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Effect of pit distance on failure probability of a corroded RC beam

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Abstract

The present paper studies effect of the variation of pit distance on structural reliability of a reinforced concrete (RC) beam, with particular emphasis on the interference of localized corrosion on adjacent tensile rebars. The research question leading the inquiry of this article is how does average distance between corrosion pits in rebars affect the probability of failure in RC beams. In this paper, by using Monte Carlo Simulation (MCS), probabilities of failure in a corroded RC beam with different pit distances are quantified. Uncertainties in material properties, geometry, loads, corrosion modelling, pit distances and pit interference are taken into account. Statistical data reported in literature regarding the extent and location of corrosion is utilized to undertake a parametric study of corresponding probability distribution functions. According to results, variation of pit distance has significant influence on probabilities of failure. This influence increases if the effect of interference of localized corrosion is taking into account.

Keywords: Concrete structure; localized corrosion; pitting; pit distance; probability of failure; interference of pits

1. Introduction

Corrosion of steel rebars embedded in reinforced concrete (RC) members, causes deterioration of concrete structures, diminishing their capacity and serviceability. There are two types of corrosion: uniform and localized (pitting) corrosion. A typical deterioration of RC structures exposed to aggressive chloride environments is localized corrosion of rebar. Pitting corrosion can lead to high degrees of cross-section area loss along the rebar [1].

Assessing reliability of RC members with corroding rebars has received increasing attention in recent years [2-9]. Early studies accounted only for uniform corrosion in rebars. However, this approach requires additional measures to take into account spatial variability of cross-section along the bar and the reduction of mechanical properties of the bar due to local stress localizations [10-12].

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Later studies have become more realistic by developing reliability assessments of RC beams by considering the effect of spatial variability of the localized corrosion on rebars. Kioumarsi et al. [13] studied the effect of interference of corrosion pits on adjacent rebars on the probability of bending failure of a corroded reinforced concrete beam. Spatial distribution of localized corrosion along a beam is considered in the analyses. These authors considered the appearance of corrosion pits as a Poison process, i.e. a process in which pits occur continuously and independently.

Different rebars, exposed to different environmental conditions would present different average values of pit distance, which in turn may affect the probability of failure of RC beams. In the present article, we follow up the work by Kioumarsi et al, drawing attention to the influence of average pit distance on reliability of corroded RC beams. The research objective leading the inquiry of this article is thus: Quantifying effect of average distance between corrosion pits in rebars on the probability of failure in RC beams. To answer the research question, we consider a case study and estimate failure probability using Monte Carlo Simulation (MCS). Uncertainties in material properties, geometry, loads, corrosion modelling, pit distances and pit interference are taken into account. Statistical data reported in literature regarding the extent and location of corrosion is utilized to undertake a parametric study of corresponding probability distribution functions.

2. Interference of localized corrosion on adjacent rebars

It is shown that the cross-section reduction varies along the tensile rebars and that the cross-section reduction differs between rebars [14]. The disparities of localized cross-section reduction between rebars may result in interference between the pits (see Fig. 1) [15].

Fig. 1. Plan view of potential pit locations and possible interference of localized corrosion between adjacent tensile rebars [15].

Kioumarsi et al. [15-17] selected an idealized case to quantify the possible interference of localized corrosion on adjacent rebars in an under-reinforced beam subjected to bending. In the idealized case two adjacent rebars were considered with one corrosion pit each. The two corrosion pits were equal in size. In a series of nonlinear finite element models the combined influence of two variables on the bending ultimate limit state (ULS) was quantified: the ratio of the distance between pits in two adjacent rebars to the distance between tensile rebars, l_p/l_r, and the ratio of the cross section reduction of the rebar due to localized corrosion to the initial cross section of rebar A_{pit}/A_0. From the numerical simulations it was found that pits interfere within a critical distance. Interference of localized corrosions reduces gradually for increasing distance between pits in two adjacent rebars (l_p). For the investigated beam with 80 mm distance between two adjacent rebars (l_r) the critical distance was 100 mm; i.e. for higher ratios of l_p/l_r > 100/80 = 1.25 no interference was observed [15-17].

Current analytical design rules cannot quantify the interference of localized corrosions for intermediate l_p/l_r ratios $\left(0 < l_p/l_r < 1.25\right)$ [17]. In order to take into account the possible interference of localized corrosions, Kioumarsi et al. [15] proposed using a modified total residual cross section of corroded tensile rebars in an analytical analysis of the strength of the cross section:

$$A_{res(mod)} = 2A_0 - (2A_{uni} + A_{pit} + \beta A_{pit}) \tag{1}$$

$$\beta = -0.76(l_p/l_r)^2 + 0.16(l_p/l_r) + 1 \tag{2}$$

where $A_{res(mod)}$ is the modified total residual cross section of two rebars after uniform and localized corrosion, A_0 is the initial cross section of a rebar, A_{uni} is the cross section reduction of a rebar due to uniform corrosion, A_{pit} is the additional cross section reduction of a rebar due to localized corrosion. The interference of localized corrosion is introduced by an interference factor β which is only a function of the ratio of the distance between pits in two adjacent rebars to the distance between tensile rebars.

3. Distribution function of localized corrosion

When inspecting naturally corroded rebars it becomes obvious that the distinction between localized and uniform corrosion is not clearly visible, and the explicit modelling of both requires some simplifying assumption. In this paper we used a different geometric description which allows modelling of localized corrosion; the original cross-section (A_0), the cross-section reduction due to uniform corrosion (A_{uni}), and the cross-section reduction due to localized corrosion (A_{pit}). See Fig. 2.

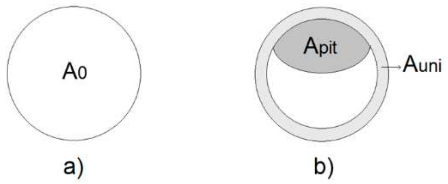

Fig. 2. (a) Original cross-section (A_0); (b) uniform (A_{uni}) and localized cross-section reduction (A_{pit}) [13].

If the uniform corrosion is assumed to be equal to the minimum cross-section loss the number of the pits is high. If the assumed uniform corrosion is increased the number of pits will gradually decrease [13]. This is exemplified in Fig. 3.

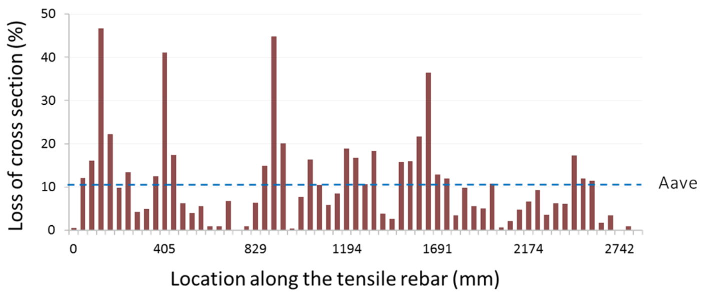

Fig. 3. Cross-section reduction of a corroded rebar. The dashed lines illustrate different assumptions of uniform corrosion, which influences the number of pits.

3.1. Distribution function of pit size ratio (A_{pit}/A_0)

Maps of steel cross-section losses of the selected corroded rebars were obtained from two recent papers [14, 18]. The average cross-section reduction was measured as mass loss for the rebars, which amounted to approximately 10%.

It is shown that the gamma distribution function represents best variation of the cross-sectional reduction along a rebar [13]. Fig. 4(a) indicates the obtained gamma functions for the selected four naturally corroded rebars. By assuming the four rebars as one long rebar, which is called "composed rebar", a gamma function is obtained, see Fig. 4(b); this is the one that will be used in this paper. It was assumed that uniform cross section reduction is equal to average cross section reduction (10%).

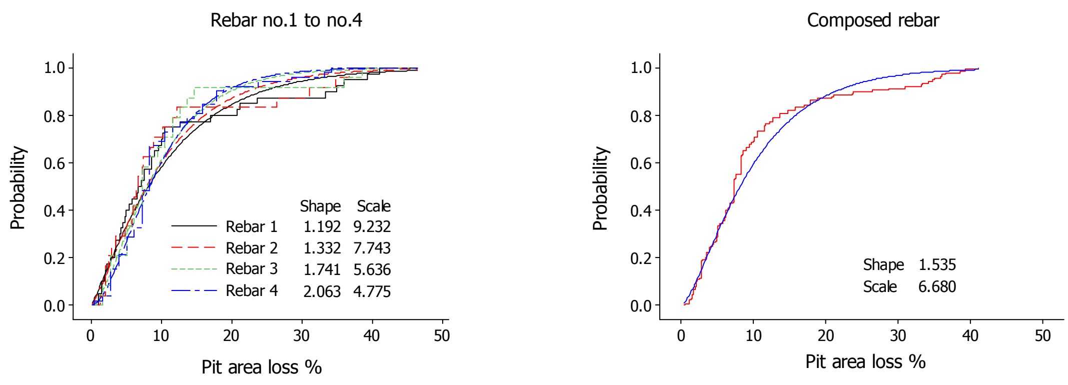

Fig. 4. Empirical and fitted cumulative distributions (using gamma distribution function) of pit size in four naturally corroded rebars, for the assumption of uniform corrosion equal to 10%.

Table 1. Statistics of the ratio of localized cross section reduction to initial cross section of rebar (A_{pit}/A_0) based on cross section loss data of four corroded rebars.

Variable	A_{uni}/A_0 (%)	Distribution	Shape parameter	Scale parameter	Reference
A_{pit}/A_0	10	Gamma	1.16	8.14	Fitted with data from [14, 19]

3.2. Distribution function of pit distance (l'_p)

The occurrence of pits along the tensile rebar can be represented by a Poisson process, i.e. the occurrence of pits is assumed statistically independent [13]. Using measured corrosion data for the same four rebars, the cumulative distribution function of the distance between pits in same rebars were fitted to the exponential probability distribution, see Fig. 5. Similar as for the distributions of pit sizes, pit distance distributions in rebars were fitted for the assumed case where the uniform cross section reduction is equal to average cross section reduction. Table 2 lists the parameters of the fitted distribution functions.

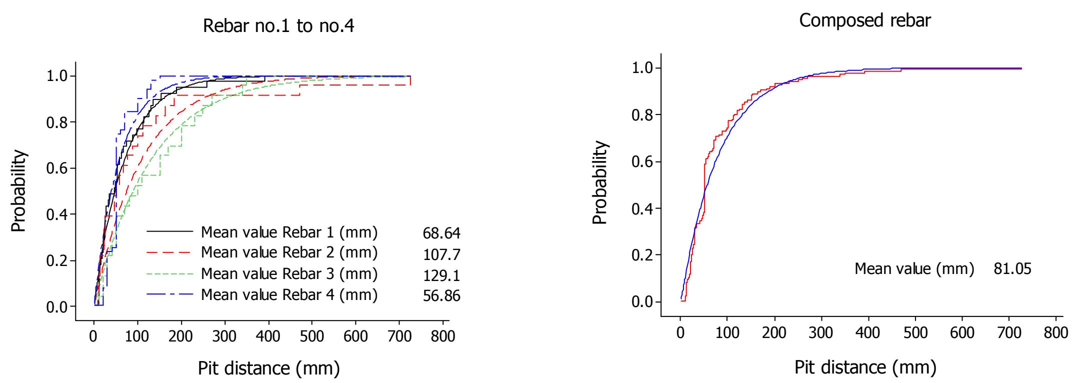

Fig. 5. Empirical and fitted (using exponential distribution function) cumulative distributions of distance between pits in same rebar in four naturally corroded rebars, for the assumption of uniform corrosion equal to 10%.

Table 2. Statistics of the distance between pits in same rebar (l'_p) based on cross section loss data of four corroded rebars.

Variable	A_{uni}/A_0 (%)	Distribution	Mean Value (mm)	Reference
l'_p (mm)	0	Exponential	47.51	Fitted with data from [14, 18]
	10	Exponential	74.48	

4. Discussion about pit distance and its effect on the distribution functions

Pit distance in corroded rebar is function of the location of the rebar and degree and type of the corrosion. It might be different from lightly to severe corroded condition. In section 3, we obtained the mean value of the pit distance for four corroded rebars. Obtained exponential distribution functions of pit distance in section 3 shows that the mean values of pit distance of the four selected rebars varies from 50 mm to 130 mm when the composed rebar has the mean value of 81 mm, see Fig. 5(a) and Fig. 6. It raises a question that "how variation of the pit distance could influences on failure probability of corroded RC beam".

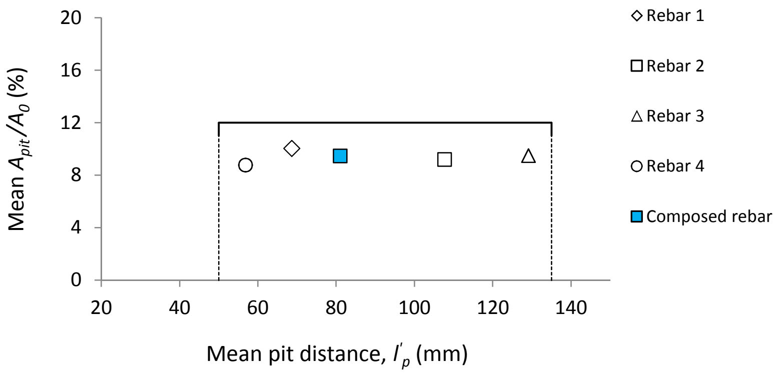

Fig. 6. Relation between mean values of ratio of localized cross section reduction to initial cross section of rebar (A_{pit}/A_0) and distance between pits in same rebar (l'_p). Uniform cross section reduction equal to (a) zero and (b) 10%.

5. Probabilistic analyses of a RC beam

The purpose of the case study presented in this section is to quantify the effect of pit distance on the probability of failure of a corroded RC beam when the interference effect of localized corrosion is taken into account.

5.1. Case study

Since under-reinforced beams are most common in practice, only this type of beam was considered. Analyses were carried out for a simply supported RC beam. The beam's dimensions are length 6 m, height 0.35 m and width 0.2 m. The beam included two tensile rebars with diameters of 24 mm and a concrete cover of 36 mm.

5.2. Statistical properties of other random variables

As it discussed earlier, the pit distance (l'_p) and ratio of localized cross-section reduction to initial cross section of rebar (A_{pit}/A_0)in each tensile rebar are represented by an exponential and gamma distribution functions, respectively. The other statistical variables of the RC beam used in the probabilistic analysis and their distribution functions are given in Table 3.

Table 3. Statistical properties of random variables.

Variable	Symbol	Distribution	Mean (μ)	CoV	References
Effective beam depth (mm)	d	Log-normal	288	0.03	[2] [19]
Beam section width (mm)	b	Normal	200	0.02	[20]
Original rebar diameter (mm)	d_0	Normal	24	0.02	[20]
Distance between adjacent tensile rebar (mm)	l_r	Normal	80	0.05	-
Self-weight (kN/m)	Q_w	Normal	1.5	0.1	[21] [7]
Live load (kN/m)	Q_L	Gamma	4.7	0.6	[7] [22]
Concrete compressive strength (MPa)	f_c	Log-normal	47.7	0.18	[2] [7, 23]
Steel yield strength (MPa)	f_y	Log-normal	592	0.1	[24] [23] [7]

5.3. Limit state function and probability of failure

To quantify the failure probability of a corroded beam with interference effect of localized corrosion, the probability of failure was estimated using Monte Carlo Simulation. The limit state function is expressed as:

$$G_M(i) = \frac{f_y A(i)_{res(mod)}}{f_c b d}\left(1 - 0.4\frac{f_y A(i)_{res(mod)}}{0.8 f_c b d}\right) f_{cd} b d^2 - \frac{(G_L + Q_L) l(i)}{2}(l - l(i)) \quad (3)$$

where f_y is the steel yield strength, f_c is the concrete compressive strength, b is the beam width, d is the effective height, $A_{res(mod)}$ is the modified total residual cross-section of two rebars after uniform and localized corrosion, ta king into account possible interference, G_L is self-weight, Q_L is live load, $l(i)$ is the location of i^{th} pit along tensile rebar and l is length of beam span.

In Monte Carlo Simulation $A_{res(mod)}$ at the location of each pit on first tensile rebar is calculated following Eq. 1 by pairing it successively with each pit in the adjacent rebar within the critical zone. The respective interference factors β_i are applied for the pits in another tensile rebar. It is assumed that the interference with the pit resulting in the smallest $A_{res(mod)}$ *is* dominant. The procedure is repeated by starting with the pits on the second rebar and identifying the potential interference with pits on the first rebar.

Monte Carlo Simulation was used to evaluate the probabilities of failure of the RC beam. At each run using the calculated cross-sectional areas of longitudinal reinforcements and the generated values of the other structural variables, the flexural strengths of the beam at all cross-sections containing pits and at mid-span were evaluated and compared with the corresponding bending moment. At each run, all variables of Table 3 are realised only once per beam. If the limit state function was violated in at least one of the verified cross-sections, the beam is considered as failed. The probability of failure, P_f, was estimated as the number of runs with a failure of the beam divided by the total number of runs.

6. Results

This section presents probabilities of failure (P_f) for two cases:

a) average cross section loss and localised corrosion are considered *including* the interference of pits,
b) average cross section loss and localised corrosion are considered *excluding* the interference of pits.

Moreover, various mean values of the distribution function of pit distance (l'_p): 20, 40, 60, 80, 100, 120, 140 and 160 mm, were used in the analyses. As it mentioned before, the ratio of average cross-section reduction to original cross section (A_{ave}/A_0) is equal to 10%.

Fig. 7 illustrates the failure probability for both cases (a) and (b) with different pit distances. According to obtained results, reducing the pit distance results in increasing P_f. For the case (b), excluding pit distance, reduction of pit distance from 160 mm to 20 mm leads to 270% increase in increase P_f. If the effect of interference of localized corrosion is taking into account this influence increases to about 700% (see Fig. 7 case a).

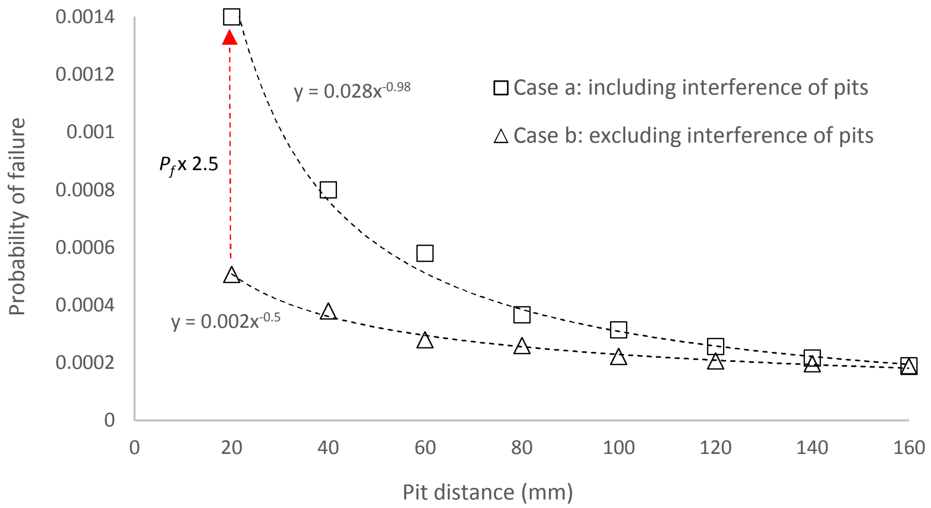

Fig. 7. Effect of the pit distance on probability of failure of the RC beam with and without considering the interference effect of localized corrosion.

The effect of the interference of pits increased by decreasing the pit distance. For example for the model with 20 mm pit distance, the interference effect increases the P_f up to 2.5 times (See the red arrow in Fig. 7), while for model with 160 mm, there is almost no change in the P_f. Less number of the pits, due to increase in the pit distance, which reduces the possibility of the interference between pits, could explain this. It could be seen that the effect of interference is substantial when the pit distances are less than 100 mm. This observed value of critical distance (100 mm) obtained by earlier studies [15, 17]. It is shown that, interaction of pits leads to a gradual reduction of the ULS for pit distance less than 100 mm.

Changing the pit distance could also influence on the reliability class suggested by EN90 or ISO13822. When the value of pit distance is larger than critical value the obtained probability of failure is in the normal reliability class with medium consequence of failure. While reducing the pit distance leads to low reliability class with low consequence of failure. Thus, considering the wrong mean value of pit distance could result in the inaccurate reliability assessment of deteriorated structures.

7. Conclusion

This paper considered the spatial variation and possible interference of localized corrosion on the reliability of a corroded RC beam. An interference model for the effect of two equal pits in adjacent rebars on the ultimate capacity and the distribution functions of spatial variation of localized corrosion were proposed earlier.

Probability of failure in a corroded RC beam with different pit distances was estimated using Monte Carlo Simulation. Uncertainties in material properties, geometry, loads, corrosion modelling, pit distances and pit interference were taken into account. The occurrence of pits along the tensile rebar was represented by a Poisson process. This assumption was supported by literature data. Based on the above assumption the following conclusions the following conclusions can be drawn:

- Reducing the pit distance leads to increase in the probability of failure. The increase will be intensified when the effect of interference of localized corrosion is taking into account.
- By increasing the mean value of pit distance, the effect of the interference of pits decreases. This is explained by the apparent lower number of pits and the reduced possibility of interference.
- Overestimating the mean value of pit distance in a corroded rebar could result in an underestimation of reliability of deteriorated RC beam.

It is emphasized that the proposed model to consider the pit distance effect on the probability of failure and application of this model was limited to one case study.

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