*Skrifter från SMDF, Nr. 14*

# *Bringing Nordic mathematics education into the future*

*Preceedings of Norma 20*
The ninth Nordic Conference on Mathematics Education
Oslo, 2021

Editors:
Nortvedt, G. A., Buchholtz, N. F., Fauskanger, J., Hreinsdóttir, F., Hähkiöniemi, M., Jessen, B. E., Kurvits, J., Liljekvist, Y., Misfeldt, M., Naalsund, M., Nilsen, H. K., Pálsdóttir, G., Portaankorva-Koivisto, P., Radišić, J. and Werneberg, A.

*SMDF* **S**vensk Förening för **M**atematik**D**idaktisk **F**orskning
Swedish Society for Research in Mathematics Education

# Contents

Preface i

Contents iii

*A framework for analyzing progress in concept knowledge in mathematics textbooks* 1
Linda Marie Ahl and Ola Helenius

*The nature of mathematical inquiry amongst kindergarten children: more than questioning and verbalisations* 9
Svanhild Breive and Martin Carlsen

*Undergraduate statistics students' reasoning on simple linear regression* 17
Marte Bråtalien and Margrethe Naalsund

*Learning modelling with math trails* 25
Nils Buchholtz and Juliane Singstad

*A crowd size estimation task in the context of protests in Chile* 33
Raimundo Elicer

*Challenges in enacting classroom dialogue* 41
Elisabeta Eriksen, Ellen Hovik and Grethe Kjensli

*An investigation activity as a means of including students in mathematical sensemaking* 49
Aleksandra Fadum, Bodil Kleve and Camilla Rodal

*Learning mathematics teaching when rehearsing instruction* 57
Janne Fauskanger

*Learning professional noticing by co-planning mathematics instruction* 65
Janne Fauskanger and Raymond Bjuland

*Spontaneous mathematical situations with young children* 73
Vigdis Flottorp & Deepika Vyas

*Milieus of learning in a Norwegian mathematics textbook* 81
Trude Fosse and Tamsin Meaney

*Investigating data collection methods for exploring mathematical and relational competencies involved in teaching mathematics* 89
Malin Gardesten

*Realization of the mathematical signifier 25 × 12* 97
Ramesh Gautam and Raymond Bjuland

*Supporting structural development in modelling at first grade* 105
Ragnhild Hansen

*Programming in the classroom as a tool for developing critical democratic competence in mathematics* 113
Inge Olav Hauge, Johan Lie, Yasmine Abtahi and Anders Grov Nilsen

*Students' productive struggle when programming in mathematics* 121
Rune Herheim and Marit Johnsen-Høines

*Teachers' views of low performances in mathematics at vocational education* 129
Karoline Holmgren

*Exploring opportunities to learn mathematics in practice-based teacher education: a Norwegian case study* 137
Gaute Hovtun, Reidar Mosvold, Raymond Bjuland, Janne Fauskanger, Åsmund Lillevik Gjære, Arne Jakobsen and Morten Søyland Kristensen

*Probability explorations via computer simulations in a Norwegian classroom: a discursive approach* 145
Antoine Julien and Kjærand Iversen

*The work of leading mathematical discussions in kindergarten: a Norwegian case study* 153
Camilla Normann Justnes and Reidar Mosvold

*"A bit uncomfortable" – preservice primary teachers' focus when planning mathematical modelling activities* 161
Suela Kacerja and Inger Elin Lilland

*Teachers' arguments for including programming in mathematics education* 169
Cecilia Kilhamn, Kajsa Bråting and Lennart Rolandsson

*Facilitating argumentation in primary school* 177
Silke Lekaus and Magni Hope Lossius

*Surveying mathematics preservice teachers* 185
Tamsin Meaney, Troels Lange, Ragnhild Hansen, Rune Herheim, Toril Eskeland Rangnes and Nils Henry W. Rasmussen

*Natural-number bias pattern in answers to different fraction tasks* 193
Pernille Ladegaard Pedersen and Rasmus Waagepetersen

*Swedish parents' perspectives on home-school communication and year-one pupils learning of mathematics* 201
Jöran Petersson, Eva Rosenqvist, Judy Sayers and Paul Andrews

*Interactive mathematical maps – a contextualized way of meaningful learning* 209
Johannes Przybilla, Matthias Brandl, Mirela Vinerean and Yvonne Liljekvist

*Exploring students' metacognition in relation to an integral-area evaluation task* 217
Farzad Radmehr and Michael Drake

*Surveying preservice teachers' understanding of aspects of mathematics teaching – a cluster analysis approach* 225
Nils Henry W. Rasmussen, Rune Herheim, Ragnhild Hansen, Troels Lange, Tamsin Meaney and Toril Eskeland Rangnes

*Using pathologies as starting points for inquiry-based mathematics education: the case of the palindrome* 233
Jan Roksvold and Per Øystein Haavold

*How does professional development influence Norwegian teachers' discourses on good mathematics teaching?* 241
Olaug Ellen Lona Svingen

*Structuring activities for discovering mathematical structure: designing a teaching sequence for grade 1* 249
Anna Ida Säfström and Görel Sterner

*Working with Euclid's geometry in GeoGebra – experiencing embedded discourses* 257
Marianne Thomsen

*A novel application of the instrumental approach in research on mathematical tasks* 265
Vegard Topphol

*Students' perceptions and challenges regarding mathematics teaching cycle in practices of historical and philosophical aspects of mathematics course* 273
Melih Turgut and Iveta Kohanová

*Whiteboards as a problem-solving tool* 281
Ingunn Valbekmo and Anne-Gunn Svorkmo

# An investigation activity as a means of including students in mathematical sensemaking

Aleksandra Fadum[1], Bodil Kleve[2] and Camilla Rodal[3]

OsloMet, Oslo Metropolitan University, Norway

[1]aleksandra.hara.fadum@oslomet.no, [2]bodil.kleve@oslomet.no, [3]camilla.rodal@oslomet.no

*We report on a classroom observation undertaken as part of a larger study of mathematics classroom practice in Norway, focusing on inclusivity. We observed a Ninth grade small-group activity in which the goal was to maximise the volume of a lidless box formed by cutting out the corners from a square piece of paper and folding up the sides. We analysed the teacher-student interactions using Schoenfeld's TRU framework. Although the investigative activity had the potential for cognitive demand and the teacher communicated intentions of exploratory work in general, as this lesson proceeded the potential for cognitive demand seemed to be scaffolded away by the teacher's direction of students' work. Thus, a tension between the teacher's intentions and the actual classroom practice was evident. Our findings suggest potentials for meaningful mathematical engagements.*

*Keywords: Investigative activities, cognitive demand, agency, representation, generalisation.*

## Introduction and literature review

Inclusion of all learners in the form of adapted education is a long-standing aim in Norwegian schooling, building on a legal requirement that every student should receive teaching appropriate to their needs.

This paper is part of the Inclusive Mathematics Teaching (IMaT)[1] project, which aims to investigate how adapted education is perceived and enacted at school and classroom level. We observed a lesson where the students worked on an investigative activity in which a mismatch between the teacher's intentions and actual practice occurred. Such mismatches have also been reported in the research literature (Kleve, 2010; Skott, 2001). In our analysis of the lesson, we consider these tensions as areas with potential for deeper engagement with mathematics.

According to Mason and Davis (2013), exploration needs to be encouraged in order to enable access to conceptual understanding in mathematics and its application in problem solving and reasoning. However, research suggests that changing teachers' practices towards more investigative ways of learning is not straightforward and even when teachers are planning for investigative activities, students' engagement in mathematical sense making is not guaranteed (Kleve, 2010). Thus, investigative activities, as such, are not necessarily an "easy fix" towards meaningful mathematics engagement for all students. According to Henningsen and Stein (1997), high-level cognitive-demand tasks are not only built on students' prior knowledge and appropriate amount of time, they rely on supportive actions by the teacher, such as scaffolding and consistently pressing students to provide meaningful explanations and make meaningful connections. Mason and Davis (2013) highlight the importance of sensitivity to students when responding to students' needs rather than depending on reactions derived from habits. Being present in-the-moment is therefore crucial for the teacher.

[1] Funded by the Research Council of Norway https://prosjektbanken.forskningsradet.no/#/project/NFR/287132

Generalisations and representations, which are widely viewed as central to mathematical problem solving, are emphasised in the renewed Norwegian Curriculum for grades 1-10 as part of the five core elements in mathematics: modelling and applications; reasoning and argumentation; representation and communication; abstraction and generalization; and exploration and problem solving (https://www.udir.no/lk20/mat01-05).

According to Mason, Graham, & Johnston-Wilder (2005), students need experiences of expressing generalities rather than just trying to reproduce techniques. They claim:

> Algebra provides a manipulable symbol system and language for expressing and manipulating that generality. The core pedagogic issue is therefore about enabling learners to employ their natural powers in using algebra to make sense of the world and of other people's use of algebra (p. 2)

In the Norwegian mathematics curriculum, it is recommended that students use various representations when learning algebra. These verbal, numerical, graphical, and algebraic representations have the potential of making the learning of algebra meaningful (Kaput, 2017). In the lesson we studied, the use of a variable became a crucial issue. This was an expression of generalisations from arithmetic and may be considered as a study of functions and relations. A research review of the flexible use of representations shows that students struggle to integrate the various representations and their relations (Nistal, Van Dooren, Clarebout, Elen, & Verschaffel, 2009).

In this paper, we study how the investigative activity developed during the lesson, and the potential for deep mathematical engagement that was provided. More precisely, we focus on how a group of students' engagements with the mathematics in the activity developed during the lesson. Our research questions are: How was the intended investigative activity implemented and enacted? How did students engage in the activity and what role did the teacher's support play?

## Setting the scene: an investigative activity

The aim of the investigative activity analysed here was to maximize the volume of a lidless box formed by cutting out the corners from square piece of paper (24cm×24cm) and folding up the sides. The teacher's presentation of the activity took place with the whole class. As a warm-up, the students were asked to reflect and then suggest how many litres a cardboard box could hold (the teacher had displayed a cardboard box with dimensions 25cm×40cm×40cm). The formula for the volume of a prism was written on the board, and the exact volume of the box was calculated after students had come up with different suggestions for the number of one litre bottles the box could hold.

Students worked in six teacher-assigned heterogeneous groups of three or four. The exact criteria for assigning groups were not clear and some assignments may have been random. They were given a piece of paper and were asked to find how much of the corners they should cut off in order to obtain the maximum volume. They were encouraged to be systematic and strategic, and as they moved into their groups, the teacher said: "What is the variable here?" He also reminded them of an activity they had completed in the previous year where the goal was to maximize a two-dimensional area, and he focused more and more on the variable when he interacted with the different groups.

## Methodology

This research was done as part of a pilot for the IMaT project. We observed three mathematics lessons with one teacher, who had volunteered to participate. The class was a mixed ability grade nine class. The students were usually seated in pairs, but in this lesson, they were placed in groups. Prior to our first classroom observation, we had a semi-structured interview with the teacher. We asked him to describe a typical mathematics lesson, his intentions in teaching mathematics and how he dealt with mixed abilities, adapted education and grouping. We also conducted a post interview with him one month after having observed this lesson.

The lesson started with a 10-minute whole class session (launch), followed by an 80-minute session with group work, and finally with a 20-minute summing-up. The lesson was videotaped, and the teacher wore a microphone. During the group work, the camera followed the teacher as he moved from one group to another. Additionally, the researchers placed an audio recorder with a randomly selected group of 2 boys and 2 girls. The three researchers did not interact with the teacher and students during the lesson and were non-participant observers in the classroom. One managed the video filming, another took general field notes, and the third used the Teaching for Robust Understanding (TRU) framework (Schoenfeld & Floden, 2014) as an observation record.

The TRU framework offers ways to reflect along five dimensions: 1-The richness of the mathematical content, 2-Cognitive demand, 3-Access to mathematics, 4-Agency, Ownership, and Identity and 5-Uses of assessment. We used the scoring rubric for small group work as an observation scheme in the classroom. The focus of the framework is when students are engaged in brainstorming, the role of the teacher is to support students in exploring and justifying. Every 15 minutes, we noted on a line segment (from one to three) a score for each dimension. This gave us the possibility to identify changes in for example cognitive demand. In advance, the researchers had prepared for the application of the framework in order to ensure reliability across the research team in the big research project.

We transcribed the video recording while making notes to obtain an overview of the lesson. Together with the observation schemes this provided us with an overall picture of the development of the teacher's support of groups. We transcribed the audio-recorded data from the group that we focused on in order to analyse how students within the group engaged in the activity. Our aim was to study the teacher's interactions with the group when he "visited" them. Transcripts from these interactions were subject to analysis. Data from pre- and post-interviews with the teacher were used to further illuminate our study and findings.

## Findings

Schoenfeld (2018) emphasises that "The main focus of TRU is not on what the teacher does, but on the opportunities the environment affords students for deep engagement with mathematical content" (p.491). In this section, we will go through the dimensions in the framework in order to identify potential for development. We report only briefly on whole class activities, focusing instead on the teacher's intentions and his interactions with the group's work. Excerpts from the audio recorded data and from the interviews illustrate the analysis.

**Mathematical content: How accurate, coherent, and well justified is the mathematical content?**

In the pre interview, the teacher emphasised that he consciously selected mathematical tasks that were meaningful for the students. Criticizing students' prior schooling he said:

> Many students have negative thoughts about mathematics when starting lower secondary. They have only experienced procedural exercises [ ], it is necessary to unlearn that that is maths. [ ] My teaching reveals a different way of thinking, solving problems in a context. [ ] I don't use a textbook, but I select a specific problem to be solved.

According to what he said here, the task selected for this lesson was well thought through. The task also had a potential for high cognitive demand. His intention was to provide students with opportunities for building a coherent view of mathematics. During the lesson launch, the students were invited to explore. The teacher illustrated the volume by displaying the dimensions of the box, drawing on the board, and calculating the box's volume.

The teacher had planned this lesson with the intention of training mathematical exploration, extending students' algebraic thinking and allowing them to connect several representations. While observing in class, we noted this in the beginning of the lesson with a score between two and three on the TRU observation record. This was confirmed when we watched and analysed the video. However, in the second half of the lesson, the implementation seemed rather procedural, which had been scored close to one on the observation scheme. We explore this further below in the analysis along the dimension of cognitive demand.

**Cognitive demand: To what extent are students supported in grappling with and making sense of mathematical concepts?**

Generally, the teacher initiated the possibilities of conceptual richness as he gave hints and was supportive. He encouraged students to understand the context (close to three on the TRU observation scheme); however, his comments were increasingly characterized by giving the students step-by-step instructions on what should be done, including how to cut out corners from a square piece of paper (close to one on the scheme). Focusing on our selected group, they had talked about cutting 12 cm, but realised that it was not possible (the paper was 24 cm and folding 12 cm would not make a box). They called on the teacher. From the video, we see that the teacher took the paper, told them to listen to him and showed them how it should be folded. He did not comment on why 12 cm was not possible.

| | |
|---|---|
| Teacher: | Ok. Listen to me now. If we cut it off, there are two centimetres here and two centimetres in there. Is it 24, then we must remove …? |
| Students: | Huh? |
| Teacher: | It is 24 cm in width, right? |
| Student 1: | 22, is it? |
| Teacher: | It is 24 cm. Then we cut away two centimetres there and two centimetres here. |
| Students: | 20 |
| Teacher: | 20. Then this [points to one side] will be 20, and this [points to the other side] will be 20. Is it 20 times 20? Will it be right then? |

Other typical teacher input both to this group and the other five groups were: “How is it going?”; “What is the height?”; “Try to be strategic and systematic”; and “What is the variable?” The inputs were directive and discussions with the groups seemed aimed at “answer getting”.

Although the given task could be described as a high-level cognitive demand task and had potential for making connections, it seemed not to be implemented as intended. Instead, most of the chances for making connections were transformed into procedural exercises.

By a process of trial and error with numerical values, the groups had no difficulties in figuring out that the maximum volume was 1024 $cm^3$ with corresponding height 4 cm. However, they were overwhelmed by the number of calculations and struggled to get an overview. The teacher told them what numbers to try out and to be strategic and systematic, which led to a decrease in cognitive demand. The teacher introduced the height as a variable, and he focused on finding a correct algebraic expression.

The task offered possibilities of productive engagement or struggle with central mathematical ideas. As observers, we recognized that the teacher’s agenda might have been to move between the four different representations (verbal, numerical, graphical and algebraic) in order to work with algebra in a meaningful and coherent way. This was not communicated clearly to the students and seemed not to be obvious. For them, the task was just to find the height providing the maximum volume and folding the paper accordingly.

**Access to mathematical content: To what extent are all students supported in meaningful participation in (group) discussions?**

Analysis of transcribed audio data from the selected group showed that group discussion, for part of the time, was devoted to sharing solution methods or ideas and making sense of them. In the pre-interview, the teacher emphasised the importance for the students to discuss mathematics with each other and then to use mathematical concept correctly: “Students never work alone, always together either in pairs or group of four. They are supposed to discuss and use the mathematical language correctly”. However, we did not identify instances where the teacher supported or encouraged students’ engagement in student-to-student discussions.

**Agency, authority and identity**: **To what extent did teacher support and/or group dynamics provide access to "voice" for students?**

In this lesson, conversations were teacher-initiated, and students’ answers were short, and sometimes seemed to be guesses. According to our field notes and TRU records, some students’ chances to explain their thinking were identified. However, the teacher was the driver of the conversations and there were no interactions between students in discussions while the teacher was present.

The teacher provided time for students to develop and express mathematical ideas and to reason during group work. In the pre-interview, he said “I try to give hints so they can think on their own until I come back”. This also characterized his interactions, and indicates his intentions were to be supportive. In the post-interview he also emphasised that students should have agency: “In a way it is their agency [] they shall have their own identity and agency and they shall experience an ownership on a meta-level. I want them to define what it implies to have flexible strategies in the subject”.

**Use of assessment**: **To what extent does the teacher monitor and help students refine their thinking within small groups?**

The teacher created a confident atmosphere in the classroom. The students felt free to express their ideas and understandings. However, questions from the teacher were mostly closed and did not seem to expect mathematical justification, and students' algebraic thinking was not challenged. Many teacher-posed questions were answered by the teacher himself. As the teacher did not consider students' reasoning, the task ended up being worked on at a lower level because teacher did not listen actively to students' reasoning and he did not try to assist them based on their pre-existing knowledge. This is exemplified in the episode below. On this fifth (and last) visit, he asked how much they suggested cutting off. They answered "four". The conversation continued:

| | |
|---|---|
| Teacher: | Fantastic! That is right! 1024 cubic centimetres? |
| Student 1: | [proudly] It will be approximately one litre? |
| Teacher: | But can we find another form of expression? Some formula … something for this. Is it possible to do? |
| Student 2: | [giggly] It must be an $x$. |
| Student 3: | [hesitantly] Should we find a formula to... How did we find out? |
| Teacher: | Look here. Now we have found the volume of a box |
| Students: | Length times width, times height. |
| Teacher: | What is the width here or the length? It's going to be... If we call it $x$, then? |
| Student 2: | Won't it be 16 then? |
| Teacher: | Then it gets here 24 minus the length from there... It will be 24 minus … How many $x$'s are we taking away? |
| Student 3: | 2 |
| Teacher: | Minus $2x$... Multiplied by that are … |
| Students: | Mmm $x$... The same |
| Teacher: | The same, correct. Times with that length. This is the length; this is the width |
| Student 3: | Then times with $x$ again. |
| Teacher: | Correct. Then the question is. We were working a bit with algebra last year. Do you remember that? Multiplying parenthesis together and so. So, 24 times 24 it is... 576. 24 times minus $2x$ it is... |

The teacher carried on transforming $(24 - 2x)(24 - 2x)x$ into $576x - 96x^2 + 4x^3$. He thus led the students down a predetermined path and did not solicit or pursue student thinking. In the end, students were left with a task that only required applying a procedure without any idea how to make connections to previous work and underlying mathematical meaning.

## Discussion and concluding remarks

The task was a high cognitive demand task with the potential for meaningful mathematical engagement. However, our findings suggest that the teacher's expressed intentions of doing investigative work were not implemented. Additionally, the students' roles throughout the lesson increasingly became to follow the teacher's instructions instead of investigating and struggling to find mathematical connections. The teacher encouraged students' thinking, but the support he gave became more and more directive. This mismatch between the intentions and implementation may be a potential area for development. This we now discuss illuminated by what the teacher said in the interviews.

In the pre-interview the teacher communicated that he did not consider himself a traditional teacher: "I never teach algorithms. I never do examples on the board followed by students doing same kinds of exercises." He also emphasised that students have to struggle: "I tell students that it takes time doing mathematics. They need to be tired in the same sense as they become tired doing sports. It is through struggling your brain develops." These expressed intentions were underpinned by his selection of a task that had the potential of a high score on all five dimensions of the TRU framework. According to our analysis above, opportunities for building a coherent view of mathematics were provided as students were invited to explore. Additionally, as emphasised by Henningsen and Stein (1997), the students were given enough time to work on the task, mostly in groups, but also when working in pairs. There was a confident atmosphere, and the teacher was available throughout the whole session.

The teacher displayed theoretical knowledge about why students should be supported in group dynamics, and he also expressed importance of students' mathematical agency. However, his interventions constrained students to produce short responses when he interacted with them, or he answered his own questions, not taking what students said into account. The teacher was the driver of the conversations and there were no interactions between students in discussions while the teacher was present.

In the last quoted interaction, in which the teacher suggested an algebraic expression with two parentheses, we see that students' contributions are not explored or built upon.

The students talked about how many litres the box contained, and were thus linking back to the introductory activity, while the teacher focused on algebra, multiplying the factors to get his desired algebraic expression. The conversation between the teacher and the students was a one-way communication. The students were not "on the teacher's track" and for them the teacher's manipulation with symbols did not make sense. The teacher displayed ownership of the mathematics and his goal seemed to be leading the students towards an algebraic representation.

The teacher provided students with possibilities for thinking, but not opportunities for argumentation and reasoning. Since the teacher knew where they should go, he was always ahead of the students, and had difficulties not telling them. The students worked with verbal and numerical representations and did not see the point with algebraic expressions and graphical representations. As Nistal et al. (2009) emphasise, this is a common struggle for students, which the teacher might have noticed. In the whole class summing up, the teacher asked for different representations from the groups and students became engaged in comparing the different representations. He made a table, wrote an algebraic expression on the blackboard and then using GeoGebra, he drew the graph on the smart board.

Our findings suggest that there was a mismatch between intentions and implementation of the task. We noticed that crucial supportive actions, such as pressing students to provide meaningful explanations or make meaningful connections were not evident. This may be due to the teacher's challenge in having to support all six groups as much as possible, so they were not left unsupported. In the pre-interview we had asked the teacher about what he saw as dilemmas or challenges in teaching mathematics: "I wish I could have smaller classes, however, not according to level, so I could have followed up more". In the post-interview he displayed self-awareness saying: "The problem is that I am so enthusiastic and may tell them too much, so they will not get time to reason, because I think it is so much fun. But I try to give hints so they can reason on their own until I come back". This underpins some conflicting issues mathematics teachers face. Although his intentions may be to give students agency and the possibility to reason on their own, time pressure and stress, and also his expressed enthusiasm in doing mathematics, tempted him to present solutions too quickly and hence he scaffolded the students' struggle away, rather than giving supportive hints as he argued for in the interview. However, his expressed theoretical knowledge and awareness of the mismatch between his intentions and actual practice, we consider a potential for development of students' deep engagement in mathematics so supportive hints will encourage mathematical struggle rather than scaffold students' struggle away.

## References


Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for research in Mathematics education, 28*(5), 524–549.

Kaput, J. J. (2017). What Is Algebra? What Is Algebraic Reasoning? In D. W. C. James J. Kaput, Maria L. Blanton (Ed.), *Algebra in the early grade* (pp. 5–18). New York: Lawrence Erlbaum.

Kleve, B. (2010). Educational reforms, how are they implemented? In B. Sriraman, S. Goodchild, C. Bergsten, C. Michelsen, G. Palsdottir, O. Steinthorsdottir, & L. Haapasalo (Eds.), *The First Sourcebook on Nordic Research in Mathematics Education: Norway, Sweden, Iceland, Denmark and contributions from Finland* (pp. 155–168). Charlotte: Information Age Pub.

Mason, J., & Davis, B. (2013). The Importance of Teachers' Mathematical Awareness for In-the-Moment Pedagogy. *Canadian Journal of Science, Mathematics and Technology Education, 13*(2), 182–197.

Mason, J., Graham, A., & Johnston-Wilder, S. (2005). *Developing thinking in Algebra*. London: Sage.

Nistal, A., Van Dooren, W., Clarebout, G., Elen, J., & Verschaffel, L. (2009). Conceptualising, investigating and stimulating representational flexibility in mathematical problem solving and learning: A critical review. *ZDM - The International Journal on Mathematics Education,, 41*(5), 627–636.

Schoenfeld, A. H. (2018). Video analyses for research and professional development: the teaching for robust understanding (TRU) framework. *ZDM, 50*(3), 491–506. doi: 10.1007/s11858-017-0908-y

Schoenfeld, A. H., & Floden, R. E. (2014). *The TRU Math Scoring Rubric*. Retrieved from Berkeley, CA & E. Lansing, MI: http://ats.berkeley.edu/tools.htlm

Skott, J. (2001). The Emergent Practices of a Novice Teacher: The Roles of His Shool Mathematics Images. *Journal of Mathematics Teacher Education, 4*, 3–28.