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"Uncertainty is the only certainty there is, and knowing how to live with insecurity is the only security"
– John Allen Paulos, 2003

ABSTRACT

Many different papers document the hedging effectiveness with the use of futures contracts, and this paper presents the analysis of the hedging effectiveness of Brent crude oil futures contracts of different estimation models and maturities. The intention is to find the most appropriate futures contract an oil producer should implement in its hedging strategy. Further, the purpose of this paper is to propose an optimal hedging strategy for the risk management that handles Brent crude oil. The hedging performances of the different models and maturities are compared, and the findings indicate that the three-monthly contracts of naïve hedge ratio model have the highest performance in reducing price risk, which is the model that also provides the lowest costs. This contradicts other empirical frameworks that find the futures contracts of shortest maturity most efficient. In the analysis, another finding is that a segmentation of the time series according to historical events, such as the financial crisis and the price fall in 2014, shows that there are changes in the hedging effectiveness. Also, an out-of-sample model that forecasts spot prices provides a prediction of the future hedging effectiveness. Despite limited information about the future, the forecasted hedging effectiveness captures approximately 90% of the actual hedging effectiveness, and gives an average deviation of only ten percentage points.

1. INTRODUCTION

Hedging with futures contracts is an essential part in risk management. The hedging strategy is measured through the performance of reducing the risk, and many different models are used in order to find the optimal hedge ratio. A number of studies focus on hedge ratio and hedging effectiveness. In this paper, the purpose is to provide new knowledge and insight about hedging with Brent crude oil futures contracts, as there exist few studies about hedging with these specific contracts. The reason for analysing Brent crude oil futures is because the production of Brent crude oil highly effects the Norwegian economy, and we question if a Brent crude oil producer can use futures contracts to reduce the risk of price changes.

Chen, Lee, and Shrestha (2003) provide a broad and detailed theoretical review of different hedge ratio estimation models, and we will use a selection of these models in this paper. However, the hedge ratio does not imply its efficiency in terms of reducing risk, and the hedging effectiveness must therefore be provided. The hedging effectiveness is the fraction that eliminates the price risk of the underlying asset. Ederington (1979) presents the importance of calculating the hedging effectiveness and how it can be achieved with an optimal hedge ratio. The hedging effectiveness makes it possible to compare the different models and maturities in order to find the most optimal model that removes most of the price risk.

The theoretical models provided in the literature assume that the prices are in a weak-form efficiency of the efficient market hypothesis, and the demand should be equal to the supply in an efficient market. However, this may not be the situation in the oil industry because the oil cartel, OPEC, influences the price by setting the quota. Golombek, Irarrazabal, and Ma (2017, p. 99) find that OPEC has substantial market power and that it is currently accounting for one-third of the global supply of oil (Vecchio, n.d., p. 3). OPEC's producers have together the biggest oil reserve with 82% of the global reserves (OPEC, 2018). Therefore, due to OPEC's market power, it is necessary to test whether the oil prices follow a random walk or not.

In this study, we examine the research question about which hedge ratio estimation model is optimal for a Brent crude oil producer to use in terms of its performance. We consider different estimation models, both static and dynamic. Some of the static models incorporate expected return and/or level of risk aversion. The focus is also on different hedging maturities of one, three, six, and twelve months.

The main findings are that the static MV hedge ratio is the best based on its simplicity and performance, and in terms of the best hedging maturity, the three-monthly contracts are the most efficient. However, a Wald test indicates that the static MV and naïve hedge ratios are not significantly different for the three-monthly contracts. Hence, these two hedge ratios are equivalent. This is an interesting result as the naïve hedge ratio model is a cheap and easy strategy, and it therefore seems to be little gain from an advanced risk management. We conclude that the naïve hedge ratio model with three-monthly futures contracts is the best model.

Nonetheless, historical events had an impact on the oil prices, and these may have had an effect on the hedge ratio and hedging effectiveness. Therefore, we consider the consequences the financial crisis of 2007 and the sharp fall in the oil prices in November 2014 had on the efficiency. This is tested by using the static MV hedge ratio model in a segmentation of the time series into subperiods. We find that there is a small decrease in hedging effectiveness of thirteen percentage points after the price fall in 2014.

Despite not finding any differences in the segmentation, there is reason to believe that the hedge ratio is not constant but fluctuates during the analysis period. Therefore, we analyse yearly estimations of the static MV hedge ratio and its hedging effectiveness. The yearly estimations show remarkable differences, which give indications of how important it is for a hedger to be aware of price changes and to adjust the hedge ratio accordingly.

Until now, the problem is that the estimation models are based on historical data, and if using these models for hedging, these will give the assumption of knowing what will happen in the future. However, a producer does not have information about the future, and there are therefore uncertainties about the future oil prices. Accordingly, an oil producer can implement an out-of-sample model to forecast future oil prices and hence the hedge ratio and hedging effectiveness. We find the out-of-sample model with a GARCH (1,1) estimation method to predict the hedge ratio and hedging effectiveness sufficiently good with capturing approximately 70% and 90% of the trend of the actual estimates, respectively, and with an average deviation of ten percentage points between the forecasted and actual hedging effectiveness. The high deviation during the financial crisis in terms of both HR and HE can indicate that the impacts from the financial crisis on the hedging with futures contracts were unpredictable.

This paper is organised as following: Section 2 presents the literature review of the different models of estimating hedge ratio and their hedging effectiveness, which is followed by Section 3 that illustrates the research methodology. Section 4 describes the data that are used and shows the tests of the models' assumptions. Section 5 presents the empirical results from the analysis, and Section 6 consists of a discussion and criticism of the paper. Finally, Section 7 presents the conclusion.

2. LITERATURE REVIEW

According to Chen et al. (2003, p. 436), the primarily objective of hedging is to create a portfolio consisting of contracts from both the spot and futures markets in order to reduce price risk. A producer invests in C_s units of long spot contracts and C_f units of short futures contracts, where:

$$C_f = HR \times C_s \tag{1}$$

and HR is the hedge ratio. The hedge ratio decides how many units of futures contracts that should be purchased for each unit of spot contract. The gain or loss of the hedged portfolio can be viewed as the change of the basis, where the basis is the difference between the spot and futures prices (Ederington, 1979, p. 159). Thus, the gain or loss on the hedged portfolio for each unit of spot contract is as following (Junkus & Lee, 1985):

$$\Pi = (S_t - S_{t-1}) - HR \times (F_t - F_{t-1}) \tag{2}$$

where Π is the profit, S_t and F_t are the spot and futures prices at time t, and S_{t-1} and F_{t-1} are the spot and futures prices at time t-*1*, respectively. Basis risk occurs when the spot and futures contracts do not have the same maturity or when the asset specified in the futures contract is not the same as the underlying asset. The change in the basis, or basis risk, can be reflected as the variance or the standard deviation. If the basis is equal to zero at the maturity of the futures contract, the hedged position is a perfect hedge. A perfect hedge is when the spot and futures contracts offset each other and give a 100% reduction of the price risk, and the hedger will therefore neither gain nor lose.

The price of a futures contract is written as $F_0 = S_0e^{rT}$, where F_0 is the future spot price calculated by the current spot price, S_0, and the risk-free rate of return, r, at maturity, T. This indicates that the futures contract is an unbiased estimate of the future spot price, or in other words, that the prices are perfectly correlated. Given the assumption of perfect correlation, HR is equal to one, and the hedger should obtain one short futures contract for each long spot contract.

Due to the hedgers' different objectives of facing risk, the theoretical models of optimal hedge ratio are calculated in various ways and consider different, relevant factors. For instance, some hedgers consider the level of risk aversion or the expected return of the futures contracts. These theoretical models include static hedge ratios, such as naïve, static minimum variance (MV), optimum mean-variance, and Sharpe hedge ratio, and a dynamic minimum variance (MV) hedge ratio. The optimal hedge ratio (HR^*) is the ratio that minimises the price risk to the given model.

2.1 Optimal hedge ratio

2.1.1 Static models

A static hedge ratio assumes that the hedge ratio is constant during the whole hedging period. However, prices are not fixed over time and new information will affect a static hedge ratio. Despite this, the static models are simple estimation models in order to calculate an optimal hedge ratio.

Naïve hedge ratio

The simplest of the five models is the naïve hedge ratio model. A naïve hedge ratio is straightforward with a hedging position in equal numbers of futures contracts as the position in the underlying asset. Therefore, the hedge ratio will always be one, as shown in the following equation (Cotter & Hanly, 2006, p. 686):

$$HR_1^* = 1 \tag{3}$$

where HR_1^* is the optimal hedge ratio.

The model assumes that the spot and futures prices are perfectly correlated and that the naïve hedge ratio will always be one. However, this is usually not an appropriate way of handling risk as the hedger can be under- or overhedged, and this is not an optimal position because it does not minimise the price risk. This is because the prices are usually not perfectly correlated.

Static minimum variance (MV) hedge ratio

The static MV hedge ratio model is the most widely used model because of its performance and simplicity, and its optimal hedge ratio gives the minimum level of variance that is achievable. It minimises the hedged portfolio risk through the following equation (Johnson, 1960, p. 143):

$$Var_H = \sigma_S^2 + HR^{*2} \times \sigma_f^2 - 2 \times HR^* \times \sigma_{sf} \quad (4)$$

where Var_H is the variance of the hedged portfolio, σ_s^2 and σ_f^2 are the variances of the returns of the spot and futures prices, respectively. HR^* is the optimal hedge ratio, and σ_{sf} is the covariance between the returns of the spot and futures prices ($\sigma_{sf} = \rho\sigma_s\sigma_f$). Thus, the static MV hedge ratio is given by (Johnson, 1960, p. 143):

$$HR_2^* = \rho_{sf} \frac{\sigma_s}{\sigma_f} \quad (5)$$

where HR_2^* is the optimal static MV hedge ratio, and σ_s and σ_f are the standard deviations of the returns of the spot and futures prices, respectively. ρ is the correlation between the returns of the spot and futures prices. When the correlation between the returns of the spot and futures is equal to one and the standard deviations are identical, the hedge ratio is one and equal to a naïve hedge ratio. When the hedge ratio is one, the futures prices perfectly reflect the spot prices.

However, the static MV hedge ratio ignores the expected rate of return of the futures prices or the hedger's level of risk aversion. The following two static models take these factors into consideration.

Optimum mean-variance hedge ratio

The optimum mean-variance hedge ratio model estimates the hedge ratio that gives the minimum level of price risk while taking the expected rate of return of the futures prices and the hedger's level of risk aversion into account. The optimal hedge ratio is the maximisation of the utility function as following (Hsin, Kuo, & Lee, 1994):

$$\max\ V\,(E(R_h), \sigma; A) = E(R_h) - 0{,}5A\sigma_h^2 \quad (6)$$

where A is the risk aversion parameter that depends on the amount of risk the individual is willing to accept. The risk aversion is an individual parameter and the risk tolerance will vary across producers. The optimal hedge ratio of the optimum mean-variance hedge ratio model is as following (Hsin, Kuo, & Lee, 1994):

$$HR_3^* = -\left[\frac{E(R_f)}{A\sigma_f^2} - \rho\frac{\sigma_s}{\sigma_f}\right] \quad (7)$$

where $E(R_f)$ is the expected return of the futures prices, A is the risk aversion parameter, σ_f^2 is the variance of the futures prices. σ_s and σ_f are the standard deviations of the returns of the spot and futures prices, respectively, and ρ is the correlation between the returns of the spot and futures prices.

With the assumption of infinite risk aversion or zero expected return of the futures prices, the optimum mean-variance hedge ratio will be equal to the static MV hedge ratio. In the condition that the risk aversion is infinite, the individual is extremely risk averse and will avoid all risk. When the expected return of the futures prices is zero, the futures prices follow a simple martingale process (i.e. a stochastic process) (Chen et al., 2003, p. 438). As a result, the optimum-mean variance hedge ratio is equivalent to the static MV hedge ratio.

Sharpe hedge ratio

An estimation model that combines both risk and return is the Sharpe hedge ratio model. Sharpe ratio is a performance measurement of the risk-return trade-off of the hedged position. This model is different from the other models as it considers expected rate of return without considering the individual's risk aversion. The optimal Sharpe hedge ratio is the maximisation of the excess rate of return of the hedged position given to its risk. The maximisation of the Sharpe ratio is given by (Howard and D'Antonio, 1984, p. 105):

$$max\ \theta = \frac{E(R_h) - R_F}{\sigma_h} \tag{8}$$

where $E(R_h)$ is the expected return of the hedged position, R_F is the risk-free rate of return, and σ_h is the standard deviation of the hedged position. Then, the optimal Sharpe hedge ratio is given by (Howard and D'Antonio, 1984, p. 106):

$$HR_4^* = -\frac{\left(\frac{\sigma_s}{\sigma_f}\right)\left[\left(\frac{\sigma_s}{\sigma_f}\right)\left(\frac{E(R_f)}{E(R_s) - R_F}\right) - \rho\right]}{\left[1 - \left(\frac{\sigma_s}{\sigma_f}\right)\left(\frac{E(R_f)\rho}{E(R_s) - R_F}\right)\right]} \tag{9}$$

In a situation where $E(R_f) = 0$, the hedge ratio is equal to the static MV hedge ratio.

2.1.2 Dynamic model

Up to now, the static models estimate hedge ratios with unconditional information and give therefore fixed hedge ratios. A dynamic model estimates a time-varying hedge ratio, which means that the hedge ratio is based on conditional information in the covariance, σ_{sf}, and in the variances, σ_s and σ_f (Chen et al., 2003, p. 440). Therefore, a dynamic hedge ratio model can be more accurate and relevant to use as an estimation model for hedge ratios. With conditional information, the dynamic MV hedge ratio is given by:

$$HR_1^* \mid \Omega_{t-1} = \rho_{sf} \mid \Omega_{t-1} \times \frac{\sigma_s \mid \Omega_{t-1}}{\sigma_f \mid \Omega_{t-1}} \tag{10}$$

where both ρ_{sf}, σ_s, and σ_f contain the conditional information Ω_{t-1}, which is the difference from the static MV hedge ratio from Equation 5.

2.2 HEDGING EFFECTIVENESS

Hedging effectiveness (HE) is the proportion of the variance that is reduced by hedging and it presents how well the hedged position performs relative to the unhedged position. If HE is equal to one, the variance is reduced by 100% compared to the unhedged position. In other words, the price risk is completely eliminated. If HE is equal to zero, the hedged position does not reduce the price risk (Cotter & Hanly, 2006, p. 680).

In general, the following equation is used for estimating the hedging effectiveness of the optimal hedge ratio (Ederington, 1979, p. 164):

$$HE = \frac{Var_U - Var_H}{Var_U} \tag{11}$$

where Var_U is the variance of the unhedged position, which means the variance of the underlying asset, and Var_H is the variance of the hedged position. The variances of an unhedged and a hedged position are estimated using the following equations (Ederington, 1979, p. 161):

$$Var_U = \sigma_S^2 \tag{12}$$

$$Var_H = \sigma_S^2 + HR^{*2} \times \sigma_f^2 - 2 \times HR^* \times \sigma_{sf} \tag{13}$$

where σ_{sf} is the covariance between the returns of the spot and futures prices ($\sigma_{sf} = \rho\sigma_s\sigma_f$), and HR^* is the optimal hedge ratio. The hedging effectiveness indicates the performance of the hedged position to the given hedge ratio. Therefore, it is possible to compare the different hedge ratio models in order to find the model that achieve the highest reduction of variance, and this model will be recommended.

3. METHODOLOGY

The different optimal hedge ratio models must be estimated with various methods in order to be used in practice. Then, the optimal hedge ratios are implemented into the formula of hedging effectiveness from Equation 11. As a result, a comparison of the hedging effectiveness of the different hedge ratio models and maturities is possible. When calculating with volatility of different times, it is important that the volatility is proportional to the square root of time. Otherwise, the results will be neither consistent nor comparable.

3.1 ESTIMATION OF THE MV HEDGE RATIO

3.1.1 Ordinary least square (OLS) method

According to the following equation, the OLS estimation method is a linear regression model that estimates the correlation coefficient for the static MV hedge ratio (Junkus & Lee, 1985):

$$R_s = \alpha + HR_2^* \times R_f + \varepsilon \quad (14)$$

where R_s and R_f are the returns of the spot and futures prices, respectively. α is the constant term and ε is the error term. HR_2^* can also be estimated through Equation 5.

Before the estimation with OLS method, the estimation criteria for OLS need to be satisfied. If not, the consequence will be that the results from the estimation are not valid. The estimation criteria for OLS are that the data have linearity in the parameters, random sampling, no collinearity, exogeneity, and homoscedasticity. These estimation criteria will be tested in Section 4.2.2.

3.1.2 Estimation using GARCH (1,1) model

A GARCH (1,1) model can be used as an estimation method to estimate a dynamic MV hedge ratio. The GARCH (1,1) model estimates the conditional variance and covariance, which gives a time-varying variance. In other words, the variance and covariance are heteroscedastic and conditional, meaning that these are non-constant and updated on new information

(Bollerslev, 1986, p. 120). For financial data, the variance of the errors is likely to not be constant over time, and the reason for using a dynamic model instead of a static model increases considerably. The conditional variance in a GARCH (1,1) model is:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \times u_{t-1}^2 + \beta \times \sigma_{t-1}^2 \tag{15}$$

where α_1 is the coefficient of autoregressive term, β is the coefficient of GARCH (1,1) term, u_{t-1}^2 is the squared residual from time t-*1*, and σ_{t-1}^2 is the conditional variance from time t-*1*.

The GARCH (1,1) model does not have the same estimation criteria as the OLS method, which means that the GARCH (1,1) model, for instance, has no criteria of having linearity in the parameters. Therefore, since the model has fewer limitations, there are fewer requirements of preparation of the model. Further, Mandelbrot (1963, p. 416) describes that, with financial data, large (small) price changes are followed by large (small) price changes. Accordingly, this conditionality is autocorrelation and contributes to volatility clustering. It is a huge benefit using a GARCH (1,1) model because it takes care of this volatility clustering issues and because the volatility is conditional, which means that the current volatility depends on the past volatility. The estimation model gives the advantage of mitigating the impact of historical volatile shocks that may lead to incorrect volatility estimates.

3.2 ESTIMATION OF OPTIMUM MEAN-VARIANCE HEDGE RATIO

In the estimation of optimum mean-variance hedge ratio, the sample parameters replace the theoretical parameters. For instance, the expected return of the futures contracts will be replaced by the average return of the futures contracts from the sample data. Further, the second term on the right-hand side of Equation 7 is equal to the calculation of the static MV hedge ratio, and the results from the estimation of the static MV hedge ratio with OLS method can therefore be used.

3.3 ESTIMATION OF SHARPE HEDGE RATIO

Equation 9 shows that sample parameters can be used to replace the theoretical parameters. The estimation of all the parameters of the spot and futures contracts will be replaced by the average of the spot and futures contracts from the sample data.

3.4 ESTIMATION OF HEDGING EFFECTIVENESS

The hedging effectiveness of the different hedge ratios described in Section 2 are calculated using Equation 11. However, with the static MV hedge ratio model, the HE can also be estimated by R^2 of the OLS regression from Equation 14. R^2 describes the fraction of the changes in the spot prices that can explain the changes in the futures prices (Johnson, 1960, p. 144; Junkus & Lee, 1985). In other words, R^2 describes the reduction of the variance of the hedged position.

4. DATA AND TESTING THE MODELS' ASSUMPTIONS

In this study, the prices from spot and futures contracts in Brent crude oil are applied in the analysis. Brent crude oil futures contracts are traded in US dollars and cents and are exchanged on the Intercontinental Exchange (ICE). The prices in the analysis are expressed as the natural logarithms of the spot and futures prices, and the returns are expressed as the natural logarithms of the differences in the prices.

Empirical studies find the nearest maturity of the futures contracts to be the most efficient to use in the hedging strategy (Chen et al., 2003, p. 448; Malliaris & Urrutia, 1991, p. 288). To find whether this is correct or not, the futures contracts with the hedging maturities of one, three, six, and twelve months are employed in this analysis. The most frequent Brent crude oil futures contracts available are the one-monthly futures contracts. Furthermore, the formulas of the MV hedge ratio models require that the data frequencies of the contracts are equal to their hedging maturities, as this reduces the number of non-overlapping price changes (Geppert, 1995, p. 4). Non-overlapping price changes follow a martingale process and are uncorrelated (Campbell, Lo, & MacKinley, 1997, p. 55). Few of these non-overlapping price changes contribute to autocorrelation in the errors of the model and it is therefore a possibility of sampling errors (Chen et al., 2003, p. 448).

The ranges of the datasets for the different contracts are different due to the availability of data. In order to collect most data and hence observations, the ranges of the datasets for the different contracts are not identical, which is shown in Table 1. All the data are accessed from Thomson Reuters Datastream.

Table 1: Description of the datasets

Contracts	Frequency	Dataset range
1-monthly contracts	Monthly	30.06.1988 – 28.02.2019
3-monthly contracts	Quarterly	30.09.1988 – 31.12.2018
6-monthly contracts	Semiannually	30.09.1989 – 30.09.2018
12-monthly contracts	Yearly	31.12.1994 – 31.12.2018

The table displays the different contracts used in the analysis with their respective frequency of the data and their dataset range.

In the estimation of the optimum mean-variance hedge ratio model, the hedger's level of risk aversion must be presented. Hanna and Lindamood (2004, p. 28) describe a moderate level of risk aversion for an investor as 5 ($A = 5$). Assuming that most investors have a moderate tolerance of handling risk, the level of risk aversion in this analysis is set to 5. However, another level of risk aversion gives a different result in terms of hedge ratio and hedging effectiveness.

A risk-free rate of return is required in the estimation of the Sharpe hedge ratio model. There exist several different risk-free rates. For instance, the Norwegian 10-year Treasury rate is viewed as risk-free. However, Hull and White (2013) argue whether London Interbank Offered Rate (LIBOR) or Overnight Index Swap (OIS) is the most appropriate risk-free rate to use when calculating with derivatives. They conclude that after the financial crisis, OIS is the most appropriate risk-free rate. Although, since most of the observations in this analysis are before the financial crisis, LIBOR is applied to this model. Further, since the futures contracts are traded in US dollars, the USD LIBOR is most relevant.

Figure 1 illustrates an increasing spread between the spot and futures prices of one-monthly contracts in Brent crude oil the last two decades. The price spread, or the basis, between the spot and futures prices can be explained by higher volatility in the oil prices. As shown in Figure 1, the oil prices have been quite stable around 20-30 dollars from 1988 to 2003. However, the oil prices have been extremely volatile after 2003, which can be explained by several crises, OPEC's market power, and the shale oil revolution. However, the spot and futures prices seem to be highly correlated as these move closely together.

Figure 1: Spot and futures prices and basis, 1988 – 2018

The figure presents the spot and futures prices of one-monthly contracts in Brent crude oil in US dollars with black and blue line, respectively. The timeline is from June 1988 to December 2018. The basis is viewed as the difference between the spot and futures prices with a green area around zero.

4.1 Descriptive statistics

The descriptive statistics of the returns of the spot and futures contracts give indications of the distributions of the different contracts, and the summary of the descriptive statistics is displayed in Table 2. Both the means and standard deviations of the returns of the prices are lowest for the one-monthly contracts and highest for the twelve-monthly contracts. There are positive kurtosis and leptokurtic distributions for all contract sizes except from the twelve-monthly contracts, which have negative kurtosis and platykurtic distribution with low peak and thinner tails. For the three-monthly spot contracts, the kurtosis is quite substantial at 13.49, which means that the distribution is quite peaked. Further, all contract sizes have negative skewed distributions, which means that the distributions are tilted towards the right-hand side. These results indicate that the returns of the prices have non-normal distributions, and can lead to possibilities of type I and type II errors in hypothesis testing. Accordingly, the testing for normal distribution is provided in Section 4.2.2. The number of observations is falling due to lower frequency when the contract maturity increases and because of the range of the dataset, as previously described in Section 4.

Table 2: Descriptive statistics of the returns of spot and futures contracts

	1-monthly contracts		**3-monthly contracts**		**6-monthly contracts**		**12-monthly contracts**	
	Spot	*Futures*	*Spot*	*Futures*	*Spot*	*Futures*	*Spot*	*Futures*
Mean	0.008	0.008	0.030	0.027	0.055	0.048	0.089	0.084
SD	0.091	0.092	0.120	0.186	0.247	0.208	0.265	0.262
Kurtosis	4.353	2.845	13.487	9.505	2.675	2.153	-0.393	-0.546
Skewness	0.434	0.373	1.929	1.496	0.761	0.316	-0.254	0.063
Counted	368	368	121	121	58	58	24	24

The table presents the descriptive statistics of the returns of the spot and futures contracts for the different maturities, which are not log transformed. It displays the mean, volatility as standard deviation, and the distribution in form of kurtosis and skewness. Counted expresses the number of observations.

4.2 TESTING THE ASSUMPTIONS OF THE ESTIMATION MODELS

In order to obtain efficient results, the assumptions from the estimation models in Section 2 must be satisfied. Therefore, it is necessary to test for stationarity and cointegration of the data, and to test the validity of the OLS assumptions before estimating the static MV hedge ratio.

4.2.1 Tests of stationarity and cointegration

In order to estimate the different optimal hedge ratios and measure their hedging effectiveness, the data have to be tested for stationarity as the prices need to follow a random walk. Further, a cointegration test between the spot and futures prices is essential due to possible spurious correlations between the spot and futures prices.

Testing for unit roots

Since imperfections exist in the oil market because of the oil cartel OPEC, these can lead to the prices being inefficient as there might not be a market equilibrium where demand equals supply. When prices are inefficient, it is an invalidation of the efficient market hypothesis where the prices are not following a random walk. In order to use OLS estimation method, the spot and futures prices must follow a random walk, or in other words, must be non-stationary. If the prices do not follow a random walk with or without drift, the estimated hedge ratio from the OLS will be invalid. Accordingly, unit-root tests such as augmented Dickey-Fuller (ADF) and

KPSS tests investigate whether the prices are stationary or follow a random walk (Dickey & Fuller, 1981; Kwiatowski, Phillips, Schmidt, & Shin, 1992).

Table 3 shows the results from the ADF and KPSS tests. The values from the ADF tests for all contract sizes do not exceed the critical values, and the null hypotheses of zero unit roots are not rejected. The series contain at least one unit root, or in other words, these prices are non-stationary and follow a random walk. The same results apply for the KPSS test. The null hypotheses of the KPSS are that the series contain at least one unit root. The values exceed the critical values and the results are statistically significant. These results imply that the prices are in a weak-form efficiency of the efficient market hypothesis. This is an evidence of that OPEC does not influence the oil price.

Table 3: Results from unit-root tests

	1-monthly contracts		**3-monthly contracts**		**6-monthly contracts**		**12-monthly contracts**	
	Spot	*Futures*	*Spot*	*Futures*	*Spot*	*Futures*	*Spot*	*Futures*
ADF test								
Constant	-1.38	-1.44	-1.75	-1.68	-1.32	-1.15	-1.30	-1.32
Constant and trend	-2.06	-2.13	-2.43	-2.25	-2.29	-2.07	-1.12	-1.02
KPSS test	4.98***	5.01***	2.04***	5.01***	1.26***	1.29***	0.71**	0.72**

The table presents the results from the different unit-root tests: augmented Dickey-Fuller (ADF) and KPSS tests. The ADF tests provide one result with a constant term and another result with both constant term and time trend. Both ADF tests and the KPSS test give the same results of the existence of unit roots in the series. ***, **, and * represent the significance at 1%, 5%, and 10% significance level, respectively.

Testing for cointegration

A cointegration test gives the answer if the spot and futures prices contain spurious correlations. If the prices are cointegrated, the error term is stationary and moves towards a long-term equilibrium, which means that the two prices share the same trend. When testing for stationarity in the residuals, we find that these are statistically significant at a level of 1% for all maturities. Further, an error correction method developed by Engle and Granger (1987) is applied in order to test if there is a cointegration relationship between the prices. While the spot and futures prices can diverge over the life of the contract, regardless of maturity, the futures prices must be equal to the spot prices. Therefore, it is reason to believe that the spot and futures prices have a long-term equilibrium relationship. A cointegration test between the two prices reveals whether this relationship exists or not. Because of the arbitrage condition, the spot and futures prices cannot drift far apart from each other in the long run. Therefore, if both the spot and futures prices follow a random walk, the two prices are expected to be cointegrated; however, a cointegration test will conclude this, and an Engle-Granger method and a Johansen test are therefore applied.

The results from the Engle-Granger method that are shown in Table 4 present that the error term corrects the spot prices back to a long-term equilibrium for all frequencies, which indicates that the error correction mechanism is operating primarily through the adjustment in the spot prices. The same results do not apply for the futures prices because the error term is not statistically significant. The cointegration test shows that there is a long-term equilibrium relationship between the spot and futures prices for all contracts.

Table 4: Results from the Engle-Granger method

	1-monthly contracts	**3-monthly contracts**	**6-monthly contracts**	**12-monthly contracts**
Dependent variable: Return of spot prices				
α	0.00	0.01	0.03	0.05
S_{-1}	-0.13	-0.27	0.66	0.10
F_{-1}	0.45***	0.34	-0.78	0.12
res_{-1}	-0.62***	-0.62***	-0.75*	-1.08**
Dependent variable: Returns of futures prices				
α	0.00	0.01	0.03	0.06
S_{-1}	-0.00	-0.22	0.60	-0.05
F_{-1}	0.19	0.19	-0.76*	-0.04
res_{-1}	-0.01	-0.15	-0.22	-0.13

The table shows the results of the Engle-Granger method using an error correcting model (ECM) as estimation method. The first part of the table shows the ECM with the return of the spot prices as the dependent variable. For all maturities, the error corrections in the spot prices are statistically significant, as the spot prices will decrease when the error terms increase. For one-monthly contracts, the future prices will lead the spot prices to a long-term equilibrium, as these are statistically significant. The second part of the table shows the ECM with the return of the futures prices as the dependent variable. However, none of the error terms are statistically significant and there is therefore no error correction through the futures prices. ***, **, and * represent the significance at 1%, 5%, and 10% significance level, respectively.

The estimation of the Johansen test is done according to Johansen's (1987, p. 4) research article. Table 5 shows the test statistics for the Johansen test, and it shows that there are long-term equilibrium relationships between the spot and futures prices for the one- and three-monthly contracts only. For the six- and twelve-monthly contracts, the prices are not cointegrated, which can be a result of few observations.

Table 5: Results from the Johansen test

	1-monthly contracts	3-monthly contracts	6-monthly contracts	12-monthly contracts
$\boldsymbol{r \leq 1}$	7.19	2.97	3.01	2.91
$\boldsymbol{r = 1}$	113.58***	22.99*	18.66	15.79

The table presents the Johansen test for cointegration. The contracts are cointegrated when the *r is less or equal to 1* are not statistically significant and when the *r is equal to 1* are statistically significant. The one- and three-monthly contracts have cointegration between the prices according to the Johansen test. ***, **, and * represent the significance at 1%, 5%, and 10% significance level, respectively.

Both the Engle-Granger method and the Johansen test imply that the prices of the one- and three-monthly contracts are cointegrated, and there are therefore long-term equilibrium relationships between the spot and futures prices for these contracts. The Johansen test cannot confirm that the six- and twelve-monthly contracts are cointegrated. However, the Engle-Granger method confirms that the contracts are cointegrated because the error correction mechanism operates through the adjustment in the spot prices. Therefore, we conclude that the spot and futures prices follow a random walk for all maturities.

4.2.2 OLS assumptions

The Gauss-Markov theorem provides five assumptions that must be present and not violated when using an OLS regression as the estimation method (Brooks, 2014, p. 90-92). When the first four assumptions are satisfied, the estimates are best linear unbiased estimators (BLUEs). When the estimator is BLUE, it means that it "has minimum variance among the class of linear unbiased estimators" (Brooks, 2014, p. 91).

The first assumption is that the mean of the disturbances must always be zero. However, it is not violated when providing a constant term in the regression, which is obtained in our analysis. Further, the assumption that the data is non-stochastic is accomplished since the prices follow a random walk as tested in Section 4.2.1. The other assumptions are tested using the tests presented in Table 6. These tests are developed by the following economists, respectively: White (1980), Durbin and Watson (1950), and Jarque and Bera (1980).

Table 6: Results from the tests of the assumptions from the Gauss-Markov theorem

	1-monthly contracts	**3-monthly contracts**	**6-monthly contracts**	**12-monthly contracts**
White test	8.667**	15.781***	12.860***	15.045***
DW test	1.954	1.175***	0.956***	2.300
JB test	271.670***	193.330***	19.155***	10.386***

The table shows the tests that are used to find any violation of the assumptions from the Gauss-Markov theorem and to detect the validity of the estimates provided from an OLS regression. The White and Jarque-Bera (JB) tests are statistically significant for all the contract maturities, which means accordingly that the data are heteroscedastic and non-normally distributed. Durbin Watson (DW) test presents that there are autocorrelations in the errors for three- and six-monthly contracts. ***, **, and * represent the significance at 1%, 5%, and 10% significance level, respectively.

The White test is statistically significant for all contract sizes and a rejection of the null hypotheses about homoscedasticity in the disturbances is present, as illustrated in Table 6. We conclude that all contract sizes contain heteroscedastic disturbances, which is a validation of the assumption of homoscedasticity in the disturbances. However, a log transformation of the data gives homoscedastic disturbances, and hence unbiased estimates.

Further, Table 6 presents the Durbin-Watson (DW) test of the assumption about no autocorrelation in the errors, and hence no pattern. The DW test is statistically significant at three- and six-monthly contracts, and we reject the null hypotheses about no autocorrelation in the errors. We conclude that one- and twelve-monthly contracts have no autocorrelation in the errors, while three- and six-monthly contracts have autocorrelation in the errors. Even though autocorrelation is present, the estimates is still unbiased.

The Jarque-Bera (JB) test is used to determine whether the assumption about normal distribution of the disturbances is violated or not. The JB test from Table 6 shows that all the contracts are non-normally distributed. This is critical because the possibility of type I and type II errors increases since non-normal distribution affects the results of hypothesis testing with *t*-test.

The results from the White, DW, and JB tests show that the assumptions are violated, and we conclude that the estimates are not BLUEs. However, these estimates are still unbiased and appropriate estimates, and we can therefore use OLS regression as an estimation method.

5. EMPIRICAL RESULTS

The optimal hedge ratios and their hedging effectiveness are estimated and compared to each other in order to find the most appropriate model an oil producer can use in its hedging strategy. Then, a segmentation of the time series in subperiods, an estimation of yearly HR and HE, and a forecasting model are presented.

5.1 RESULTS OF HEDGE RATIO AND HEDGING EFFECTIVENESS

Table 7 displays the results of HR and HE of the different hedge ratio models described in Section 2. These are the static hedge ratios and a point estimate of the dynamic MV hedge ratio. The HE in the table make it possible to compare the different hedge ratio models and maturities.

Table 7: Results of hedge ratio (HR) and hedging effectiveness (HE)

	1-monthly contracts		**3-monthly contracts**		**6-monthly contracts**		**12-monthly contracts**	
	HR	*HE*	*HR*	*HE*	*HR*	*HE*	*HR*	*HE*
Naïve	1.000	0.467	1.000	0.876	1.000	0.863	1.000	0.205
Static MV	0.722	0.545	1.012	0.896	1.111	0.874	0.458	0.205
Optimum mean-variance	0.299	0.359	0.653	0.766	0.579	0.673	-0.217	-0.288
Sharpe	0.743	0.547	1.031	0.876	1.105	0.871	1.234	-0.014
Dynamic MV	0.718	0.547	0.999	0.876	0.978	0.859	-	-

The table shows the different hedge ratio and hedging effectiveness of the five different models of one-, three-, six-, and twelve-monthly contracts. When it comes to hedging maturity, the three-monthly contracts perform best overall as these eliminate most risk, measured by their hedging effectiveness.

5.1.1 Naïve hedge ratio

Table 7 shows that the naïve hedge ratio is equal to one for all maturities. According to the theory, the futures contracts are in the same amount as the opposite position of the underlying asset, and the hedge ratio will therefore always be one. Nonetheless, HE varies for the different contracts. The three-monthly contracts reduce most of the price risk with 88%, which is quite a difference compared to the twelve-monthly contracts that reduce only 21% of the price risk. Even though the naïve model is the simplest model to use for hedging, the choice of hedging maturity must be considered in order to reduce most price risk.

5.1.2 Static minimum variance (MV) hedge ratio

The static MV hedge ratio is one of the models that performs overall the best, and this model is therefore the most efficient model, as presented in Table 7. The three-monthly contracts perform the best with an HE approximately equal to 90% compared to both maturity and the other hedge ratio models. A three-monthly contract using static MV hedge ratio estimation reduces the most of the risk and is therefore the most appropriate model. Further, the static MV hedge ratio model is not a complex model and is easy to understand compared to the other models, except from the naïve hedge ratio model.

It is interesting to test if there is a statistically significant difference between the static MV and naïve hedge ratios by using a Wald test. If the result from the Wald test indicates that both of the models give the same results, it will be a discussion of costs contributed to the models, as the naïve hedge ratio model requires fewer resources. The static MV hedge ratio model is the best performed model overall and is quite similar to the naïve hedge ratio for some of the maturities. A Wald test uses hypothesis testing for a single parameter, and if the single parameter is statistically significant, it adds value to the model (Brooks, 2014, p. 452). In this analysis, the null hypothesis of the Wald test is that $HR_2^* = 1$, while the alternative hypothesis is that $HR_2^* \neq 1$. As Table 8 presents, the Wald test is statistically significant for one-, six-, and twelve-monthly contracts, and we can conclude that the static MV hedge ratio is not like the naïve hedge ratio for these contracts. However, the three-monthly contracts are not statistically significant different from the naïve hedge ratio, and we fail to reject the null hypothesis. We conclude that the static MV hedge ratio for three-monthly contracts is similar to one and hence the naïve hedge ratio. Further, this means that the two related HE is equivalent. Therefore, since the static MV hedge ratio model is not better than the naïve hedge ratio model for three-monthly contracts and because of the simplicity of the naïve hedge ratio model, this is a more suitable hedge ratio model.

Table 8: Wald test of static MV hedge ratio with the null hypothesis of $HR_2^* = 1$

	1-monthly contracts	**3-monthly contracts**	**6-monthly contracts**	**12-monthly contracts**
***t*-stat**	-8.0814***	0.3625	2.2289***	-2.6979***

The table displays the Wald test used to find out if there is a statistically significant difference between the static MV hedge ratio and 1. The Wald test is statistically significant for one-, six-, and twelve-monthly contracts. The three-monthly contracts are not statistically significant different from 1, and we conclude that static MV hedge ratio is equal to 1. ***, **, and * represent the significance at 1%, 5%, and 10% significance level, respectively.

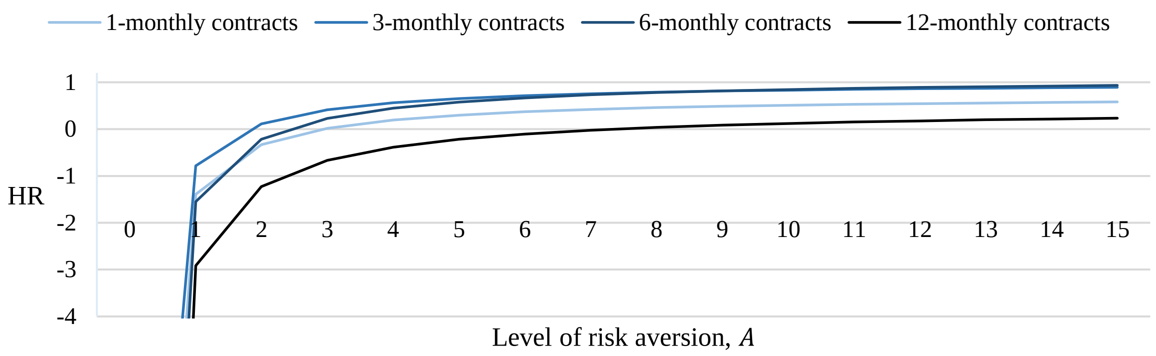
5.1.3 Optimum mean-variance hedge ratio

The optimum mean-variance hedge ratio model is the model that performs the poorest of the models, and the HR and HE of the twelve-monthly contracts are negative, as presented in Table 7. Chen, Lee, and Shrestha (2001, p. 592) describe a negative hedge ratio as "lower expected profit on the hedged portfolio". The reason for these negative values is that the optimum mean-variance hedge ratio model incorporates both return and level of risk aversion. Therefore, the optimum mean-variance result in less hedging than the static MV hedge ratio model. Consequently, the HE becomes negative, and there is not optimal to hedge during the analysis period with twelve-monthly contracts. Comparing the maturities of the optimum mean-variance hedge ratios, the three-monthly contracts reduce most of the price risk.

The limitation to the optimum mean-variance hedge ratio model is that the level of risk aversion is fixed. As previously discussed, A is set to 5 in the estimation as it reflects a moderate risk aversion. The level of risk aversion varies across individuals, and using another level of risk aversion will therefore give different HR and HE. As shown in Figure 2, the hedge ratios increase asymptotically with the increasing level of risk aversion. The optimum mean-variance hedge ratio will be equal to the static MV hedge ratio if A is infinite.

Figure 2 illustrates that the twelve-monthly contracts require a higher level of risk aversion in order to produce a positive hedge ratio. Compared to the twelve-monthly contracts, the other contracts do not require the same level of risk aversion. At the lowest levels of risk aversion, the three-monthly contracts have the highest HR for all levels. However, a breakeven point between the three- and six-monthly contracts occurs when the level of risk aversion is approximately 6, and these have the same HR beyond this point. Beyond the level of risk aversion of 2, the one-monthly contracts diverge from the six-monthly contracts.

Figure 2: Optimum mean-variance HR relative to the level of risk aversion, *A*



The figure presents the optimal mean-variance hedge ratio for one-, three-, six-, and twelve-monthly contracts with different level of risk aversion. The twelve-monthly contracts require higher level of risk aversion to provide a positive hedge ratio compared to the other contracts. Further, the figure illustrates that higher level of risk aversion gives higher hedge ratio. However, after a given level of risk aversion, the hedge ratio trends asymptotically and converges to the static MV hedge ratio.

5.1.4 Sharpe hedge ratio

The Sharpe hedge ratios and their HE are surprisingly high and perform relatively close to the static MV hedge ratio model despite the fact that the Sharpe hedge ratio model incorporates both return and risk. Compared to the static MV hedge ratio model that considers only the risk, the Sharpe hedge ratio model should hold a higher risk due to the risk-return trade-off. However, the model shows a high hedging effectiveness, which means that the trade-off is higher when reducing the risk. Compared to the optimal mean-variance model that also takes return into account, the results are dividing and can be explained by the fact that the Sharpe ratio incorporates return and risk and not return and level of risk aversion.

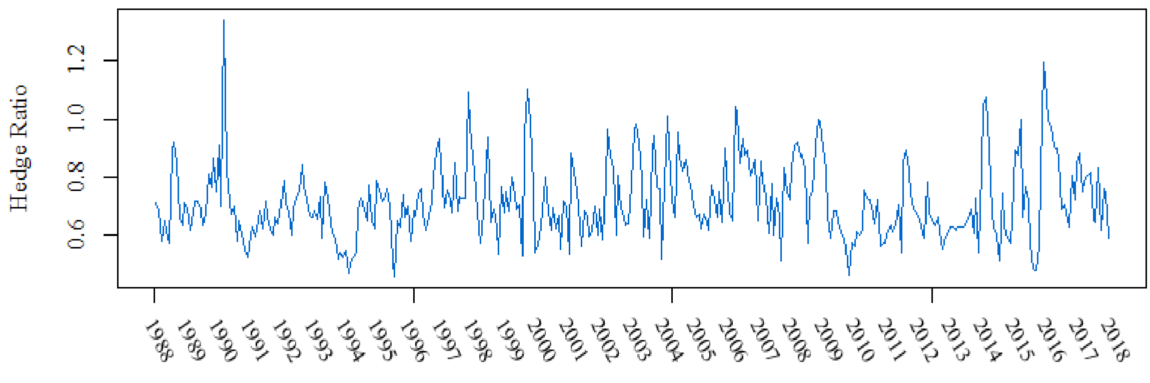
Again, the three-monthly contracts reduce most risk as these eliminate 88% of the variance in the oil price, as presented in Table 7. This is equivalent to the HE of the naïve hedge ratio, and when the Wald test is taken into consideration, it is also equal to the HE of the static MV hedge ratio. For one-monthly contracts, both the Sharpe and the dynamic MV hedge ratio models perform the best of all the models. However, the twelve-monthly contracts have a negative HE despite a highly positive HR, which means that the optimal hedge ratio is not effective and the hedged portfolio increases the risk compared to an unhedged portfolio.

5.1.5 Dynamic minimum variance (MV) hedge ratio

It is easy to assume that the dynamic MV hedge ratio model performs better than the static models. This is because a dynamic model is updated by new information due to the conditional variance. However, when comparing the point estimate from the dynamic MV hedge ratio with the other models' hedge ratios, the dynamic MV hedge ratio performs close to the static MV and Sharpe hedge ratio models in terms of HE for one-, three-, and six-monthly contracts. In this case, it seems indifferent to use static or dynamic MV, or Sharpe hedge ratio model to estimate HR, which also means that it does not matter whether you use a dynamic or a static model. However, the dynamic MV hedge ratio model is a very complex and difficult model to use for estimation, which means that to estimate a hedge ratio with this model requires a lot of resources in terms of risk management. This model is not beneficial compared to the other simpler models, such as the naïve and static MV hedge ratio models, because these will reduce costs and be less time consuming.

The dynamic MV hedge ratio for one-monthly contracts is illustrated in Figure 3. The HR varies substantially through the years, with the maximum HR of approximately 1.35 in 1990, and with the minimum HR of approximately 0.45 between 1995 and 1996 and in 2010. The huge variation in the HR shows how important it is to always update the HR through time, which the dynamic hedge ratio model does.

Figure 3: The dynamic MV hedge ratio for one-monthly contracts



The figure illustrates the dynamic MV hedge ratio during the analysis period of 1988-2018. The hedge ratio fluctuates quite a lot, where the fluctuation in the hedge ratio is within a range of 1.35 at highest and 0.45 at lowest.

The information criteria reveal the information lost by the given model, and the lower the information criteria, the less information is lost. We find that the GARCH (1,1) model for all maturities is supported by the different information criteria, and the information criteria state that the GARCH (1,1) estimation method is rightfully specified according to the appropriate number of lags.

For the three-monthly contracts, the dynamic MV hedge ratio is equal to one, which is the same as the naïve hedge ratio, and when the Wald test is taken into consideration, it is also equal to the static MV hedge ratio. When comparing the different maturities, the three-monthly contracts perform the best as these eliminate 88% of the price risk, as presented in Table 7. However, a problem occurs when estimating the twelve-monthly contracts because of the few observations. This results in not being able to estimate the hedge ratio using a GARCH (1,1) estimation method. The problem of few observations is also a problem for the six-monthly contracts; however, we are able to estimate a point estimate of the hedge ratio using GARCH (1,1) estimation method. Nonetheless, we cannot be certain if this estimate is valid.

5.1.6 Overall results of the hedge ratios and hedging effectiveness

Overall, the hedge ratios of the different hedge ratio models do not give a perfect hedge because their hedging effectiveness are not one, or in other words, give a 100% reduction of the price risk. Therefore, the hedge contains some basis risk in the form of nonmatching maturities of the spot and futures contracts or in the form of other factors that may contribute to the basis risk. However, it is known that a perfect hedge is difficult to obtain.

5.2 A SEGMENTATION OF THE TIME SERIES

Historically, there have been many different events during the analysis period that have contributed to affect the oil prices, such as the financial crisis of 2007 and the rapid fall in the oil price of 2014. As shown in Figure 1, the financial crisis had the most impact on the oil prices, where the oil price fell from 133 dollars per barrel in June 2008 to 43 dollars per barrel in February 2009. This is a price fall of almost 68% in eight months.

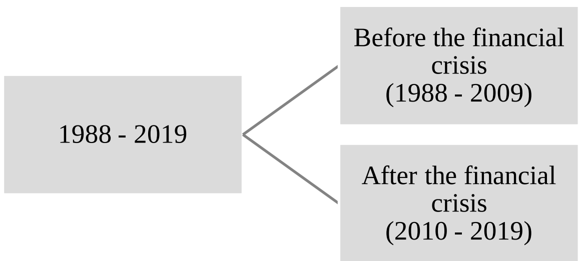
It can be interesting to investigate if different time periods have different outcomes in terms of HR and HE, and dividing the whole time series into subperiods is therefore appealing. The whole time series will undergo a segmentation of different approaches. The first approach manages only the financial crisis, and the time series is divided accordingly into two subperiods: before and after the financial crisis. This is an interesting segmentation as the prices before the financial crisis are quite stable; however, after the financial crisis the oil prices become very volatile, which means that the hedging will be less effective after the financial crisis. The second approach manages both the financial crisis and the rapid fall in the oil price in 2014, and the time series is divided accordingly into three subperiods. Another big, historical event beside the financial crisis alone can make a considerable effect on HR and HE, and it is therefore appealing to examine if this event makes results that differ from the results of the first approach.

The estimation model that is employed in the segmentation of the time series is the static MV hedge ratio model because this is the most efficient model as described in Section 5.1.2. However, only the one-monthly contracts are used as these achieve the highest level of observations.

5.2.1 First approach

In the first approach of the segmentation, the two subperiods are before the financial crisis and after the financial crisis. The segmentation from the first approach is shown in Figure 4. The subperiod before the financial crisis is estimated before and during the financial crisis in the time period from 1988 to the end of 2009. It can be discussed when the financial crisis ended since it differs across countries; however, in Norway the financial crisis ended in 2009 (Notaker, 2018). Since the Brent crude oil is extracted from the North Sea, the decision of when the subperiod ends is therefore set at the end of 2009, which is also supported by Figure 1 that illustrates that the oil prices do not start to increase before the year of 2010.

Figure 4: Segmentation of the time series in the first approach



The figure shows the segmentation of the analysis period 1988-2019. The first subperiod is the period before and during the financial crisis, which has the timeline 1988-2009. The second subperiod is the period after the financial crisis with the timeline 2010-2019.

The estimates of HR and HE for the subperiod before the financial crisis from Table 9 are quite similar to the estimates of HR and HE for the whole sample period from Table 7. In the subperiod after the financial crisis, the estimates of HR and HE are lower, which can be explained by the lower correlation that again can be explained by lower observations. However, the differences between the subperiods before and after the financial crisis are small, and the results are statistically significant at a level of 1%. The hedging is less effective after the financial crisis. Nonetheless, it can be explained by more volatile oil prices, which make it inconvenient to hedge, or it can be explained by fewer observations.

Table 9: Results of the segmentation from the first approach

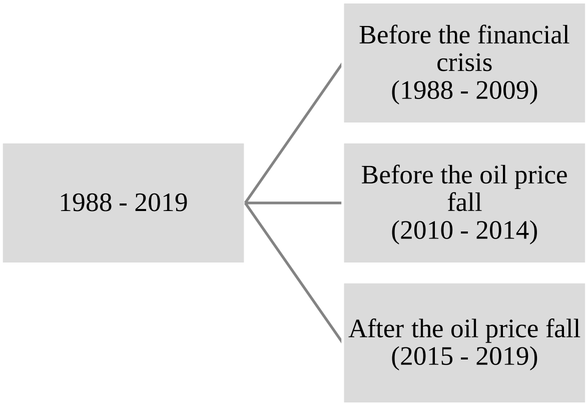
	Before the financial crisis	*After the financial crisis*
HR	0.725	0.696
HE	0.558	0.484
***t*-stat**	17.942***	9.966***
Correlation	0.747	0.696
Counted	267	108

The table displays the results of the segmentation from the first approach. The time series is divided into two periods, before and after the financial crisis. The HR and HE before the financial crisis are higher compared to the HR and HE after the financial crisis. However, the correlation and the number of observations are lower in the period after the financial crisis than before the financial crisis. ***, **, and * represent the significance at 1%, 5%, and 10% significance level, respectively.

5.2.2 Second approach

The whole time series is divided into three subperiods in the second approach, as shown in Figure 5. The subperiod before the financial crisis is obviously defined as the same as in the first approach. Further, the subperiod after the financial crisis is now divided into two new subperiods: the period before and the period after the fall in the oil price in 2014. The subperiod before the price fall includes the price drop, while the last subperiod consists of the time after the price drop.

Figure 5: Segmentation of the time series in the second approach



The figure shows the segmentation of the analysis period from 1988-2019. The first subperiod is the period before and during the financial crisis, which has the timeline 1988-2009. The second subperiod is the period before the oil price fall, which includes the price fall and has a timeline between 2010 to the end of 2014. The third subperiod is the period after the oil price fall with the timeline 2015-2019.

In 2014, it was an oversupply of oil production that contributed to a drop in the oil prices. The oversupply of oil was due to the shale oil revolution in US and to OPEC's decision of not cutting the oil production that they usually do. The two last subperiods are divided by the end of 2014. This is because the oil price was 112 dollars in June 2014 but it fell to 48 dollars by January 2015, and then the price becomes more volatile as the price starts to increase before it decreases again. This can be viewed in Figure 1. Therefore, it is optimal to split the time series at the end of 2014.

Table 10 displays the results of the segmentation from the second approach, and the results are statistically significant at a level of 1%. The subperiod before the price fall shows a lower HR but a higher HE than the whole subperiod after the financial crisis from Table 9. Further, the subperiod after the price fall shows a higher HR but a lower HE than the whole subperiod after the financial crisis from Table 9. The difference in HE between before and after the price fall is approximately thirteen percentage points, which means that hedged position from the subperiod before the price fall reduces approximately thirteen percentage points more of the price risk. The correlations between the spot and futures prices of the two last subperiods from Table 10 have changed compared to the whole subperiod after the financial crisis from Table 9. The correlation of the subperiod before the price fall has increased with the reason of the prices being more stable during this subperiod as Figure 1 illustrates. Further, in the subperiod after the price fall, the correlation decreases due to more volatile prices. The correlations affect the results of HE.

Table 10: Results of the segmentation from the second approach

	Before the financial crisis	*Before the price fall*	*After the price fall*
HR	0.725	0.662	0.715
HE	0.558	0.577	0.451
***t*-stat**	17.942***	8.902***	6.144***
Correlation	0.747	0.760	0.671
Counted	267	60	48

The table displays the results of the segmentation from the second approach. The HE and HR are quite similar for before the financial crisis and before the price fall; however, these are a bit lower before the financial crisis. After the price fall, the HE is the lowest and can be explained by the lower correlation and fewer observations. ***, **, and * represent the significance at 1%, 5%, and 10% significance level, respectively.

5.3 Yearly static MV hedge ratio

As previously reported, the oil price has been very volatile, and it is therefore reasons to believe that the HR and HE are not equally effective for each year. There were no large differences in the HR and HE when dividing the time series into different subperiods. However, it will be interesting to examine if there are any significant differences in the yearly estimations of the HR and HE.

The static MV hedge ratio model is the overall best performed model as stated in the results from Table 7. Further, the yearly HR and HE are estimated from 1988 to 2018 with one-monthly contracts since these provide the highest level of observations. The year 2019 is excluded because of few observations. The yearly estimations of HR and HE are presented in Table 11 and illustrated in Figure 6.

Table 11: Yearly HR and HE using a static MV hedge ratio model

	HR	HE = R^2	σ_s	σ_f	F-test
1988 – 2019	0.7220	0.5449	0.0906	0.0927	438.207 ***
1988	0.6350	0.4803	0.0345	0.0377	3.700
1989	0.5296	0.2873	0.0222	0.0225	4.030 *
1990	0.9213	0.7078	0.0570	0.0521	24.230 ***
1991	0.6519	0.6588	0.0276	0.0343	19.310 ***
1992	0.6076	0.3175	0.0143	0.0133	4.650 *
1993	0.4706	0.2181	0.0165	0.0164	2.789
1994	0.4846	0.4993	0.0188	0.0274	9.972 **
1995	0.7697	0.5331	0.0176	0.0167	11.420 ***
1996	0.4163	0.2325	0.0191	0.0221	3.030
1997	0.6655	0.4644	0.0227	0.0232	8.669 **
1998	0.6242	0.4653	0.0316	0.0346	8.703 **
1999	0.5950	0.5633	0.0355	0.0448	11.610 ***
2000	0.6883	0.5438	0.0351	0.0376	11.923 ***
2001	0.5965	0.3389	0.0276	0.0269	5.126 **
2002	0.6137	0.4590	0.0243	0.0269	8.483 **
2003	0.7702	0.6317	0.0249	0.0257	17.150 ***
2004	0.4065	0.1744	0.0231	0.0237	2.112
2005	0.8801	0.7421	0.0193	0.0189	28.768 ***
2006	0.7860	0.5616	0.0183	0.0174	12.808 ***
2007	0.7792	0.3711	0.0182	0.0142	5.901 **
2008	0.9283	0.7767	0.0353	0.0335	34.787 ***
2009	0.4470	0.1881	0.0184	0.0178	2.317
2010	0.3864	0.3316	0.0114	0.0170	4.961 **
2011	0.4961	0.3489	0.0111	0.0132	5.36 **
2012	0.8068	0.6160	0.0145	0.0141	16.040 ***
2013	0.5411	0.3625	0.0074	0.0082	5.687 **
2014	0.8200	0.7968	0.0160	0.0174	39.212 ***
2015	0.5858	0.3252	0.0306	0.0298	4.819 *
2016	0.7889	0.3919	0.0294	0.0233	6.445 **
2017	0.7461	0.5296	0.0129	0.0126	11.261 ***
2018	0.7744	0.6967	0.0197	0.0212	22.967 ***

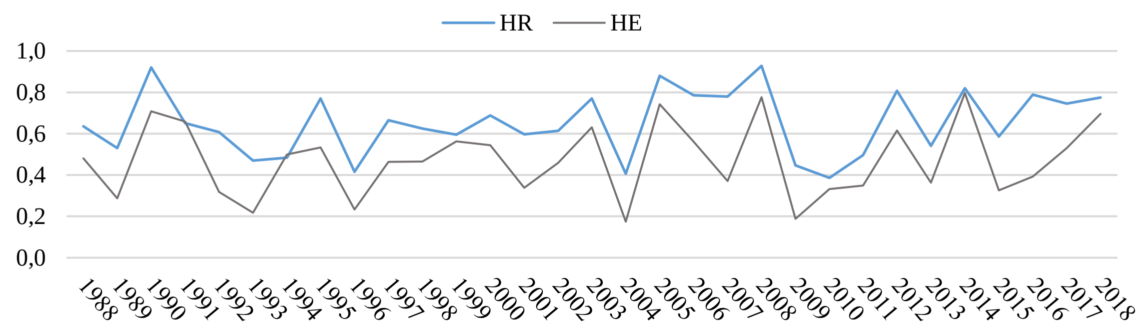
The table describes the yearly estimates of hedge ratio, hedge effectiveness, and standard deviations of the spot and futures prices. The estimates are estimated with a static MV hedge ratio model of the one-monthly contracts. The hedge ratios vary within a range between 0.39 and 0.93 during the 20 years. This huge variation in hedge ratio affects the hedging effectiveness, which vary within a range between 17% and 80%. The standard deviations for both the spot and futures prices vary quite during the analysis period, which are reflected in the hedge ratios; however, the spot and futures prices correlate quite well. ***, **, and * represent the significance at 1%, 5%, and 10% significance level, respectively.

The standard deviations for both the spot and futures contracts are very similar to each other for every year due to the high correlation between them. However, we see that the standard deviations change considerably from year to year.

From the regression analysis, α is the cost difference between the spot and futures markets. However, none of the α in the analysis are statistically significant, which can be viewed as no cost difference between these two markets. We are not discussing this further.

Interestingly, the hedge ratios from the yearly estimation change substantially through the analysis period. The highest hedge ratio is 0.93 in 2008, which is very close to a naïve hedge. However, this is the only occasion during the analysis period where the hedge ratio is above 0.9. The hedge ratio in 2010 is the lowest with 0.39. The difference between the hedge ratios of 2008 and 2010 means that an oil producer in 2010 needs to reduce the hedge ratio by approximately 60 per cent compared to 2008 in order to stay in an optimal position. This shows how volatile the oil market is and how important it is for an oil producer to be in a hedged position and to stay put about the rapid changes in the market. The hedge ratio in 2009 is not statistically significant and it may therefore not be an optimal HR. It can be questioned if the financial crisis had an effect on this HR, where the oil prices increased massively prior to the crisis and fell dramatically after the crisis.

Figure 6: Yearly HR and HE using a static MV hedge ratio model



The figure is an illustration of the yearly hedge ratios and their hedging effectiveness during the analysis period 1988-2018. The illustration gives an indication that the hedge ratios and hedging effectiveness have a highly positive correlation. Both the HR and HE have huge variations in their estimates through the years.

Since there are a lot of fluctuations in the HR, the HE will change accordingly. The year of 2004 is the worst year in terms of hedging effectiveness. Only 17% of the variance is reduced by using short futures contracts compared to the best year. The best year is the year of 2014 with a hedging effectiveness of 80%. This is remarkable as this is the year the oil price fell dramatically; although, it is a positive sign for the producers that hedged against this high price risk. They managed to get the highest hedging effectiveness through the optimal hedge ratio, which means that they eliminated a considerably large fraction of the price risk. However, the HR and HE in 2004 are not statistically significant. Nonetheless, the analysis of the yearly HE shows that in some periods, the oil producer is better off not hedging due to a low HE, and in some periods the producer should absolutely be in a hedged position.

5.4 FORECASTING HR AND HE

Until now, every calculation is based on historical data and the results shows that the HR and HE vary quite a lot for each year. The estimates of HR and HE are adaptable in the future given the assumption of knowing what will happen in the future. However, the future is uncertain and an oil producer that has the desire to hedge does not currently have the knowledge of the future spot price. By using only historical data, the estimation models conduct with more information than an oil producer actually has, and the hedge ratios may therefore not be as efficient as the results indicate. To get a more appropriate hedge ratio that an oil producer can correspond to given the current situation the oil producer is in, a forecasting model of the spot price can be valuable. Since the prices of futures contracts are calculated by $F_0 = S_0 e^{rT}$ and the future spot prices are uncertain, a forecasted standard deviation of the returns of the spot prices is needed in order to forecast HR. Then, the forecasted HR can be compared to the actual HR in order to investigate if it is able to capture the actual HR.

When comparing the results of HR and HE between the static and dynamic MV hedge ratio models from Table 7, the results are quite similar. Even though a static MV hedge ratio estimation is used to estimate yearly HR and HE, we find a dynamic MV hedge ratio with GARCH (1,1) estimation method better suited for the forecasting because of the conditional variance it provides. GARCH (1,1) minimises the errors in the forecasting by re-estimating for prior errors in the forecasting, and in this way, the prediction gets more accurate than using an OLS estimation method.

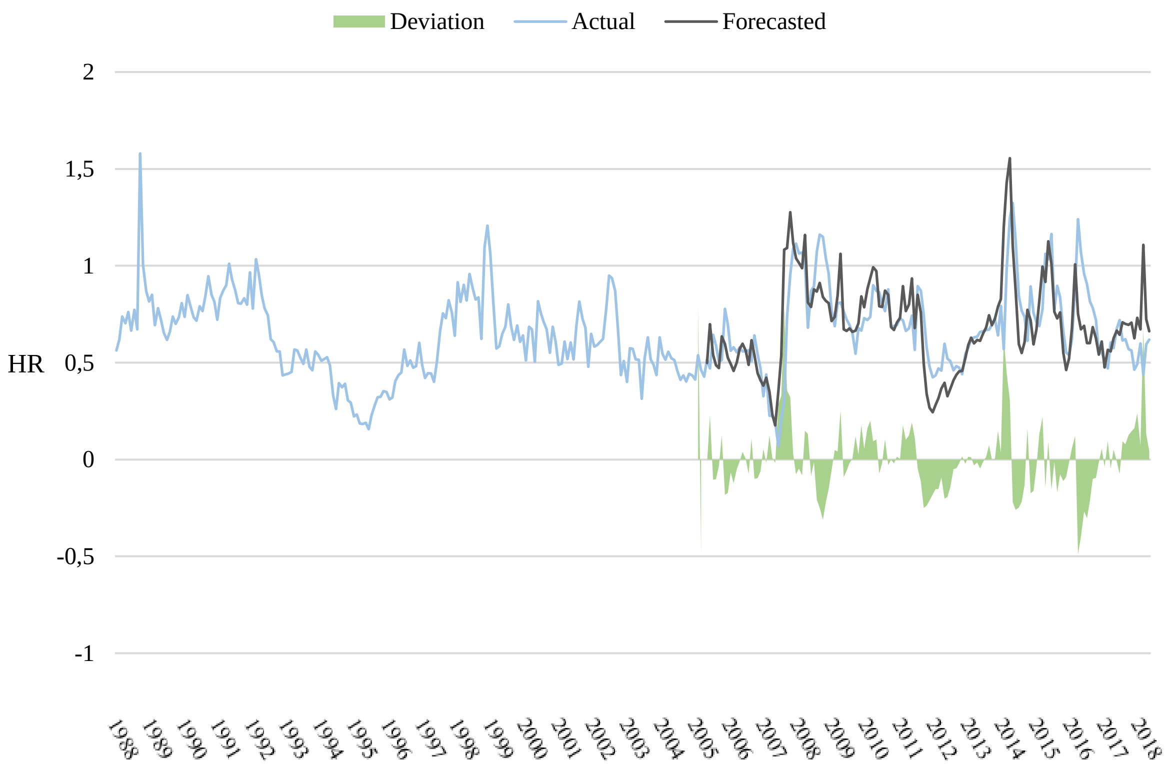
The forecasted spot prices of one-monthly contracts are predicted using a rolling window in the GARCH (1,1) estimation method, where the rolling window is forecasting a 1-month ahead conditional variance and is updated for each month in the out-of-sample period. The in-sample period is from 1990 to 2006 and the out-of-sample period is from 2006 to 2018. In the period before the financial crisis, the volatility of the spot prices has been stable, but after the financial crisis, the increasing volatility of the spot prices has continued and stayed high. It will be interesting to notice if the volatility in the prices before the financial crisis can predict a more unstable volatility using a GARCH (1,1) estimation method with conditional variance.

The black line in Figure 7 represents the forecasted HR, which is calculated by using the forecasted standard deviation of the returns of the spot prices. The actual HR with GARCH (1,1) estimation method is shown as the blue line in Figure 7. To calculate the dynamic MV hedge ratio using GARCH (1,1), the correlation between the spot and futures prices is also forecasted using a similar rolling window. Both the forecasted and actual HR have unstable volatilities; however, the Pearson's correlation between the forecasted and actual HR is 0.69, which indicates that the forecasted HR captures almost 70% of the actual HR, and it is statistically significant at a level of 1%.

Further, the deviation between the forecasted and actual HR illustrates how good the predicted forecast is, and it is shown as the green line in Figure 7. The deviation is at the highest in October 2008 where the forecasted HR is much higher than the actual HR. The deviation between the forecasted and actual HR is as high as 0.78. It can be explained by the absence of the forecasting model's ability to forecast the lower HR when the financial crisis hit. Moreover, in January 2017, the deviation of the actual HR compared to the forecasted HR is the highest with a deviation of 0.49. The forecasted HR underperforms compared to the actual HR, and the producer is therefore exposed to higher risk in this period.

The period of July 2012 to September 2013 has the longest deviation in horizon, where the forecasted HR underperforms compared to the actual HR. During these fourteen months, the forecasting model of the HR is not able to predict the much higher price changes as the actual HR does. Therefore, the producer will have higher risk during this period because of a lower hedged position than optimal. As illustrated in the Figure 7, there are no long periods where the deviation is zero over time. This means that the forecasting model of the HR is not able to predict the actual HR by 100%; however, the forecasting model is great when it comes to capturing the trend, and it is therefore a good model.

Figure 7: Forecasted hedge ratio (HR) using GARCH (1,1) estimation method

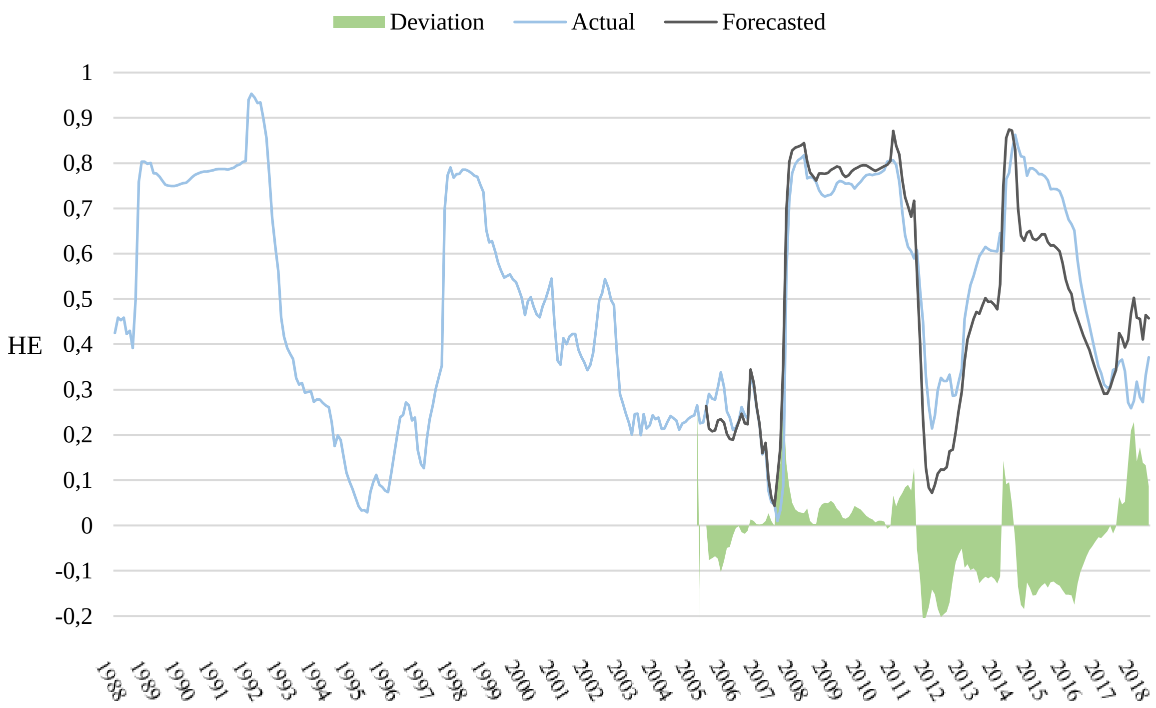


The figure presents the actual and forecasted hedge ratio with a GARCH (1,1) estimation method. The blue line is the actual hedge ratio during the analysis period. The grey line is the forecasted hedge ratio that is estimated using a rolling window in the GARCH (1,1) estimation method. The rolling window is updated for each month ahead. The figure shows that the forecasted hedge ratio captures the actual hedge ratio rather well. In some periods, there are higher deviations than in other periods, which are shown as the green area that fluctuates around zero.

Figure 8 shows an actual and a forecasted HE, where the Pearson's correlation is 0.92 and statistically significant at a level of 1%. This indicates that an oil producer without knowledge of the future spot price can conduct a forecasting model and find the trend of the actual data with approximately 90% accuracy.

The deviation between the forecasted and actual HE measures how good the predicted forecast is, and it is presented as the green line in Figure 8. The HE has the highest deviation with 27 percentage points in October 2008, which is the same period the HR has the highest deviation. This is due to the fact that the forecasting model is not able to predict the financial crisis. The oil producer is overhedged in this period of time, which means that the producer gets less profit. In almost the whole period from July 2012 to February 2018, the forecasted HE underperforms relatively to the actual HE, which means that the forecasted HE is less efficient than the actual since it is not able to eliminate as much of the price risk. This is a limitation to the forecasted HE as the producer will be exposed to more risk than necessary. Nonetheless, this deviation is not huge since the average deviation in this period are ten percentage points. However, the forecasted HE is able to capture the trend of the actual HE substantially.

Figure 8: Forecasted hedging effectiveness (HE) using GARCH (1,1) estimation method



The figure shows the actual and forecasted hedging effectiveness with a GARCH (1,1) estimation method. The blue line is the actual hedging effectiveness, while the grey line is the forecasted hedging effectiveness. There are some deviations between the forecast and actual hedging effectiveness, which is illustrated by the green line. This can be viewed as the forecasted hedging effectiveness is better to reduce risk than the actual, while in other periods it does not.

Dynamic estimations models for hedge ratio are very difficult estimation models and require a deep knowledge of both mathematics and econometrics in order to calculate correct HR and HE. If the risk management estimates wrongly, they can be in a situation where they are over- or underhedged, which can lead to a massive loss for the corporation. Therefore, it is much more important to use a simple static model correctly rather than missing the mathematics and getting wrong estimates from a dynamic model. The static models assume that the HR is held constant during the whole hedging period. On the other hand, it should not stop the risk management to update the HR with new information that comes along, which is a limitation to the static model. However, if the producer has access to an advanced risk management, it is a good solution to use GARCH (1,1) estimation method in a forecasting model. In the analysis, the conclusion of the forecasting of the spot price is very good using GARCH (1,1). The producer is able to capture the trend despite some deviations; nonetheless, these are marginal.

6. DISCUSSION

The analysis in this paper contains some shortages mainly due to few observations. It is ideal to have more observations, and it is reasons to believe that the results will be more satisfiable with more observations. The paper from Chen et al. (2003) is the main inspiration to this analysis, and they state that the data frequency has to match the hedging maturity, which had a huge effect on the results of the twelve-monthly contracts with only 24 observations in our analysis. When we use the GARCH (1,1) as the estimation method, the results for the six-monthly contracts is not reliable, and we are not able to get an estimate of the twelve-monthly contracts because the number of observations is too low. Therefore, we cannot conclude anything with the GARCH (1,1) model with six- and twelve-monthly contracts. Further, it does not exist daily or weekly futures contracts for Brent crude oil, and the number of observations is therefore limited to the observations that we have collected.

Because of the violation of the assumptions from the Gauss-Markov theorem, we choose to exclude the significance of the values of the results of HR and HE in Section 5.1.1. The data are not normally distributed, and testing the statistically significance levels will therefore be inappropriate because of the high risk of type I and type II errors. Most commodity futures are not normally distributed. Choudhry (2009, p. 61) tests seven different commodities for normal distribution, where all the commodities are not normally distributed. Then, it can be discussed whether our results are valid or not. However, transforming the variables to be normally distributed will indicate that the results do not reflect the futures market.

One considerable thing that accompanies hedging with commodities, which has been excluded in this paper, is the producer's ability to store the commodity. By storing the oil, the producer has the option to wait to sell the commodity, which means that the producer can wait to sell the oil until the oil price is high or to store the oil when the price is low. There are two ways to store the oil: The oil can be extracted from the reserves and then stored or the producer can wait to extract the oil. The first way to store the oil requires some caution, for instance, the oil needs to be carefully stored because it is highly combustible. The second way to store the oil does not have this problem. However, there are huge fixed costs that come with an oil production, and this solution will therefore be difficult to conduct. To find the optimal way to store the oil is complicated, and hedging with futures contracts will therefore be a more efficient solution.

Entering into a futures contract is free. However, there are some transaction costs that occur when entering and closing the contract, such as transaction costs that are brokerage fees. Another limitation in this article is that these charges are excluded. This is an important factor the oil producer needs to take into account before hedging. The more frequent the producer hedges, either buying or selling futures contracts, the more expensive the hedging will get. Therefore, the rollover strategy with short hedging frequencies can be an expensive strategy. However, we find that the twelve-monthly contracts were not efficient, which means that the most efficient way to hedge is to use futures contracts with higher frequencies, such as the three-monthly contracts, and hedge more often despite the higher costs.

7. CONCLUSION

This paper analyses and compares hedge ratio estimation models of different objectives within the Brent crude oil futures market, and futures contracts of different hedging maturities are employed: one-, three-, six-, and twelve-monthly contracts. The hedging effectiveness is used as a measurement of their performance, and it gives the result of which model is most suited for a Brent crude oil producer to use in its hedging strategy.

According to Chen et al. (2003, p. 440), the dynamic estimation method GARCH (1,1) should give a better estimation than the static MV hedge ratio due to conditional information and non-constant volatility. However, this complex method requires experience and mathematical skills, and not all risk managements have the resources. Small calculation errors can lead to massive losses. Our results show that the static MV hedge ratio model overall performs the best. Nonetheless, the naïve hedge ratio model underperforms slightly compared to the static MV hedge ratio model. The naïve hedge ratio model is a good and simple estimation model for hedgers with limited resources and experience, but the danger of being under- or overhedged with this strategy is possible. However, when considering a Wald test, the static MV hedge ratio for the three-monthly contracts is equal to the naïve hedge ratio. Therefore, as a result of the simplicity of the model, the naïve hedge ratio model is a more suitable model because it seems to be little gain from an advanced risk management.

Further, our results initiate that it is not efficient for a risk management to use futures contracts with a maturity of shorter or longer than three months. This contradicts with several empirical frameworks where futures contracts with the shortest maturity are the most optimal to use (Chen et al., 2003, p. 448; Malliaris & Urrutia, 1991, p. 288). For a hedging strategy with longer maturity, the risk management can provide a rolling strategy, meaning that a new futures contract is entered as the old one matures.

Many historical events in the economy have left their marks on the oil prices. A segmentation of the time series consisting of two approaches considers historical events. Between the subperiods used in the approaches, there are no large differences in the HR as the subperiods perform quite similar. In the second approach, we find that the HE after the price fall in 2014 decreased with thirteen percentage points than before the price fall. The reason for the fall in the HE can be explained by that the futures contracts did not capture the oil price movements when the oil market changed rapidly, or another explanation, the subperiods contain few observations and have lower correlations.

Further, the oil market is very volatile and it gives the reason to believe that the HR and HE vary from time to time. The yearly estimations of HR and HE capture the short-time volatility for each year instead of a single volatility for the whole analysis period. Compared to the segmentation, the estimates vary quite a lot from year to year. For instance, the minimum reduction in the price risk is in 2004 with 17% and the maximum is in 2014 with 80%.

Until now, every calculation is based on historical data, and these calculations assume that the estimates contain information about the future, which the oil producer does not have the knowledge of. In order to implement the estimation models in the future, a forecasting model of spot price is required in order to test if it gives similar HR and HE estimates as the historical ones. The conclusion from the forecasting model is that the forecasted HR deviates from the actual HR, and the correlation between them is approximately 70%. The forecasted HE captures surprisingly 90% of the trend of the actual HE. This demonstrates that an oil producer without the knowledge of the future spot price can interpret a forecasting model and achieve a prediction of the trend by 90%. However, there are some deviation between the forecasted and actual HR and HE. The high deviations during the financial crisis in terms of both HR and HE can indicate that the impacts from the financial crisis on hedging with futures contracts were unpredictable.

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