\mathcal{P}^{\text{sources}}$)	p_Y	$\mathcal{P}^{\text{uses}}$	w	r	st. dev.
0.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	2.50	3.33	2.67	2.50	2.50	0.30
0.5	0.0	-0.26	-0.35	-0.28	-0.26	-0.26	0.03	2.50	3.33	2.67	2.47	2.54	0.30
1.0	0.0	-0.51	-0.68	-0.54	-0.50	-0.53	0.06	2.50	3.33	2.67	2.45	2.58	0.30
-0.5	0.5	0.53	0.71	0.56	0.54	0.51	0.07	2.50	3.33	2.67	2.55	2.42	0.31
0.0	0.5	0.26	0.35	0.28	0.26	0.26	0.03	2.50	3.33	2.67	2.53	2.46	0.31
0.5	0.5	0.00	0.00	0.00	0.00	0.00	0.00	2.50	3.33	2.67	2.50	2.50	0.30
1.0	0.5	-0.25	-0.34	-0.27	-0.25	-0.26	0.03	2.50	3.33	2.67	2.48	2.54	0.30
-0.5	1.0	0.79	1.05	0.84	0.81	0.75	0.10	2.50	3.33	2.67	2.58	2.39	0.31
0.0	1.0	0.52	0.69	0.55	0.53	0.50	0.06	2.50	3.33	2.67	2.55	2.43	0.31
0.5	1.0	0.25	0.34	0.27	0.26	0.25	0.03	2.50	3.33	2.67	2.53	2.46	0.31
1.0	1.0	0.00	0.00	0.00	0.00	0.00	0.00	2.50	3.33	2.67	2.50	2.50	0.30

Note: Since $\gamma_L = \gamma_K$, $\theta_{XL} = \theta_L$ and therefore the index p_X leads to the same results as the index $\mathcal{P}^{\text{sources}}$. The standard deviation is calculated accounting for both indices.

our simulations show that it need not be. To further illustrate this point, online appendix D.1 derives the explicit tax increases that, if applied to each candidate price or price index, would be equivalent to a 10% increase in the tax anchored to $\mathcal{P}^{\text{uses}}$.

Note that the choice of p_Y as the anchoring price could seem natural to a modeler. Indeed, when modeling the effects of taxes on market goods, e.g., the capital tax considered in Harberger (1962), the default choice would be to anchor the tax to the market price of the good subject to taxation (i.e., an *ad valorem* tax on the own price).[28] In the absence of a market price for the dirty input, one could easily be tempted to tie the pollution tax to the price of the dirty good—the only good produced using that input. For instance, one could imagine anchoring a tax on SO_2 emissions to the price of electricity. The results of Table 2 would caution against that choice if the policy being modeled is one that indexes the pollution price on, say, the CPI.

Effects on incidence mirror those on pollution, although the size of the effects themselves is smaller. Still, for some model parameterizations, the incidence of the tax is shown to differ qualitatively between a model that uses w or r as anchoring price and one that uses p_Y. For instance, for $e_{KZ} = -0.5$ and $e_{LZ} = 1.0$, using r yields a predicted increase in $\frac{w}{r}$ (resp. $\frac{p_Y}{p_X}$) of 0.75% (resp. 2.39%), versus an increase of 1.05% (resp. 3.33%) when using p_Y. On the uses side, the largest source of variation in predicted effects is the choice of anchoring price, not the choice of Allen cross-price substitution elasticities.

An intuitive explanation as to why the choice of p_Y leads to pollution and incidence

[28]In online appendix C, we show that such a choice, while innocuous when modeling the effect of a marginal tax, can become problematic when modeling an increase in an existing tax.

effects that are larger in magnitude than those from other prices, notably $\mathcal{P}^{\text{uses}}$, is that the tax causes p_Y to rise relative to $\mathcal{P}^{\text{uses}}$ ($\hat{p}_Y > \hat{\mathcal{P}}^{\text{uses}}$). As a result, a given rise in the *ad valorem* tax rate will have larger real effects if the anchoring price is p_Y rather than $\mathcal{P}^{\text{uses}}$. Mathematically, with p_Y as anchoring price the relative change in the pollution price is $\hat{p}_Z = \hat{\tau}_Z + \hat{p}_Y$, versus $\hat{p}_Z = \hat{\tau}_Z + \hat{\mathcal{P}}^{\text{uses}}$ with $\mathcal{P}^{\text{uses}}$ as anchoring price. Relative to $\mathcal{P}^{\text{uses}}$, the pollution price thus rises by $\hat{\tau}_Z + \hat{p}_Y - \hat{\mathcal{P}}^{\text{uses}}$ in the first instance versus $\hat{\tau}_Z + \hat{\mathcal{P}}^{\text{uses}} - \hat{\mathcal{P}}^{\text{uses}} = \hat{\tau}_Z$ in the second one. Note, however, that p_Y need not rise relative to $\mathcal{P}^{\text{uses}}$, even if it does so for the parameterizations presented here. As explained in Section 4.2, p_Y may actually *decrease* relative to p_X with the pollution tax. Whenever this happens, it is also the case that $\hat{p}_Y < \hat{\mathcal{P}}^{\text{uses}}$, because $\hat{p}_Y - \hat{\mathcal{P}}^{\text{uses}} = \hat{p}_Y - (\theta_X \hat{p}_X + \theta_Y \hat{p}_Y) = \theta_X (\hat{p}_Y - \hat{p}_X) < 0$. As a result, effects can be more pronounced with $\mathcal{P}^{\text{uses}}$ than with p_Y as anchoring price.

In online appendix D.2, we report additional results that hold constant all Allen substitution elasticities but allow factor intensities, as captured by $\gamma_K - \gamma_L$, to vary. Again we follow the model parameterizations investigated in Fullerton and Heutel (2007a). Our results confirm the importance of the choice of anchoring price for predictions on pollution outcomes and tax incidence. Notably, the variation in predictions induced by such choice is not dwarfed by that arising from the choice of factor intensities, and even exceeds it in the case of incidence on the uses side.

In online appendix D.3, we report results for a synthetic model that borrows the parameter values from Table 1, but assumes that pollution taxation is higher in the baseline, perhaps reflecting more advanced stages of environmental policy. Specifically, we assume that environmental taxes represent a cost share of 50% instead of 25% in industry Y. We keep the same relative cost shares for labor and capital, i.e., $\theta_{YL} = 0.3$ and $\theta_{YK} = 0.2$. For a given choice of Allen cross-price elasticities, the standard deviation of predicted pollution impacts across anchoring prices rises above 30% of the impact estimate obtained with the CPI. A notable insight from these parameterizations is that anchoring the pollution tax to p_X no longer produces comparative statics that are close to those obtained using the CPI, even if the largest discrepancies are still found with the use of p_Y. The reasons are that the magnitude of the change in the relative prices of the two consumption goods is larger (Table D.5 in online appendix D vs. Table 3), and the expenditure share of good Y, which is inversely related to the cost shares θ_{YL} and θ_{YK} for given values of the factor intensities,[29] is relatively large ($\frac{1}{3}$ vs. $\frac{1}{4}$), implying a substantial weight on p_Y in the CPI.

Finally, in online appendix D.4 we report results of a calibration relevant for a US carbon tax and adapted from the recent study by Fullerton and Ta (2019). This calibration

[29]Specifically, $\theta_Y = \frac{\gamma_L \gamma_K}{\gamma_L \gamma_K + \gamma_K \theta_{YL} + \gamma_L \theta_{YK}}$.

also leads to large discrepancies across anchoring indices. Specifically, because the dirty sector is relatively small in terms of its use of labor and capital, and its expenditure share on the polluting input is substantial, using p_Y as the anchoring index gives results very different from those obtained from the alternative indices.

6 Conclusion

This paper argues that numeraire choice can matter for assessing the general equilibrium effects of environmental policy on pollution levels and relative prices, both analytically and numerically, if the pollution tax is modeled as a *nominal* rather than *ad valorem* tax. Anchoring the pollution tax to the equilibrium prices of other goods, as we have done here, renders the choice of numeraire innocuous, but comparative statics then depend on the choice of anchoring price, a choice that reflects different varieties of environmental policy. Although our model is extremely stylized, the same remarks would hold in richer models with many goods and many inputs. They would also hold if one considered pollution taxation in conjunction with other policies, e.g., a tax swap under revenue neutrality.

Further, we have shown that if one anchors the pollution tax to inflation, then pollution always decreases with an increase in the tax rate, the resulting equilibrium is always Marshallian stable, and tax incidence results previously derived for specific model parameterizations in fact hold quite generally. Further, because indexing the tax on inflation eliminates the need to focus on special cases, we have derived general incidence results on the uses side that were not fully discussed in prior studies. These results highlight an interesting possibility, namely that an increase in the tax on a polluting input may decrease the price of the good produced using that input relative to that of the "clean" good. This counterintuitive result can arise if the factor used more intensively by the clean industry is a better substitute for pollution than the other factor, and the price of the former rises relative to that of the latter as a result of the pollution tax.

One may legitimately ask whether our result that the use of the CPI, or its sources-side equivalent, guarantees satisfaction of a law of input demand for the polluting input would survive generalization to the many-factors and many-goods case. Both indices would still be defined, and it seems quite plausible to us that the property would carry over. Whether it would also hold if one introduces imperfect competition into the model seems more doubtful. Indeed, the possibility of pollution-increasing environmental taxation in oligopolistic settings has already been shown to exist in partial equilibrium (Levin, 1985; Requate, 2006). We leave the formal arguments for future research.

Ultimately, our analysis implies that analytical and numerical models aimed at capturing the general equilibrium effects of environmental taxes that are actually indexed on inflation should use the relevant inflation index as numeraire, or explicitly anchor these taxes to inflation, so that comparative static results have the correct sign, and the correct magnitude. The fact that even in the pared-down model we use, the pollution and incidence outcomes of a nominal tax can vary widely with the choice of numeraire suggests that the same would be true in larger, more detailed models of the economy with a large set of candidate numeraire goods.

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