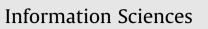
Contents lists available at ScienceDirect





journal homepage: www.elsevier.com/locate/ins

A personality-aware group recommendation system based on pairwise preferences



Roza Abolghasemi^{a,*}, Paal Engelstad^a, Enrique Herrera-Viedma^{b,c,*}, Anis Yazidi^{a,*}

^a Department of Computer Science, Oslo Metropolitan University, Oslo, Norway

^b Department of Computer Science and AI, Andalusian Research Institute in Data Science and Computational Intelligence, University of Granada, Granada, Spain ^c Faculty of Engineering, The School of Computing, Universiti Teknologi Malaysia, Johor Bahru, Malaysia

ARTICLE INFO

Article history: Received 26 April 2021 Received in revised form 9 February 2022 Accepted 18 February 2022 Available online 23 February 2022

Keywords: Group recommendation system Pairwise preferences Group decision-making Personality traits Reaching consensus

ABSTRACT

Human personality plays a crucial role in decision-making and it has paramount importance when individuals negotiate with each other to reach a common group decision. Such situations are conceivable, for instance, when a group of individuals want to watch a movie together. It is well known that people influence each other's decisions, the more assertive a person is, the more influence they will have on the final decision. In order to obtain a more realistic group recommendation system (GRS), we need to accommodate the assertiveness of the different group members' personalities. Although pairwise preferences are long-established in group decision-making (GDM), they have received very little attention in the recommendation systems community. Driven by the advantages of pairwise preferences on ratings in the recommendation systems domain, we have further pursued this approach in this paper, however we have done so for GRS. We have devised a three-stage approach to GRS in which we 1) resort to three binary matrix factorization methods, 2) develop an influence graph that includes assertiveness and cooperativeness as personality traits, and 3) apply an opinion dynamics model in order to reach consensus. We have shown that the final opinion is related to the stationary distribution of a Markov chain associated with the influence graph. Our experimental results demonstrate that our approach results in high precision and fairness.

© 2022 The Authors. Published by Elsevier Inc. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).

1. Introduction

Recommendation Systems (RS) aim to find and recommend a set of items to a single user, and are commonly used in various domains, such as movies, music, travel, e-commerce, and so on. While a classic RS tries to recommend a suitable set of items for an individual user based on their preferences, group recommendation systems are concerned with recommending a set of items that appeal to a group of people. There are numerous applications of GRS in real-life settings. Application scenarios for GRS include examples such as a group of friends who want a recommendation for a movie to watch together, passengers in a car who want to listen to the same music while driving, etc.

https://doi.org/10.1016/j.ins.2022.02.033 0020-0255/© 2022 The Authors. Published by Elsevier Inc.

^{*} Corresponding authors at: Department of Computer Science and Artificial Intelligence, University of Granada, Granada, Spain (E. Herrera-Viedma) Department of Computer Science, Oslo Metropolitan University, Oslo, Norway (A. Yazidi) (R. Abolghasemi).

E-mail addresses: rozaabol@oslomet.no (R. Abolghasemi), viedma@decsai.ugr.es (E. Herrera-Viedma), anisy@oslomet.no (A. Yazidi).

This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).

There are different types of recommendation systems: Collaborative Filtering (CF) RS [1], content-based RS [2], demographic-based RS, utility-based RS, knowledge-based RS [3], and hybrid RS, which is a combination of other methods.

The CF models have shown promising results as compared to the different recommendation systems. The datasets used in the CF models include hundreds of thousands of item ratings given by users. These ratings are used to compute recommendations for target users based on two main methods: 1.) KNN-based CF [4,5], which generates recommendations based on the ratings given by the k most similar users, 2.) Model-based CF [6], which builds the model based on a rating matrix. The most popular model-based CF is Matrix Factorization (MF) [7], which decomposes the rating matrix into a product of two smaller matrices containing latent factors, namely a user matrix, and an item matrix. In matrix factorization, the data are usually the ratings of the items given by the users. However, some authors claim that comparing items or pairwise preferences can yield more accurate preferences than the rating in a predefined scale [8]. For instance, in a normal rating system, if a user gives two movies five stars, we cannot know which of the two they prefer. For this reason, a predefined rating scale consisting of discrete values is not considered to be very precise. Instead, by using pairwise preferences, two-by-two comparisons of movies can be made, and thus users' movie preferences can be better expressed.

Although pairwise preferences are long-standing in group decision-making (GDM) [9–11], they have, with a few exceptions [12–14], received very little attention from the recommendation system community. Driven by the recently reported advantages of pairwise preferences on ratings in the field of recommendation systems, in this paper we have further pursued this approach, however we have done so for GRS.

In [14], a new RS was introduced which used pairwise preference scores instead of pure rating data. Relying on pairwise preference scores led to better performance in terms of Normalized Discounted Cumulative Gain (NDCG) and better precision than that obtained by the legacy methods based on single item ratings. Among the most popular approaches to matrix factorization based on pairwise preference scores are Bayesian Personalized Ranking (BPR) [15] and Multiple Pairwise Ranking (MPR) [16], which formulate the matrix factorization problem as a maximum likelihood estimation problem and use stochastic gradient descent to deduce the latent factors.

Pairwise matrix factorization also aims to find an embedding of items and users. The advantage of pairwise preference rating methods compared to other single rating methods is that they are more precise and can yield better predictions due to their pairwise comparison nature [12,15,16]. In this paper, we calculated personalized item scores based on the previously mentioned pairwise preference rating methods, and then we computed a final group score for each item based on opinion dynamics theory [17]. The weights of the influence graph were computed from the personality values collected from the TKI test. Finally, according to the stationary distribution of a Markov chain, we have proven that the group members will reach consensus. Moreover, we have provided an alternative proof and interpretation of the convergence results.

A brief explanation of the contribution of our work is detailed as follows. We have resorted to opinion dynamics theory (social influence) in order to model how users influence each other's opinions within a group based on defining mutual influence relations derived from the TKI personality test. We prove that the group will reach a consensus under some mild conditions on the weight matrix describing mutual influences. We also show a link between our approach to aggregating user ratings based on opinion dynamics and the widely used I-OWA approach to group decision-making.

Another contribution of this paper is that while most of the group recommendation systems use single rating scores, we have used pairwise preferences instead. Using pairwise preferences is known in the literature to provide more precise recommendations due to the derivation of more implicit feedback from users compared to a single rating [12,15,16]. We have also tested our approach by using three pairwise preference ranking approaches and different experimental results have been reported in different scenarios.

The rest of this paper is organized as follows. In Section 2, we first outline a state-of-the-art review of work where personality has been used for RS and GRS systems. Section 3 explains the three pairwise ranking methods that we adopted to predict the personalized item ranking scores. Moreover, fuzzy preference aggregation is explained in a subsection. We then describe our proposed model for a personality-based group recommendation system in Section 4. Next, Section 5 introduces our experimental settings and evaluation metrics and Section 6 reports our experimental results and our main findings in light of the existing studies. Finally, Section 7 discusses the findings further and includes some of their managerial implications.

2. Related work

2.1. Personality-based RS

In this section, we focus on the articles concerning personality-based RS and GRS systems.

Personality-based RS refers to a class of RS that takes the personality characteristics of a person into account when making a recommendation. We will first provide some examples of studies of the role personality plays in RS for a single user before shedding light on the role personality plays in GRS. The work in [18] concerns recommendations for a single user (as opposed to a group) based on a personality profile derived from information about their professional activities. The role of personality in tourism destination recommendations is highlighted in [19]. This article classifies users based on travel personality categories, which leads to particular travel behavior. To address the recommendation redundancy and cold start problems, Dhelim et al. [20] proposed a personality-aware product RS based on metapath discovery and user interest. The proposed method aspires to incorporate the users' personality traits in the associated items. The main personality theory used in that paper is the Five-Factor Model (FFM), which is based on the following traits: neuroticism, openness to experience, extraversion, agreeableness, and conscientiousness. The work in [21] investigated the role of Twitter users' personality traits when recommending followers. The role personality plays is often ignored in follower recommendation methods, since most RS systems only use graph topology and user-generated content for this task. For a comprehensive overview of personality-based RS, we refer the reader to [22]. This paper discusses some challenges and future research directions concerning this subject.

2.2. Group recommendations

Although single-user recommendation systems have received significant research attention and have gained a lot of popularity due to their use by Internet giants such as Google, Amazon, and Netflix, studies of GRS systems are rather sparse. A GRS aims to provide group recommendations that maximize group members' satisfaction and minimize inequality among users. GRS systems are usually more complex than single RS systems. For instance, Xiao et al. [23] have shown that solving a GRS problem is usually NP-hard in different semantics. They mapped the group recommendation problem to a multiple objective optimization problem and used Pareto optimality as the criterion for the solution. Recently, remarkable work has been carried out with GRS based on deep learning. For instance, in 2020, Zhenhua et al. proposed a Multi-attentionbased Group Recommendation Model (MAGRM) [24], which uses multi-attention-based deep neural network structures to achieve accurate group recommendations. To achieve this goal and to capture the internal social features of groups, a vector representation of group features was used to capture each of the groups' deep semantic features. Then, group preferences for items were inferred using a neural attention mechanism that takes the preference interaction into account among group members.

Group recommendations based on implicit feedback (like or buy) have attracted much attention in services such as Facebook and Amazon. An example of such a recommendation is Group Preference Based Bayesian Personalized Ranking (GBPR) [25], which was introduced by Pan and Chen.

There is little research on GRS systems that takes the psychological nature of interactions between group members into account when making a joint decision. One of the first prominent studies to address this issue was carried out by Recio-Garcia et al. [26], who proposed a personality-aware group recommendation model based on the Thomas-Kilmann Conflict Mode Instrument (TKI) [27]. This test is based on conflict management and its effect on personal and group dynamics. According to this test, there are five different styles of conflict management: Competing, Collaborating, Avoiding, Accommodating, and Compromising. For a more detailed explanation of these styles, we refer the reader to Section 4.1.

In this paper, we have applied the concept of TKI to represent the group members' personality scores. In practice, our method works well when all the users are able to participate in the TKI test, which is not always a convenient option, especially for large groups. For large groups in social media, however, it is possible to automatically predict the personality traits of users based on their information and activity in social media without requiring a TKI test [28]. The TKI metaphor is another alternative to the TKI test [29] that consists in showing the users two movie characters that have opposite personalities, and requiring the user to decide which character has a personality that is more similar to theirs.

To improve group recommendations, some papers have combined the influence of personality and social trust. In [30], personality factors were derived from TKI and, based on these factors, a value called Conflict Mode Weight (CMW) was calculated to reflect the influence of personality on group decisions. Moreover, mutual-trust relationships between group members were extracted from the underlying social network that connects them. The trust value is based on various network measures, combining their distance in the social network, the number of mutual friends, duration of friendship, shared pictures, etc. In [31], agreeableness was used as a personality factor that models altruistic behavior and is based on the Big-Five model [32].

The majority of the studies in the field of group recommendation systems consider the opinion of all members equally. While in real life, personality traits influence the group's final decision significantly. Although there are some studies that take the impact users' personalities have on group decisions into the consideration, they lack a sound theory that quantifies group dynamics. Furthermore, few group recommendation works use pairwise preference. As previously mentioned, it has been shown that, in general, pairwise preference methods in recommendation systems show greater precision than individual rating methods [12,15,16].

At this point, we will mention some notable works that are more relevant to our method. In a group recommendation system, it is very important that group members negotiate with each other to be able to reach a final decision. In some works, a coordinator interacts with members to find out their opinion. Wang et al. [33] created a virtual user that acted as a coordinator who tried to solve the members' conflicts in a group in order to reach a final decision. This was done based on the mutual trust that existed between the coordinator and members, and not their personalities. In other words, users' trust-relations created some sort of personal influence, which had an impact on the virtual coordinator's opinion.

There are some works in group recommendation systems like [34] which used the results of the TKI method in both social relationships and social behavior, not only to infer a group's preference, but also to model the tolerance and altruism characteristics of the group members. Quijano et al. used personality traits in [29] to design a system called HappyMovie, which recommends movies to Facebook users based on their level of trust derived from the social network and the TKI metaphor, which serves as an alternative to the TKI test. They expressed the adjustment of a user rating based on the ratings of users

they trust using a simplistic formula where the influence of other users is inversely proportional to the user personality. The formula is similar to the memory-based approach for recommendation systems, where the authors use trust instead of similarity between users. The main shortcoming of their method is that it does not take the personalities of other users into account, and only applies the personality of one user at a time.

From the literature, we see that the vast majority of works that take personality traits into account by using the TKI test employ some sort of ad hoc formula (heuristics) to find the group score. In contrast to all of these works, we rely on the social influence theory in order to accurately simulate the effect of other users in the final group rating. Another personality-aware group recommendation based on the TKI test was proposed by Recio-Garcia et al. [26]. Their recommendation is based on existing collaborative filtering techniques and considers group personality composition.

Among the models that used the TKI test, we find the work of Guo et.al. [35] who introduced a group recommendation model using individual personality, the impact preference similarities between users, susceptibility, intimacy, and expertise factor have on the ability to improve the recommendation system. Their approach, however, is not suitable for large-scale applications, while our proposed method works well on any group size. Moreover, their work did not use pairwise preferences, and its efficiency on heterogeneous data has not been studied. We, however, use random personality values derived from different probability distributions in our work to show the applicability of our model to any type of data.

Additionally, although group recommendation systems based on TKI and personality traits have been used in these papers, as noted, none of them applied pairwise preference to ratings in their methods. In our paper, besides using personality traits to understand the impact has user on the final group decision, we have applied three pairwise preferences methods in order to calculate the item ratings more accurately compared to single-item ratings.

3. Preliminaries

3.1. Item Ranking based on pairwise preferences

In this section, we present three popular methods for ranking items based on the pairwise preferences that we have used in our experiments.

3.1.1. Bayesian Personalized Ranking (BPR)

Rendle et al. [15] proposed a generic optimization criterion, BPR-OPT, for personalized ranking that converts a user-item matrix into a set of per-user item-to-item matrices and tries to maximize the likelihood of per-user pairwise preferences. To this end, they considered two assumptions: 1. the observed item i by user u is preferred over all unobserved items. 2. The likelihood of pairwise preference of user u is independent of the others. The likelihood of BPR is formulated as:

$$BPR = \prod_{u \in U_i \in I_u^+} \prod_{j \in I - I_u^+} pr(r_{uj} > r_{uj}) \times \left[1 - pr(r_{uj} > r_{ui})\right] \tag{1}$$

where the set of all users is represented as *U* and the set of all items as *I*. In this equation, $I_u^+ \subset I$ denotes items that received positive feedback from the user and r_{ui} is user *u*'s preference as regards to item *i*. $pr(r_{ui} > r_{uj})$ is defined as the individual probability that the user *u* prefers item *i* over *j*. This is obtained using the logistic sigmoid function:

$$pr(r_{ui} > r_{uj}|\Theta) = \frac{1}{1 + e^{-x_{uj}(\Theta)}}$$
(2)

where $x_{uij}(\Theta)$ is an arbitrary real-value function of the model parameter vector Θ , which captures the special relationship between user *u*, item *i*, and item *j* from the matrix factorization model [15].

This method solves the matrix factorization problem using stochastic gradient descent with the aim of maximizing the Area Under the Curve (AUC).

3.1.2. Multiple Pairwise Ranking (MPR)

There are two main reasons that can explain unobserved items $I - I_u^+$ in BPR: a user either dislikes the unrated items or has not seen them. To account for these two reasons, Yu et al. [16] introduced a new version of BPR called Multiple Pairwise Ranking (MPR). They divided unobserved items into two subsets I_u^- and I_u^* , then defined three subsets of items I as:

- I_u^+ : The items that user u has seen and expressed positive feedback on.
- I_u^- : The items that user u has seen but has not expressed feedback on.
- I_u^* : The uncertain negative items that user u has not seen.

MPR states that items for which the user has given positive feedback (I_u^+) have a higher probability of being preferred by the user than items that the user has not seen (I_u^*) . Furthermore, the items that the user has not seen (I_u^*) have a higher probability of being preferred by the user than the items that they have seen but have not provided feedback on (I_u^-) . Considering $r_{uij} = r_{ui} - r_{uj}$, the likelihood of MPR among items can be given as follows:

Information Sciences 595 (2022) 1-17

$$\begin{cases} MPR = \prod_{u \in U_{i,p,p' \in I_u^+; q' \in I_u^-; q \in I_u^+}} \prod_{pr(r_{uij} \ge r_{uqq'}, r_{uqq'} \ge r_{upp'}) \\ \times \left[1 - pr(r_{uij} < r_{uqq'}, r_{uqq'} < r_{upp'}) \right] \end{cases}$$
(3)

where $i, p, p' \in I_u^+$, while $j, q' \in I_u^-$, and $q \in I_u^*$.

3.1.3. Matrix factorization pair-score prediction (MFP)

MFP, introduced by Kalloori et al. [12], provides pairwise scores for a set of items that indicate how much a user prefers one item over another. By integrating the pairwise scores, personalized item scores are computed that indicate how much a user prefers an item. Let **R** be a matrix with elements r_{uij} , where r_{uij} indicates how much a user u prefers item i over item j. MFP factorizes a matrix **R** into two smaller d-dimensional matrices **X** and **Y** such that **R** = **X**.**Y** is the dot product of the two. Here, $R \in R^{users* items}$ is the user-item rating matrix, $X \in R^{users* latentfactors}$ is user matrix containing the user's latent factors (x_u) and $Y \in R^{items* pair-itemslatentfactors}$ is the item matrix that contains the latent factors of the pair item (y_{ij}) . The pair-score of a user u for the pair-item i, j is:

$$f_{uij}^* = \mu + b_u + b_{ij} + x_u^i y_{ij}$$
 (4)

where μ is the average of all pair scores in the matrix **R** and b_u and b_{ij} are the baseline parameters for modeling the deviation from the average score for a user u and item pair (i, j), respectively. In MFP, the model parameters are learnt using stochastic gradient descent, which minimizes the prediction error on the training data $(r_{uij} - r_{uij}^*)$. The result is a matrix **R** with elements consisting of predicted missing pair scores r_{uij}^* . The final personalized item score v_{ui} is calculated according to the following equation:

$$v_{ui} = \frac{\sum_{j \in I \setminus \{i\}} r_{uij}^*}{|I|}$$
(5)

3.2. Fuzzy preference aggregation

In this section, we focus on preliminaries about GDM and preference aggregation as a method for reaching consensus. A GDM problem consists of a group of *m* members $G = \{g_1, g_2, ..., g_m\}$ expressing their preferences for a set of items $X = \{x_1, x_2, ..., x_n\}$ to reach a common solution. These preferences might be fuzzy preference relations (FPR), which are pairwise preferences of items. An FPR *P* on a set of items *X* can be represented as a matrix $P = (p_{ij})$, where $p_{ij} = \mu_P(x_i, x_j)$ is the membership function $\mu_P : X \times X \to [0, 1]$, such that [36]:

$$\mu_{p}(x_{i}, x_{j}) = \begin{cases} 1 & \text{if } x_{i} \text{ is definitely preferred to } x_{j}, \\ x \in (0.5, 1) & \text{if } x_{i} \text{ is slightly preferred to } x_{j}, \\ 0.5 & \text{if } x_{i} \text{ and } x_{j} \text{ are equally preferred,} \\ y \in (0, 0.5) & \text{if } x_{j} \text{ is slightly preferred to } x_{i}, \\ 0 & \text{if } x_{j} \text{ is definitely preferred to } x_{i}. \end{cases}$$

$$(6)$$

In a group with *m* members, there are *m* FPRs P_1, \ldots, P_m , where $P_k = (p_{kij})$ for $k \in \{1, \ldots, m\}$ and $i, j \in \{1, \ldots, n\}$. To obtain a combined FPR, an aggregation rule called Ordered Weighted Average (OWA) [37] [38] is often used. An OWA is defined as:

$$OWA(p_1,\ldots,p_m) = \sum_{k=1}^m w_k p_{\sigma(k)}$$
(7)

where $W = (w_1, \ldots, w_m) \in [0, 1]^m$ is a list of weights such that $\sum_{k=1}^m w_k = 1$ and (p_1, \ldots, p_m) is a list of preference values. In this equation, $\sigma : \{1, \ldots, m\} \rightarrow \{1, \ldots, m\}$ is a permutation function, such that $p_{\sigma(k)} \ge p_{\sigma(k+1)}$ for each $k \in \{1, \ldots, m-1\}$. Therefore, $p_{ij} = OWA(p_{1ij}, \ldots, p_{mij})$.

The behaviour of OWA strongly depends on the weight vector. A non-decreasing proportional fuzzy quantifier is proposed by Chiclana et al. [39] to initialise the weight vector inspired by the behaviour of *soft majority*. A non-decreasing proportional fuzzy quantifier can be defined by a membership function as:

$$\mu_{Q}(y) = \begin{cases} 0 & \text{if } y < a \\ (y-a)/(b-a) & \text{if } a \le y \le b \\ 1 & \text{if } y > b. \end{cases}$$
(8)

Depending on the chosen quantifier Q, the values of a and b are different (see [40]). The weights of the OWA operator can be calculated as follows [37]:

R. Abolghasemi, P. Engelstad, E. Herrera-Viedma et al.

$$w_{k} = \mu_{Q}(\frac{k}{m}) - \mu_{Q}(\frac{k-1}{m}); k \in [1, \dots, m]$$
(9)

If we extend the notation to matrices, then the aggregated FPR P is $P = OWA_Q(P_1, ..., P_m)$, where the weights of OWA_Q are initialized with the quantifier Q. Another way to obtain the weight vector is to use each group member's contribution to the final decision, which means how much each member's opinion influences the final decision. This goal can be achieved by assigning an *importance degree* $u_k \in [0, 1]$ to each individual member in the group ($g_k \in G$). The concept of *Induced OWA* (IOWA) introduced by Yager et al. [41] relies on reordering the set of values and weighting them using some order-dependent weights. Later, in [42] *importance IOWA* (I-IOWA) used the same concept as IOWA and considered the importance of each preference. Therefore, the I-IOWA is defined as:

$$I - IOWA_{Q}((p_{1}, u_{1}), \dots, (p_{m}, u_{m})) = \sum_{k=1}^{m} w_{k} p_{\sigma(k)}$$
(10)

where u_k and p_k are the importance degree and preference values of user k, respectively. Q is a non-decreasing proportional fuzzy quantifier and σ is a permutation function where $(u_{\sigma(k)} \ge u_{\sigma(k+1)})$. The weight vectors in 10 can be obtained as:

$$w_k = \mu_{\mathbb{Q}}(\frac{S(k)}{S(m)}) - \mu_{\mathbb{Q}}(\frac{S(k-1)}{S(m)}); k \in [1, \dots, m]$$
(11)

In this equation, $S(k) = \sum_{l=1}^{k} u_{\sigma(k)}$. Extending the notation to matrices, for *m* individual FPRs and user's importance degree U, the aggregated FPR P is:

$$P = I - IOWA_Q((P_1, u_1), \dots, (P_m, u_m))$$
(12)

4. Proposed personality-based GRS

This section describes our proposed personality-based group recommendation system. The first subsection deals with personality traits and how we used the personality values to develop an influence graph in order to reach a consensus between the group members. In Section 4.2 an overview of the proposed method has been described in detail.

4.1. Decision-making based on Personality traits

The reason for using personality traits in group recommendation systems is due to the fact that, when a group of people want their preferences to converge to a common item, such as a movie to watch together, their individual impact on the final decision often varies depending on their individual personalities. Some people in the group may have a stronger personality, be more assertive, and have or display a confident and forceful personality. On the other hand, some people are cooperative and rely on mutual assistance when working towards a common goal. The more assertive a person is, the greater the effect they will have on the final decision. Psychological tests can be used to help us perceive people's different personalities. The Thomas-Kilmann Conflict Mode Instrument (TKI) [27] is a test designed to measure the personality of people in conflict situations. Based on this test, five personality styles can be identified, namely,

- Competing: a person who wants to be the winner, stands up for their rights and defends the position that they think is right.
- Collaborating: a person who is concerned with finding an appealing solution that completely satisfies all the group members as well as themselves.
- Avoiding: an unassertive and uncooperative individual who postpones an issue to a more suitable time.
- Accommodating: a very generous and selfless individual who obeys others.
- Compromising: neither a very cooperative nor a very assertive person who attempts to find an expedient, acceptable decision for both parties

Along the first dimension in Fig. 1, we can observe that a person will be assigned a high cooperativeness value if their personality style is very collaborative and accommodating. Similarly, a person with a very competitive and collaborative personality style will be given a high assertiveness value. Further explanations of how these values are calculated are provided in [26]. To mathematically formulate the result of the TKI test, we consider per_u to be the personality value of user u and compute it using the following equation [43]:

$$per_u = \frac{1 + Assertiveness(u) - Cooperativeness(u)}{2}$$
(13)

In this equation, per_u is a number in the range of [0, 1] and 1 will be assigned to a very selfish person and 0 to a very easygoing person.

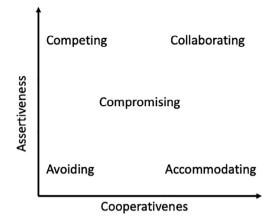


Fig. 1. TKI personality modes inspired by [43].

In [44], social influence is defined as changing someone's thoughts, feelings, attitudes, or behavior as a result of interacting with other people or belonging to a group. Therefore, a decision made by a group is the result of the group members' interaction. From a psychological point of view, this decision strongly depends on the personality of the group members. To model this influence, we propose using a graph that represents the different mutual influences between groups members (see Fig. 2). In this graph, the nodes are users in the group, and each arc (g_i, g_j) represents the strength of the influence of user *j* on user *i* in the group by $w_{ij} \in [0, 1]$. According to the normalization property, the influence of peers on each user should sum to one: $\forall i \in \{1, 2, ..., m\}, \sum_{j=1}^{m} w_{ij} = 1$ (*m* is the number of users in the group). So, we define the pairwise influence of peers *j* on *i* ($w_{i,j}$) as:

$$w_{ij} = \begin{cases} \frac{1}{m-1} \frac{per_j}{per_i + per_j} & \text{if } i \neq j \\ \\ 1 - \sum_{j=1, j \neq i}^m w_{ij} & \text{if } i = j \end{cases}$$
(14)

where per_i is the personality value of user *i* as defined in Eq. (13). It is worth noting that the formula is quite intuitive: $w_{i,j}$ is proportional to $\frac{per_j}{per_{i+per_i}}$ which reflects the influence *j* has on *i*, while the factor $\frac{1}{m-1}$ acts as a normalization factor.

If we consider $G = \{g_1, g_2, \dots, g_m\}$ as a group of *m* members, then $y_x^{(1)} = [v_{g_1x}, v_{g_2x}, \dots, v_{g_mx}]$ will be a vector, representing the group members' score for item *x* (see Eq. (5)). It is assumed that, after the group members interact, their opinions will change based on the influence they have on each other. Mathematically, $y_x^{(2)} = Wy_x^{(1)}$ where $W = (w_{i,j})$ [40] is an $m \times m$ weight matrix (see Eq. (14)). By iterating the process, after *t* iterations, the group members opinion will be:

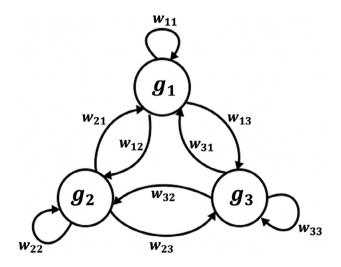


Fig. 2. A graph indicating the pairwise influence of peers in a group of three users.

R. Abolghasemi, P. Engelstad, E. Herrera-Viedma et al.

$$y_{x}^{(t)} = W y_{x}^{(t-1)}$$
(15)

which is equal to:

$$y_x^{(t)} = W^{t-1} y_x^{(1)} \tag{16}$$

Since matrix W is a square stochastic matrix, it can be considered as being the transition probability matrix of a Markov chain with *m* states and stationary transition probabilities. Based on [17], if a positive integer *l* exists such that every element in at least one column of the matrix W^l is positive, then *m* opinions are expected to converge to the same value. Accordingly, in our model, as the group members interact more, their opinions will eventually converge to the same value. So, after infinite iterations reaching a consensus is guaranteed:

$$y_x^{(\infty)} = W^{\infty} y_x^{(1)}$$
(17)

Since, in reality, interaction between group members over an infinite number of iterations is not feasible, we show how we obtain the final consensus value based on the stationary distribution of the associated Markov chain without applying infinite iterations. Furthermore, in practice, as our simulation results have shown, we can usually approach a consensus value very rapidly with only a few iterations. According to [45], the stationary distribution of a Markov chain is a vector $\pi = [\pi_1, \pi_2, \dots, \pi_m]$ that satisfies the following conditions:

1.
$$\pi W = \pi$$

2. $\forall i \in \{1, 2, ..., m\} : \pi_i \ge 0$
3. $\sum_{i=1}^{m} \pi_i = 1$

where W is the transition matrix and m is the number of states (in our model, m is the number of group members). From the first condition ($\pi W = \pi$), we generate $\pi W^{\infty} = \pi$. Therefore, the final group score S_{G_x} for item *x* will be calculated using the following equation.

$$S_{G,x} = \pi y_x^{(1)} \tag{18}$$

Note: At this point, it is worth mentioning that the above results can be also viewed from a GDM point of view. Therefore, we provide an alternative proof and interpretation of the convergence results. If we consider $P_k^{(1)} = (p_{kj}^{(1)})$ as the FPR of the *k*th member in the group (in our paper, $p_{kj} = r_{uij}^*$ in Eq. (4) or a pairwise preference score in the BPR and MPR model given by the member k), then, after t iterations, it is possible to compute k's user FPR as:

$$p_{kij}^{(t)} = I - IOWA_Q((p_{1ij}^{(t-1)}, w_{k1}), \dots, (p_{mij}^{(t-1)}, w_{km}))$$
(19)

where w_{ki} is the pairwise influence of peers i on k. Extending the notation to matrices, the previous equation changes to:

$$P_k^{(k)} = I - IOWA_Q((P_1^{(k-1)}, w_{k1}), \dots, P_m^{(k-1)}, w_{km}))$$
⁽²⁰⁾

In the Appendix, it is demonstrated that m FPRs converge to the same FPR if there is a positive integer l, so that every element in at least one column of W^{l} is positive. The convergence proof relies on demonstrating that our influence model coincides with a special case of a GDM model with a particular I-IOWA operator.

In the next section, we will show that by applying this method to the MovieLens dataset, the individual scores of the group members $\forall g_i \in G : v_{g,x}$ for every item will change after some iterations and converge to the same group score $S_{G,x}$. Note that our consensus model and the corresponding theoretical result can be seen as a special case of the model reported in the research carried out by one of the authors of this article [40].

4.2. Summary of the proposed method

This section describes the different steps of the proposed method shown in Fig. 3.

In order to be able to use the BPR and MPR methods, the dataset described in Section 5.1 should be changed to implicit feedback. In fact, both BPR and MPR rely on the fact that an observed item that is not chosen represents a form of negative implicit feedback (see Section 3). Generally, the implicit feedback is click, purchase, etc.

Here, we follow the same experimental methodology as [25], which considers ratings higher than 3 as the observed positive feedback.

The personalized item score v_{ui} for MFP is calculated using Eq. (5). When it comes to BPR and MPR, $v_{ui} = x_u^T y_i$ where x_u and *y*_i are *u*th row in matrix **X** and *j*th row in matrix **Y**, respectively. It is worth noting that **X** and **Y** are user and item factorized matrices in the matrix factorization process.

The personality-based weights for the group members have been calculated using Eq. (13) and Eq. (14). Since the users' personality traits in the aforementioned dataset was not available, we tested the proposed method using synthetic personality numbers (per_u) . In this way, we were able to demonstrate the influence of each group member's personality on the final item ratings, which could serve as an example of a real-life situation.

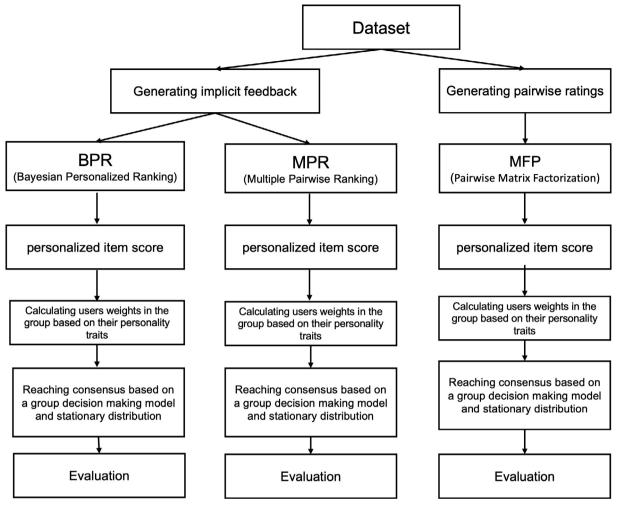


Fig. 3. Steps of the proposed method.

5. Experimental settings, and evaluation metrics

5.1. Dataset

The dataset for the MFP method was acquired from an online experiment performed by Blèdaitè et al. [14] to collect users' pairwise preferences. The authors developed an online interface that allows users to compare different movie pairs and enter their pairwise scores. In this experiment, a total of 2,262 pairwise scores related to 100 movies from the MovieLens dataset were collected based on feedback from 46 users. In addition, 73,078 movie ratings from 1,128 users in the MovieLens 100 K dataset were used. These movie ratings were converted into pairwise scores using the equation:

$$r_{uij} = r_{ui} - r_{uj}$$

where $r_{ui} \in [1, 5]$ is user *u*'s rating for item *i* and $r_{uij} \in [-4, +4]$ is user *u*'s pair score for items *i* and *j*, indicating how much user *u* prefers *i* over *j*. The dataset is summarized in Table 1. The dataset used for the BPR and MPR methods is MovieLens 100 K.

Table 1Dataset used for MFP method.

Dataset	#Users	#Movies	#Pair-scores
Online interface Dataset	46 #Users	100 #Movies	2262 #Ratings
MovieLens 100 K	1128	100	73078

5.2. Evaluation metrics

In order to check the quality of the proposed method, we used precision, consensus, and fairness. Precision is the fraction of the number of relevant recommended items (true positives) in relation to the total number of recommended items.

$$precision_G = \frac{\#TP_G}{\#(TP_G \cup FP_G)}$$
(22)

where TP_G and FP_G denote the true positive and false positive, respectively. They are defined as:

$$TP_G = \{i \in R_G | \exists g \in G \text{ such that } r_{g,i} \neq \bullet \text{ and } \forall u \in G r_{u,i} \neq \bullet \to r_{u,i} \geq \theta\}$$
(23)

$$FP_G = \{i \in R_G | \exists g \in G \text{ such that } r_{g,i} < \theta\}$$

$$\tag{24}$$

Here, the set of items recommended to group *G* is denoted R_G , while the rating of user *u* for item *i* is $r_{u,i}$. To measure whether a user likes or dislikes an item, we used a threshold $\theta = 4$. Note that the user test-ratings are on a scale from 1 to 5. In Eq. (23) the dot point (•) means that the rating is missing (not given by the user).

Consensus is a measure used to evaluate the extent to which the group members reached agreement [46]. In collaborative filtering, consensus is defined as the pairwise distance between the final item-x ratings $r_{g_i,x}^{(*)}$ of each group member g_i (see Eq. (25)). To normalize the result, the maximum possible rating r_{max} is used.

$$consensus = 1 - \frac{\sum_{\substack{(g_i,g_j) \in G, (i \neq j) \\ |G| \times (|G| - 1)/2 \times r_{max}}}{|G| \times (|G| - 1)/2 \times r_{max}}$$
(25)

Fairness can be regarded as a measure that evaluates how much the group members are satisfied with the final recommended items. In this work, fairness for every recommended item $x \in R_G$ is the fraction of group members $g \in G$ such that their rating $r_{g,x}$ for item x is greater than a threshold $\theta = 3.5$.

$$fairness(G, x) = \frac{\left| \bigcup_{g \in G} : r_{g,x} > \theta \right|}{|G|}$$
(26)

6. Results and evaluation

In Section 6.1, we provide some proof of concept experiments to illustrate how consensus is reached in our model. Then, in Section 6.2 we provide a more thorough evaluation of the performance of our approach on varying group sizes.

6.1. Reaching consensus using personality-based opinion dynamics

In this section, we will provide a few examples that show how our approach can reach consensus using the personalitybased opinion dynamics model presented in Eq. (17). These examples will help to illustrate the effect of personality (Eq. (13)) on the achieved consensus.

6.1.1. Reaching consensus

Personality-based weights.

Table 2

In order to explain the procedure whereby consensus is reached, we generated six random numbers $PER = [0.53, 0.84, 0.12, 0.41, 0.22, 0.30] \in [0, 1]$ as personality values of six individuals in the group, then converted them into personality-based weights using Eq. (14) (see Table 2). Each row *i* in the table indicates the weight of influence of others on person *i* (w_{ij} for $j = \{1, ..., m\}$). It is obvious that the diagonal entries in this matrix w_{ii} contain the influence each individual has on themselves, and the higher the *per_i* the higher the w_{ii} . This matrix *W* is considered to be a transition matrix of the Markov chain. According to the DeGroot opinion dynamics model [17], the item rating of the individuals changes after they

	Person 1	Person 2	Person 3	Person 4	Person 5	Person 6
Person 1	0.62	0.12	0.03	0.08	0.05	0.07
Person 2	0.02	0.74	0.03	0.08	0.03	0.07
Person 3	0.16	0.18	0.23	0.16	0.13	0.14
Person 4	0.11	0.13	0.04	0.56	0.06	0.08
Person 5	0.14	0.16	0.07	0.13	0.38	0.11
Person 6	0.13	0.15	0.05	0.12	0.08	0.47

interact due to the influence these individuals might have on each other, until they eventually agree on a rating after some iterations (see Fig. 4).Fig. 5 illustrates how, after few iterations, a group of six individuals can reach a consensus. In this example, $per_2 = 1.0$ indicates a very selfish person and $per_5 = 0.0$ indicates a completely obedient person. As expected, the final item score is very close to person 2's score and very different from person 5's score. In this sense, person 5 changed their opinion dramatically after just one interaction, while in all the interactions between the group members, person 2's opinion only changed slightly. Moreover, in the beginning, person 5 and person 6 started with the same opinion, namely, they both gave the item a score of 0.0. Although they finally agreed on 3.5 as the item score, it took more time for the less obedient person to converge to the final score. This figure confirms that adopting the weights according to the pairwise influence of peers given in Eq. (14) could model the role of group members' personality traits in making decisions and reaching consensus.

Item ranking and recommendation We applied the influence model described in Section 6.1.1 to all of the items in the dataset in order to deduce the group's final personalized item scores for every item as given by Eq. (5). Then, we sorted them in descending order and recommended top-10 items to the group. Fig. 6 illustrates an example of convergence to final group personalized item scores in the case of six experts, each with a different initial score for each of the four items. We observe that, after some iterations, all group members reach consensus on every item score. The higher the final item score is, the better the rank in the final order will be. Please note that each of the four colors in the graph corresponds to a different item.

6.2. Evaluation under varying group sizes

In Tables 3–5, the evaluation results for three models, with and without considering members' personalities, are reported. The results correspond to the average evaluation of four group sizes: small (i.e., $\{\forall G \subseteq U : |G| \in [2,4]\}$), mid-size (i.e., $\{\forall G \subseteq U : |G| \in [5,8]\}$), large (i.e., $\{\forall G \subseteq U : |G| \in [5,8]\}$), large (i.e., $\{\forall G \subseteq U : |G| \in [13,20]\}$). Interestingly, for all of the models, the consensus is 1, which means that the preference score for the recommended items given by all members in every group converged to the same number. This is a consequence of the influence model we adopted. Furthermore, since assertive members have more influence on the final decision, it seems that taking personality traits into consideration in item recommendations will result in an unfair recommendation. To investigate this, we adopted 1,000 random configurations for every group size. Interestingly, our results summarized in the tables show that the precision and fairness in our models are almost the same with and without taking personality into consideration. Although some members are more influential than others, on average, our recommendation model is still able to find a set of top items that appeal to all members.

Moreover, in order to show different personality scenarios and the impact they have on precision and fairness, we repeated the experiment with a group of four users. The results can be seen in Table 6. According to this table, one possibility is that all the users have the same personality, for instance, all are strongly assertive (first row). Therefore, all personality values are 1 ($per_i = 1, \forall i \in G$) (see Eq. (13)). Then according to Eq. (14), $w_{ij} = w_{j,i}, \forall i, j \in G$. This means that all users have the same influence on the final decision. The weights will be the same when all users are strongly easy-going or when they all have equal personality values. Since every user in this scenario has the same impact on the final decision, we expect to get a fair recommendation. The results reported in the table confirm this expected result and maximum fairness (0.7) value is achieved in this scenario.

The second scenario in the table is when there is a very assertive person and the others are highly easy-going, like a leader and his followers (second row in the table). In such a situation in the real world, all users would follow the "leader" and the final decision would be as much closer to the opinion of the leader than to the opinions of the followers. The results reported in the table confirm this and the lowest fairness (0.6) value is reported in this scenario.

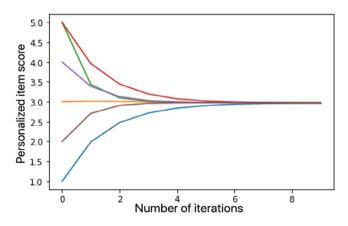


Fig. 4. Reaching consensus in a group of 6 members (15,16,17).

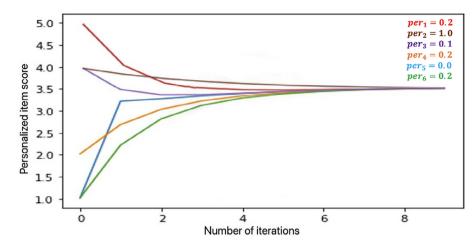


Fig. 5. Reaching consensus in a group of six members – *per* values in the upper right corner of the figure indicate personality traits of the group members ((15)–(17)).

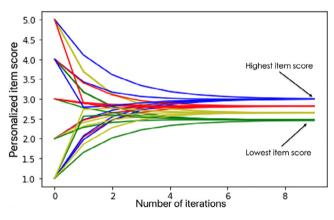


Fig. 6. Illustration of convergence to group final personalized item scores corresponding to four different items for six experts ((15)-(17)).

Table 3Comparing precision and fairness with and without consider-
ing members' personalities from the model based on BPR.

Personalities	Group Size	Precision	Fairness
Same	Small	0.68	0.76
Random	Small	0.69	0.76
Same	Mid-size	0.81	0.76
Random	Mid-size	0.82	0.76
Same	Large	0.87	0.76
Random	Large	0.87	0.76
Same	Very large	0.91	0.77
Random	Very large	0.91	0.77

We repeated the experiment with different random personalities. The results show slight changes in precision and fairness, on different types of distributions. However, the closer the personality values are to the second model (leader and followers), the higher the precision and the lower the fairness.

Figs. 7–9 indicate the precision vs. fairness of three models based on MFP, BPR, and MPR, respectively, in two different states: a) personality traits are not considered, b) random personalities are assigned to group members. Models were run in groups of sizes varying from 2 to 20. In the figures, the sizes of the bubbles are proportional to the group sizes.

Information Sciences 595 (2022) 1-17

Table 4

Comparing precision and fairness with and without considering members' personalities from the model based on MPR.

Personalities	Group Size	Precision	Fairness
Same	Small	0.71	0.79
Random	Small	0.72	0.79
Same	Mid-size	0.85	0.79
Random	Mid-size	0.85	0.79
Same	Large	0.90	0.79
Random	Large	0.91	0.79
Same	Very large	0.93	0.80
Random	Very large	0.93	0.80

Table 5

Comparing precision and fairness with and without considering members' personalities from the model based on MFP.

Personalities	Group Size	Precision	Fairness
Same	Small	0.74	0.72
Random	Small	0.75	0.70
Same	Mid-size	0.84	0.69
Random	Mid-size	0.84	0.67
Same	Large	0.89	0.68
Random	Large	0.89	0.68
Same	Very large	0.92	0.68
Random	Very large	0.92	0.68

Table 6

Different personalities in a group of four members result in different fairness.

	Personality Traits	personality values	Precision	Fairness
1	All assertive (All the same personality)	1.00, 1.00, 1.00, 1.00	0.80	0.70
2	One assertive, rest easy-going	1.00, 0.00, 0.00, 0.00	1.00	0.60
3	Three assertive, one easy-going	1.00, 1.00, 0.00, 1.00	0.80	0.68
4	Four random personalities	0.50, 0.10, 0.30, 0.20	0.80	0.69
5	Four random personalities	0.40, 0.28, 0.65, 0.84	0.80	0.69
6	Four random personalities	0.24, 0.57, 0.97, 0.24	0.80	0.69
7	Four random personalities	0.54, 0.52, 0.48, 0.38	0.80	0.70
8	Four random personalities	0.05, 0.12, 0.50, 0.52	0.81	0.67
9	Four random personalities	0.04, 0.11, 0.47, 0.62	0.79	0.68
10	Four random personalities	0.12, 0.60, 0.51, 0.04	0.78	0.67
11	Two assertive personalities	1.00, 1.00, 0.00, 0.00	0.88	0.63

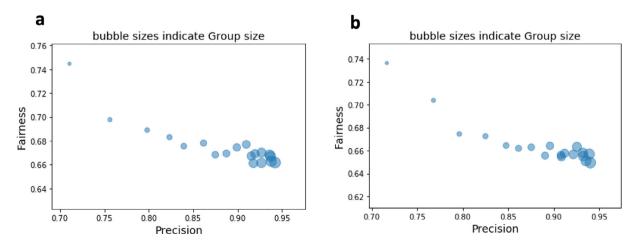


Fig. 7. Precision vs fairness from the model based on MFP in two different states: a) same personality traits, b) random personality traits (Bubble sizes indicate the group sizes).

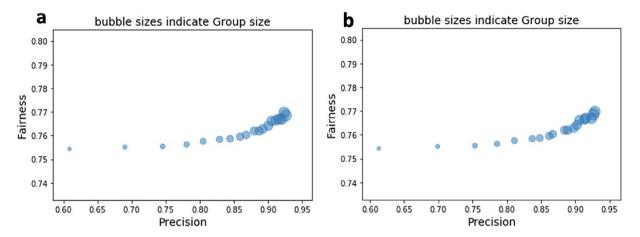


Fig. 8. Precision vs fairness from the model based on BPR in two different states: a) same personality traits, b) random personality traits (Bubble sizes indicate the group sizes).

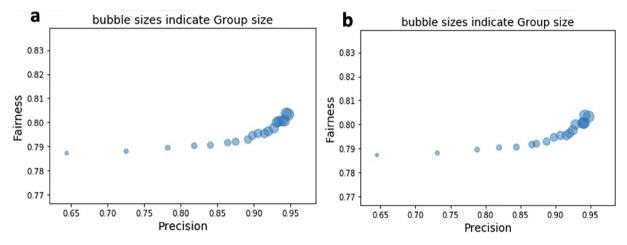


Fig. 9. Precision vs fairness from the model based on MPR in two different states: a) same personality traits, b) random personality traits (Bubble sizes indicate the group sizes).

7. Managerial implications and discussions

By comparing Figs. 7–9, primarily, we observe that the average fairness in the methods based on BPR and MPR is better than it is with methods based on MFP. This might be due to the fact that in BPR and MPR based methods, items with positive feedback have been pushed up and those without feedback pushed down. Therefore, the proposed method could recommend items that appeal to more users in the group and as a result, the average fairness is higher.

As stated in [47,48], generally, in group recommendation system, fairness decreases as the group size increases, which is the same trend as observed in Fig. 7. However, in Figs. 8 and 9, we observe that the larger the group size, the fairer the model. This is because the influence of the personality score on the outcome is greater for smaller group sizes. For instance, in a group of two users, the opinion of the stronger personality would dominate the weaker personality. Thus, if the result is not preferred by the non-dominant user, it would be unfair to 50% of the group members (one of the two), while, in larger-sized groups, the influence of the personality has diminished. This is because the outcome is less likely to be dominated by one personality in larger groups as compared with smaller groups. Consequently, the recommended items would be appealing to more members.

By the discussion above, we have presented new models for group recommendation, in which the pairwise preference is focused on achieving stronger results. We have also focused on a real-life scenario in which members not only discuss their preferences, but also influence each other's decisions through a factor we call 'personality'. We have also observed that when there is a strong desire for fairness, BPR and MPR based models perform better. Similarly, when the group size is large, it is better not to use the MFP based model.

8. Conclusion

This paper proposes an approach to group recommendations that takes the personalities of group members into account. In the proposed approach, three pairwise scoring methods (BPR, MPR, and MFP) were used to predict item scores. We also designed a consensus model based on personality traits that results in a joint group recommendation, and we evaluated the fairness and precision of the proposed GRS for different member personalities using real-life datasets.

One of the limitations of the proposed method is the way the personality traits are computed. In fact, our method requires users to fill in a TKI test composed of 30 questions. As performing such a test might not be so practical in real-life scenarios or for large groups, as future work we would like to investigate other more lightweight alternatives to assess users' personality traits. For instance, in [28], personality values were predicted from each user's social media content, while in [29], a TKI metaphor was used to replace the TKI test.

Moreover, in our paper, users were considered to have 'fixed' personality traits. However, in practice, users may change their attitude if they start to understand the impact of their personality value. An open research question is whether it is possible to design an approach that gives incentives to the users to report their personality traits truthfully.

In the future, in addition to the personality traits of the users used in this work (assertiveness and cooperativeness), we could also take into account the level of curiosity of the individuals. It might be of interest to investigate whether individuals with a high level of curiosity are more interested in receiving recommendations that are different from their previous experiences, while individuals with a low level of curiosity tend to do the same things they did in the past, with no interest in new or different areas.

CRediT authorship contribution statement

Roza Abolghasemi: Conceptualization, Methodology, Software, Validation, Formal analysis, Writing - original draft, Writing - review & editing, Visualization. **Paal Engelstad:** Conceptualization, Methodology, Writing - review & editing. **Enrique Herrera-Viedma:** Conceptualization, Methodology, Supervision, Writing - review & editing, Project administration.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

The work of the third author is supported by both Spanish State Research Agency Project PID2019-10380RBI00/AEI/10. 13039/501100011033 and Andalusian Government Project P20_00673.

Appendix A. Appendix

In this section, we prove the convergence of the influence model as a special case of the GDM model with a particular I-IOWA operator, which works similarly to [40]. In Eq. (8), if we consider a = 0 and b = 1 for the quantifier Q and a positive integer *l* exists such that at least one column in the weight matrix has positive elements, then all FPRs converge to the same FPR.

Combining the definition of FPR in Eq. (19) with the I-IOWA operator (Eq. (11)) and weight Eq. (10), the elements of FPR P are defined as:

$$p_{k}^{(t)} = I - IOWA_{Q}((p_{1}^{(t-1)}, w_{k1}), \dots, (p_{m}^{(t-1)}, w_{km}))$$

$$= \sum_{i=1}^{m} (\mu_{Q}(\frac{S(i)}{S(m)}) - \mu_{Q}(\frac{S(i-1)}{S(m)}))p_{\sigma(i)}^{t-1}$$
(27)

In this equation, $S(i) = \sum_{j=1}^{i} w_{k\sigma(j)}$ and σ is a permutation function. If we consider a = 0 and b = 1 in quantifier Q, then according to Eq. (8), for $0 \le y \le 1$, $\mu_Q(y) = y$. Since $S(i) \le S(m)$ then it is clear that $0 \le \frac{S(i)}{S(m)} \le 1$. Therefore, $\mu_Q(\frac{S(i)}{S(m)}) = \frac{S(i)}{S(m)}$. Substituting this into the previous equation gives:

$$p_{k}^{(t)} = \sum_{i=1}^{m} \frac{S(i)}{S(m)} - \frac{S(i-1)}{S(m)} p_{\sigma(i)}^{(t-1)}$$

$$= \sum_{i=1}^{m} \frac{\sum_{j=1}^{i} w_{k\sigma(j)} - \sum_{j=1}^{i-1} w_{k\sigma(j)}}{\sum_{j=1}^{m} w_{k\sigma(j)}} p_{\sigma(i)}^{(t-1)}$$

$$= \sum_{i=1}^{m} \frac{w_{k\sigma(i)}}{\sum_{j=1}^{m} w_{k\sigma(j)}} p_{\sigma(i)}^{(t-1)}$$
(28)

According to the paper, $\sum_{j=1}^{m} w_{kj} = 1$. Since σ is a permutation function, it only changes the order of the items. Therefore, $\sum_{k=1}^{m} w_{k\sigma(i)} = 1$, as well. By replacing this in the previous equation, we get:

$$p_{k}^{(t)} = \sum_{i=1}^{m} w_{k\sigma(i)} p_{\sigma(i)}^{(t-1)} = \sum_{i=1}^{m} w_{ki} p_{i}^{(t-1)}$$
(29)

Extending the notation to the matrix W, we can generalize the preceding equation to $p^{(t)} = Wp^{(t-1)} = W^{t-1}p^{(1)}$. As explained in Section 4.1, W can be considered as the transition probability matrix of a Markov chain with m states and stationary transition probabilities. Based on [17], if a positive integer l exists such that every element in at least one column of the matrix W^l is positive, then a value p exists such that m opinions are expected to converge to it ($\forall k \in \{1, ..., m\} \lim_{t\to\infty} p_k^{(t)} = p$). This means that after t iterations, users' preferences will converge to the same value. Extending the notation to the FPRs, after t iterations, all FPRs converge to the same FPR.

References

- [1] M.A. Hameed, O. Al Jadaan, S. Ramachandram, Collaborative filtering based recommendation system: A survey, Int. J. Comput. Sci. Eng. 4 (5) (2012) 859.
- [2] M.J. Pazzani and D. Billsus, Content-based recommendation systems, in The adaptive web. Springer, 2007, pp. 325-341..
- [3] S. Bouraga, I. Jureta, S. Faulkner, C. Herssens, Knowledge-based recommendation systems: A survey, Int. J. Intell. Inform. Technol. (IJIIT) 10 (2) (2014) 1– 19.
- [4] J. Bobadilla, F. Serradilla, J. Bernal, A new collaborative filtering metric that improves the behavior of recommender systems, Knowl.-Based Syst. 23 (6) (2010) 520–528.
- [5] Y. Park, S. Park, W. Jung, S. Lee, Reversed CF: A fast collaborative filtering algorithm using a k-nearest neighbor graph, Expert Syst. Appl. 42 (8) (2015) 4022–4028.
- [6] C.C. Aggarwal, Model-based collaborative filtering, in Recommender systems. Springer, 2016, pp. 71-138..
- [7] D. Bokde, S. Girase, D. Mukhopadhyay, Matrix factorization model in collaborative filtering algorithms: A survey, Procedia Comput. Sci. 49 (2015) 136– 146.
- [8] N. Jones, A. Brun, A. Boyer, and A. Hamad, An exploratory work in using comparisons instead of ratings, in E-Commerce and Web Technologies 12th International Conference, ser. Lecture Notes in Business Information Processing, vol. 85. Springer, 2011, pp. 184–195..
 [9] E. Herrera-Viedma, S. Alonso, F. Chiclana, F. Herrera, A consensus model for group decision making with incomplete fuzzy preference relations, IEEE
- Trans. Fuzzy Syst. 15 (5) (2007) 863–877.
- [10] Y. Dong, Q. Zha, H. Zhang, G. Kou, H. Fujita, F. Chiclana, E. Herrera-Viedma, Consensus reaching in social network group decision making: Research paradigms and challenges, Knowl.-Based Syst. 162 (2018) 3–13.
- [11] J.A. Morente-Molinera, G. Kou, I.J. Pérez, K.E. Samouylov, A. Selamat, E. Herrera-Viedma, A group decision making support system for the web: How to work in environments with a high number of participants and alternatives, Appl. Soft Comput. 68 (2018) 191–201.
- [12] S. Kalloori, F. Ricci, M. Tkalcic, Pairwise preferences based matrix factorization and nearest neighbor recommendation techniques, in: Proceedings of the 10th ACM Conference on Recommender Systems, Boston, ACM, 2016, pp. 143–146.
- [13] S. Kalloori, F. Ricci, R. Gennari, Eliciting pairwise preferences in recommender systems, in: Proceedings of the 12th ACM Conference on Recommender Systems ACM, 2018, pp. 329–337.
- [14] L. Blédaité, F. Ricci, Pairwise preferences elicitation and exploitation for conversational collaborative filtering, in: Proceedings of the 26th ACM Conference on Hypertext & Social Media ACM, 2015, pp. 231–236.
- [15] S. Rendle, C. Freudenthaler, Z. Gantner, L. Schmidt-Thieme, BPR: bayesian personalized ranking from implicit feedback, in UAI 2009, in: Proceedings of the Twenty-Fifth Conference on Uncertainty in Artificial Intelligence, 2009, pp. 452–461.
- [16] R. Yu, Y. Zhang, Y. Ye, L. Wu, C. Wang, Q. Liu, E. Chen, Multiple pairwise ranking with implicit feedback, in: Proceedings of the 27th ACM International Conference on Information and Knowledge Management ACM, 2018, pp. 1727–1730.
- [17] M.H. DeGroot, Reaching a consensus, J. Am. Stat. Assoc. 69 (345) (1974) 118-121.
- [18] C. Bologna, A.C.D. Rosa, A.D. Vivo, M. Gaeta, G. Sansonetti, and V. Viserta, Personality-based recommendation in e-commerce, in Late-Breaking Results, Project Papers and Workshop Proceedings of the 21st Conference on User Modeling, Adaptation, and Personalization, vol. 997, 2013.
- [19] U. Gretzel, N. Mitsche, Y. Hwang, D.R. Fesenmaier, Tell me who you are and I will tell you where to go: Use of travel personalities in destination recommendation systems, Inform. Technol. Tourism 7 (1) (2004) 3–12.
- [20] S. Dhelim, H. Ning, N. Aung, R. Huang, J. Ma, Personality-aware product recommendation system based on user interests mining and metapath discovery, IEEE Trans. Comput. Soc. Syst. (2020).
- [21] A. Tommasel, A. Corbellini, D. Godoy, S.N. Schiaffino, Exploring the role of personality traits in followee recommendation, Online Inform. Rev. 39 (6) (2015) 812–830.
- [22] M.A.S.N. Nunes, R. Hu, Personality-based recommender systems: an overview, in Sixth ACM Conference on Recommender Systems. ACM, 2012, pp. 5– 6..
- [23] X. Lin, M. Zhang, Y. Zhang, Z. Gu, Y. Liu, S. Ma, Fairness-aware group recommendation with pareto-efficiency, in: Proceedings of the Eleventh ACM Conference on Recommender Systems ACM, 2017, pp. 107–115.

- [24] Z. Huang, X. Xu, H. Zhu, M. Zhou, An efficient group recommendation model with multiattention-based neural networks, IEEE Trans. Neural Networks Learn. Syst. 31 (11) (2020) 4461–4474.
- [25] W. Pan, L. Chen, GBPR group preference based bayesian personalized ranking for one-class collaborative filtering, in IJCAI 2013, in: Proceedings of the 23rd International Joint Conference on Artificial Intelligence, 2013, pp. 2691–2697.
- [26] J.A. Recio-Garcia, G. Jimenez-Diaz, A.A. Sanchez-Ruiz, B. Diaz-Agudo, Personality aware recommendations to groups, in: Proceedings of the third ACM conference on Recommender systems, 2009, pp. 325–328.
- [27] R.H. Kilmann, K.W. Thomas, Developing a forced-choice measure of conflict-handling behavior: The mode instrument, Educat. Psychol. Meas. 37 (2) (1977) 309–325.
- [28] R. Gao, B. Hao, S. Bai, L. Li, A. Li, T. Zhu, Improving user profile with personality traits predicted from social media content, in: Proceedings of the 7th ACM conference on recommender systems, 2013, pp. 355–358.
- [29] L.Q. Sánchez, J.A. Recio-García, B. Díaz-Agudo, Happymovie: A facebook application for recommending movies to groups, in: IEEE 23rd International Conference on Tools with Artificial Intelligence, 2011, pp. 239–244.
- [30] L.Q. Sánchez, Personality and social trust in group recommendations, in 22nd IEEE International Conference on Tools with Artificial Intelligence. IEEE Computer Society, 2010, pp. 121–126..
- [31] F. Barile, F. Cervone, and S. Rossi, Evaluating user's personality and social interactions for groups recommendations, in Proceedings of the 2nd International Workshop on Decision Making and Recommender Systems, vol. 1533, 2015, pp. 17–20..
- [32] P.T. Costa Jr, R.R. McCrae, The Revised NEO Personality Inventory (NEO-PI-R), Sage Publications Inc, 2008.
- [33] X. Wang, Y. Liu, J. Lu, F. Xiong, G. Zhang, Trugrc: Trust-aware group recommendation with virtual coordinators, Future Gener. Comput. Syst. 94 (2019) 224-236.
- [34] L. Sun, X. Wang, Z. Wang, H. Zhao, W. Zhu, Social-aware video recommendation for online social groups, IEEE Trans. Multimedia 19 (3) (2017) 609-618.
- [35] J. Guo, Y. Zhu, A. Li, Q. Wang, W. Han, A social influence approach for group user modeling in group recommendation systems, IEEE Intell. Syst. 31 (5) (2016) 40-48.
- [36] Y.-M. Wang, Z.-P. Fan, Fuzzy preference relations: Aggregation and weight determination, Comput. Ind. Eng. 53 (1) (2007) 163–172.
- [37] R.R. Yager, Families of owa operators, Fuzzy Sets Syst. 59 (2) (1993) 125-148.
- [38] J. Malczewski, Ordered weighted averaging with fuzzy quantifiers: Gis-based multicriteria evaluation for land-use suitability analysis, Int. J. Appl. Earth Observ. Geoinform. 8 (4) (2006) 270–277.
- [39] F. Chiclana, F. Herrera, E. Herrera-Viedma, Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations, Fuzzy Sets Syst. 97 (1) (1998) 33–48.
- [40] N. Capuano, F. Chiclana, H. Fujita, E. Herrera-Viedma, V. Loia, Fuzzy group decision making with incomplete information guided by social influence, IEEE Trans. Fuzzy Syst. 26 (3) (2017) 1704–1718.
- [41] R.R. Yager, D.P. Filev, Induced ordered weighted averaging operators, IEEE Trans. Syst., Man, Cybern. Part B (Cybernetics) 29 (2) (1999) 141-150.
- [42] F. Chiclana, E. Herrera-Viedma, F. Herrera, S. Alonso, Some induced ordered weighted averaging operators and their use for solving group decisionmaking problems based on fuzzy preference relations, Eur. J. Oper. Res. 182 (1) (2007) 383–399.
- [43] L. Quijano-Sanchez, J.A. Recio-Garcia, B. Diaz-Agudo, G. Jimenez-Diaz, Social factors in group recommender systems, ACM Transactions on Intelligent Systems and Technology (TIST) 4 (1) (2013) 1–30.
- [44] Q. Liang, X. Liao, J. Liu, A social ties-based approach for group decision-making problems with incomplete additive preference relations, Knowl.-Based Syst. 119 (2017) 68–86.
- [45] E. Seneta, Computing the stationary distribution for infinite markov chains, Linear Algebra and Its Applications 34 (1980) 259-267.
- [46] A. Felfernig, L. Boratto, M. Stettinger, M. Tkalčič, Group recommender systems: An introduction, Springer, 2018.
- [47] M. Kaya, D. Bridge, N. Tintarev, Ensuring fairness in group recommendations by rank-sensitive balancing of relevance, in: Fourteenth ACM Conference on Recommender Systems, 2020, pp. 101–110.
- [48] D. Sacharidis, Top-n group recommendations with fairness, in Proceedings of the 34th ACM/SIGAPP Symposium on Applied Computing. ACM, 2019, pp. 1663–1670.