Propositional Logic and Formal Codification of Behavioral Operations

Gunnar Salthe and Jon A. Lokke
Ostfold University College
Erik Arntzen
Oslo and Akershus University College

ABSTRACT: A formal symbolic language for behavioral operations is proposed, based on propositional logic. The system describes how an experiment changes an organism's physical environment. With few exceptions, the codification system results in statements reduced to the truth-conditions of observable events. The main purpose is clarification of key concepts used in behavior analysis by describing the logic of behavioral operations. Using the system, we explain, for instance, the difference between positive and negative reinforcement and how differential reinforcement contains an extinction procedure. Use of a well-established formal language may also facilitate co-operation across disciplines as behavior analysis, biology, and economy.

Key Words: formal codification, logic of operations, key concepts, behavior analysis, behavioral operations

Propositional Logic and Formal Codification of Behavioral Operations

A few notation or codification systems have been proposed in behavior analysis. The most elaborated is that of Mechner (1959, 2008, 2011). Mechner’s (1959) purpose was to describe the pre-designed rules for experiments with a codification system based on symbolic diagrams using flow-chart notation in computer programming, Boolean algebra, and mathematical notation. The rules are called behavioral procedures, reinforcement contingencies, and behavioral contingencies in Mechner’s article from 2011. Lokke, Arntzen, and Lokke (2006) describe a notation system for behavioral operations intended as a pedagogic tool, based on Mechner’s article from 1959, and the presentation in Pierce and Cheney (2004). The initiating background for this system was a survey of the behavior analytic literature indicating inconsistent and incomplete ways of writing operations in short forms (Lokke, Lokke, & Arntzen, 2008).

Here we suggest a codification system based on *systematic* use of logical connectives combining basic statements which are, as far as possible, about public events. We thus describe the *logic* of all basic behavioral operations, reduced to the truth-conditions of the basic statements. The aim is to clarify basic concepts used in behavior analysis.

Since logic is a well-established formal language, use of logic may also promote collaboration across disciplines and might pave the way for full formalization of behavior analysis, including the anticipated measured effects on the dependent variable.

Mechner (2011) suggests how formal symbolic languages may be helpful through visualization, communication, teaching, abstraction, identification of parameters, and conceptualization. We focus on the objective of conceptualization at the end of Mechner's list. Unlike most of the existing notation, the proposed language is more than shorthand. By formulating the logic of behavioral operations, we may explain, for instance, the difference between arranging for positive and negative reinforcement. We may also use the proposed formalization to examine the difference between classical and operant conditioning, and we may demonstrate that differential reinforcement contains elements of extinction. Furthermore, if the formalization makes the difference between classical (respondent) and operant conditioning clear, we might contribute to the question of what the unit of selection is in behavior analysis (Donahoe & Palmer, 1994).

Like that of Mechner (2011), our system focuses on the theory's independent variables. Mechner has a different focus by codifying everyday examples of operations, without implying their effects though. The scope of the present manuscript is formalization of operations, thus, we want to describe procedures for stimulus presentations. We do not describe observable behavior processes, or formulate laws. In other words, we formalize the plans of experiments; but we do not codify the effects on the dependent variables of implementing the plans.,.

Choice of Formal Language

Terms do not assert anything on their own, so they cannot be true or false. To say something substantial, we must combine terms, forming statements, therefore, we use propositional logic. Propositional logic starts by symbolizing simple statements and then combines them into complex statements using logical connectives. Terms are symbolized indirectly, by statements about their valence, appearance, and sequence. In order to attach valence to particular stimuli, and specify how stimuli and response are sequenced, we had to accept, among the basic statements of our system, some that are not quite as simple as a statement might be. The truth and falsity of the basic statements may still be controlled by observation to a satisfactory extent as long as we restrict the scope of the codification to plans for experiments, particularly when the plans succeed.

Alternatively, we could have chosen predicate logic, symbolizing the elements of a statement. Terms would be symbolized directly, as predicates. We could then characterize something as a stimulus by attaching a predicate to a variable. That a stimulus is aversive or appetitive requires yet a predicate, as well as something being a response. A sequence could be symbolized as a relation between two variables. Predicate logic would therefore start by identifying all the variables as stimuli or responses, then add whether stimuli are appetitive or aversive, specify the sequence as relations among variables, and finally combine all these elements by parentheses and logical connectives. Propositional logic is used because this language provides simpler formulae.

The main advantage of the proposed codification system is due to logical connectives and parentheses being substituted for ill-defined symbols like the colon and ambiguous use of arrows (pointed out by Mechner, 2011). Logical connectives are defined by the truth and falsity of the combinations they form between elementary statements. Their meanings are completely and unequivocally reduced to the truth and falsity of the elementary statements they combine. That is why propositional logic is *truth-functional*, which means that the truth or falsity of the resulting complex statements is a function of the truth or falsity of the elementary statements. Because we use basic statements formed such that their truth and falsity as far as possible may be checked by observation and combine them by use of logical connectives and parentheses, we achieve the rigor that should be required for codification of behavioral operations.

There are many theories of truth (Künne, 2003); but given the behavior analytic conception of verbal behavior, we should not proceed by defining the meaning of the word “true.” We suggest a pragmatic approach where truth means acceptance by the audience of a tact or an interverbal of the type “Caesar crossed the Rubicon” (see Skinner, 1957/1992, p.129). The efficiency of a science then depends on how precisely the scientific community may settle whether an utterance should be accepted, by use of scientific methodology. We may help the behavior analytic audience (including the experimenter) reject contradictory formulae. However, since behavior analysis is an empirical science, we cannot require unconditional truth (logical validity) as a criterion for acceptance. In this regard, our contribution is specification of the truth-conditions of behavioral operations.

In spite of reference to the metaphysical notion 'proposition', propositional logic does not require belief in propositions. We will interchangeably use 'statement' and 'assertion' to replace that word.

Before we start formalizing operations, we describe the constituent parts of our suggested system—connectives first, then the basic statements we need. We will thereafter use the formal language to discuss the logic of operations by clarifying how their truth-conditions differ.

Connectives

Six logical connectives will be used, as shown in Table 1. They may be symbolized in various ways. We present our choice of symbols and the more technical symbols used in most textbooks.

Table 1
Language Elements 1: Connectives

Name	**Natural language expressions**	**Technical symbols**	**Suggested symbols**
negation	not …	¬	~
conjunction	… and …	∧	&
inclusive disjunction	… or …	∨	or
exclusive disjunction	ether ... or …	$\underline{\vee}$	$\underline{\text{or}}$
conditional	if ... then …	→	if
biconditional	if and only if ... then …	↔	iff

We avoid use of arrows as symbols of conditionals, since arrows are used inconsistently in behavior analytic textbooks. They sometimes indicate time, sometimes causation, sometimes conditionals (even in Mechner 1959, but in 2008 and 2011 arrows seems to be used solely as logical connectives). We want pure, unambiguous, logical connectives. For all connectives, we chose symbols close to the natural language English. This might help readers who are unfamiliar with logic. The symbols we suggest are also easy to write on a keyboard, and they are within current use.

Regarding conditionals, the condition comes first in normal logic notation (if P then Q), but the sequence must be reversed when "if" is substituted for "if … then...". We then have Q if P, which is also customary in the natural language English. For convenience, we do the same with biconditionals.

Not just signs, but also concepts used in logic, sometimes take on a richer meaning when used in behavior analysis. It is not always clear whether the expression "contingency" used in behavior analysis is derived from what is called "conditionals" in logic. Sidman (1986), for instance, describes three-term, four-term and five-term *contingencies* while he uses expressions as "if … and no other … then", "only if" or "but only" when explaining his Tables 1, 3 and 10 (pp. 217, 223–224, and 238–239). We will later suggest why there are good reasons for switching between conditionals and biconditionals, but left uncommented, it might lead to misunderstandings. We still use expressions like classical and operant *conditioning*, as customary within behavioral analysis.

In logic, an *antecedent* signifies the condition in a conditional ("P" in "if P then Q"), Q is called the consequent. In behavior analysis, antecedents are stimuli that precede responses. In describing the connectives, we will use the word *antecedent* as in logic; but to avoid confusion, we switch to "condition" when we later introduce the basic statements. When we, in that context, speak about stimuli presented before the response, we will use "antecedent" in combination with the word "stimulus".

The reader should note that in logic the inclusive disjunction means one or the other or both, while the exclusive disjunction means one or the other but not both.

Most natural languages do not make this distinction. Logic does not allow such ambiguities.

Defining the Connectives

Let us introduce P and Q as placeholders for whatever assertions we want. They may be true or false, abbreviated T and F. These are the truth-values statements can take. They comprise all the semantic we need for a clear definition of the connectives. The definitions are given by a truth-table, Table 2.

Table 2

Truth-table Defining Logical Connectives

P	**Q**	**~P**	**P or Q**	**P & Q**	**Q if P**	**Q iff P**	**P <u>or</u> Q**
T	T	F	T	T	T	T	F
T	F	F	T	F	F	F	T
F	T	T	T	F	T	F	T
F	F	T	F	F	T	T	F

Note. P and Q are placeholders for whatever statement we want to use. T denotes *true* and F denotes *false*. These are the truth-values statements can take.

A vertical line is essential to truth-tables. To the left of the vertical line, we write all possible permutations of truth-values (T, F) for two statements (P, Q). To the right, they are combined by the logical connectives. The meaning of the connectives is *defined* by the truth-values of the resulting combinations, listed to the right of the vertical line (Tomassi, 1999). The result is that the truth-values of the complex statements are completely and unequivocally referred back to those of the elementary statements (left of the vertical line).

As a consequence of this semantic, different complex statements are logically equivalent when they are verified and falsified by the same combination of observations, specified by the elementary statements. Regarding truth and falsity, they have the same meaning.

As defined by Table 2, the connectives connect only two statements. If we want to combine more statements, parentheses may be needed to avoid ambiguities.

About the Connectives

Table 2 shows that the disjunction P or Q and the conditional Q if P are true in 3 of 4 possible combinations of truth-values for P and Q. The conjunction P & Q is true in just one of them. With respect to P and Q, we are thus told exactly how the world must be for the conjunction to be true. Disjunctions and conditionals are far less informative. Hence, conditionals and disjunctions produce weak statements while conjunctions form strong statements. Let us look closer at conditionals.

Conditionals are *false* only when the antecedent is true and the consequent false. When the antecedent is false, conditionals are defined as true, no matter what truth-value the consequent takes. Some have found this unsatisfactory (Edgington, 2001).

One reason is that in everyday language we use conditionals to assert our belief that the consequent will be true on the condition that the antecedent is true: *If* P is true, then the assertion is that Q is also true. Table 2, however, defines the meaning of the conditional for *all* combinations of truth-values for P and Q. We should therefore read Table 2 as follows: If P is false, nothing in particular is said about Q—therefore Q can be true or false (Quine, 1952). This is as in the warning: “If you eat this mushroom, you will die.” If you do not, you can live, or you can die for some other reason.

Another reason for discontentment is that classical logic defines conditionals through an extensional conception of truth. This is close to holding that the meaning of some utterance is defined through observations, which should suit behavior analysts well. However, if 'the sun rises every day' is substituted for P and 'human beings are mortal' for Q, the complex statement 'if the sun rises every day, human beings are mortal' is accepted. We should, therefore avoid arbitrary use of conditionals (Quine, 1952). Logic is just a language. Theories must be formulated and tested before we can learn about the world. What we learn through experiments will therefore help selecting interesting conditionals.

Because conditionals are weak statements, they may be appropriate for describing natural processes. Mechner (2011) seems to hold this view, but then describes the conditionals as if they were counterfactual conditionals (p. 94) in order to denote dispositions. Counterfactual conditionals are often preferred in discussions of causation (Collins, Hall, & Paul, 2004), but involve a shift to modal logic.

Plans for an experiment require statements stronger than classical conditionals. We will therefore use biconditionals and conjunctions to express plans for stimulus presentation under experimental control. (When Mechner 2008 on page 126 specifies that “Only As or Ts can consequent Cs” he may be opting for the same with his arrows and brackets.) Before we codify operations, however, we have to introduce the other constituent part of the proposed codification system, basic statements about relevant events.

Basic Statements

In Table 3, we introduce 14 symbols for basic assertions about the appearance and sequence of terms, stimuli valence, and motivational operations. S denotes that a particular stimulus is presented; M denotes that motivational operations are established, R that an instance of the target response is observed. Later, 8 symbols are added to write delayed reinforcement, intermittent reinforcement and differential reinforcement.

Table 3

Language Elements 2: Basic Statements

Symbol	**The symbolized basic statement**
R	An instance of the target response is observed
S^{A}	A neutral stimulus is presented *anterior* to (superscript A) R.
S^{U}	An *unconditioned* stimulus (superscript U) is presented anterior to R.
S^{P+}	An *appetitive* stimulus (superscript +) is presented immediately *posterior* to (superscript P) R.
S^{P-}	An *aversive* stimulus (superscript -) is presented immediately *posterior* to (superscript P) R.
S^{A+}	An *appetitive* stimulus (superscript +) is presented *anterior* to (superscript A) R.
S^{A-}	An *aversive* stimulus (superscript -) is presented *anterior* to (superscript A) R.
$S^{A(+)}$	A stimulus is presented, *anterior* to some R, with an established history of subsequent S^{P+} if R.
$S^{A(-)}$	A stimulus is presented, *anterior* to some R, with an established history of subsequent S^{P-} if R.
S^{AS}	A stimulus is presented *anterior* to some other stimulus (superscript AS).
S^{CoS}	A stimulus is presented *concurrent* with some other stimulus (superscript CoS).
S^{PS}	A stimulus is presented *posterior* to some other stimulus (superscript PS).
$M^{\uparrow}$	Motivational operations are established, *increasing* the valence (superscript ↑) of some stimulus.
$M^{\downarrow}$	Motivational operations are established, *decreasing* the valence (superscript ↓) of some stimulus.

Note. S means that a stimulus is presented, M that motivational operations are planned for. Numbers in subscript may be used to identify different terms. Superscript “A” means *anterior* to, superscript “P” means *posterior* to, and superscript “Co” means *concurrent* with some other element in the scheme. S_1^{CoS2} thus means that stimulus number 1 is presented concurrently with stimulus number 2. When superscript *A* and *P* are used without specification of which element they precede or follows, the meaning is *anterior* or *immediately posterior* to the target *response*. Superscript “+” means positive valence, superscript “-” means negative valence. Signs of stimuli valence in *parentheses* indicate that the stimulus, to which this superscript is added, is neutral, but has a history of correlation with an appetitive or aversive stimulus provided that the target response is emitted. If several appetitive or aversive stimuli are presented, superscript may be added inside the parentheses of $S^{A(+)}$ or $S^{A(-)}$ to indicate the stimulus to which they have an established correlation if R.

We have tried to use as few symbolic elements as possible and build basic statements saying as little as possible. The idea is that the main part of the description of behavioral operations should be achieved by the way logical connectives combine the basic statements. How plans for experiments differ logically is thus exposed, enabling theoretical clarification.

Most of our basic statements are less than elementary, however, because superscripts are used to add properties to individual events and determine the sequence. Formally we stay within propositional logic, because even elementary statements say something about an object, but our statements contain *several* predicates and relations. This is acceptable because the basic statements are still as observable as can be, and the combination of statements picture the logic of behavioral operations without hiding important aspects within the basic statements.

Several superscripts in current use are avoided. As Mechner (2011) argues, we should avoid confounding the fact of independent variables with their anticipated effect on the dependent variable (p. 96). Care has therefore been taken to formulate the basic statements about terms such that they, as far as possible, state observable facts established *before* the experiment starts. The *planned function* of terms should appear as a consequence of how basic statements are combined. This makes the logic of specified operation explicit. Symbols like S^R, S^D, S^C, and S^Δ are therefore rejected, while S^U is retained.

Basic statements about *different* stimuli are identified by numbers attached to S in subscript. The same will be done to other basic statements when several terms of the same type are used in one behavioral operation. Superscript may then be added to M, specifying the stimulus whose valence (hypothetically) will be increased or decreases. $M^{\uparrow S3}$ will then mean that motivational operations are established in order to increase the valence of stimulus number 3.

Superscript is also used to notate sequence, valence, and established correlation with an appetitive or aversive stimulus following the target response. The letters A and P denote *anterior* and *posterior*. The terms are thus situated within the sequence, but except for the notation S^P, there is no information about the intervals involved. Sequence information is sufficient to differentiate between the logic of the basic behavior operations. Resources for specifying the intervals are introduced later, in Table 9. In our notation of valence (+, -), we follow Mechner (2008, 2011).

Combined by classical propositional logic, the 14 basic statements are sufficient for the formulation of all basic forms of behavioral operations. "M" and ~M may serves as an example. Because M denotes that motivational operations are planned, ~M means that they are abolished (whether $M^\uparrow$ or $M^\downarrow$).

Given few and inevitable assumptions, the basic statements may, through observation, be accepted as true or rejected as false by persons present during the experiment and appropriately educated. The concept of neutral stimuli and unconditioned stimuli are theoretical constructs, but such is all scientific terms (Quine, 1960), and because they are part of his or her plan, the experimenter will be able to check by observation also S^A and S^U. When the plan is explained to a third party, the occurrence of the stimulus is publicly available even to this party; but neither this party, nor the experimenter, may observe how a stimulus is perceived by the organism in the experiment.

Public Events

How precisely facts are expressed depends on the basic statements. It is not always easy to capture the reference of the basic terms response, stimulus and valence. When Mechner (2008, 2011) attempts to codify cases from every-day life, outside the carefully controlled environment of an experiment, he faces this problem to the full extent. In experiments, however, the experimenter defines the target response, and the experimenter controls the consequential stimulus. If the planned behavioral operation succeeds, the behavior of the organism exposed to the plan will therefore end by satisfying the experimenter's definitions. The extent to which this happens is directly observable. That is sufficient for the practical purposes of an experiment. The basic statement R should then not be a source of ambiguities. The same reasoning applies to the term stimulus.

The results of an experiment shows the extent to which the organism under study adapts to exactly those stimuli the experimenter has included in the plan. The experimenter cannot know directly whether the organism under study attends to exactly those events and properties of events as stated in the plan; but the experimenter's only solution is to stick to the definitions in the plan. If the experiment fails, the experimenter may adjust his or her theory of what the organism under study responds to or revise the hypothesis under study. To learn more about what the organism under study attends to, the experimenter may vary the experiment.

Relevant questions regarding the organism's private behavior include: Are the organism's internal states and events a part of the prevailing stimulus complex? Is a stimulus the same stimulus when presented multiple times? These questions pertain to a full account of the causal process that implementation of an experiment initiates. Answers to such questions are beyond the scope of the present codification, but might have to be addressed in a complete codification of behavior analysis.

$S^{A(+)}$ and $S^{A(-)}$ assume established correlations between the antecedent stimulus and S^{P+} or S^{P-} if R. These correlations may be established by preceding behavioral operations or gradually as a consequence of the experiment, if the experiment succeeds.

The experimenter might learn more from use of a narrow rather than a broad definition of the target response, but the proposed codification is neutral on that issue.

Valence

Valence is the main source of ambiguities. As explained by Mechner (2011, p. 97), in procedures for stimulus presentation, stimuli valence is based on conjectures. If some design for stimulus presentation increases the frequency of future instances of the target response, we say that the response class is reinforced. If, on the other hand, the response rate decreases, we say that the response class is punished. To make the difference between punishment and reinforcement comprehensible, we assume stimuli valence—that the consequential stimulus is perceived as attractive or aversive by the organism under study.

We should refrain from further speculation on the nature of those experiences. Postman (1947) recommends that valence be identified with pleasure and pain; but hedonism entails a dogmatic attitude to an empirical issue. As argued by Tonneau (2008), we may reject Postman's proposal and still avoid tautological explanations of the result of behavior analytic experiments. Admittedly, to say that a response increases in frequency because of reinforcement does not say much unless we specify the stimuli involved in the process; but the utterance denies that the increase is caused by classical conditioning, for instance. When we then specify the stimuli involved and their valence, a gain in explanatory power depends on the addition being non-circular.

To prevent that the added statements of stimuli valence become void of empirical content, it is sufficient that we learn from *other* sources than the designed experiment what positive and negative valence stimuli may have for the organism under study. Assumptions about stimulus valence should therefore accord with functional analysis (Hanley, Iwata, & McCord, 2003; Skinner, 1953) of *earlier* observation of public behavior; see Mechner's (2011, p. 96) remarks regarding the three-terms contingency including defined discriminative and reinforcing stimulus as empirical constructs based on prior contact with the independent variables. Assumptions of stimulus valence may also be based on recognized biological facts.

Assumptions about stimulus valence are still hypotheses in the present experiment and during the functional analysis. We cannot avoid this exception from the claim that everything about the basic statements should be directly observable; but confirmation by empirical studies increases the plausibility of each instance of the assumptions and confer to these assumptions empirical content, though indirectly.

The truth of the hypotheses depends on the observed reactions of the organism subjected to the experiment. As long as the planned operations succeed, the experimenter has most reasons to believe that his or her assumptions are true; but if the experiment fails to produce the expected results, the experimenter may react by revising the hypothesis used to predict the effect of the planned operation, by adjusting his or her beliefs about what the organism attends to, or by adjusting his or her beliefs about stimulus valence, taking all available and relevant evidence into consideration.

Valence comes in degrees, it may be strong or weak, and may thus be altered via motivational operations; stimuli perceived as attractive might change valence in direction of neutral or aversive valence due to satiation. Degrees of valence strength are, however, not symbolized in the suggested codification system. Table 3 allows for notation of neutral and unconditioned stimuli but does not imply that theories hypothesizing such entities are true.

Symbolizing Events

Note that indication of sequence is built into the basic assertions, which are all denoting *events*. The reason is that all connectives are defined by the same time-ignoring truth-table. Conditionals, for instance, express timeless conditionals. The translation from formal language to the natural language English is therefore 'if …

then …', not 'first … then …'. As already mentioned, existing notation systems use arrows and are often ambiguous on this point.

While $S^{A(+)}$ and $S^{A(-)}$ merely signalize that an appetitive or aversive stimulus will (or could) follow if a particular response is emitted, some behavioral operations presuppose the initial presence of a stimulus that by itself is appetitive or aversive. The same stimulus then disappears as a consequence of the response—thus S_1^{A+} and $\sim S_1^{P+}$, for instance, denote the appearance and disappearance of the *same* stimulus within a single schedule before and after the target response. Different symbols are used because they denote different events. Temporal logic handles this differently, allowing that the truth-values of elementary statements may change from one time to another, while in classical propositional logic the same truth-table is used indiscriminately for all events (Venema, 2001). Use of temporal logic would require that behavioral analysts become well versed in formal languages, however. We therefore stick to classical logic. Our problem is then that S_1 and $\sim S_1$ (the presence and disappearance of the same stimulus) cannot both be true within the same complex statement.

Time is therefore built into the basic assertions, resulting in four different symbols for the appearance of aversive and appetitive stimuli before and after the response; S^{A+}, S^{A-}, S^{P+}, and S^{P-} describing four different events. The order of presentation of antecedent stimuli is likewise indicated in the superscript, for the same reasons, S_1^{AS2} symbolizing that S_1 is presented before S_2. Interval information will allow for more precision and will be introduced later, when necessary.

Negation

Generally, $\sim S^{P+}$ is not the opposite of S^{P+}. The meaning of $\sim S^{P+}$ is anything but S^{P+}. If we admit the existence of neutral events, negation of some appetitive stimulus may mean an aversive or neutral event, or both. Moreover, when more than one posterior stimulus is relevant, $\sim S_1^{P+}$ includes all other posterior stimuli, neutral, aversive and appetitive alike. Hence $\sim S^{P+}$ is not logically equivalent to S^{P-}. Both types of valence are therefore symbolized.

Similarly, $\sim S_1^{A(+)}$ is normally not logically equivalent to the stimulus picked out to signal that the target response will be ineffective (S_2^{A}).

Describing Operations

Having introduced the language, we will continue by formalizing all basic behavioral operations. Each formula is numbered, and we subsequently refer to the numbers in the text.

We will start by introducing the main difference between the logic of classical conditioning and that of the circumstances for operant conditioning. We will then formalize the most basic behavioral operations for operant conditioning, before returning to procedures for classical conditioning.

The Difference between Classical and Operant Conditioning

Reflexes could be described by the biconditional R iff S^U, saying that whenever S^U, R will result, but never in the absence of S^U. This might be true in some cases; but for organisms capable of learning by classical conditioning, it is too strong. For these organisms, the target response may also come under the control of other, conditioned stimuli, and we do not want to preclude this possibility. We therefore suggest the conditional R if S^U. The stimulus *controls* the response whenever the stimulus appears. We might intuitively say that the stimulus *elicits* the response (Catania, 2007). To the extent that empirical research helps us avoid arbitrary use of conditionals, R if S^U and R if S^A may both replace the intuitive expression because we can read right out of both conditionals what happens, without further information. The conditionals then explain what *elicit* means.

In operant conditioning, the organism learns that S^{P+} if R. This conditional may come under control by an antecedent stimulus. We express this by the nested conditional (S^{P+} if R) if S^A. The logical difference between operant and classical conditioning is not explained by just adding a second, appetitive stimulus following the response as a consequence, saying that if S^A then R then S^{P+}. That would be ambiguous. The parenthesis is necessary because the truth-conditions of the nested conditionals (S^{P+} if R) if S^A and S^{P+} if (R if S^A) differ. The nested conditional S^{P+} if (R if S^A) would be wrong. We do not want to say that S^{P+} selects a conditioned response.

As long as S^A is true, the two nested conditionals give the same result (in terms of truth-values), but not when S^A is false. Then the conditional R if S^A will always be true, whether R is true or false. For this reason, when S^A is false, the nested conditional S^{P+} if (R if S^A) will be false whenever S^{P+} is false. The nested conditional S^{P+} if (R if S^A) is therefore false when all the basic statements are false. That is clearly unacceptable. We should accept S^{P+} being false when R and S^A are false as well. Moreover, outside the laboratory, a response may produce the appetitive stimulus under other circumstances than S^A, but that is also denied by S^{P+} if (R if S^A).

We may now examine the difference between classical and operant conditioning. The nested conditional (S^{P+} if R) if S^A is logically very different from the conditional R if S^U. In the proposed codification system, the antecedent stimulus does *not* control the *response*; it controls the parenthesis. This makes it clear that established expressions like ‘stimulus-control’ cannot be taken literally when we talk about operant conditioning. The nested conditional (S^{P+} if R) if S^A says that the *parenthesis* will be true when S^A is true, and the parenthesis says that the response will be favorable for the organism. Operant conditioning means that the organism tends to repeat similar responses more frequently in the future under the circumstances specified by (S^{P+} if R) if S^A. We cannot leave out the influence of the posterior stimulus, saying that the behavior has come under the control of a discriminative stimulus.

Intuitively, we may say that S^A *sets the occasion for* the *parenthesis* and that the organism *emits* the response because of the conditional S^{P+} if R (Catania, 2013). In an expression like S^D: R => S^{R+}, the arrow is undefined, and the symbol ‘:’ reads “sets the occasion for"; it simply repeats the intuitive description. Because the logic is concealed, it is necessary to tell the reader, by superscript, how the function of the two stimuli differ. Logical connectives and parentheses do the job, unambiguously. The nested conditional (S^{P+} if R) if S^{A} may replace the intuitive descriptions, thus explain them, and may do so precisely because S^{P+} and S^{A} are silent about the *function* of the two stimuli. Admittedly, we then assume that R is reinforced because of S^{P+} if R. That has proved correct numerous times before.

The entire process of continuous positive reinforcement is not symbolized by the chosen nested conditional. The nested conditional (S^{P+} if R) if S^{A} is seen as a cause of behavioral change, as S^{U} is seen as the cause of reflexive behavior in the conditional R if S^{U}. To describe the entire process would require a formal language with more resources than the proposed formalization—probability calculus, for instance.

Because we do not formalize the entire process in operant conditioning, we cannot contribute directly to the issue of what the selection unit is in operant conditioning. We have formalized the logical structure of the cause, however. That might be of some help because, within the limits set by anatomy and physiology, learning is a favorable adaption to the environment. The main clues to learning are therefore outside, not inside the body. Description of the public events that cause operant conditioning should then contain the clues.

As just stated, the effect of reinforcement cannot be R if S^{A}. The selected unity might be R, because of the conditional S^{P+} if R. Against this conclusion, we might argue that, although there are universal laws, we are not always in a position to use them. It is therefore unlikely that any instance of the conditional S^{P+} if R is universally true. Is reinforcement possible although the organism does not discriminate between situations where the response is effective and those in which it is ineffective? Could an organism be influenced by the effect of a response before it has detected the conditions under which the effect comes? The correct answer to questions like these is of course an empirical issue. What we may suggest is that *if* S^{A} is inevitable, S^{A} & R might be the selected unity. The nested conditional (S^{P+} if R) if S^{A} is logically equivalent to S^{P+} if (S^{A} & R).

The conjunction S^{A} & R is false unless S^{A} and R are both true. Therefore the conditional S^{P+} if (S^{A} & R) can only be false when S^{A} is true, R is true, and S^{P+} false. Let us compare this with our chosen nested conditional (S^{P+} if R) if S^{A}. The conditional S^{P+} if R is false only when R is true and S^{P+} false. The nested conditional can then only be false when S^{A} is true, R is true and S^{P+} false. These two complex expressions are therefore true under the same combination of truth-values for the basic statements.

In natural environments, S^{P+} may be produced by more than one type of response, and each response may be effective under different circumstances. There might be a large set of effective S^{A} & R conjunctions and it is not easy to determine when an organism has achieved mastery of the entire set. Outside the laboratory, it might therefore not be quite appropriate to conceive of the end product of operant conditioning on the model of the conjunction S^{A} & R.

We will now turn to designs for presenting the consequential stimulus in experiments. Conditionals are too weak to describe designs, at least for most experiments. In the conditional Q if P, Q may be true although P is false. The conditional S^{P+} if R and the nested conditional (S^{P+} if R) if S^{A} therefore allow that S^{P+} may be achieved in numerous ways, not just by the target response, and that the parenthesis may be controlled by other stimuli than S_1^{A}—that is, by S_2^{A}… S_n^{A}. This may be appropriate for description of the cause of positive reinforcement outside the laboratory, but is too liberal for description of experimental designs.

Positive Reinforcement; One Option

In an experiment, the experimenter wants to control the presentation of stimuli contingent on the organism's responses. There is no room for alternative ways of producing the posterior stimulus; it should appear exactly as designed. To express planned presentation of stimuli within an experiment, biconditionals are therefore better than conditionals. Use of arrows is then misleading, not just ambiguous.

We suggest the following expression for planned *continuous* positive reinforcement in designs with successive presentation of antecedent stimuli:

(1) S_3^{P+} iff (S_1^{A} & ~S_2^{A} & R).

Formula (1) has a rather simple structure. It is a biconditional between S_3^{P+} and a parenthesis. The parenthesis lists all the conditions for presenting the consequential stimulus, and states that they must *all* be true. Being a biconditional, (1) requires that the posterior stimulus should *never* appear unless the parenthesis is true and should *always* take place when it *is* true. An interior parenthesis is not required because the complex conjunctions (S_1^{A} & ~S_2^{A}) & R and S_1^{A} & (~S_2^{A} & R) are logically equivalent. By entering two antecedent stimuli, discrimination is symbolized and S_1's function as discriminative stimulus becomes apparent.

Most of the formulae that follow will be built on the same simple structure. All of them will be biconditionals between some consequential event and a parenthesis stating the conditions for that event. In most of them, the parenthesis will simply list the conditions, as in (1).

Since the connectives are truth-functional, the truth-conditions for (1) are a function of the truth-conditions for the basic statements. Since the basic statements do not contain information of the function of the stimuli, (1) explains what it is to arrange for positive reinforcement, within an experiment, by statements that, as far as possible, may be controlled by observation. The biconditonal (1) does not presuppose what should be explained.

All possible combinations of the basic statements are represented in Table 4. Experiments with successive presentation of antecedent stimuli will not make use of the four first lines in the truth-table. There will be a second rule for presentation of antecedent stimuli in these experiments: ~S_2^{A} iff S_1^{A}. If someone should want to allow simultaneous presentation of antecedent stimuli, (1) still applies, but prohibits presentation of the consequential stimulus in those cases, as shown by the first four lines in Table 4.

Table 4

Truth-table Testing Variants of Positive Reinforcement Operations

S_1^A	**S_2^A**	**R**	**S_3^{P+}**	**S_3^{P+} iff S_1^A & ~S_2^A & R)**	**[S_3^{P+} if (S_1^A & ~S_2^A & R)] & [~S_3^{P+} if (~S_1^A or S_2^A or ~R)]**
T	T	T	T	F (F)	(F) [T] F [F] (T)
T	T	T	F	T (F)	(F) [T] T [T] (T)
T	T	F	T	F (F)	(F) [T] F [F] (T)
T	T	F	F	T (F)	(F) [T] T [T] (T)
T	F	T	T	T (T)	(T) [T] T [T] (F)
T	F	T	F	F (T)	(T) [F] F [T] (F)
T	F	F	T	F (F)	(F) [T] F [F] (T)
T	F	F	F	T (F)	(F) [T] T [T] (T)
F	T	T	T	F (F)	(F) [T] F [F] (T)
F	T	T	F	T (F)	(F) [T] T [T] (T)
F	T	F	T	F (F)	(F) [T] F [F] (T)
F	T	F	F	T (F)	(F) [T] T [T] (T)
F	F	T	T	F (F)	(F) [T] F [F] (T)
F	F	T	F	T (F)	(F) [T] T [T] (T)
F	F	F	T	F (F)	(F) [T] F [F] (T)
F	F	F	F	T (F)	(F) [T] T [T] (T)

Note. The symbols S and R with all their sub-and superscripts are defined in Table 3. The connectives are introduced in Table 1 and defined in Table 2. T denotes true and F denotes false. These are the truth-values statements can take. The truth of the inner parentheses is determined first, then the outer parentheses, and lastly the entire complex statement.

The conditional S_3^{P+} if (S_1^A & ~S_2^A & R) is logically equivalent to the nested conditional (S_3^{P+} if R) if (S_1^A & ~S_2^A); but (1) is a stronger statement and cannot be transformed accordingly. The biconditional (1) is appropriate for the well-regulated circumstances of learning within an experiment. A successful experiment described by (1) has a unique end-product. In this situation, we may therefore expect that the conjunction (S_1^A & ~S_2^A & R) will be selected as the outcome of the experiment.

Adequate and Inadequate Reformulations of (1)

Since the target response is a stochastic variable, we expect full variation (R or ~R). The design should specify how the experimenter should administer the posterior stimulus regarding *all* possible combinations of the anterior stimuli both if R and if ~R. The conjunction (1)' does so in full detail, every possible combination of antecedent stimuli and R is used to specify how the experimenter should react:

(1)' [S_3^{P+} if (S_1^A & ~S_2^A & R)] & [~S_3^{P+} if (~S_1^A or S_2^A or ~R)]

The first bracket spells out under what conditions S_3^{P+} should be administered. The second bracket states that if any of the conditions listed in the parenthesis are true, the posterior stimulus should not appear. As shown by Table 4, the conjunction (1)' is logically equivalent to the biconditional (1).

The formula (1) is a biconditional between S_3^{P+} and the parenthesis. In order to complete Table 4, we must therefore first find the truth-value for the parenthesis and may then find the truth-values for the entire expression. We proceed similarly for the conjunction (1)', starting with the inner parentheses and working our way outwards.

By filling out a truth-table, we may inspect the distribution of truth-values of complex statements for all possible combinations of truth-values for its elementary statements. It is thus possible to check whether a complex statement says exactly what we want to say, and whether different complex statements are logically equivalent. This is possible because all the connectives we use are truth-functional.

In line 5–6 in Table 4, the antecedent stimuli are as required by the design and the response is observed. In line 5, the posterior stimulus is correctly presented; the plan has become true. In line 6, the experimenter fails to present the consequential stimulus, so the design is violated. In all other lines, at least one of the conditions is false. Under these circumstances, the design is violated when S_3^{P+} and satisfied when not. Such is a plan for continuous positive reinforcement where an organism learns to discriminate between two antecedent stimuli.

The conjunction (1)' gives us the same result. (1) and (1)' are thus logically equivalent—the biconditional (1)' iff (1) is true in all possible cases (logically valid). Using logic, we may thus reformulate complex statements and know whether we preserve their truth-conditions.

We prefer (1), the simpler formula. We could also simplify by turning the second bracket in (1)' into a biconditional; but $\sim S_3^{P+}$ iff $(\sim S_1^A$ or S_2^A or $\sim R)$ is impractical because of $\sim R$.

Dropping $\sim S_2^A$ from the first bracket in (1)' and $\sim S_1^A$ from the second will be too weak. Setting up additional truth-tables will show that this simplified conjunction $[S_3^{P+}$ if $(S_1^A$ & $R)]$ & $[\sim S^{P+}$ if $(S_2^A$ or $\sim R)]$ accepts presentation of the consequential stimulus also when S_1^A and S_2^A are both true (as in Table 4, line 1 and 2). The organism under study may then simply ignore S_2^A. That could be part of a design. It is unacceptable, however, that the simplified conjunction also allows reinforcement of the response although S_1^A is false, provided that S_2^A is also false. That is too weak even for an experiment designed to reinforce attending to S_1^A, ignoring S_2^A. Someone might want a design where the target response is reinforced when S_1^A, whether S_2^A is true or not. We may describe such a design by simply skipping $\sim S_2^A$ from (1), since S_2^A should be ignored.

The ability to test expressions by the simple means of a truth-table shows why the proposed formalization does better than simple shorthand.

Positive Reinforcement; Two Options

In some experiments, there are two levers or buttons. R_1 denotes that push on one of them is observed, R_2 that push on the other is observed. S_1^A signals that R_1 will be effective. In one possible design, S_2^A signals that neither response will be effective. Simply ignoring R_2 and S_2^A would then pay off. As already explained, we may describe this variant by the biconditional S_3^{P+} iff $(S_1^A \& R_1)$.

In more interesting designs, S_1^A signals that R_1 will be effective while S_2^A signals that R_2 will be effective. If the antecedent stimuli were presented simultaneously, both actions would be signaled as effective. When the consequences of the two target responses are the same, the experimenter should therefore reject joint presentation of both antecedent stimuli as an occasion for reinforcement. We are then back to an elaboration of (1).

Reinforcement should then take place on two conditions, one including R_1, the other containing R_2, as follows:

(2) S_3^{P+} iff $[(S_1^A \& {\sim}S_2^A \& R_1 \& {\sim}R_2)$ or $({\sim}S_1^A \& S_2^A \& {\sim}R_1 \& R_2)]$

The biconditional (2) requires presentation of the consequential stimulus when the conditions in one of the inner parentheses are true and prohibits such presentation when neither of them are true.

In more complex designs, the consequences of the two responses may differ. We then need two rules. The bracket in (2) will be split; each inner parenthesis will form a separate rule, one for each type of consequence. In those cases, simultaneous presentation of antecedent stimuli may be admitted (by removing ${\sim}S_2^A$ from the first rule and ${\sim}S_1^A$ from the second). We may also accept simultaneous presentation of discriminative stimuli in experiments on conditional discrimination.

Other Basic Behavioral Operations

We may now present procedures for all the basic forms of continuous operant conditioning operations. The formulae (3)–(10) are biconditionals, constructed on the model of (1). They are presented and numbered in Table 5 together with (1) and (2). Because only one of the stimuli in (10) has valence, there is no need to specify, by superscript, that $M^{\uparrow}$ should increase the valence of S_3.

Table 5

Basic Operations: Continuous positive and negative reinforcement, punishment and extinction

	Verbal description	**Notation**
1	Positive reinforcement, one response	S_3^{P+} iff $(S_1^{A} \ \& \sim S_2^{A} \ \& \ R)$
2	Positive reinforcement, two responses	S_3^{P+} iff $[(S_1^{A} \ \& \sim S_2^{A} \ \& \ R_1 \ \& \sim R_2)$ or $(\sim S_1^{A} \ \& \ S_2^{A} \ \& \sim R_1 \ \& \ R_2)]$
3	Negative reinforcement (avoidance)	$\sim S_3^{P-}$ iff $(S_1^{A(-)} \ \& \sim S_2^{A} \ \& \ R)$
4	Negative reinforcement (escape)	$\sim S_1^{P-}$ iff $(S_1^{A-} \ \& \ R)$
5	Positive punishment	S_3^{P-} iff $(S_1^{A} \ \& \sim S_2^{A} \ \& \ R)$
6	Negative punishment	$\sim S_1^{P+}$ iff $(S_1^{A+} \ \& \ R)$
7	Extinction of positively reinforced behavior	$\sim S_3^{P+}$ iff $(S_1^{A(+)} \ \& \sim S_2^{A} \ \& \ R)$
8	Extinction of avoidance behavior	S_3^{P-} iff $(S_1^{A(-)} \ \& \sim S_2^{A} \ \& \ R)$
9	Extinction of escape behavior	S_1^{P-} iff $(S_1^{A-} \ \& \ R)$
10	Establishing motivational operations for positive reinforcement	S_3^{P+} iff $(M^{\uparrow} \ \& \ S_1^{A} \ \& \sim S_2^{A} \ \& \ R)$

Note. The symbols S, R, and M with all their sub- and superscripts are defined in Table 3. The connectives are introduced in Table 1 and defined in Table 2.

The structure of all designs (1)–(10) is fairly simple and the main structure remains the same: Like (1), the formulae (3)–(9) say that when the parenthesis is true, the corresponding specific form of appearance or disappearance of the posterior stimulus should result, otherwise not. In (2), the same applies to S_3^{P} and the bracket. The bracket contains two conditions, one for each target response.

Assume that positive reinforcement (1) has been repeated sufficiently many times for S_1^{A} to signal an occasion for positive reinforcement. Then extinction of positive reinforcement (7) is true when (1) is false. The experimenter withholds the consequential stimulus in (7) on the occasions where he would present them in (1). Conversely, (1) is true and (7) false when, under the same conditions, the experimenter makes the appetitive stimulus appear. Similarly, avoidance (3) is false when extinction of avoidance (8) is true, and vice versa. The same pattern holds for escape (4) and extinction of escape (9). Hence, designs for reinforcement and designs for extinction have exactly opposite truth-values. This is hardly surprising; it is still one of the merits of the proposed codification system that we can demonstrate it.

In escape (4) and in negative punishment (6), the appearance of a, respectively, aversive or appetitive antecedent stimulus is an event terminated by the disappearance of the same stimulus in the consequent. In extinction of escape (9), the last event is that the aversive stimulus persists despite the response. Hence, the connective negation is quite sufficient for formulation of escape and avoidance behavior. A particular sign for blocking (Mechner, 1959, 2008, 2011) is unnecessary. In (1) S_3^{P+} is blocked whenever S_2^{A}.

The consequence of emitting the target response is negative in avoidance (3), escape (4), negative punishment (6), and extinction of positive reinforcement (7). In (4) and (6), the environmental change is represented, thus obvious. It is less obvious in (3) and (7), how a negative fact can have an effect on an organism's future behavior. Formula (7) presupposes that the organism has previously been exposed to (1). The statement $S_1^{(A)}$ is replaced by $S_1^{A(+)}$, but the target behavior and antecedent stimuli are the same. The event that follows the response is different, however; it is this change that explains the effect of shifting from (1) to (7). In the same way, avoidance (3) presupposes positive punishment (5); but in (3), the organism learns to emit a response different from the one in (5). By a shift in response, the organism learns to avoid (5).

In extinction of avoidance (8), nearly the same pattern is established as in (5), but the target response in (8) is the same as in (3), not the one in (5). The main difference between (5) and (8) does not appear unless the target responses are specified. To recapitulate, in (5) the organism learns that the target response will be punished when S_1^{A} is true and S_2^{A} false. In (3), because of earlier exposure to (5), S_1^{A} has turned into $S_1^{A(-)}$, and the organism now learns to avoid punishment by responding *differently* under these conditions. In (8), the response learned through (3) is no longer effective.

By comparing the formulae (1) and (3)–(9), we can see straight away how extinction resembles punishment. The resemblance is rather close between positive punishment (5) and extinction of negatively reinforced behavior, (8) and (9). The same holds for extinction of positively reinforced behavior (7) and negative punishment (6). Being exposed to (6) is more severe than being subjected to (7), however. The end situation is the same; but in (6), the difference in stimulus valence is larger between the organism's situation before and after the target response. This difference is also larger in (5) and (8) than in (9); but the end situations in (5), (8), and (9) are all worse than in (6) and (7).

The Difference between Positive and Negative Reinforcement and Punishment

Our suggested formulae show that if, in some situation, the biconditional S_3^{P+} iff $\sim S_3^{P-}$ is true, the biconditional (3) iff (1) would also be true. In such situations, the logical difference between positive reinforcement and avoidance disappears. It is perhaps less obvious that when the biconditional S_3^{P+} iff $\sim S_1^{P-}$ is true, the logical difference between (1) and escape (4) would vanish completely. In (4) the antecedent stimulus is aversive, not so in (1). It would still be the case that when the biconditional S_3^{P+} iff $\sim S_1^{P-}$ is true, (4) could be reformulated as the biconditional S_3^{P+} iff (S_1^{A-} & R), and that is certainly a positive reinforcement schedule. Similarly, if in some situation the biconditional S_3^{P-} iff $\sim S_1^{P+}$ is true, we would no longer be able to distinguish between positive and negative punishment. Moreover, if we ignore the subscripts, S_3^{P+} iff $\sim S_1^{P-}$ is logically equivalent to the biconditional S_3^{P-} iff $\sim S_1^{P+}$. The proposed codification system thus allows us to describe exactly what is involved in the issue of differentiating between positive and negative reinforcement, opened by Michael (1975) and reopened by Baron and Galizio (2005).

Logic cannot decide an empirical issue, but may guard against the fallacy of holding a priori that negation produces the opposite of what is negated. Let us imagine that we were color-blind and could only discern white from black. *Not white* would then mean gray or black; but as things are, *not white* means every other color than white. This point is important in experiments where several posterior stimuli are combined. We cannot simply ignore the subscripts. When, however, only one posterior stimulus is involved and its valence-conferring properties vary between values making the stimulus appetitive to values making it aversive, then regarding dichotomies, it will be true that S^{P-} and $\sim S^{P+}$ are equivalent, as are also S^{P+} and $\sim S^{P-}$. That is not always true. Like Sidman (2005) and Iwata (2005), we therefore want to retain the distinctions between positive and negative reinforcement and between positive and negative punishment.

The offered codification system is based on statements about the public events of presentation or removal of physical stimuli. Our reason for formalizing what the experimenter does to an organism's environment is that organisms learn by adapting to physical events. Behavior analysts should analyze how they do so. In an experiment where motivational operations have established the aversive bodily state hunger, we therefore insist on thinking about the experiment as presentation of food rather than removal of hunger. To improve their situation, pigeons have to find food. They are not much helped by focusing on hunger-avoidance.

Moreover, when *plans* for stimulus presentations are formed, stimulus valance is assumed. Since valence is assumed, it is difficult to capture, and cannot be confirmed until the results of the experiment appear. An event's valence to the organism is important; we should still postpone conclusions on that issue until interpretation of the results.

Catania (2013) suggests that when the frequency of an organism's response increases while the response *produces* the stimulus, we call the schedule positive reinforcement. The schedule is defined as negative reinforcement when the frequency of the response increases while the organism responds *after* having been exposed to the stimulus and the response removes that stimulus or prevents the appearance of a stimulus correlated with it. Catania's view postpones the difficult issue of stimulus valence until the results are known. He describes behavioral operations by combining statements of public events, as we do in Table 5.

Such are our arguments for formal codification of public events. Our codification system does not provide these arguments; it is built on them. The rest of our argument for rejecting Michael's position relies on systematic use of truth-functional connectives, in particular the formal properties of negation.

To illustrate Michael's point, let us *suppose* that hunger could be described as S^{A-}. For the sake of the argument, we then accept that the result of motivational operations may qualify as an antecedent stimulus. We can then skip (10) by adding S^{A-} to (1). We will then have:

(1)'' S_4^{P+} iff (S_1^{A-} & S_2^{A} & $\sim S_3^{A}$ & R)

In terms of valence, Michael holds the rather strong statement that S_4^{P+} iff $\sim S_1^{P-}$, due to the empirical fact that food neutralize hunger. If we accept his view, we may substitute the one for the other and achieve:

(1)''' $\sim S_1^{P-}$ iff (S_1^{A-} & S_2^{A} & $\sim S_3^{A}$ & R)

(1)''' is sufficiently alike (4) to save Michael's argument. Positive reinforcement can no longer be distinguished from escape-behavior. Accordingly, Michael may achieve his point by focusing directly on valence as the crucial matter rather than on how an experiment changes an organism's physical environment.

We find it unnatural, however, to see hunger as a stimulus. The physical event is that the experimenter presents food. We therefore hold that the pigeons are exposed to formula (1) and that food is attractive to pigeons when they are hungry. They learn how to find food.

Michael's point may also concern physical events, however. When an organism stays in a cold chamber and may turn on some heating devise, modification of the physical stimulus may be described on a temperature scale. To avoid low temperatures is to increase them; on the temperature scale $+2 = -(-2)$. We insist nevertheless that the organism under study has to find the material answer, to turn on the heating devise.

There are cases, however, where we should accept that the biconditional (3) iff (1) is true. Can anyone ever tell whether a student reads to achieve a good grade or to avoid a low one? Grades are values on a continuous variable; but in the proposed codification system, we have to categorize it as good, bad or neutral, which makes $\sim S^{P+}$ more than S^{P-}. In the future, however, someone might come up with a formal language for formalizing operations allowing for degrees of valence. In this improved language, good grades may always be rewritten as not bad grades. Students want to achieve high and avoid low grades. If we still try to investigate whether the results of reading to achieve S^{P+} differ from those of reading to avoid S^{P-}, the only result we might find is that students are encouraged by climbing upwards on the scale and discouraged when their results fall. The difference between reinforcement and punishment persists, but it might be difficult to tell whether students read to climb upwards or to avoid falling down.

In a comment on Baron and Galizio (2005), Michael (2006) repeats his focus on valence rather than public events. He suggests that in experiments combining aversive and appetitive stimuli, we may characterize the situations before and after the response by measuring and comparing the organism's net value balance in each case. This suggests that stimulus valence may be measured along one single continuous utility scale across all stimuli. Every possible case will then be like the student example. For this utility approach to be a possible and reasonable reduction procedure, we must succeed in measuring all stimuli valence reliably along one single utility scale. A unique scale for each organism is sufficient (Resnik, 1987).

Establishing such a scale, however, requires public data. As with all assumptions about valence, utility measurement should be established independently of the experiment.

Until someone comes up with procedures for reliable utility measurement of deprivation and stimulus valence on a single scale, the difference between positive and negative reinforcement remains, as does also the difference between positive and negative punishment. Meanwhile, we insist on symbolizing whether the experimenter presents or removes this or that physical stimulus—although it might sometimes be difficult to know what the experimenter then does to the organism under study regarding stimulus valence.

Motivational Operations

We have already rejected that deprivation may be symbolized as presentation of an aversive stimulus. The term stimulus is reserved for physical objects and publicly available properties of physical objects. Deprivation increases the value of an appetitive stimulus. However, and depending on earlier learning history, an organism may become increasingly sensitive to an antecedent stimulus. Since deprivation affects the organism's behavior, it should be symbolized and represented in formalized designs. Its place in the proposed symbolic system is as a motivational operation, symbolized as M↑. A procedure for continuous positive reinforcement with motivational operations and successive presentation of antecedent stimuli is expressed by (10) in Table 5. The formula (10) completes the description of the basic continuous operant schedules. We are aware of the problem with deprivation and satiation as fairly rude labels, but for our purpose the labels are anticipated as sufficient. We believe that the individual’s history and biological makeup influence the effects operations will have, but the influence is often unobservable and hence out of our control. We codify controllable independent variables. More elaborated versions of the language might specify motivational operations in greater detail.

Learning may also be affected by reducing the value of some appetitive posterior stimulus, and behavior may change when the value of an aversive antecedent stimulus is reduced or increased. Thus M↓ is necessary as well. The term motivational operations are thus reserved for public acts performed by the experimenter assumed to change the organism's physical structure in ways we may classify as deprivation or satiation. These assumptions should be based on earlier empirical evidence, as with all assumptions about stimulus valence.

If we want to test how efficient motivational operations are, we may compare the results of (1) and (10).

Having expressed the basic schedules for continuous operant conditioning, let us try express designs for classical conditioning.

Classical Conditioning

In one type of design for classical conditioning, the stimulus picked out to become a conditioned stimulus is presented before the unconditioned stimulus. The unconditioned response should then always follow. The two forms of stimuli may also be presented simultaneously, or the sequence may be reversed. The differences in time structure are included in the basic statements. Otherwise, the logical structure of the procedures is similar. The three designs are presented in Table 6.

Table 6

Basic Operations: Classical conditioning

	Verbal description	**Notation**
11	Classical conditioning, anterior presentation of conditioned stimulus	S_2^{AS1} iff [(R if S_1^U) & S_1^U]
12	Classical conditioning, concurrent presentation of conditioned stimulus	S_2^{CoS1} iff [(R if S_1^U) & S_1^U]
13	Classical conditioning, posterior presentation of conditioned stimulus	S_2^{PS1} iff [(R if S_1^U) & S_1^U]

Note. The symbols S and R with all their sub-and superscripts are defined in Table 3. The connectives are introduced in Table 1 and defined in Table 2.

In Table 6, the bracket describes the two conditions we should require for administering S_2. The first condition states that *when* S_1^U is the case, the R is also true. The second condition states *that* S_1^U is the case. If S_2 is administered only when these conditions are true, it is reasonable to hope for the desired effect in due time, that R if S_2 even in the absence of S_1^U.

In (11)–(13), the first condition is *assumed*. The reason is that in all three cases, the unconditioned stimulus is presented *before* the response is observed—i.e. before we know whether the stimulus will be effective. The truth of the conditional R if S_1^U should therefore be established first. The experimenter should also know under which conditions it is likely that the assumption R if S_1^U will be true—that the organism under study is healthy, fit, and awake, for instance. If there are reasons to doubt that R if S_1^U, the plan says that S_2 should not be presented.

Appearance of the response is symbolized by R indiscriminately, whether it is a conditioned or an unconditioned response, for the simple reason that the response remains the same. Whether it is conditioned or unconditioned depends on the functional relation to the preceding stimulus. That should be shown by the logical structure of the complex statements.

The biconditionals (11)–(13) are rather strong statements. They do not accept that S_2 is true when S_1^U is false. To test the efficiency of the designs, we must therefore reverse the second condition, as shown by (11)'. Then S_1^U in the first requirement will be false, however; the inner parenthesis will therefore always be true; so there is no longer any point in the first requirement. We simply present S_2 and test whether it has become effective in eliciting R, even when S_1^U is false:

(11)' R if (S_2^{AS1} & $\sim S_1^U$)

The conditionals (12)' and (13)' are formed correspondingly.

The biconditionals (11)–(13) are plans for how to cause (11)'–(13)', so do not contain any observation of the target behavior. They are designed to cause the target response and cannot contain their effect. In this they are alike the biconditionals (1)–(10). The biconditionals (1)–(10) contain the target response, but may do so because they are not designed to cause it; they are designed to cause a future change in frequencies of the response class.

The conditionals (11)'–(13)' describe both cause and effect. They are false when S_2 is true, S_1^U false and R false. In Table 7, this is represented by line 6. In line 5, the parenthesis (S_2 & $\sim S_1^U$) is true and so is R. The lines 5 and 6 are the test conditions for (11)'–(13)'. In all the other lines, the parenthesis is false. Since (11)'–(13)' are conditionals, they are true in all these cases whether R is true or false. They are formed as conditionals because we cannot require that R fail to appear when S_2 is false and S_1^U true. There might even be other conditioned stimuli that could elicit R.

Table 7

Truth-table Testing Designs for Classical Conditioning

S_1^U	**S_2**	**R**	**S_2 iff [(R if S_1^U) & S_1^U]**	**R if (S_2 & $\sim S_1^U$)**
T	T	T	(T) [T] T	(F) T
T	T	F	(F) [F] F	(F) T
T	F	T	(T) [T] F	(F) T
T	F	F	(F) [F] T	(F) T
F	T	T	(T) [F] F	(T) T
F	T	F	(T) [F] F	(T) F
F	F	T	(T) [F] T	(F) T
F	F	F	(T) [F] T	(F) T

Note. The symbols S and R with all their sub-and superscripts are defined in Table 3. The connectives are introduced in Table 1 and defined in Table 2. T denotes *true* and F denotes *false*. These are the truth-values statements can take. The truth of the inner parentheses is determined first, then the outer parentheses, and lastly the entire complex statement.

The truth-conditions for (11)–(13) are also listed in Table 7. In lines 1–4, the second condition is true. In lines 1 and 3 the first condition is also true. In line 1, the conditioned stimulus is administered so the plan is satisfied; not so in line 3, where the experimenter fails to present it. In lines 2 and 4, the first condition is violated. If the conditioned stimulus is administered nevertheless the design is violated. In lines 5–8, the second condition is false. If the conditioned stimulus is administered nevertheless, as in lines 5 and 6, the design is violated.

The biconditionals (11)–(13) are plans for how a neutral antecedent stimulus can be made to control the target response directly, as shown by the tests (11)'–(13)'. In contrast, the main issue in the biconditionals (1)–(10) is stimulus change as a *consequence* of the target response. In (1)–(10), the antecedent stimuli do not control the response as in (11)–(13). They either specify additional conditions for the change caused by the response, as in (1), or participate in defining the effect, as in (4). All the biconditionals (1)–(7) are plans for how some response will be reinforced or punished by its effects on the environment.

We have now described basic operations. The proposed formal language is, however, not limited to classical conditioning and the basic forms of continuous operant conditioning. We may expand the analytical unit.

Conditional Discrimination

In 1986, Sidman made suggestions about the need for expanding the analytical unit by introducing a formal system for describing four- and five-term contingencies. We may write conditional discrimination by simply adding conditional stimuli to (1) such that each conditional stimulus "determines the control which other stimuli exert over responses" (Sidman, 1986, p. 225). The simplest version is presented in Table 8 as (14). If we want what Sidman calls a balanced experiment, the formulae might be like (15) or (16) in Table 8. By adding even more antecedent stimuli, we may write second order conditional discrimination as in (17). The formula (16) is built on (2).

Table 8

Basic Operations: Conditional Discrimination

	Verbal description	Notation
14	Simple conditional discrimination	$S_5^{P+} \text{ iff } (S_1^{AS3} \mathbin{\&} {\sim}S_2^{A} \mathbin{\&} S_3^{A} \mathbin{\&} {\sim}S_4^{A} \mathbin{\&} R)$
15	Balanced conditional discrimination	$S_5^{P+} \text{ iff } [(S_1^{AS3} \mathbin{\&} {\sim}S_2^{A} \mathbin{\&} S_3^{A} \mathbin{\&} {\sim}S_4^{A} \mathbin{\&} R)$ $\text{or } ({\sim}S_1^{A} \mathbin{\&} S_2^{AS4} \mathbin{\&} {\sim}S_3^{A} \mathbin{\&} S_4^{A} \mathbin{\&} R)]$
16	Balanced conditional discrimination, simultaneous presentation of discriminative stimuli	$S_5^{P+} \text{ iff } [(S_1^{AS3} \mathbin{\&} {\sim}S_2^{A} \mathbin{\&} S_3^{A} \mathbin{\&} R_1 \mathbin{\&} {\sim}R_2)$ $\text{or } ({\sim}S_1^{A} \mathbin{\&} S_2^{AS4} \mathbin{\&} S_4^{A} \mathbin{\&} {\sim}R_1 \mathbin{\&} R_2)]$
17	Balanced second order conditional discrimination	$S_7^{P+} \text{ iff } [(S_1^{AS3} \mathbin{\&} {\sim}S_2^{A} \mathbin{\&} S_3^{AS5} \mathbin{\&} {\sim}S_4^{A} \mathbin{\&} S_5^{A} \mathbin{\&} {\sim}S_6^{A} \mathbin{\&} R)$ $\text{or } (S_1^{AS4} \mathbin{\&} {\sim}S_2^{A} \mathbin{\&} {\sim}S_3^{A} \mathbin{\&} S_4^{AS6} \mathbin{\&} {\sim}S_5^{A} \mathbin{\&} S_6^{A} \mathbin{\&} R)$ $\text{or } ({\sim}S_1^{A} \mathbin{\&} S_2^{AS3} \mathbin{\&} S_3^{AS6} \mathbin{\&} {\sim}S_4^{A} \mathbin{\&} {\sim}S_5^{A} \mathbin{\&} S_6^{A} \mathbin{\&} R)$ $\text{or } ({\sim}S_1^{A} \mathbin{\&} S_2^{AS4} \mathbin{\&} {\sim}S_3^{A} \mathbin{\&} S_4^{AS5} \mathbin{\&} S_5^{A} \mathbin{\&} {\sim}S_6^{A} \mathbin{\&} R)]$

Note. The symbols S and R with all their sub-and superscripts are defined in Table 3. The connectives are introduced in Table 1 and defined in Table 2.

In all the formulae (14)–(17), the individuality of each antecedent stimulus is denoted by subscript, the superscript indicates sequence, and the function of each stimulus is determined by the construction of the complex statement. Regarding prohibited antecedent stimuli, sequence notation may be simplified; it is sufficient to note that they should not appear before the target response.

In (14) there are only one out of sixteen possible combinations of antecedent stimuli that signals S_5^{P+} if R. That is, why we may use the simple structure of (1) and just add the new elements. Balanced experiments require more symbols to make sufficient description.

In (15) and (16), S_1 and S_2 function as selectors of the discriminative function of the stimuli S_3 and S_4. While (15) requires successive presentation of all antecedent stimuli, (16) describes an experiment with two target responses, like (2). The formula (16) differs from (2) in allowing for presentation of the consequential stimulus also when the discriminative stimuli appear simultaneously; but (16) still requires successive presentation of conditional stimuli. That might be sufficient as a clue for choosing the effective response.

The two inner parentheses in (15) describe under which conditions it will be correct to present the posterior stimulus. Because each of the parentheses in (16) mention only one of the two discriminative stimuli, (16) accepts four situations in which the consequential stimulus should be presented.

In (17) S_1 and S_2 are second order conditional stimuli controlling variations in the relations between S_3 and S_4 on the one hand and S_5 and S_6 on the order. The four inner parentheses describe exactly the conditions under which the consequential stimulus should be presented.

Intermittent Reinforcement

By adding symbols for the lengths of intervals, for response rates, and three new symbols in superscript, we enlarge the system allowing for eight new basic statements, listed in Table 9. We may thus write schedules for intermittent reinforcement, differential reinforcement and delayed reinforcement. Sequence intervals are specified.

Table 9

Language Elements 3: Additional Elementary Statements for Intermittent reinforcement

Symbol	**The symbolized elementary statement**
#T	A specified number (#) of time units (T) have passed.
$\underline{\#}T^{L\#}$	A variable number of time units (T) have passed, with the mean number of units ($\underline{\#}$) and the upper limit for the variation (superscript L#) specified.
$\#T^{P}$	A specified number (#) of time units (T) have passed, posterior to R.
S^{AT}	A stimulus is presented when T starts (immediately anterior to T).
S^{PT}	A stimulus is presented when #T (immediately posterior to #T).

anonym 1/27/15 18:50

Comment: The table needs to be moved up or down, perhaps the authors have a suggestion on how to solve this?

#R	The target response is observed a specified number (#) of times.
R^{PT}	The target response is observed after #T (immediately posterior to T).
$\underline{\#}R^{L\#}$	The target response is observed a variable number of times with the mean number ($\underline{\#}$) and the upper limit for the variation (superscript L#) specified.

Note. S means that a stimulus is presented, R that an instance of the target response is observed. Numbers in subscript may be used to identify different terms. Superscript 'A' means *anterior* to, superscript 'P' means *posterior* to. The symbol # indicates intervals when preceding “T” and rates when preceding “R”. It is a constant to be specified by a specific number for each experiment. Time units for the measurement of intervals should be specified by subscript (sec = seconds or min = minutes, for instance). The upper limit for variation for intervals and rates is symbolized by superscript “L#” (variation *limit*). The value of “L#” is specified by a number substituted for #. The symbol “$\underline{\#}$” is used to note the mean for variable rates or intervals.

The symbol # indicates intervals when preceding “T” and rates when preceding “R”. Variable rates or intervals are characterized by the mean ($\underline{\#}$) and the upper limit of variation (L#). Underlining is used to indicate the variation mean because it is easier to write on a computer than the conventional symbol, combining # with a macron (and M is already used to denote that motivational operations are established).

As shown in Table 10, the language can now express the logic of all basic behavior operations for intermittent reinforcement, differential reinforcement, and delayed reinforcement. The formulae are listed as (18)–(30).

Table 10

Basic Schedules of Intermittent Reinforcement and Differential Reinforcement

	Verbal description	**Notation**
18	Fixed interval	S_3^{P+} iff (S_1^{AT} & ~S_2^{A} & #T & R^{PT})
19	Fixed time	S_1^{PT+} iff #T
20	Variable time	S_1^{PT+} iff $\underline{\#}T^{L\#}$
21	Variable interval	S_3^{P+} iff (S_1^{AT} & ~S_2^{A} & $\underline{\#}T^{L}$ & R^{PT})
22	Limited hold	S^{P+} iff (... & ~#T_2)
23	Fixed ratio	S_3^{P+} iff (S_1^{A} & ~S_2^{A} & #R)
24	Variable ratio	S_3^{P+} iff (S_1^{A} & ~S_2^{A} & $\underline{\#}R^{L\#}$)
25	Differential reinforcement of other behavior	S_3^{PT+} iff [($S_1^{AT(+)}$ & ~S_2^{AT}) & (~R or #T)]
26	Differential reinforcement of alternative behavior	S_3^{P+} iff ($S_1^{A(+)}$ & ~S_2^{A} & R_1 & ~R_2)

27	Differential reinforcement of low rate	S_3^{PT+} iff ($S_1^{AT(+)}$ & ~S_2^{A} & #T & ~#R)
28	Differential reinforcement of high rate	S_3^{PT+} iff (S_1^{AT} & ~S_2^{A} & #T & #R)
29	Differential reinforcement of paced responding	S_3^{PT+} iff ($S_1^{AT(+)}$ & ~S_2^{A} & #T & $\#_1$R & ~$\#_2$R)
30	Delayed reinforcement	S_3^{PT+} iff (S_1^{A} & ~S_2^{A} & R & #T^{P})

Note. The symbols S, R, T and #, with all their sub-and superscripts are defined in the Tables 3 and 9. The connectives are introduced in Table 1 and defined in Table 2.

In fixed interval (18) and variable interval (21), the codification R^{PT} cannot be avoided by writing the requirements for reinforcement in (18) as the conjunction (S_1^{AT} & ~S_2^{A}) & (R iff #T), for instance. Again, the truth-tables are timeless; the biconditional does not mean "before and only before". What is more, the biconditional R iff #T says that ~R is an acceptable occasion for reinforcement provided that ~#T. This is clearly unacceptable. In (18), S_1^{AT} is necessary to situate S_1 at the start of #T. S_2, however, should not be true, at any moment before R, thus ~S_2^{A} (not before the response).

In fixed ratio (23), reinforcement requires that a fixed number of responses are observed. In variable ratio (24), the required number of responses varies around a mean number ($\underline{\#}$) with an upper limit specified (superscript L and a number). In variable time (20) and variable interval (21), the variation of time units that should pass before a reinforcer is presented is similarly determined by a mean number ($\underline{\#}$) and the upper limit of the variation (superscript L and a number). The formula (20) contains no response-specifications. The same holds for fixed time (19), where the requirement for reinforcement simply is that the predefined interval has passed.

Limited hold (22) means that a basic assertion about another time-dependent condition (~#T_2) is added to some formula already containing a time-dependent condition (indicated by three dots). The different intervals are identified by subscripts. If (22) is added to fixed interval (18), we will arrive at the biconditional S_3^{P+} iff (S_1^{AT} & ~S_2^{A} & #T_1 & R^{PT1} & ~#T_2). This means that although the response should come after some specified number of time units; the experimenter is not required to wait forever.

In delayed reinforcement (30), the interval is situated after the response. In the other schedules including an interval, it comes before the response. In these other schedules, when the interval has passed and the other conditions are satisfied, the consequential stimulus should follow, without another delay.

Differential Reinforcement

In differential reinforcement of other behavior (DRO) (25), there are two complex conditions for allowing reinforcement; one concerning discriminatory stimuli ($S_1^{AT(+)}$ & ~S_2^{AT}), the other regarding types of responses within a specified interval. Within the interval, the response-requirement is negative, thus rather unspecified. ~R denotes other behavior, which is all behavior except the target response. We are specifically asked not to reinforce the target response for a specified period; to reinforce any other response is permitted; but to *require* reinforcement of all other responses would be impracticable. After the interval, the target response is accepted as an occasion for reinforcement. Unlike fixed time (19), (25) thus contains *some* response-requirement; but, compared to fixed interval (18), it is rather unspecified.

The second condition could be described as the complex disjunction (~R & ~#T) or #T, but this is logically equivalent to the simpler disjunction ~R or #T, and we prefer the simplest formula. We could avoid the negative response-requirement by using the conditional #T if R; but since the conditional is true when ~R or when #T, this amounts to the same. It just looks like a positive response-requirement.

In differential reinforcement of paced responding (DRP) (29), there are two response rates, identified by subscript. Within the period #T, the response rate should not exceed $\#_2$R but be equal or higher than $\#_1$R. The subscripts are attached to the rate symbol because the response remains the same. The formula (29) says that as soon as the specified period has passed, reinforcement is administered if the response rate $\#_1$ is achieved, but not if the response rate $\#_2$ is achieved. Thus (29) presupposes that $\#_2$ is higher than $\#_1$. If not, (29) is a contradiction.

Except for differential reinforcement of high rate (DRH) (28), all the other schedules for differential reinforcement contain a negative or partly negative response-requirement. The formula for differential reinforcement of alternative behavior (DRA) (26) contains a positive response-requirement as well. In differential reinforcement of low rate (DRL) (27) and of paced responding (DRP) (29), the response rate should not exceed a predefined level. It is possible to practice the negative response-requirements in (25), (27) and (29) because the requirement is limited in time.

We have listed these formulae as reinforcement schedules, because they are known under this name, but DRO (25) in particular is better characterized as an extinction schedule. We can show this by reformulating (25) such that the response-requirement becomes positive.

To find this reformulation, we may look at (1)'. Formula (1) is a reformulation of the first bracket in (1)'—turning the conditional into a biconditional. As already remarked, we might have done the same with the second bracket. Choosing the last option when reformulating (25), we will have the biconditional $\sim S_3^{PT+}$ iff [$\sim(S_1^{AT(+)}$ & $\sim S_2^{AT})$ or ~(~R or #T)]. The negation ~(~R or #T) is logically equivalent to the conjunction R & ~#T, and the negation $\sim(S_1^{AT(+)}$ & $\sim S_2^{AT})$ is logically equivalent to the disjunction $\sim S_1^{AT(+)}$ or S_2^{AT}. By substitution, we obtain:

$$(25)' \qquad \sim S_3^{PT+} \text{ iff } [(\sim S_1^{AT(+)} \text{ or } S_2^{AT}) \text{ or } (R \,\&\, \sim\#T)]$$

The formulae (25) and (25)' are logically equivalent.

We have now a positive response-requirement and may compare the result to the formulae (1)—(9). It is easy to see that (25)' is like (7), restricted to the interval #T. In both cases, the consequence of R is $\sim S_3^{PT+}$ even when $S_1^{AT(+)}$ is true. The formula (25) is therefore a limited extinction schedule.

In this conclusion, we assumed that for the organism now exposed to (25), S_1 has previously signaled reinforcement if R. This will normally be the case for most of the differential reinforcement schedules. Hence, the codification (25)' tells the experimenter that the effect of (1) should be extinguished within the interval #T.

It might be that DRO is used in situations where the antecedent stimulus is appetitive. We should then write the design as follows:

(25)'' S_1^{PT+} iff [S_1^{AT+} & (~R or #T)]

which is equal to:

(25)''' ~S_1^{PT+} iff [~S_1^{AT+} or (R and ~#T)]

The design (25)'' is more severe than (25). Because (25)'' and (25)''' are logically equivalent, the design (25)'' can be characterized as limited negative punishment. Even in case of S_1^{AT+}, the consequence of R is ~S_1^{PT+}, unless #T.

The formula (27) (DRL) may be reformulated to contain positive response-requirements using the same procedure as for (25). We then achieve:

(27)' ~S_3^{PT+} iff (~$S_1^{AT(+)}$ or S_2^{AT} or ~#T or #R)

or:

(27)'' ~S_1^{PT+} iff (~S_1^{AT+} or ~#T or #R)

This is a result parallel to the one we found for DRO. If the target response exceeds a certain level within a predefined period, it is subjected to extinction (27)' or negative punishment (27)''.

The formula (26) (DRA) also contains a negative response-requirement, but in this schedule, there is a positive response-requirement as well. The first target response is reinforced. Hence this is at least partly a plan for positive reinforcement. It is similar to (2), without the second inner parenthesis. Before (26) was implemented, however, the second target response was often reinforced precisely by the consequential stimulus now used in (26). The second target response is then subjected to extinction or negative punishment.

Differential reinforcement of incompatible behavior (DRI) is simply (26) used to reinforce a response that happens to be incompatible with another. The conditional ~R_2 if R_1 does not tell when to present the consequential stimulus. It is just that the experimenter knows it and selects target response number one for this reason.

It is established knowledge that DRO is improperly characterized as a reinforcement schedule. By formalizing the theory for behavioral operations using logical connectives, we may reconstruct formulae with negative response-requirements such that they contain positive response-requirements while we preserve their truth-conditions. We may then compare the result with the basic formulae (1)—(9). Thus the true nature of negative response-requirements is clearly exposed. The same conclusion therefore applies to DRL as well as to DRO, and partly also to DRP, DRA and DRI, but not to DRH. Such are the merits of codifying behavioral operations based on formal logic.

Conclusion

We have codified examples of basic behavioral operations, a formal codification that can easily be expanded.

Our main concern has been the systematic use of logical connectives. We have written the basic statements in such a way that the logic of behavioral operations is exposed through the use of these connectives. To the extent that the basic statements denote public events, everyone may keep track of the truth-conditions of behavioral operations.

We have shown that use of a well known, established formal language promotes discussions of theoretical issues. Empirical issues cannot be solved by formal means; but when the logic of empirical theories is clearly exposed, that might be of help in deciding empirical issues.

The proposed codification system is sufficiently elaborated for avoiding ambiguities and clearing up several issues. For the basic behavioral operations, the simple structure of a biconditional is sufficient, connecting the presentation of the consequential stimulus to a conjunction of the conditions required for presenting it.

It is presupposed that all formulae should be repeated. When the experimenter has employed one instance of planned change of a stimulus, the procedure is iterated in order to obtain the predicted type of behavioral change. A next step might therefore be the introduction of a formal language that allows for describing the effect of iteration. We then embark on description of behavior processes, which requires incorporation of mathematics. The concept of conditional probabilities might be a link to our codification system. Mathematics might allow for a more economical and elegant formalization; we leave that for future research.

References

Baron, A., & Galizio, M. (2005). Positive and negative reinforcement: Should the distinction be preserved? *The Behavior Analyst, 28,* 85–98.

Catania, A. C. (2013). *Learning* (5th ed.). NJ: Sloan Publishing LLC.

Collins, J., Hall, D., & Paul, L. A. (2004). Counterfactuals and causation: History, problems, and prospects. In J. Collins, D. Hall, & L. A. Paul. (Eds.) *Causation and counterfactuals* (pp. 3–58). Cambridge, MA: The MIT Press.

Donahoe, J. W., & Palmer, D. C.(1994). Learning and complex behavior. Boston: Allyn & Bacon.

Edgington, D. (2001). Conditionals. In L. Goble (Ed.) *The Blackwell Guide to Philosophical Logic* (pp. 385–415). Malden, MA: Blackwell Publishers Ltd.

Hanley, G. P., Iwata, B. A., & MCCord, B.E. (2003). Functional analysis of problem behavior: a review. *Journal of Applied Behavior Analysis, 36,* 147–185.

Iwata, B. A. (2005). On the distinction between positive and negative reinforcement. *The Behavior Analyst, 28,* 121–123.

Künne, W. (2003). *Conceptions of Truth.* Oxford: Clarendon Press.

Lokke, J. A., Arntzen, E., & Lokke, G. (2006). *Notation in behavior analysis: some tentative proposals.* Poster presented at the annual convention of the Association for Behavior Analysis, Atlanta, GA.

Lokke, J., Lokke, G., & Arntzen, E. (2008). *Notation of behavioral operations in behavior analysis: status in modern textbooks.* Poster presented at the annual convention of the Association for Behavior Analysis, Chicago, IL.

Mechner, F. (1959). A notation system for the description of behavioral procedures. *Journal of the Experimental Analysis of Behavior, 2,* 133–150.

Mechner, F. (2008). Behavioral contingency analysis. *Behavioral Processes, 78,* 124–144.

Mechner, F. (2011). Why behavior analysis needs a formal symbolic language for codifying behavioral contingencies. *European Journal of Behavior Analysis, 12,* 93–104.

Michael, J. (1975). Positive and negative reinforcement, a distinction that is no longer necessary; or a better way to talk about bad things. *Behaviorism, 3,* 149–155.

Michael, J. (2006). Comment on Baron and Galizio (2005). *The Behavior Analyst, 29*, 117–119.

Pierce, W. D., & Cheney, C. D. (2004). Behavior analysis and learning. (3^{ed} ed.) Mahwah, New Jersey: Lawrence Erlbaum Associates.

Postman, L. (1947). The history and present status of the law of effect. In *Psychological Bulletin, 44*, 489–563.

Quine, W. V. (1952). *The methods of logic*. London: Routledge and Kegan Paul.

Quine, W. V. (1960). *Word and Object*. Cambridge, Massachusetts: The MIT Press.

Resnik, D. M. (1987). *Choices: An Introduction to Decision Theory*. Minneapolis: University of Minnesota Press.

Sidman, M. (1986). Functional analysis of emergent verbal classes. In T. Thompson & M. D. Zeiler (Eds.), *Analysis and integration of behavioral units* (pp. 213–245). Hillsdale, NJ: Lawrence Erlbaum.

Sidman, M. (2005). The distinction between positive and negative reinforcement: Some additional considerations. *The Behavior Analyst, 28,* 135–139.

Skinner, B. F. (1957/1992). *Verbal Behavior.* Cambridge, Massachusetts: B. F. Skinner Foundation.

Skinner, B. F. (1959). *Science and human behavior.* New York: Macmillian.

Tomassi, P. (1999). *Logic*. N.Y.: Routledge.

Tonneau, F. (2008). The concept of reinforcement: Explanatory or descriptive? In *Behavior and Philosophy, 36*, 87–96.

Venema, Y. (2001). Temporal logic. In L. Goble (ed.) *The Blackwell Guide to Philosophical Logic* (pp. 203–224). Malden MA: Blackwell Publishers Ltd.

Author's Note

There is no conflict of interest to declare concerning the three authors. Please send correspondence concerning this article to Jon A Lokke , Department of Health and Social Studies, Ostfold University College, 1757 Halden, Norway. E-mail: jon.lokke@hiof.no