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Calibration of interest rate models in Solvency II

Impact on capital requirements

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Abstract

Solvency II is the new solvency regulations for European insurers and reinsurers that will replace the current regulations. Solvency II has been postponed several times, but implementation is now scheduled for January 2016.

Life insurance companies manage funds over a long time period, the return on assets and the development of size of the liabilities determine if they manage to fulfill the future liabilities. Life insurance companies distribute the return to shareholders and policyholders after a profit sharing model. Contracts with interest rate guarantee are a product that is challenging for life insurance companies. They are responsible for meeting an annual interest rate over the lifetime of the contract, and they are left with the downside risk associated with this guarantee. The life insurance company must in a worst-case scenario cover the guarantee by taking from the equity. Interest rate guarantees increases risk for the company's equity if the interest rate is low, as the situation is today.

In this thesis, we have programmed, calibrated and simulated interest rate models to see how capital requirements under Solvency II are affected. We use three different interest rate models, Vasicek, Cox, Ingersoll and Ross, and Libor Market Model. A profit sharing model is used to distribute returns to the various stakeholders on the liability side in the balance sheet. Capital requirements are calculated from the simulated balance sheet. We have run analysis with different scenarios where we have adjusted one parameter to see what effects this gives. Our analysis shows that both choice of model and calibration affects capital requirements to a greater extent than initially assumed.

Sammendrag

Solvens II er det nye solvensregelverket for europeiske forsikringsselskaper som skal erstatte det nåværende solvensregelverket. Solvens II har blitt utsatt flere ganger, men implementeringen er nå planlagt til januar 2016.

Livselskapene forvalter verdier over en lang tidsperiode, og avkastningen til eiendelene og størrelsen på forpliktelsene avgjør om de kan oppfylle de fremtidige forpliktelsene. Livselskapene fordeler avkastning til eiere og forsikringstakere etter en profittdelings modell. Kontrakter med garantert avkastning, fripoliser, er et produkt som er utfordrende for livselskapene. De er ansvarlig for å garantere en årlig rente på kontrakten gjennom hele dens livsløp, og de sitter med hele nedside risikoen forbundet med den garanterte avkastningen. Livselskapene må i verste fall dekke den garanterte avkastningen ved å ta fra egenkapitalen. Kontrakter med garantert avkastning øker risikoen for selskapets egenkapital når renten er lav.

I denne oppgaven har vi programmert, kalibrert og simulert rentemodeller for å se hvordan kapitalkrav under Solvens II blir påvirket. Vi benytter oss av tre ulike rentemodeller, Vasicek, Cox, Ingersoll og Ross, og Libor Market Model. Profittdelings modellen blir brukt til å fordele avkastningen til de ulike interessentene på passiva siden i balansen. Kapitalkravene er beregnet fra den simulerte balansen. Vi har kjørt analyser med ulike scenario hvor vi har justert en parameter for å se hvilken effekt dette gir. Våre analyser viser at bade valg av modell og kalibrering påvirker kapitalkravet i større grad enn først antatt.

Høgskolen i Oslo og Akershus, Fakultet for Samfunnsfag
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Preface

This thesis concludes our Master of Science education in Business and Administration at Oslo and Akershus University College of Applied Science.

This theme is chosen based on our interest in risk management. Solvency II is a relevant topic in today's market. The new regulations offer many issues to specialize in. We have chosen to look into interest rate modeling and calibration. We wanted to examine how calibration of different interest rate models affects life insurance companies' capital requirements.
It has been interesting and educational to work with the new regulations that will have major impact on the insurance industry.

We would like to thank our supervisor, Helge Nordahl, for valuable ideas, good guidance and constructive feedback throughout the work with this thesis.

Oslo, 30 May 2014

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1 Introduction

1.1 History and development of insurance

The development of insurance is an ongoing process that started thousands of years ago. From Hamnurabi's Code in Babylon through the medieval Coverage and eventually at Lloyd's Coffee house in London in the late 17th century, people have sought and found ways for reducing their individual risk. The underwriting started at Lloyd's with the new shipping industry, shipments between the new world and the colonies. Risk seeking investors guaranteed for part of the cargo by writing under for part of the cargo list for a premium. Further in 1654 when Blaise Pascal and Pierre de Fermat laid the foundation of probability of Against the Gods theory, underwriters, by using Pascal Triangle, were able to start quantifying risk and thereby reasonable risk premiums. After the London fire in 1666, fire insurance reach the market and short after that mortality table set the basis for life insurance.

The Society of Assurance for Widows and Orphans was the first mutual life insurance company, started in 1699, but did not survive long. Some of the first subsequent companies used an equal fee for people under the age of 45. Natural, this increased the need and development of mortality tables and better risk measures. Through the twentieth century the industry flourished and had a major influence on the global trade evolution. Since the consequences of failure of insurance companies grew with size, the need for national regulations for preventing insolvency had come.

In Europe, the current regulations regime is Solvency I. The regime was developed over several stages through the second half of the twentieth century, before it was implemented in 2002. EU's Solvency regime first started in the 1970s and is a principal based system. Especially after the turbulent financial crises of 2008 the need for a new regulative, which is able to incorporate the importance effective risk management and corporate governance, appeared. The new Solvency II regulative set out with the purpose of creating a market based system that would increase the competitiveness of European insurance companies and the safety of the customers. They aim to do this by creating a more rigorous risk management framework with demanding adequacy capital reserves, greater transparency and disclosure requirements.

The new Solvency regime is enormous and developed for the EU members. Since this regulative also will be law in Norway through EEC- treaty, we aim to analyze some of the potential effects this regime will have on the Norwegian market. Some life insurance products offer a guaranteed return to policyholders. The historically low risk free rate today makes it unusually hard to fulfill this promise without taking additional risk. Stronger capital requirements from Solvency II make this an acceptable challenge for the insurance industry today.

In this thesis we are trying to make estimates of an insurance company value of equity and the corresponding risk, using the Value at Risk measure, and compare it up against the Solvency capital requirements. This is done in several stages: first we estimate the short interest rate by using different one factor short rate models, like the Vasicek 1977 model. This model was further developed by Cox, Ingersoll and Ross as the CIR model. The Libor Market Model is also used for interest rate estimations, by simulating forward rates and their corresponding spot rates. Further we use these rates to calculate bond prices and simulate stock prices. The assets are combined for creating an approximately realistic portfolio for a Norwegian company. We also developed an insurance model for distribution of the return for the portfolio between owners and policyholders, a profit sharing model. Our goal is to see how different calibrations of the models will affect the capital requirements.

We find that the calibration and choice of interest rate model, to a large extent can affect the presumed risk in the portfolio and the required capital.

In Chapter 2 we will introduce the new regulative and some of its relevant and specialties. The theory and techniques behind the different assets simulating are presented in Chapter 3. The properties of how an insurance contract is designed, and the return distributions are set up in chapter 4. In Chapter 5 we calibrate all the models used, together with setting up all the levels used in our base case. The results of the base case are presented in chapter 6. Further in chapter 7 we experiment how different scenarios will influence the Value at risk level. Our conclusions are presented in chapter 8.

2 Solvency II

Solvency II is the new solvency regulations for European insurers and reinsurers that will replace the current regulations. The new regulations will take into account various risks, to encourage transparency and market discipline. All insurers shall comply with the new regulations, which will improve the competitive situation since they all will have to follow the same rules.

The Solvency II *Directive 2009/138/EC of the European parliament and of the Council of 25 November 2009,* requires insurers to focus on managing all the risks facing their organization. It offers European insurers a real opportunity to improve their risk-adjusted performance and operational efficiency, which is likely to be good news for policyholders, the insurance industry, and the European Union (EU) as a whole (KPMG 2011, 1). Solvency II has been postponed several times, and is now scheduled for implementation from January 2016. There are a multitude of reasons as to why these directives are delayed. Here are a select two:
(1) The immense scope of the regulative that requires massive negotiations and implementations challenges throughout the European Union member states.
(2) The turbulent decade has drawn all capital reserves in the market to other troublesome areas. The timing for a regulative demand for higher capital requirements in the great insurance industry may postpone the recovery in the European member states.

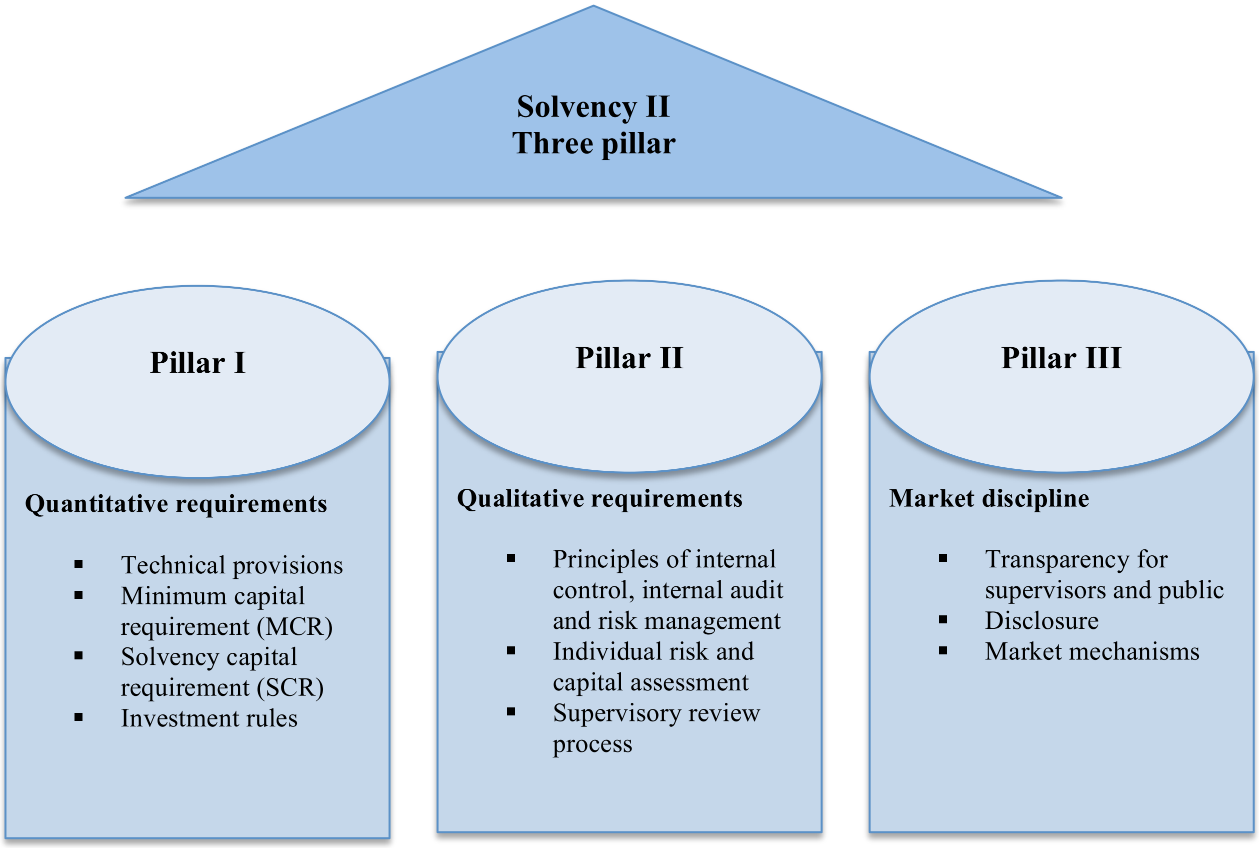
The main objective of the regulations is to ensure that insurers have sufficient assets to cover its future obligations. Insurers manage large values over a long period of time. The return on savings and developments in the size of insurance company liabilities determines their ability to meet future liabilities. The risk that liabilities are not meet can be attributed to assets and liabilities in the insurers balance. The new regulation takes into account the risk of both the asset - and liability side of the balance.

2.1 Three Pillar approach

Solvency II and Basel II, which is the regulation for the banking sector, have similarities in their structure. Both regulations are split into three pillars, which include quantitative and qualitative requirements and market discipline as well as specific components that focus on capital, risk, supervision, and disclosure. While there are similarities between these two regulations, there are also differences since insurance and banking are distinctly different industries.

The three pillar approach gives Solvency II an orderly layout, as we see from the figure below it aims to categorize related risk aspects of the business into different pillars (Isden 2010). Most practitioners focus on Pillar I with the quantitative requirements. This thesis also has the focus here.

Figure 2-1: The three pillars



2.2 Pillar I – Quantitative requirements

The quantitative requirements in Pillar I is the pillar that has received the most attention. It includes Technical provisions, Minimum Capital Requirement (MCR), Solvency Capital Requirement (SCR) and Investment rules. This pillar aims to secure that firms are adequately based with risk-based capital. All valuation should be market-consistent. Companies may use either the Standard Formula approach or an internal model approach.

2.2.1 Minimum capital requirements

The MCR is primarily described in section 5, article 128-131 in the Solvency II directive. The MCR is the minimum of what insurance and reinsurance must hold of capital eligible to the basic own funds. This corresponds to an absolute lower level of what is acceptable risk for the policyholders. If the MCR is not met, it will lead to supervisor intervention and the firm will be restricted for writing any more business. SII directive Art. 129 § 3 states that the MCR are intended to be in the middle of 25% and 45 % of the solvency capital requirements (SCR), see below. In accordance with Solvency II directive Art, 129 § 2 the MCR "shall be calculated as a linear function of a set or sub-set of the following variables: the undertakings technical provisions, written premiums, capital-at-risk, deferred tax and administrative expenses". As an absolute minimum floor for reinsurance, states the capital should not be less than EUR 3.200.000.

2.2.2 Solvency capital requirements

The solvency capital requirements shall represent the level of capital insurers and reinsurers are required to cover the quantifiable risk. The calculation will be calculated as an ongoing concern basis, and should also include the expected business one year in the future. The risk shall correspond to a Value at Risk measure with confidence level of 99,5%, meaning they should hold enough capital to resist a bad event occurring every other century. The more comprehensive calculations in Solvency II compared to Solvency I will not change the risk itself, but the SCR may be higher due to the fact that it measure more quantifiable risk than before. The Basic SCR shall cover at least the following risks modules:

Risk Modules

(a) Non-life underwriting risk;
(b) Life underwriting risk;
(c) Health underwriting risk;
(d) Market risk;
(e) Credit risk;
(f) Operational risk

The risk attributed form each sub module is aggregated after allowing for effects from diversifications. Here market risk is clearly the largest risk driver. The Basic SCR can be calculated this way:

$$BSCR=\sqrt{\sum_{i=1}^{5}\sum_{j=1}^{5}\rho_{i,j} * SCR_i * SCR_j}$$

Formula 2-1

Where ρi,j is the correlation between *i* and *j,* for example where market risk is *i* and credit risk is *j,* and SCR *i* and *j* is each sub models SCR.

2.2.3 Standard Formula approach or Internal model approach

Standard Formula

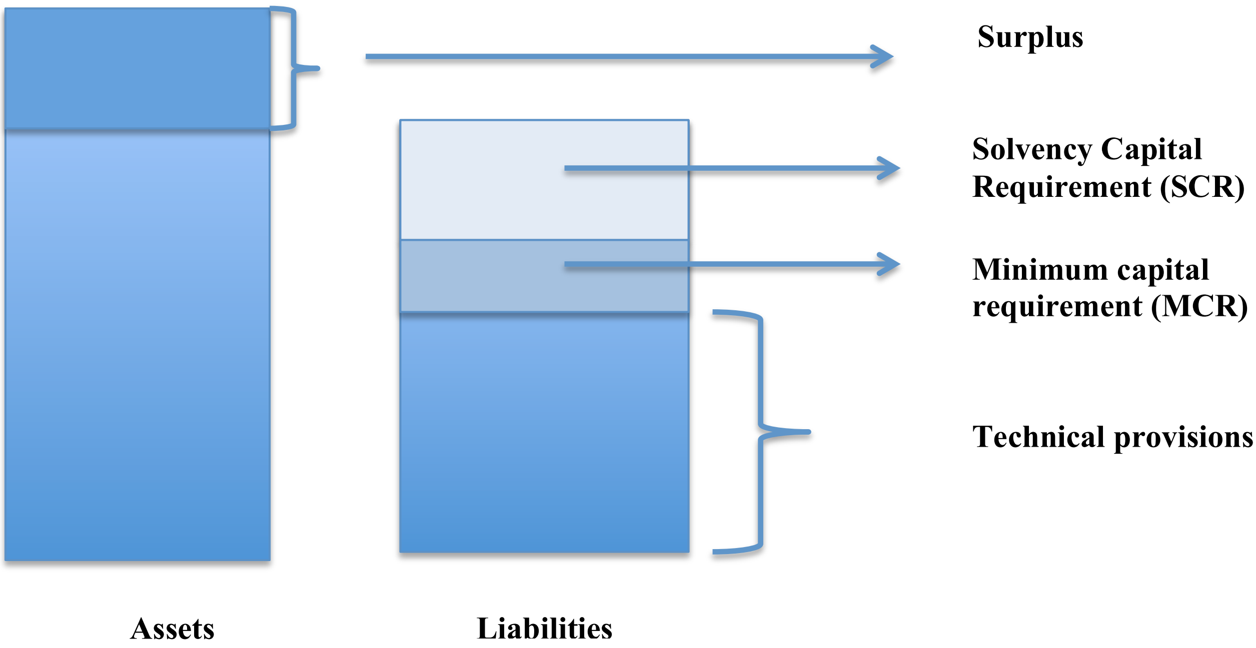
Life insurance companies can determine the Solvency capital requirement (SCR) with the European Standard Formula. The standard formula is a specified set of stress tests or factor based formulas that life insurance companies have to apply to their assets and liabilities for the various risk modules. The European Standard formula is calibrated for the EU market as a whole, and may not be suitable for all life insurance companies.

Internal approach

Life insurance companies can under Solvency II use an internal approach to calculate the solvency capital requirement. Companies that wish to use their own internal model have to get regulatory approval. Internal models have to meet all the requirements in the SII Directive of 2009. Internal models can be calibrated with different time period or risk measure than the standard formula, however all choices have to be justified. It is important that the time period is justified with regard to the duration of the liabilities. Partial internal models are also allowed for calculating the solvency capital requirement, these models requires approval too. Companies can use a partial internal model for different risk categories or risks that are not covered by the standard formula.

Technical provisions

Technical provisions are quantitative requirements insurers have to meet, both under the current solvency regulations and under Solvency II. Technical provisions should cover expected future claims from policyholders. Under Solvency II it should be equivalent to the amount another insurer would be expected to pay in order to take over and meet the insurers obligations to policyholders.



In this thesis we will focus on Pillar I, and the risk module for market risk, more specifically interest rate risk.

2.3 Pillar II Qualitative requirements

The qualitative requirements in Pillar II consist of principles of internal control, internal audit and risk management, individual risk and capital assessment, and supervisory review process. Higher standards of risk and governance will give supervisors a new focus.
The requirements in this pillar give supervisors more powers to challenge their firms on risk management issues.

2.4 Pillar III Market discipline

Pillar III consists of transparency for both the supervisor and the public, disclosure, and support of risk-based supervision through market mechanisms. There are more pressure from capital markets and rating agencies. This pillar should ensure market discipline through disclosure requirements to the public and reporting requirements to supervisory authorities.

2.5 Own Risk and Solvency Assessment (ORSA)

Under Solvency II companies are required to produces an ORSA report. Article 45 in the directive says that as a part of the risk management system, insurance and reinsurance companies should conduct its own risk and solvency assessments. This is not a new capital regime, but it is a method for linking the management's strategy and development plans into long term company wide risk assessments. For instance what effects new products have on SCR.

2.6 Criticism of Solvency II

The enormous reporting requirement demanded by Solvency II is a major transition from the current regulations. Especially for smaller companies who do not have the necessary manpower to comprehend and deliver all the demands. Large companies have the prerequisites necessary to develop their own internal model to better fit the capital demands for their unique situation. Smaller companies do not have is opportunity, and thereby is left with all the faults the standard model presents.

Stefan Mittnik (2011) argues that the correlations between classes may be biased, because the calibrations procedure used to estimate correlations between equity classes might lead to spurious correlations. The spuriousness mainly originating from the annualization procedure used to calculate yearly return from daily.

3 Theory

In this chapter we will elaborate the theoretical part of the models we use to estimate interest rates, bonds and stocks. First we describe some general processes used in several models.

3.1.1 Wiener process and Brownian motion

A Wiener process, also referred to as Brownian motion, is a continuous-time stochastic process W(t) with the following properties (Gatarek 2006)

- W(0)=0
- W has continuous paths

- W(s) and (W(t)-W(s)) are independent random variables for any 0 < s < t
- W(t) has Gaussian distribution with mean 0 and variance t

3.1.2 Generalized Wiener Process

The *drift rate* is known as the mean change per unit time for a stochastic process, while the *variance rate* is known as the variance per unit time.
A generalized Wiener process for a variable *x* can be defined in terms of dz as (Hull 2012)

$$dx = a\,dt + b\,dz$$

Formula 3-1

where *a* and *b* are constants and dz is as a Wiener process

The generalized Wiener process has an expected drift rate (i.e., average drift per unit of time) of *a* and a variance rate (i.e., variance per unit of time) of b^2.
For a proof that an insurer's asset portfolio follows a Geometric Brownian Motion, see: Insurance and Mathematics and Economics 49 (2011) 115-125 Appendix A.

3.1.3 Random number generation

In Monte Carlo simulations a generation of random numbers for the error term is needed. Generating random numbers is normally done through algorithms embedded in statistical packages with a *pesudo* type draw. In the statistical program R this is done by the function *rnorm*. The function picks random numbers for the normal distribution with a specified mean and standard deviation N~(u,sd). The characteristics of the Bell shaped curve appears when we increase the number of simulations.

In our case we need to be able to generate correlated random numbers. In our estimations we are simulating multiple assets simultaneously and need to incorporate the effect the different assets have on each other. In R this is done by the function *mvrnorm* from the package "MASS". The function uses our correlations matrix as a base for the simulations. Alternative method would be a Cholesky decomposition (Hull 2012).

3.2 Interest Rate Models

A term structure model is a model describing the evolution of all zero-coupon interest rates. We will focus on term structure models constructed by specifying the behavior of the short-term interest rate, *r.* There are especially three theories about the term structure of interest rates, *Expectation theory, Market segmentation theory* and *Liquidity theory* (Hull 2012).

The expectation theory states that for a given time horizon, all investments with different maturity should give the same expected return. This means that the forward rate over the second year is set to the spot rate that people expect to prevail over the second year. An *i* year forward rate must equal the expected difference between an *i* year spot rate and an *i+1* year spot rate. An example is that an investment in a bond with 2 years maturity should give the same expected return as an investment in a bond with 1-year maturity, and at maturity you rebalance the amount in a new bond with 1-year maturity.

There are three separated markets with their own corresponding supply and demand that decides the zero rates. The *Market segmentation theory* assumes that there is no relationship between the short-, mid- and long- term interest rates.

Liquidity preference theory argues that investors prefer to invest funds for shorter periods and borrowers wants to lend funds for longer periods at fixed rates. It then follows that the forward curve usually is higher than the corresponding zero curve.

3.2.1 Equilibrium Models

Equilibrium models usually start with assumptions about economic variables and derive a process for the short rate, *r.* You can assume that the economy tends toward some equilibrium, based on such fundamental factors as the productivity of capital, long-term monetary policy, and so on. Short-term rates will be characterized by *mean reversion* – interest rates appear to be pulled back to some long-run average level over time. When the short-term is above its long-run equilibrium value, the drift is negative, driving the rate down towards this long-run value. When the rate is below its equilibrium value, the drift is positive, driving the rate up toward this value. In addition to being a reasonable assumption about short rates, mean reversion enables a model to capture several features of term structure behavior in an economically intuitive way (Tuckman and Serrat 2012).

3.2.2 The Vasicek Model

The Vasicek Model is a one-factor short rate model used to estimate interest rate derivatives, first developed by Oldrich Vasicek in 1977. The model is formulated in continuous time and assumes a no-arbitrage argument. This was the first one-factor short rate model that incorporated the mean reversion factor in a closed form solution. The short rate is assumed to satisfy the stochastic differential equation (Vasicek 1977)

$$dr(t) = k\big(\theta - r(t)\big)dt + \sigma dW(t)$$

Formula 3-2

where W(t) is the random market risk, represented by the Wiener process,
t is time,
$k(\theta - r(t))$ is the expected change in the interest rate at t, this is also called the drift factor,
k is the speed of reversion,
θ is the long-term level of the mean,
and σ is the volatility at the time.
k, θ and σ are constants.

A drawback of the model is that it is theoretically possible for the interest rate to become negative, which is intuitively wrong. There have been situations where this has happened. At the peak of the Euro crisis in 2012 the German treasury bills was traded at a negative yield, meaning that investors was willing to pay to lend money to Germany. Due to this crisis and the following recovery, interest rate are held unnaturally low for an unnaturally long time. When the rates are so low it causes problems in the simulations, since the range of the estimations paths go below zero. We will in our model assume that the rate cannot become negative. Other drawbacks of the model are constant local volatility, which is most likely unrealistic, and challenges of adapting it to the yield curve with a good fit. The fact that this is a one-factor model prevents from incorporating more complex shifts in the term structure.

Zero-coupon pricing with Vasicek

The Vasicek model can be used to price a zero-coupon bond at time t with maturity T under a risk neutral measure (Hull 2012, 685). Using the Markow property and partial differential equation (PDE) techniques, the bond price can be estimated with the following equations:

$$B(t,T) = \exp\,(-A(t,T)r(t) + D(t,T))$$

$$A(t,T) = \frac{1-e^{-k(T-t)}}{k}$$

$$D(t,T) = (\theta - \frac{\sigma^2}{2k})[A(t,T) - (T-t)] - \frac{\sigma^2 A(t,T)^2}{4k}$$

Formula 3-3: Zero-coupon pricing with Vasicek

For a closer elaboration of this model, see Vasicek`s original paper from 1977.

3.2.3 The Cox, Ingersoll, and Ross Model

Cox, Ingersoll, and Ross (1985) (CIR) made an alternative model where the short-term interest rates are always positive. In this one factor model, the short rate is assumed to satisfy the stochastic differential equation

$$dr(t) = k\big(\theta - r(t)\big)dt + \sigma\sqrt{\sigma}dW(t)$$

Formula 3-4: CIR Model

The process above has the same mean reverting drift as Vasicek; the difference is that the standard deviation of the change in the short rate in a short period of time is proportional to $\sqrt{\sigma}$. This indicates that, as the short-term interest rate increases, the standard deviation increases.

Zero-coupon pricing with Cox, Ingersoll, and Ross Model
The CIR model gives the same general form for bond pricing as Vasicek does (Hull 2012)

$$B(t,T) = \exp\,(-A(t,T)r(t) + D(t,T))$$

but the functions A(t, T) and D(t, T) are different:

$$A(t,T) = \frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + k)(e^{\gamma(T-t)} - 1) + 2\gamma}$$

$$D(t,T) = [\frac{2\gamma e^{(k+\gamma)(T-t)/2}}{(\gamma + k)(e^{\gamma(T-t)} - 1) + 2\gamma}]^{2k\theta/\sigma^2}$$

where $\gamma = \sqrt{k^2 + 2\sigma^2}$

Formula 3-5

Bond pricing with CIR compared with Vasicek is more difficult as the model is not Gaussian. This makes it less analytically attractive.

3.3 The LIBOR Market Model

The LIBOR rate stands for *London Interbank Offered Rate*. This is a reference rate for reflecting at which rate banks are willing to lend money between each other (Hull 2012). LIBOR rates have maturities up to 12 months and are quoted in all the major currencies. The London Interbank Bid Rate (LIBID) on the other hand is the rate banks will accept deposits from other banks (Hull 2012, 76). The LIBOR rate is traditionally used as an estimate of the risk free rate. Alternatively Treasury Bills are used as an approximation of the risk free rate, but T-Bills may be artificially low, LIBOR rates may be a better reflection of the risk free rate. In this thesis we need to estimate forward rates of the LIBOR rate 30 years in the future and the corresponding spot rates.

The lognormal forward LIBOR Market Model (LMM) is the first model of interest rate dynamics consistent with the market practice of pricing interest rate derivatives. The model was created in 1994 by Kristian Miltersen, Klaus Sandmann and Dieter Sondermann and then developed in 1995 to a form applicable in practice by Alan Brace, Dariusz Gatarek and Marek Musiela (Gatarek, Bachert, and Maksymiuk 2006). The LMM is also called BGM after Brace-Gatarek-Musiela. This model is built on short rate models – where the dynamics of all interest rates was determined by the dynamics of the overnight rate.

The LMM is mostly used for pricing of exotic interest rate derivatives. LMM applique interest rates as they are observed in the market. When modeling with simple or compounded rates the LMM gives us formulas for the current market rates.

Forward Rates

A forward rate is the rate on a forward loan, which is an agreement to lend money at some time in the future to be repaid some time after that (Tuckman and Serrat 2012, 75).

The forward rates in the LMM are assumed to be log normally distributed. In our case we are interested in the following spot rates.

Spot Rates

A spot rate is the rate in a spot loan, an agreement in which a lender gives money to the borrower at the time of the agreement to be repaid at some single, specified time in the future (Tuckman and Serrat 2012, 74).

As an approximation of the model for forward rates, a simple Euler approximation is used (Gatarek 2006):

$$L_n^E(t) = L_n(0)\exp\,(-\sum_{j=n+1}^{k} K_j(0)C_{jn}(0,t) - \frac{1}{2}C_{nn}(0,t) + M_k^n(t))$$

Formula 3-6

where

$L_n^E(t)$ - forward LIBOR rates

K_j - volatility component

C_{nn} - BGM covariance

M_k^n - BGM martingale

We use a variant of the formula shown above

$$F_i(t) = F_i(t-1) * \exp\,(X)$$

$$X = \sigma_i(t)\sum_{k=t}^{i}\frac{F_k(t-1)\rho_{ki}\sigma_k}{1+F_k(t-1)} - \frac{1}{2}\sigma_i(t)^2 + \sigma_i(t) * \varepsilon_i(t)$$

Formula 3-7

where

$F_i(t)$ - forward rates

$\sigma_i(t)$ - standard deviation of the log changes to the forward rates

ρ_{ki} – correlation between the rates of return from the forward rates

σ_k - standard deviation

The relationship between forward rates and spot rates are as follows:

$$F_{t+i}(t) = \frac{[1 + R_{i+1}(t)]^{i+1}}{[1 + R_i(t)]^i} - 1$$

Formula 3-8: Relationship between forward rates and spot rates

where, F is the forward rate and R is the spot rates.
From the simulated forward rates in the Libor market model, we can get back to the corresponding spot rates using the following formulas:

$$R_1(t) = F_t(t)$$
$$R_2(t) = [\big(1 + F_t(t)\big)\big(1 + F_{t+1}(t)\big)]^{\frac{1}{2}} - 1$$
$$R_i(t) = [\prod_{k=0}^{i-1}\big(1 + F_{t+k}(t)\big)]^{1/2} - 1$$

Formula 3-9: Spot rats from forward

Bond pricing with LMM

The LMM has no separate bond pricing formulas; we therefore use the following formula to price the zero-coupon bonds in our portfolio

$$B = 100e^{-r*t}$$

Formula 3-10: Bond pricing with LMM

B = bond price
r = spot rate, calculated from the LMM
t = duration

Volatility

There are two parameters in the LMM that needs to be specified: volatility and correlation. The specification of the volatility function is an important issue in calibrating the Libor Market Model (Jong, 2000). There are different ways to specify the volatility function:

1. The volatility function is depending on time to maturity: $\sigma_t(t) = \sigma_{i-t}$
2. The volatility function is constant; the volatility of the forward Libor rate is constant until maturity: $\sigma_i(t) = \sigma_i$
3. The volatility is different for different Libor rates; the volatility is different for all maturities and times: $\sigma_i(t)$

In the first specification the volatility is assumed to depend only on the distance between *t* and *i,* and not the values of *t* and *i*. The volatility will usually increase as the time to maturity decreases. This volatility function is consistent with mean-reverting behavior of interest rates, because mean-reversion typically implies that interest rates close to maturity have a larger volatility than interest rates that are far from maturity (Jong, 2000).
The second specification says that the volatility does not depend on *t*, and is therefore constant for the forward rate until maturity.

Correlation

Packham (2005) describes the following properties correlation should have in the LMM:

1. Correlation, $\rho_{i,j}(t)$, must be positive for all *x, y* and *t*
2. Correlation, $\rho_{i,j}(t)$, tends to decrease for increasing maturity intervals
3. Correlation, $\rho_{i,i+p}(t)$, increases with increasing *i*

Equilibrium rate

The equilibrium interest rate is the rate that ensures that capital accumulation corresponds to saving in the economy. This rate is determined by long-term phenomena related to the structure of the economy. Fundamental structural issues in the economy, such as consumer impatience and the economic growth rate determine the long-term equilibrium interest rate. The economic situation varies over time. Monetary policy will alternately set a rate that is above and below the neutral rate. Therefore it is unlikely that interest rates differ widely from the long-term equilibrium rate over time (Bergo 2003). Storebrand ASA used an assumption

of a long-term equilibrium rate of 4,5 % in 2009 (Årsrapport 2009. Storebrand ASA). This long-term equilibrium rate is based on 2 percent real rate and 2,5 percent inflation.
In this thesis we have chosen to use a long-term equilibrium rate of 4 %.

3.4 Stock model

The term stochastic process is often used to describe prices of financial assets that evolve over time (Tsay 2010). By modeling of financial assets there are two types of stochastic processes, discrete-time and continuous- time. The discrete-time process describes a situation where variables can only take values within a specific area. The continuous-time process variables can take an infinite number of values. The distinctions are also made concerning time. A stock price is somewhat in between as the stock exchanges are only open in a given time period. We are assuming that the stock prices follow a continuous stochastic process.
The simulations of stock prices in our stock model are created with a Wiener process.

$$S_{t+1} = S_t e^{(\mu - \frac{1}{2} * \sigma^2 + \sigma dw_1)}$$

Formula 3-11: Stock price model

where
S_{t+1} = stock price tomorrow
S_t= stock price today
μ = expected return
σ = volatility measured by standard deviation
dw_1= the standard Wiener process

3.5 Correlation between assets

The correlations between stocks, bond and interest rate will mainly be incorporated through the error term. We will use different type of correlations based on the necessary inputs of the models. For assets we have calculated a correlation matrix P(*i,j*), where *i* and *j* is respectively the different assets. The matrix is shown in Table 5-4 and Table 5-5

For the forward rates used in LMM, we have calculated a correlation matrix where P(*i,j)* is the correlations between a forward rate in year t+*i* and forward rate in year t+*j, for all times t*

Norwegian life insurance companies have investments in both foreign and domestic's securities. But the weights of investments are in global stocks, so in this thesis we are most interested in correlations between global indexes.

From the stress scenarios, different assumptions about the correlations are taken by CEIOPS calibrations paper from 15.april 2010 pt. 3.1291. page 347/384. Respectively different assumptions about the correlations between equity and interest rate are taken based on an interest rate increase, fall or status quo.

CEIOPS advise from Consultation Paper nr. 74 (3.74) will be used as inputs in addition to our historical correlations estimations.

3.6 Value at Risk

Value at Risk (VaR) is a widely used risk management tool in all over the world. Analysts working at JP Morgan were the first to develop VaR (Hull 2012 RM). JB Morgans Chairman Dennis Weatherstone required an easier to understand risk measure tool for a large portfolio, compared to reports with Greek letters of the different positions. VaR summarize the portfolio risk in a single number that is easy to understand. Hull (2012) expresses VaR as "we are X percent certain that we will not loose more than V dollars in time T". Where V is the VaR level. Definitions are functions of time and the confidence level. VaR is assuming that the returns are normally distributed. The VaR level gives the user an insight of how bad things can get. For portfolio management we can calculate how much capital needed to resist bad events.

$$\text{VaR}=X * \sigma * \alpha * \sqrt{T}$$

Formula 3-12: Value at risk

Were X is the portfolio value, σ is the standard deviation, α is the confidence level and T is time.

In Solvency II the incorporations of VaR 99,5 works so that the insurance companies basic own funds should be able to resist an extreme event occurring every 200th year. The accuracy of the VaR estimations will increase with when the number of simulations increases.

3.6.1 VaR in the short-rate models

Stress scenarios for interest rates and stocks

For calculating Value at risk with Solvency II requirements, we have used a two steps method. We wish to see how much the market value of equity and liability can change in one year. Step one is to look at a normal case (the base case). Step two is to see how much the values change when we introduce stress over one year.

The inputs for the stress scenarios for the interest rate and stock market over one year are retrieved from “CEIOPS Calibration paper Solvency II”. The Calibration paper provides background information for calibrations for the standard formula approach connected with Quantitative impact study 5 (QIS5). The CEIOPS use scenarios with a large drop in the interest rate and the stock market for the first year. We made a simplification of the stress scenarios. The interest rate stress scenario has a drop of 58% in the term structure, calculated in the start of the second year, thereafter a normal drift. In the stock market a 45 % drop in prices calculated and thereafter a normal drift is assumed. The correlation parameter between is set to 0.5 from the CEIOPS calibration paper.

Value at risk is termed Solvency Capital Requirement (SCR) and is calculated as the difference between the discounted equity in the base case less than the discounted equity after a stress scenario.

The required capital is calculated as the difference between the entry level of the liability side of the balance sheet and the base case results after one year. The required capital is interpreted as the required capital in addition to book equity.

The procedures are done for both interest rate and stocks, and the formula for total SCR is:

$$SCR_{Total}= \sqrt{SCR_{Stocks}^2 + SCR_{Interest\ rate}^2 + (2 * \ \rho * SCR_{Stocks} * SCR_{Interest\ rate})}$$

Formula 3-13

where, ρ is the correlation parameter. SCR_{Stocks} are the SCR from the stress case on stock prices and $SCR_{Interest\ rate}$ the part form the stress case on interest rate.

And the Required Capital is:

$$\text{Required Capital} = (\text{Entry value} - \text{Base case value of equity}) + SCR_{Total}$$

Formula 3-14

where, the base case is simulated without stress.

3.6.2 VaR in LMM

Stress scenario – forward rates Libor Market Model

To determine the stress scenario for forward rates in the Libor Market Model, we have looked at CEIOPS Calibration Paper Solvency II. In 3.20 there is described proposed stresses, both up and down, for different maturity in years (0.25-30). There are no stress scenarios for forward rates, so we made a simplification of the stress scenarios for interest rates. We have used the proposed stress for a down scenario, where the rate is dropping, in our case a drop in forward rates. We used maturity from 1-30 years. We assume that there is just a drop in year 2. The following drop is calculated as follows:

$$Proposed\ stress\ down_{t+1} * Maturity_{t+1} - Proposed\ stress\ down_{t} * Maturity_{t}$$

We use a drop in forward rates of 58 %.

4 Life Insurance companies

A life insurance company is an insurance company engaged in life insurance. Life insurance is a generic term for insurance that is paid out in connection with death, disability and pension.

4.1 Investment universe

Forsikringsvirksomhetsloven §9-7 states that the company's assets will be divided into collective portfolio, investment portfolio and corporate portfolio. Each portfolio can be divided into several sub-portfolios. The collective portfolio should consist of assets that offset the insurance reserves to cover contractual obligations. The investment portfolio should consist of assets that offset the insurance reserves to cover liabilities related to the value of the investment portfolio separately. The corporate portfolio should consist of assets that offset the company's capital and any debt other than insurance liabilities.

4.1.1 The asset allocation

"Forskrift om kapitalforvaltning i livsforsikring" §3-1 - 3-6 gives insurance companies placement constraints in the management of the collective portfolio. The collective portfolio usually invests in stocks, bonds, interest rate instruments and real estate. There are tons of different investment opportunities, but due to the risk aversion from both regulators and customers insurance companies are usually prevented from investing in more sophisticated products like options and other derivatives.

The underlying positions in our fictive portfolio are investments in stocks, bonds and short rate securities. For simplicity reasons, we disregard investments in real estate and lending portfolios.

4.1.2 Interest rate guarantees

A pension capital certificate is accrued pension capital from a collective defined benefit pension contract. These pension capital certificates have an interest rate guarantees. In these contracts there are linked a guaranteed return to ensure a minimum level of future payments. A pension capital certificate is accrued pension from previous employments; you must have worked for 12 months with an employer to receive the pension capital certificate. The accrued pension will be paid out at retirement.

There are two types of interest rate guarantees (Graf 2011):

- Point-to-point guarantee – a guarantee that is only relevant at maturity of the contract
- Cliquet-style (or year-by-year) guarantee – the policyholders have an account to which at least a certain guaranteed rate of return has to be credited.

The guaranteed interest return, or the base rate, cannot be higher than a maximum rate established by the Financial Supervisory Authority (Nordal 2012). There is no retroactive effect if there is a change in the base rate, meaning that a new base rate will only apply on newly established schemes and for new entitlements under existing schemes.
The guaranteed interest rate in Norway is a year-by-year guarantee. Base rate for new schemes from 2011 is set to 2.5 percent. The Financial Supervisory Authority of Norway (Norwegian FAS) suggested reducing the base rate to 2.0 percent from 1.1.2014, but the Ministry of Finance decided to keep the base rate unchanged at 2.5 percent.

4.2 Insurance Model:

The balance sheet of life insurance companies is intuitively a lot like a limited liability company, but with some different characteristics of the posts that will be explained here. The return distributions will be explained under. A simplified balance sheet at time *t* in our model consists of the following elements:

Figure 4-1: **Balance sheet of a life insurance company**

Assets (A)	Liabilities
Stocks (t_0)	Equity (t_0)
Bonds (t_0)	Premium Reserve (t_0)
Money market (t_0)	Additional statutory reserve (t_0)
	Market value reserve (t_0)

A special characteristic of insurance companies is buffer funds, which consists of market value adjustment reserve and additional statutory reserve. Details of the investments on the asset side are described above. Specifications of the liability side are:

Equity: The shareholders capital. We have not done any further deviations of sources of equity financing.

Premium reserves: The premium reserve represents the present value of its total insurance liabilities including future management costs according to the individual insurance less the present value of future premiums.

Market value adjustment reserve ("kursreguleringsfond")
Market value adjustment reserves are unrealized profit for stocks. These reserves can cover both the guaranteed return and losses. This applies only to stocks, we assume that the bond portfolio is realized every year.

§ 9-20 Forsikringsvirksomhetsloven
The market value adjustment reserve shall equal the sum of unrealized gains on financial assets included in the collective portfolio, see § 9-7 second paragraph. The market value adjustment reserves shall not be assigned contracts in the collective portfolio.

Additional statutory reserve ("tilleggsavsetninger")
The company sets additional statutory reserves each year. These reserves can cover the guaranteed return, but are not allowed to cover losses.

§ 9-17 Forsikringsvirksomhetsloven
To secure their obligations under contracts with contractual obligations, the company can at the end of each year make additional statutory reserves beyond the minimum premium reserve. The Financial supervisory authority of Norway may, when it finds that solvency considerations indicates this, order the company to make additional statutory reserves.
The additional statutory reserves shall be allocated among the individual contracts.
The company determines the year`s additional statutory reserves, see § 9-9 first paragraph, as a percentage of the premium reserve to each contract. If the return attributable to a contract under § 9-9, first paragraph in a year is not large enough to cover the annual provision required by § 9-16, third paragraph, the provision requirement is fulfilled by transfer of additional allocations assigned to the contract. If the additional statutory reserves allocated to a contract one-year cause the total additional statutory reserves to exceed an amount equal to

12 percent of the premium reserves relating to the contract, the excess amount is assigned to the contract as profit. The total of the premium reserve and additional statutory reserves relating to a contract with contractual obligations may otherwise not be reduced in any other way than by payment by the insured.

Hidden reserve

Hidden reserve is unrealized gains for bonds. For simplicity we are excluding hidden reserves in this thesis.

4.2.1 Profit sharing model

A profit sharing model distributes the allocations of profit for a life insurance company. The distributions of returns between the shareholders, customers, additional statutory reserve and market value adjustment reserve are as follows.

Solvency II and national guidelines describes the distributions of return. Different scenarios dependent on the level of return will affect equity and the customers return. Additional statutory reserve and market value adjustment reserve serves as a buffer to adjust the value for both recipients. Life insurance companies are obliged to give the customers the guaranteed return, g, which is currently 2,5 %.

Life insurance companies have a profit sharing model for allocating profit between policyholders and shareholders. Forsikringvirksomhetsloven § 9-12 states that the shareholders are entitled to up to 20 percent of the profits on investment returns that are assigned to the contract.

$$Market\ value\ adjustment\ reserve_{t+1} = \max(MVAR_t + Unrealized\ gains - Unrealized\ loss, 0) * \pi$$

If the market value adjustment reserves or additional statutory reserves are negative, it is set to zero in our model.

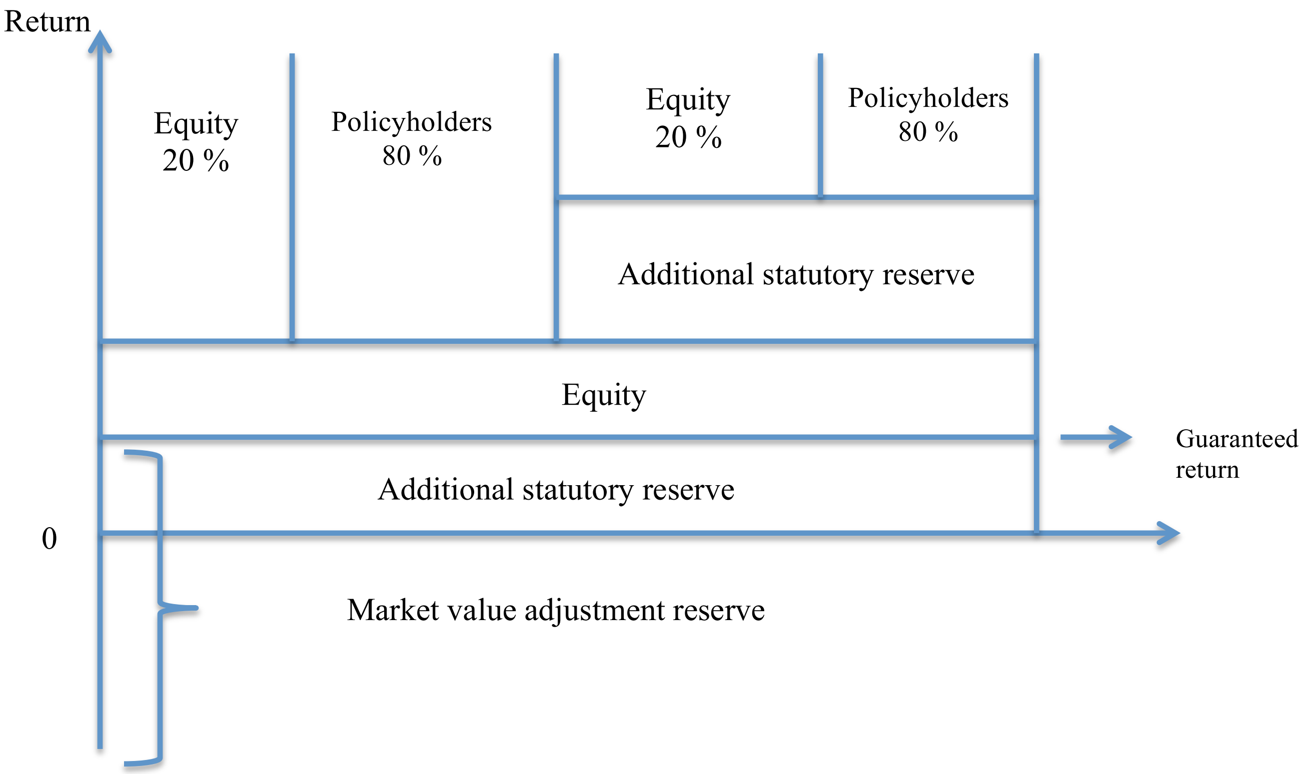
The value of L (customers value) is given in a specific manner:

- $If\ R_p \leq g, L = g$
- $If\ R_p > g, L = g + (R_p - g) * W_L$

If the assets are less than the liabilities, the company is presumed bankrupt, L=A. The return to the shareholders is more complicated. Depending on the level of portfolio return, the additional statutory reserve will be used to cover up the guaranteed return, but is not allowed to cover losses. Market value adjustment will cover up the guaranteed return, plus potentially losses.

The return to shareholder and the evolutions of MVAR and ASR:

Figure 4-2: **Profit sharing model**



Notations in insurance model:

$A_t = Assets\ at\ time\ t$

$E_t = Equity\ at\ time\ t$

$L_t = Liabilities\ at\ time\ t$

$R_{P,t} = Portfolio\ return\ at\ time\ t$

$R_{S,t} = Shareholder\ return\ at\ time\ t$

$g = Guaranteed\ return\ (in\ \% - terms)$

$W_L = Profit\ sharing\ to\ customer\ fund, after\ ASR$

$MVAR_t = Market\ value\ adjustment\ reserve$

$ASR_t = Additional\ statutory\ reserve$

$R_{k,t} = Return\ after\ allocation\ to\ MVAR$

$R_{A,t} = Return\ before\ allocaton = R_p$

$r_{k,t} = Profit\ after\ allocation\ to\ MVAR$

$\pi = Unrealized\ proportion\ of\ profits\ from\ shares, allocated\ to\ MVAR$

$t = Bonus\ for\ shareholders$

***Scenario 1*:** The return is negative, $R_k < 0$

For shareholders: In principle the guaranteed return (L*g) has to be covered from the equity. If they have a market value adjustment reserve fund, they can use from this fund to cover losses up to the guaranteed return. If they have additional statutory reserve, this can be used to cover the guaranteed return between 0 and g. There are no profit to share between customers and shareholders.

$Additional\ statutory\ reserve_{t+1} = \max\ (0, ASR_t - g)L_t$

***Scenario 2*:** The return is positive, but lower than the guarantee,

$0 < R_k < g * A_t$

For shareholders: To cover the guaranteed return, equity is charged if they do not have buffer. If they have a buffer, the insurance company will use their current additional statutory reserve to cover up the guaranteed return. There are no additional profit to share between customers and shareholders. There are no new deposits to the additional statutory reserve.

$Additional\ statutory\ reserve_{t+1} = \max\ (0, ASR_t - (g - r_k)L_t$

Scenario 3: The insurance company meets the guarantee, $R_k = g * A_t$

For shareholders: Return is equal zero, so there are no additional profit to share between customers and shareholders. No movements in MVAR and ASR are necessary if R_k is equal g*A.

***Scenario 4*:** The return is above the guarantee, $R_k > g * A_t$

If the return is above the guaranteed interest, they will use a profit sharing model for allocating profit between policyholders and shareholders, where shareholders is entitled to up to 20 percent of the profits on investment returns that are assigned to the contract. Allocation to additional statutory reserve has to be done before profit sharing between customers and shareholders. In this scenario the insurance company can build up the additional statutory reserve. Customers will get their guaranteed interest, and the insurance company will also be allocating to the additional statutory reserve.

$$Additional\ statutory\ reserve_{t+1} = ASR_t + (1 - W_L - t)(R_k - g * L_t)$$

5 Data, modeling and calibration

5.1 Data

We have used Thomson Reuters Datastream to obtain data for use in this thesis.
There are four time series, 20-year data on a weekly and monthly basis, 10-year data on a weekly and monthly basis. 20-year data starts at 1.4.1994 and ends at 1.4.2014 for the monthly time series, the weekly time series ends at 2.5.2014. 10-year data starts at 1.4.2004 and have the same end date as for 20-year data.

The 10-year time period is justified partly because Norway in 1999 went to an inflation control target system. This time period covers both the boom in the real estate market and the following financial crisis (2008-2009). To a certain degree this time period could be representative for future estimations since the future most likely will consist of both booms and crashes.

The specific time series used are as follows.

Table 5-1 Data

Data	Currency	Symbol (Reuters)
MSCI WORLD U$ - PRICE INDEX	USD	MSWRLD$
OSLO EXCHANGE BENCHMARK - TOT RETURN IND	NOK	OSLOBMI
NORWAY INTERBANK 3MTH (NOMINAL) - MIDDLE RATE	NOK	NWBIK3M
NORWAY INTERBANK 12MTH(NOMINAL) - MIDDLE RATE	NOK	NWBIK12M
NORWAY T BILL 3 MONTH - RED. YIELD	USD	NWTBL3M
NORWAY T BILL 1 YEAR - RED. YIELD	USD	NWTBL1Y
UK INTERBANK 3 MONTH - MIDDLE RATE	UK	LDNIB3M
UK INTERBANK 1 YEAR - MIDDLE RATE	UK	LDNIB12M
US INSTANT. FWD RTE CONT. COMP. 1-30Y - MIDDLE RATE	USD	FRTNF01-FRTNF30

A Norwegian life insurance company invests in both Norwegian and foreign stocks. In our fictive portfolio we are assuming that the majority of stocks consist of foreign stocks.
MSCI World is chosen for the international stocks. We have different types of rates for the short-term interest rate in Vasicek and CIR. NIBOR 3 month is used in our base case, while NIBOR 12 month, T Bill and LIBOR are used for calibration in different scenarios. The data series for T Bills starts in January 2003, we have chosen to use this data in the time series for 20 years, although this is incorrect. These rates are also used as a base for bonds and money

market securities in our portfolio. For the LMM we have used American forward rates, specifically US Instant Forward rate continuous compounding. The forward rate runs from a one-year forward to a thirty-year forward, for a 20-year period. Ideally we would use Norwegian swap rates in LMM, but we were unable to retrieve them from Thomsen Reuters Datastream. The instant forward rates are a good alternative that hopefully would not change any conclusions.

Figure 5-1: Historical development of interest rates, monthly

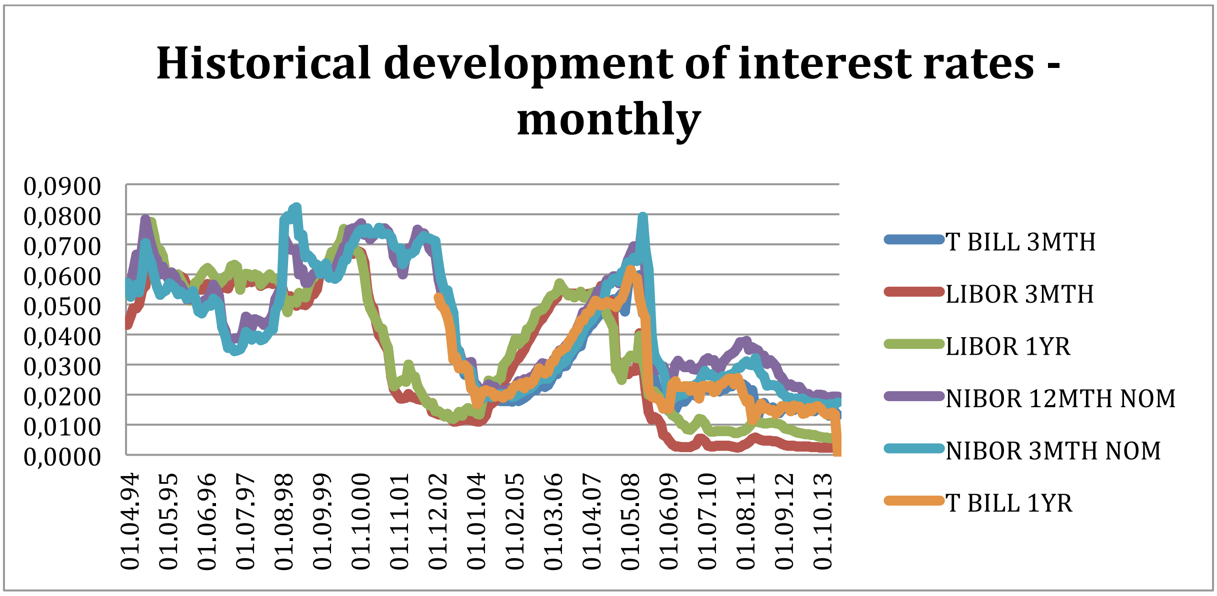
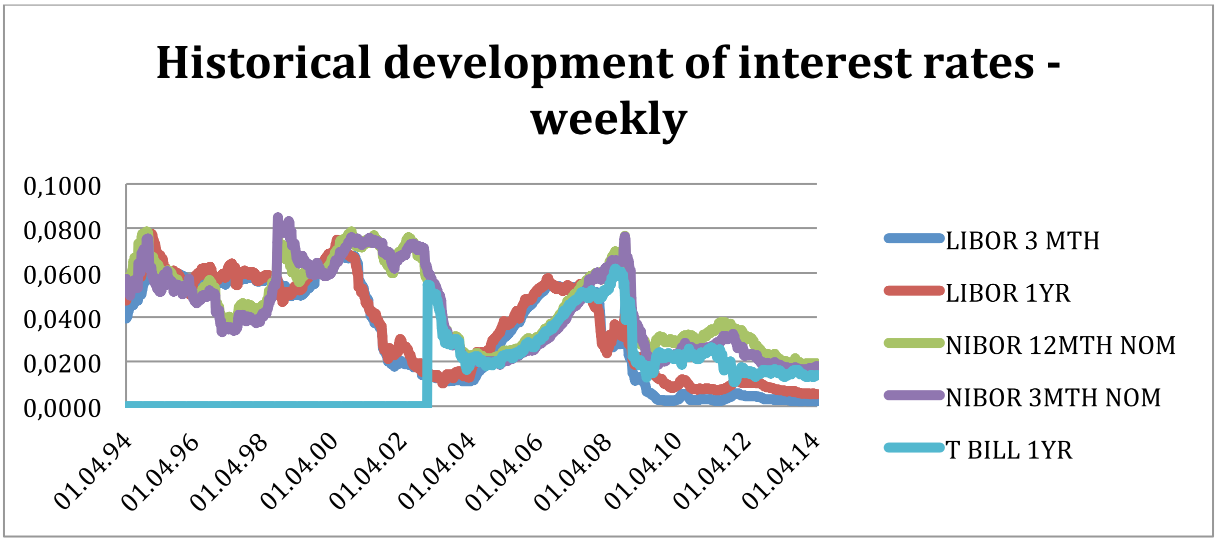


Figure 5-2: Historical development of interest rates, weekly



5.2 Summary of method

Method and procedure used in this thesis are described here.

The short rate models Vasicek and CIR are programed in R for simulation of interest rates for 30 years ahead. The Libor Market Model is used to simulate forward rate and the corresponding spot rates. Theory around this model is described in Chapter 3. The interest rate and spot rates provides the basis for the bonds and money market in our fictive portfolio.

We are also programming a stock model for simulation of stock prices, this is the basis for the stocks in our portfolio. The interest rates created from Vasicek and CIR are used to calculate stock prices, see Formula 3-2 and Formula 3-4. Spot rates are used in LMM. Bond prices are calculated with the corresponding formulas from the models, see Formula 3-3 for Vasicek and Formula 3-5 for CIR. Formula 3-7 is used for LMM. We assume constant bond duration of 6 years. This is achieved by holding the bond for one time period before we sell it, and buy a new bond with duration of 6, resulting an approximation of a constant duration.

After we have simulated the stock prices, short rates and bond prices, we calculate the return for the assets.

$$r_t^s = ln\frac{S_{t+1}}{S_t}$$

Formula 5-1

$$r_t^{sr} = r_t$$

Formula 5-2

$$r_t^b = ln\frac{B_{t+1}}{B_t}$$

Formula 5-3

where r_t^s is stock return, S_t is stock price at time *t,* r_t^{sr} is short rate return, r_t is rate at time *t,* r_t^b is bond return and B_t is bond price

Portfolio return is calculated with this formula

$$r_p = LN(e^{r_t^s} * \omega_s + e^{r_t^{sr}} * \omega_{sr} + e^{r_t^b} * \omega_b)$$

Formula 5-4

where, r_p is the portfolio return, ω is weight

The returns are further distributed through the profit sharing model described in chapter 4. We use this formula for calculations of stock profit

$$profit\ stock_{t+1} = A_t * \omega_s * (e^{r_t^s} - 1)$$

Formula 5-5

where, A_t is assets at time t

Stock profit is used to calculate MVAR, as described in chapter 4.

For calculations of ASR, we compute "profit invest", this is our return after allocation to MVAR ($R_{k,t}$) in the profit sharing model

$$profit\ invest_t = A_{t+1} - A_t - (MVAR_{t+1} - MVAR_t)$$

Formula 5-6

where, $MVAR_t$ is the market value adjustment reserve at time t

Profit invest is then the basis used to distribute returns to the ASR fund.

At the end of the estimation period, any value left in MVAR and ASR fund are left for the policyholders. If A in a year is less than L, a bankruptcy has occurred and E is set to zero. We are allowing the company to reappear with value in a later year. This is not reasonably in real life, and fortunately this miss specification did not affect the results, since the number of bankruptcies was low. The simulated values for assets, liabilities, and equity at time **t** ***are discounted back with the simulated corresponding interest rate or spot rate at time*** **t-1.** ***MVAR and ASR are added to the liabilities. Solvency Capital Requirement and Required capital are than calculated on the basis of formula***

Formula 3-13 and

Formula 3-14.

5.3 Calibration of Vasicek

There are four parameters that need to be specified in the Vasicek model: theta, kappa, beta and the start rate R_0. For calibration of these parameters we will use inputs from historical data. We used 3 month NIBOR (monthly basis, 10 years time series) in our base case scenario. We have calculated interest rate changes this way

$$r_{t+1} - r_t$$

Formula 5-7

There are different methods for calibrating the model parameters in Vasicek, in this thesis we have used the least square regression method (Brigo 2007). Another method is the use of autoregressive models.

$$y = ax + b$$

Formula 5-8

To perform regression analysis (Allison 1999), we have used excel. a is the slope, this is our kappa (k). b is the intercept.

The relationship between the following observations r_t and r_{t+1} is linear

$$r_{t+1} - r_t = (a-1)r_t + b + \varepsilon$$

Formula 5-9

Calibration with least square regression for kappa, theta and beta

Kappa (k):

Kappa is the speed of mean reversion (drift). Kappa is calculated as the slope in the regression, where the interest rate change is the independent variable and the interest rate is the dependent variable. To obtain annual value, kappa is multiplied with 12.

Theta (θ):

Theta is the mean reversion parameter; this is the long-term mean that the short-term interest rate in Vasicek will drift to in the future. This formula can be used for calibration of theta

$$\theta = -\frac{b}{a}$$

Formula 5-10

For simplicity reasons we have decided to keep theta constant at the same level as the start rate.

Beta (β):

Beta is the interest rate standard deviation. It is estimated as the standard deviation of the interest rate changes less the drift (kappa).

$$Interest\ rate\ change_t = Interest\ rate\ change_{t+1} - k(\theta - rate_t)$$

Formula 5-11

Monthly standard deviation is calculated as the standard deviation of the new interest rate changes. Annual standard deviation is calculated with this formula:

$$\sigma = sd(\varepsilon)\sqrt{\frac{-2\ln a}{\delta(1-a^2)}}$$

Formula 5-12

Start rate (R_0):

The NIBOR 3 month at 1.4.2014 is 0.7246. As stated earlier, the Vasicek model allows negative interest rates. This is historically an unusually low rate, which will cause the Vasicek model to produce several negative interest rates. The start rate is therefore set to 0.04. In our simulations negative interest rates are set to zero, and than allowed them to further drift to the long-term mean.

5.4 Calibration of CIR

There are four parameters in CIR that needs to be calibrated: theta, kappa, beta and the start rate R_0. For calibration of these parameters we will use inputs from historical data. We used the same data set as for Vasicek. Interest rate changes are calculated in this way

$$\frac{r_{t+1} - r_t}{r_t^{0.5}}$$

Formula 5-13

Calibration with least square regression for kappa, theta and beta

Kappa (k):

Kappa is the speed of mean reversion (drift). This parameter is calculated in the same manner as for Vasicek. The difference is how interest rate changes are calculated.

Theta (θ):

Theta is the mean reversion parameter; this is the long-term mean that the short-term interest rate in CIR will drift to in the future. To calibrate the mean reversion parameter the formula used for Vasicek may be used.

Beta (β):

Beta is the standard deviation. The drift had to be subtracted from the interest rate changes before the standard deviation was calculated. This formula differs from Vasicek

$$Interes\ rate\ change_t = Interest\ rate\ change_{t+1} - k(\frac{\theta}{r_t^{0.5}} - r_t^{0.5})$$

Formula 5-14

Monthly standard deviation is calculated as the standard deviation of the new interest rate changes. Annual standard deviation is calculated with the same formula as for Vasicek.

Start rate (R_0):

The start rate is the same as for Vasicek, the entrance parameter is set to 0.04. CIR does not produce negative interest rates. For a better comparison of the models, the interest rate, R_0, is set at the same value as for Vasicek.

Table 5-2: Vasicek and CIR parameters base case

NIBOR 3MTH (2004-2014)	Vasicek		CIR	
	Monthly	Annual	Monthly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,052927	0,183344	0,052927	0,183344
Std. Dev rate	0,003065	0,010728	0,014934	0,053721
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,020799	-0,249588	-0,073514	-0,882174

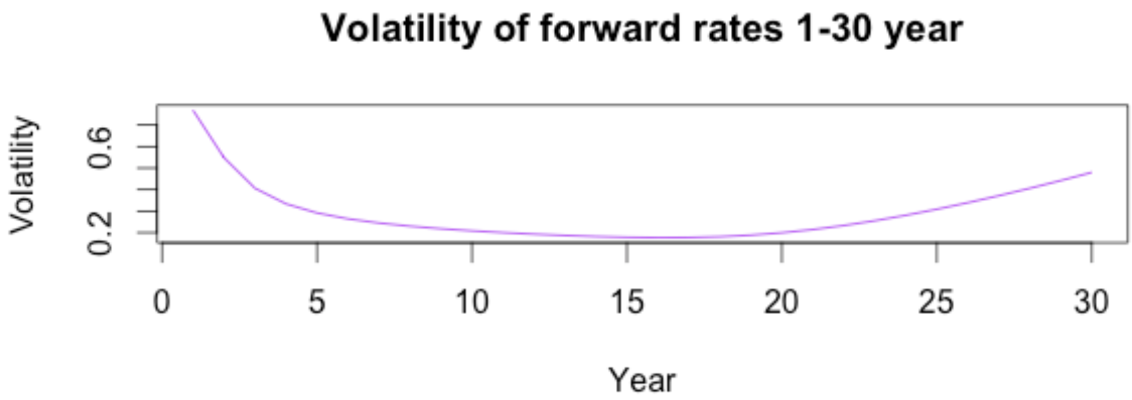
5.5 Calibration of the Libor Market Model

There are two parameters in the LMM that needs to be specified: volatility and correlation. We used forward rates from year 1-30, data are on a monthly basis and a 20-year time series for our base case.

5.5.1 Calibration with historical volatility of forward rates

Volatility is estimated from historical data. It is estimated as the standard deviation of the log changes to the forward rates. We assume that the volatility takes the form as mentioned in specification 1 in chapter 3. This means that the volatility has the form (i-t), which means that the volatility observed at time *t* for the forward rate running between time *i* and time *i+1* only depends on the distance between *t* and *i*, and not by the values of *i* and *t*. Monthly and weekly data are converted to annual values by multiplying by the square root of 12 and square root of 52.

Figure 5-3: Historical volatility of forward rates



The volatility is highest for the 1-year forward, it then decreases until the 16-year forward, before it increases for the remaining forwards. We have used the whole standard deviation curve in the simulations. An alternative would be to use the volatility for the 16-year forward for the remaining forwards.

5.5.2 Calibration with historical correlations of forward rates

We have computed a matrix of historical correlations of forward rates. This matrix of historical correlations was computed by using monthly US forward rates. The time series starts at 1.4.2004 and ends at 1.4.2014. We calculated correlations between the rates of return from the US forward rates. One assumption taken is that the correlation between forward rates and stocks is zero. Correlation matrix can be seen in the Table 13-1: Correlations of forwards 1-30 Years and Table 13-2 Standard deviations forwards.

5.6 Calibration of stock model

There are parameters in the stock model that have to be calibrated: correlation and standard deviation.

Calibration with historical standard deviation of stocks

We calculated the logarithmic return for MSCI World, and the volatility of stocks is the standard deviation to the logarithmic return.

Table 5-3: Standard deviation stock

	Vasicek		**CIR**	
	Monthly	Annual	Monthly	Annual
Std. Dev stocks	0,052927	0,183344	0,052927	0,183344

Calibration with historical correlation of stocks, bonds and short rates

Correlation is calculated between stocks (MSCI World), bonds (NIBOR 12MTH) and short rates (NIBOR 3MTH). Correlation is different for Vasicek and CIR, since the interest rate changes are calculated different for Vasicek and CIR.

Table 5-4: Correlation Vasicek stocks - bond -short rate

Correlation	**MSCI World**	**NIBOR 3MTH NOM**	**NIBOR 12MTH NOM**
MSCI World	1	-0,046657	0,127868
NIBOR 3MTH NOM		1	0,895886
NIBOR 12MTH NOM			1

Table 5-5: Correlation CIR stocks - bonds- short rate

Correlation	MSCI World	NIBOR 3MTH NOM	NIBOR 12MTH NOM
MSCI World	1	-0,077017	0,100149
NIBOR 3MTH NOM		1	0,853904
NIBOR 12MTH NOM			1

5.7 Entry allocations

For the base case asset allocation in the entry balance sheet, we have looked at two major insurance companies in Norway: Storebrand and DNB Life Insurance. The accounting data are obtained from their annual reports and the market data published by Finance Norway ("Finans Norge"). In addition we have studied accounting data for all Norwegian life insurance companies (TNC) also published by Finance Norway, as a comparison and reference point. The Data is from 2013 and 2012.

5.7.1 The Assets side

The asset allocation in 2013 and 2012 in the relevant companies are presented in the table under:

Table 5-6: Asset allocation 2012 and 2013

	Storbrand		DNB Life Insurance		Total of Norwegian Companies	
	2012	**2013**	**2012**	**2013**	**2012**	**2013**
Stocks	7 %	6 %	7 %	11 %	11 %	13 %
Bonds fixed return	32 %	29 %	36 %	30 %	31 %	30 %
Bonds HTM	40 %	39 %	37 %	44 %	39 %	39 %
Money market	1 %	7 %	1 %	1 %	1 %	2 %
Loans	3 %	3 %	2 %	2 %	3 %	3 %
Real Estate	18 %	16 %	17 %	11 %	15 %	14 %
Total	100 %	100 %	100 %	100 %	100 %	100 %

Real estate are mainly reported as subsidiaries in Finans Norge

For simplicity reasons we have excluded real estate and loans, so that only stock, bonds at amortized cost, HTM bonds and money markets are relevant for our base case. Which led to a new distribution:

Table 5-7: New asset allocation 2012 and 2013

	Storebrand		DNB		Total all companies		Mean	Mean
	2012	**2013**	**2012**	**2013**	**2012**	**2013**	**2012**	**2013**
Stocks	16 %	14 %	15 %	17 %	19 %	20 %	17 %	17 %
Bonds	81 %	77 %	81 %	80 %	77 %	75 %	80 %	78 %
Money markets	3 %	9 %	3 %	3 %	4 %	5 %	3 %	6 %
	100 %	100 %	100 %	100 %	100 %	100 %	100 %	100 %

Asset allocation in both companies and TNC are approximately around the same levels. We use the national level as a comparison for the estimation of the entry level. We will assume that all bonds are available for sales bonds with a given duration specified below. Since real estate is excluded from this thesis, we distribute real estate 50/50 between stocks and bonds. Other loans are considered as money markets investments. These assessments give this distribution:

Table 5-8: Asset allocation base case

Stocks	16 %
Available for sales Bonds	79 %
Money markets	5 %
Total	100 %

5.7.2 The Liability side

The same data source for the entry levels for the liability side is used. Our focus is limited to Equity ASR, MVAR and the Liability/premium reserves. Historical numbers for 2013 and 2012 are listed in the table below to give an impression of a normal level in Norway.

Table 5-9: Historical liability allocations

	Storebrand		DNB Life insurance		Total of Norwegian Companies	
	2012	**2013**	**2012**	**2013**	**2012**	**2013**
Equity	16 122 (6%)	17 849 (6.1%)	17 892 (7.4%)	20 096 (8.1%)	59 781 (6%)	66 723 (6.1%)
ASR	4 874 (1.8%)	4 916 (1.7%)	5 489 (2.3%)	4 279 (1.7%)	25 464 (2.6%)	24 786 (2.3%)
MVAR	1 085 (0.4%)	2 735 (2.3%)	1 027 (0.4%)	3 823 (1.5%)	15 304 (1.5%)	24 740 (2.3%)
Liability	207 705 (76.9 %)	216 271 (74.4%)	162 269 (67.4%)	165 873 (66.6%)	729 360 (73%)	775 821 (71.4%)
***Total**	270 061	290 652	240 595	249 227	990 507	1 086 264

*The total is the of the total balance of the companies

The percentage share of the total portfolio is listed in parentheses, when assuming the liability side only consists of those four elements.

Table 5-10: Liability allocations respectively to 100 %

	2012	2013	2012	2013	2012	2013
Equity	7,00 %	7,40 %	9,60 %	10,40 %	7,20 %	7,50 %
ASR	2,10 %	2,00 %	2,90 %	2,20 %	3,10 %	2,80 %
MVAR	0,50 %	1,10 %	0,60 %	2,00 %	1,80 %	2,80 %
Liability	90,40 %	89,50 %	86,90 %	85,50 %	87,90 %	87,00 %
	100 %	100 %	100 %	100 %	100 %	100 %

Based on these allocations we have chosen these levels for our base case:

Table 5-11: Entry levels liability side, base case

Equity	ASR	MVAR	Liability
7 %	2.5 %	2.5 %	88 %

5.7.3 Duration

The portfolios bond duration, defined as a measure of how long the bondholder has to wait before receiving back the investment (Hull 2012), are set at Storebrand and DNB's past two years level. In their annual reports of 2013 the bond duration of their portfolio for 2012 is

5.86 and 5.12. For 2013 it was 6.02 and 5.92. Based on this, we set the average duration to 6 for our base case.

6 Results

In this chapter we will present results from our analysis. The main features are that the various interest rate models provide different results for the required capital. We find that calibration and selection of parameter has a surprisingly great impact on the results.

First, we will review our standard base case for our three models, and then we will look at analysis with calibration of different interest rates, meaning different time periods and different frequencies of the data. Results from the three scenarios, base case, stress scenario for interest rates and stress scenario for stocks, in the tables below. We present figures for the results from the three models. The figure **"Development of Assets, Equity and Liabilities"** ***shows the evolution of assets, liabilities and equity over the 30 years estimation period. This is calculated with mean values of the 100 000 simulations. The LMM is not a mean reversion model, and some of the simulation gives unnaturally high forward - and spot rates, so it is not quite correct to use the mean for this model. The figure*** **"SCR and Required Capital"** ***shows the SCR for interest rates, SCR for stocks, total SCR and Required capital. See***

Formula 3-13 for calculation of SCR and

Formula 3-14 for calculation of required capital. Mostly of the figures that show development of assets, equity and liabilities for the stress scenarios are shown in the appendix. The graphs in these figures are calculated with the results from the stress scenarios.

We have chosen to look at selected results and show these results here, results that are not shown in this chapter are included in the appendix.

6.1 Standard Base case

Our standard base case for Vasicek and CIR is the 3 month Nibor with a time series of 10 years, on a monthly basis with 100 000 simulations. Standard base case for LMM is time series of 10 years on a monthly basis, with 30 000 simulations (ideally, 100 000 here to, but due to capacity problems only 30 000 is used). Entry levels used are described in chapter 5. Parameters can be seen in table 6.1.

Table 6-1: Parameters Vasicek and CIR - Base case

NIBOR 3MTH (2004-2014)	Vasicek		CIR	
	Monthly	Annual	Monthly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,052927	0,183344	0,052927	0,183344
Std. Dev interest rate	0,003065	0,010728	0,014934	0,053721
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,020799	-0,249588	-0,073514	-0,882174

After estimating the value development of assets, equity and liability 30 years ahead, and discounted it back with the corresponding interests rates at the start of each period. We should expect the value of assets to be exactly what we started with, 100. We see that the models produce major differences in the value of equities, liabilities, the SCR levels and required capital.

If deviations from assets =100, we can use a statistical measure to see what we could expect in accordance with the portfolio risk. If we assume that the returns are normally distributed, we can use portfolio theory to calculate the portfolio standard deviation, confidence interval and standard error. Normal portfolio standard deviation is calculated this way (Brealey 2011).

$$\mathrm{Std} = \sqrt{\mathrm{W}_\mathrm{i}^2 * \mathrm{Var}_\mathrm{i}^2 + \mathrm{W}_\mathrm{j}^2 * \mathrm{Var}_\mathrm{j}^2 + (2 * \rho_{\mathrm{i,j}} * \mathrm{W}_\mathrm{i} * \mathrm{W}_\mathrm{j} * \mathrm{Var}_\mathrm{i} * \mathrm{Var}_\mathrm{j})}$$

Formula 6-1

Where, W i and j is the ratio of stocks and bonds, Var^2 is the standard deviation and ρ is the correlation parameter. We use an approximation to calculate the standard deviation to bonds, by multiplying the interest rate times the bond duration. Further,

$$\text{Standard error} = \frac{Std}{\sqrt{N}} * \sqrt{T}$$

Formula 6-2

Used with a confidence level, an upper and lower interval for the acceptable margin of error is achieved.

6.1.1 Vasicek

Table 6-2: Results Vasicek Base Case

Vasicek			
A	100,9412	**E**	5,9811
		L	94,9600
	100,9412		100,9412

SCR base case	1,0189

Stress scenario interest rates			
A	106,5180	**E**	5,4436
		L	101,0744
	106,5180		106,5180

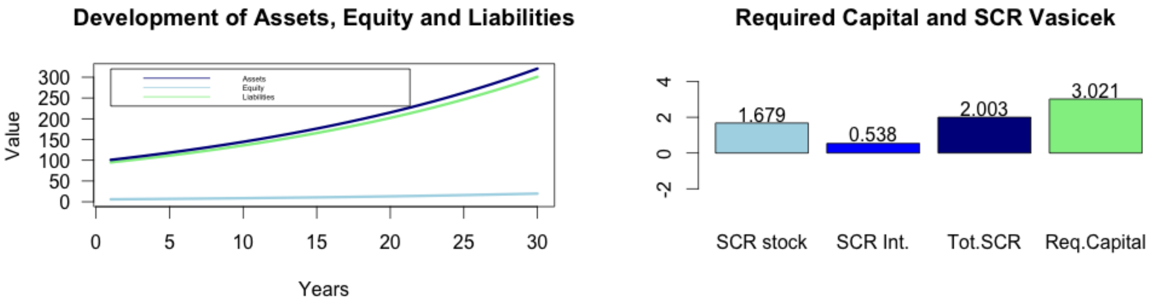
SCR stress case	0,5376

Stress scenario stocks			
A	93,6787	**E**	4,3022
		L	89,3764
	93,6787		93,6786

SCR stress case	1,6789

Total SCR	2,0026
Required capital	3,0215

Figure 6-1: Results from Vasicek Base Case



In the base case the portfolio SD is:

$$\text{Std} = \sqrt{0{,}16^2 * 0{,}18334^2 + 0{,}79^2 * (0{,}010728 * 6)^2 + 2 * 0{,}16 * 0{,}18334 * 0{,}79 * (0{,}010728 * 6) * (-0{,}046657)} = 0{,}0575$$

$$\frac{0{,}0575}{\sqrt{100000}} * \sqrt{30} = 0.000996$$

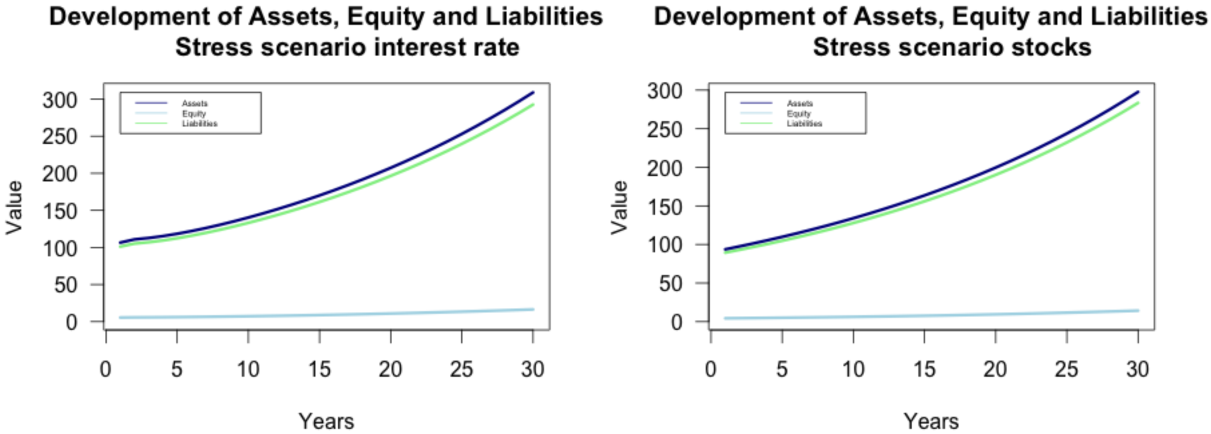
Inputs are, duration of 6, assets allocations, 5 % in risk free asset are zeroed out. ρ is the correlation between, MSCI World and monthly 3 month Nibor 10 years.

If we use a 99 % confidence level we get an interval for the value of assets between [100-0.000996*2.58 ; 100+ 0.000996*2.58] = [99.9990 ; 100.0009].

With 100 000 simulations, this leaves a minimal margin of error. The value of assets in the base case is 100.9412. Unfortunately, beyond the margin of error. We observe that our results are over the limit, but assume the error is small enough to pass. This can have several explanations. Especially assumptions of normal distributions of returns may have explanatory power. The Vasicek models shortcomings when it comes to valuing bonds may have influenced the deviation since the bond ratio is 79 %, or the deviation may originate from randomness.

When the Vasicek model is used, we see that after one year the value of equity is reduced from 7.00 to 5.98, a value reduction of 14.56 %. We see that the reduction of the value of equity is more than one could expect, but with a 16 % stock ratio it is possible.
Liabilities increase with 2.1 % from 93 to 94.96. We see that standard base case parameters have a positive effect for the policyholder at the expense of shareholders.

Figure 6-2: Results from stress scenarios Vasicek



The stress scenario for interest rates causes the value of assets to increase. Value of equity is reduced with 8.98 %, whereas value of liabilities has an increase of 6.43 %. When interest rates fall, bond prices will increase. This will in turn lead to an increase in value of liabilities. Interest rate changes will influence the assets value substantially, especially through the bonds. The stress scenario individually contributes with 0.5375 to the SCR.

There is a decrease in assets when conducting stress scenario for stocks. There is a reduction in the value of both equity and liabilities, but relatively the shareholders take the largest loss. With a loss of 28 % for the value of equity compared with 5.87 % reduction in the liabilities. The 45 % fall in stock prices contributes with 1.6789 to the SCR.

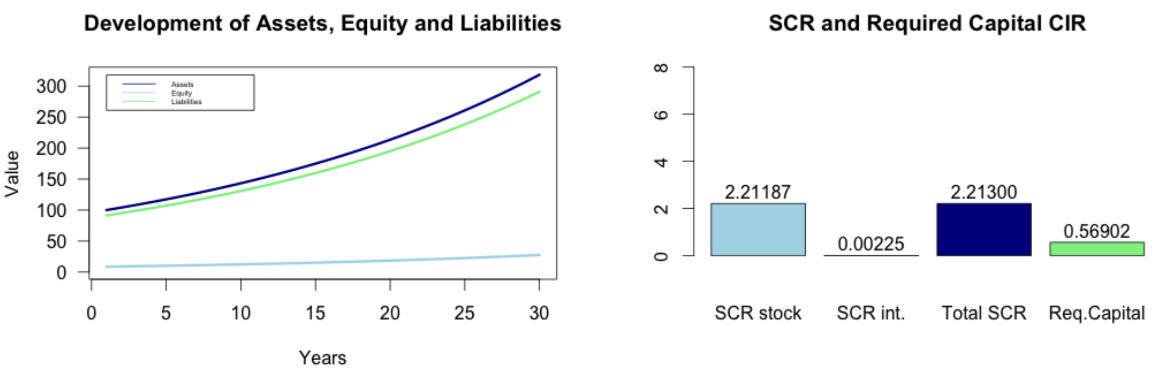
The Vasicek model gives a required capital, which is higher than SCR. Since the value of equity is reduced, it will lead to a higher required capital. The life insurance company should have required capital in addition to book equity to cover the risk of the company. SCR represents the risk the insurance company has in their portfolio, this risk increases when equity decreases.

6.1.2 CIR

Table 6-3: Results from CIR - Base Case

CIR			
A	99,99227	E	8,643973
		L	91,34829
	99,99227		99,992263
E.value		-1,643973	
Stress scenario interest rates			
A	102,0735	E	8,641719
		L	93,4318
	102,0735		102,073519
VaR stress case		0,002253	
Stress scenario stocks			
A	92,79579	E	6,432105
		L	86,36369
	92,79579		92,795795
VaR stress case		2,211868	
Total VaR		2,212996	
SCR		0,5690231	

Figure 6-3: Results from CIR



CIR model gives a more precise discounting of the assets. The assets are valued at 99.992, which is above our margin of error with 0.003, something we will accept. If we had used a different random numbers in the model, we might have gotten a result within our margin of error.

$$\text{Std} = \sqrt{0{,}16^2 * 0{,}18334^2 + 0{,}79^2 * (6 * 0{,}056121)^2 + 2 * 0{,}16 * 0{,}18334 * 0{,}79 * (6 * 0{,}056121) * (-0{,}077017)}$$
$$= 0.2653$$

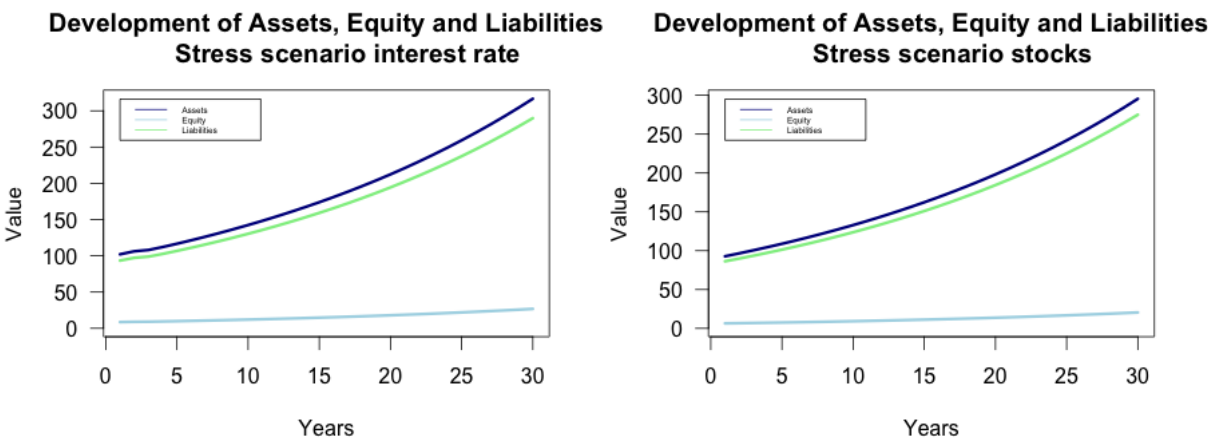
$$\frac{0.2653}{\sqrt{100000}} * \sqrt{30} = 0.00459$$

Inputs are: duration of 6, assets allocations, 5 % in risk free asset are zeroed out. ρ is the correlation between, MSCI World and monthly 3 month Nibor 10 years.

When we use a 99 % level, we get a confidence interval for the value of assets between [100-0.00459*2.58 ; 7+ 0.00459*2.58] = [99.995 ; 100.0045]

The calibration of CIR shows an increase in the value of equity of 23.49 %, while the value liabilities have a reduction of 1.78 % compared to the standard base case.

Figure 6-4: Results from stress scenario CIR



The stress scenario for interest rates do not have a large effect on the balance sheet compared to the base case. This is an unexpected finding, and ideally we should have done a stress scenario where the interest rates rises. Assets and liabilities have a slight increase in value, while value of equity has increased slightly. Value of assets and liabilities are going up because of the fall in interest rates, as a result of the bonds increase in value.

The stress scenario for stock with CIR causes a fall in equity of 25.58 %, and a 5.45 % reduction in the liabilities. This scenario accounts for most of the total SCR with 2.211.

From CIR we get a required capital, which is smaller than SCR, the reason for this that the market value of liabilities is higher than book value. When the life insurance company has a high value of equity, they are better positioned to cover the risks the company is exposed to.

6.1.3 Libor Market Model

Parameters used in the LMM can be seen in Table 13-1: Correlations of forwards 1-30 Years in appendix D.

Table 6-4: Results from LMM Base case

LMM

A	101,2352	**E**	2,8096
		L	98,4256
	101,2352		101,2352
SCR base case		4,1904	

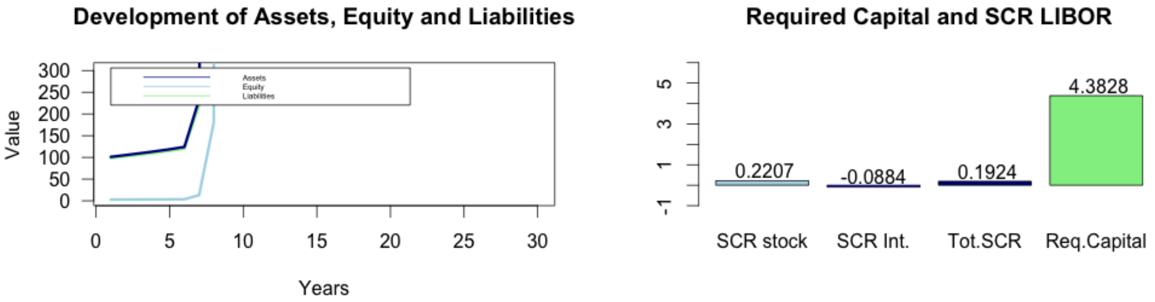
Stress scenario interest rates

A	106,0747	**E**	2,8980
		L	103,1767
	106,0747		106,0747
SCR stress case			-0,0884

Stress scenario stocks

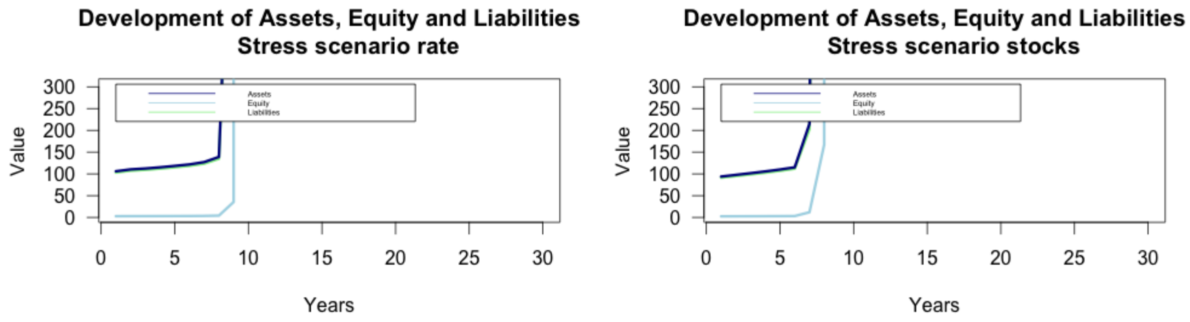
A	93,9450	**E**	2,5889
		L	91,3562
	93,9450		93,9450
SCR stress case			0,2207
Total SCR			0,1924
Required capital			4,3828

Figure 6-5: Results from LMM Base case



The LMM has a reduction of 59.86 % in value of equity, while there is an increase in the value of liabilities by 5.83 %.

Figure 6-6: Results from stress scenario LMM



Assets, liabilities and equity have an increase in value in the stress scenario for interest rates, but the increase in value of equity is minimal compared to equity in the base case. Stress scenario for stocks provides a decrease in the whole balance sheet, where the value of liabilities has the greatest change.

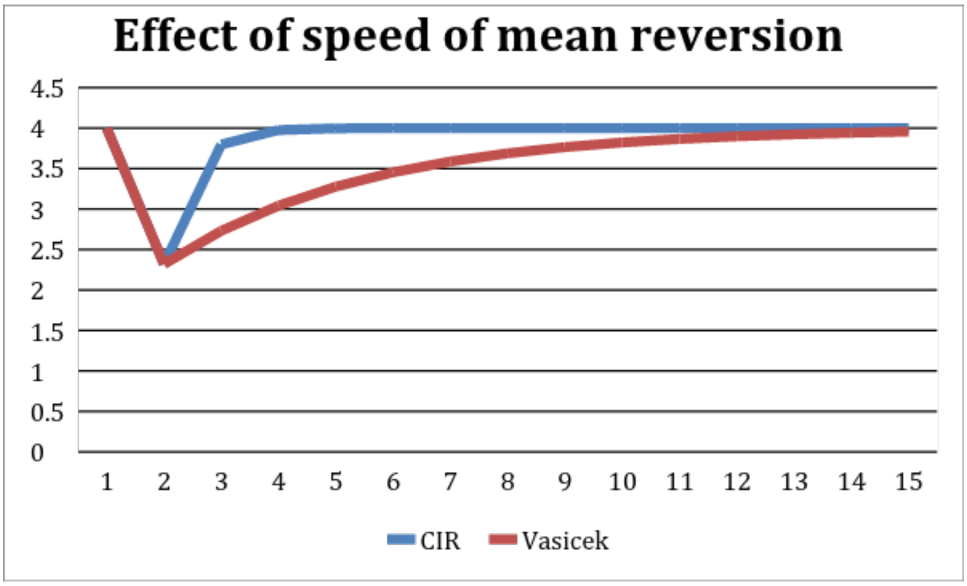
Between year 5 and 10 all the simulations in LMM produces enormous forward and corresponding spot rates, unthinkable high. This could jeopardize the model, but we use the corresponding spot rates to discount the value back, so we end up with a reasonably value in year one. Our first thought of the gigantic spot rates was model error or unreasonably results. From another point of view we ask the reader to think back to year 1900 in Germany. Who would then say that within the next 45 years, Germany would experience 2 world wars, depression and a rocket sky-high inflation of approximately 25 billons. So when LMM suddenly produces one spot rate on several thousand out of 30 000 simulations it is hopefully unthinkably, but not totally unreasonable.

Required capital is higher than SCR for the LMM. The value equity is small in the first scenario, base case, and the stress scenarios do not affect the value of equity significant since it was already low. SCR is as mentioned calculated from the value of equity in the base case. When the life insurance company has a low value of equity it causes the in addition to book equity to increase. The reason for this is that the company should have capital sufficient to cover the risk

6.1.4 Comparison of the models

The three interest rate models give different results. This may be due to differences in the actual models or calibration of them. The LMM is calibrated on different terms compared to the other two models. The calibration is done on the basis of other data, and the calibration is also different. There are differences between the parameters, including standard deviation and speed of mean reversion (kappa), used in Vasicek and CIR. As shown in table 6.1 the standard deviation for CIR is much higher than for Vasicek. The speed of mean reversion is also much stronger for CIR compared to Vasicek, which affects the results. Since the speed of mean reversion determines how fast the interest rate drifts against the long term mean, the major differences in kappa between Vasicek and CIR explain much of the differences in values. As we see from figure 6.7**,** after a stress scenario, CIRs large kappa means that this model will more quickly reach back to the interest rate long term mean, almost 3.5 times as fast.

Figure 6-7: Effect of speed of mean reversion



CIR produces higher equity for all scenarios compared to Vasicek and LMM, while the LMM has higher liabilities for all scenarios. LMM is therefore a more favorable model for policyholders. Seen from the shareholders' side is this model not especially beneficial, due to the low equity.

Of the three models it is LMM, which has the highest value of required capital, while CIR has the highest SCR. Results from the standard base case shows that CIR has a higher value of equity, which leads to the smaller required capital. The life insurance company has already a high value of equity, and does not have the same need for in addition to book equity.

6.2 Results with different calibration inputs

6.2.1 3 month Nibor – weekly 10 year

Parameters used in the analysis with weekly 3 month Nibor for a 10 year time period are shown in table 6.5. We wanted to see if it gave impact on the results if we used weekly data instead of monthly data.

Table 6-5: Parameters 3 month Nibor - Weekly

NIBOR 3MTH (2004-2014)	Vasicek		CIR	
	Weekly	Annual	Weekly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,026122	0,188369	0,026122	0,188369
Std. Dev interest rate	0,001151	0,008315	0,006945	0,050513
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,003129	-0,162709	-0,017047	-0,886428

Table 6-6: Results Vasicek 3m Nibor Weekly

Vasicek

A	101,3077	**E**	6,568282
		L	94,73945
	101,3077		101,3077
SCR base case		6,568282	

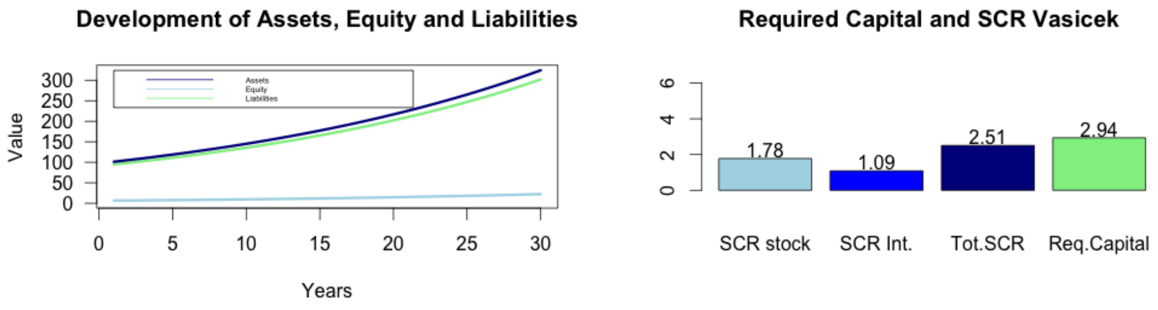
Stress scenario interest rates

A	108,1771	**E**	5,481427
		L	102,6956
	108,1771		108,177027
SCR stress case		1,072603	

Stress scenario stocks

A	94,02151	**E**	4,789219
		L	89,23229
	94,02151		94,021509
SCR stress case			1,779062
Total SCR			2,505972
Required capital			2,937691

Figure 6-8: Results Vasicek 3 m Nibor Weekly



The weekly data has a higher standard deviation for interest rates, but a smaller kappa. The main differences we find when using weekly instead of monthly data is rather large in some of the results. Value of equity is higher for the weekly scenario, it increases with 9.8 % compared to the standard base case. SCR increases by over 25 %. Required capital is smaller for the weekly scenario.

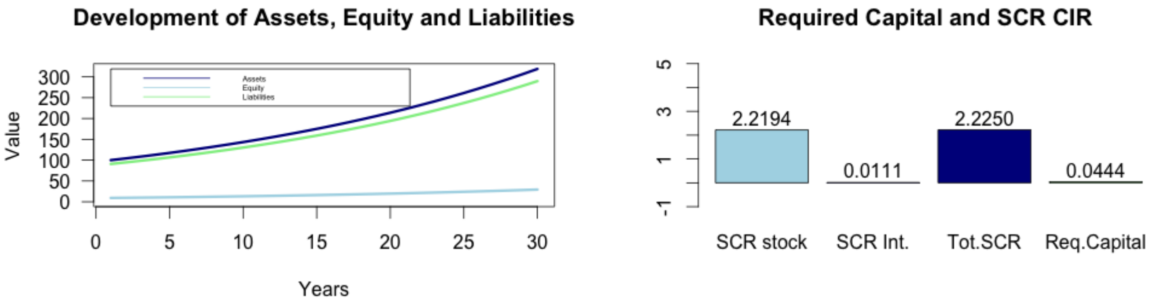
Values of equity and liabilities in the stress scenarios have only some small changes. Appendix B 1 shows the stress scenarios development. When we perform stress on the interest rate, we see the effect of the lower kappa. The asset value increases more with weekly data, because the kappa causes the interest rate to stay smaller for a longer time, which is favorable for the bonds.

Seen from the shareholders' perspective it would be beneficial to calibrate this model with weekly data, since the market value of equity is higher and the required capital is reduced. It is an interesting finding that the required capital decreases even though the portfolio risk increases.

Table 6-7: Results CIR 3 m Nibor Weekly

	CIR		
A	99,99312	**E**	9,180612
		L	90,8125
	99,99312		99,993112
SCRbase case			-2,180612
	Stress scenario interest rates		
A	102,07	**E**	9,169482
		L	92,90048
	102,07		102,069962
SCR stress case			0,01113023
	Stress scenario stocks		
A	92,79774	**E**	6,961213
		L	85,83653
	92,79774		92,797743
SCR stress case			2,219399
Total SCR			2,224985
Required capital			0,04437342

Figure 6-9: Results CIR 3 m Nibor Weekly



There are no major differences in the parameters used in this scenario compared to the standard base case. The final balance is almost identical when we compare the monthly and weekly scenarios. When weekly data are used on the CIR model, the main differences are changes in value of equity in the base case and capital requirements. Value of equity increase with 6.25%, and the capital requirement increase with as much as 52 %, compared to the standard base case. Figure Appendix B 2 in the appendix show the stress scenarios development.

It is interesting to see that the equity and liabilities values change more than expected. We would not expect any relative change since the parameters only have small differences. CIR calibrated with weekly data is also advantageous for shareholders, value of equity increases and the required capital decreases. Seen from the policyholders' side it is more beneficial to use weekly data in the calibration of CIR, as this provides higher liabilities.

6.2.2 3 month Nibor monthly - time period 20 years

Another comparison we did was to see what impact a longer time period would have on the results. Parameters used in the analysis with monthly 3 month Nibor for a 20 years time period are shown in table 6.7.

Table 6-8: Parameters 3 m Nibor 20 years

NIBOR 3MTH (1994-2014)	Vasicek		CIR	
	Monthly	Annual	Monthly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,047876	0,165849	0,047876	0,165849
Std. Dev interest rate	0,003381	0,011839	0,015154	0,053586
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,021249	-0,254992	-0,040529	-0,486343

Table 6-9: Results Vasicek 3 m Nibor 20 years

Vasicek

A	100,9227	**E**	5,4312
		L	95,4916
	100,9227		100,9227

SCR base case	1,5688

Stress scenario interest rates

A	106,3877	**E**	4,8968
		L	101,4909
	106,3877		106,3877

SCR stress case	1,5691

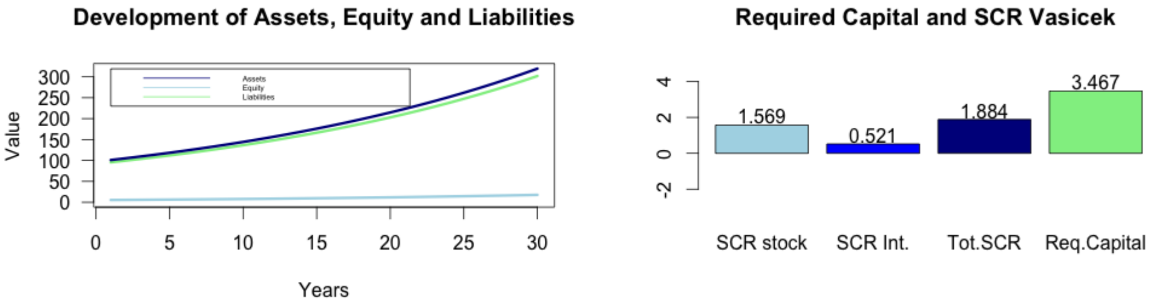
Stress scenario stocks

A	93,6893	**E**	3,8483
		L	89,8409
	93,6893		93,6892

SCR stress case	1,5691

Total SCR	1,8937
Required capital	3,4763

Figure 6-10: Results Vasicek 3 m Nibor 20 years



There are no major differences in the parameters used here contra the monthly case. Although the final balance sheet is almost identical to the standard base case, there are differences for values of equity and liabilities, this could be due to coincidences.

When we double the time period used to calibrate the parameters for Vasicek, the value of equity decreases with 9 %. Value of equity is decreasing in all scenarios compared to the monthly case, this means that the riskiness of equity increases, and thereby raises the required capital. The life insurance company would need in addition to book equity to cover the risk. The SCR contribution from the stress case for interest rate triples and the capital requirements increases by 13 %. Figure Appendix B 3 in the appendix show the stress scenarios development.

Table 6-10: Results CIR 3 m Nibor M 20 years

CIR

A	99,98997	**E**	8,212707
		L	91,77726
	99,98997		99,989967
E,value			-1,212707

CIR

Stress scenario interest rates

A	103,5421	**E**	8,168654
		L	95,3734
	103,5421		103,542054
VaR stress case			0,0440524

CIR

Stress scenario stocks

A	92,79312	**E**	5,975736
		L	86,81739
	92,79312		92,793126
VaR stress case			2,236971
Total VaR			2,259319
SCR			1,046612

Table 6-11 Results CIR 3 m Nibor M 20

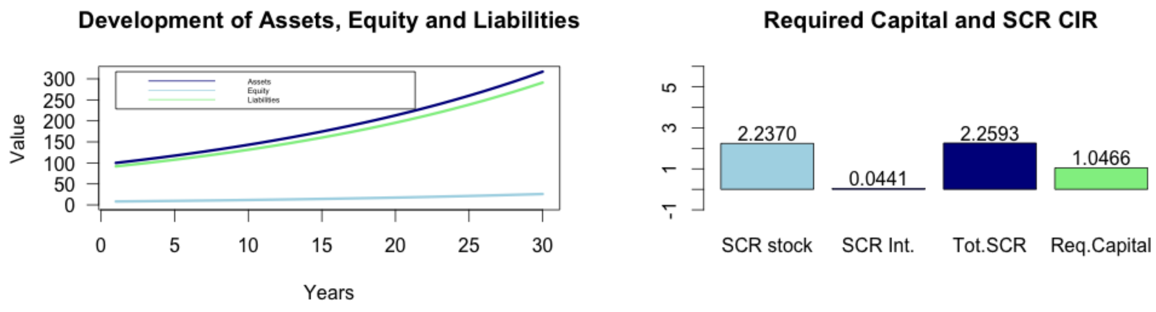


Figure Appendix B 4 in the appendix show the stress scenarios development. Kappa is reduced for the data with longer time period, but it does not seem as though this reduction gives a great effect on the results. The effects of increasing the time period on the CIR model have some similarities with Vasicek. Value of equity decreases a bit and the contribution from stress test on interest rate increases relatively much, but the absolute value is still smaller than the contribution from stress on stocks. The required capital increases relatively much with 67.8%

6.2.3 12 month Nibor - time period 10 years

We have examined whether the use of a different rate interest rate for calibration would be reflected in the results. We wanted to see if there are major differences between 3 month Nibor and 12 month Nibor. Parameters used in the analysis with monthly 12 month Nibor for a 10 years time period are shown in table under.

Table 6-12: Parameters 12 month Nibor M 10 year

NIBOR 12MTH (2004-2014)	Vasicek		CIR	
	Monthly	Annual	Monthly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,052927	0,183344	0,052927	0,183344
Std. Dev interest rate	0,003112	0,010913	0,015427	0,056121
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,024457	-0,293479	-0,094717	-1,136606

Table 6-13: Results Vasicek 12 m Nibor M 10 years

Vasicek			
A	100,7497	**E**	6,6114
		L	94,1383
	100,7497		100,7497

SCR base case	0,3886

Stress scenario interest rates			
A	105,8516	**E**	6,1805
		L	99,6711
	105,8516		105,8516

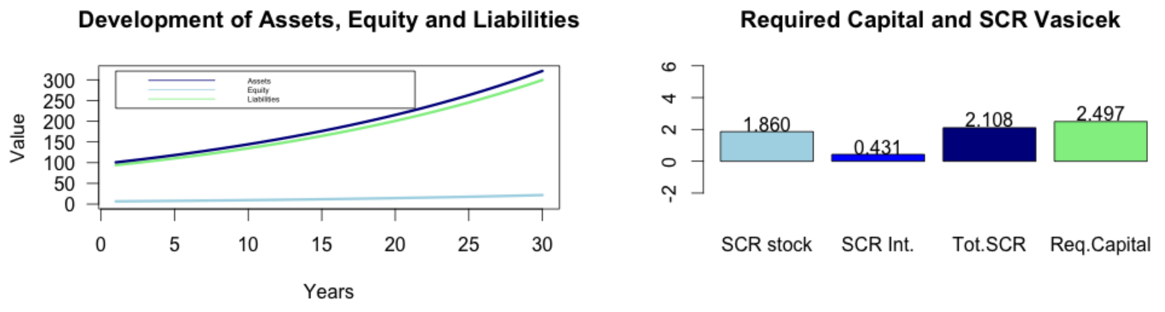
SCR stress case	0,4309

Stress scenario stocks			
A	93,5007	**E**	4,7517
		L	88,7490
	93,5007		93,5007

SCR stress case	1,8597

Total SCR	2,1084
Required capital	2,4970

Figure 6-11: Results Vasicek 12 m Nibor M 10 years

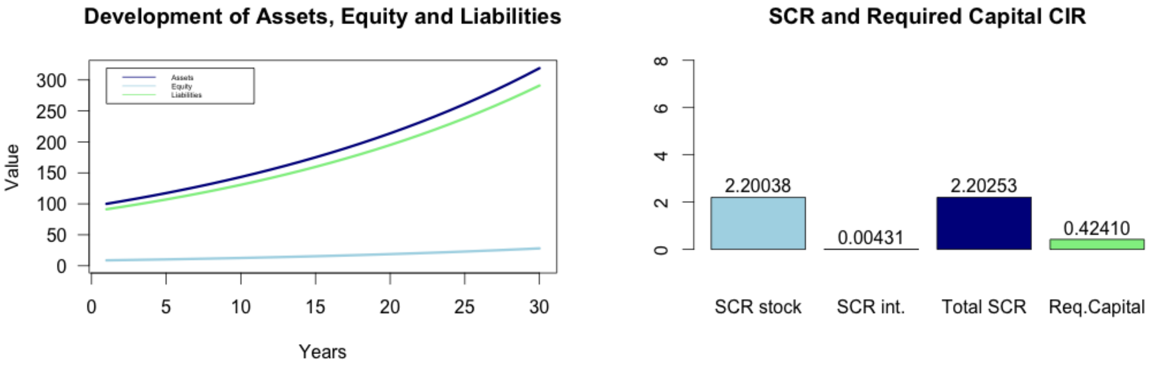


Using 12 month Nibor versus 3 month Nibor in the standard base case gives no appreciable effect on the results. This applies for both Vasicek and CIR. Results from CIR can be seen in the Appendix B 5. Appendix B 6 and Appendix B 7 in the appendix show the stress scenarios development.

Table 6-14: Results CIR 3 m Nibor M 10 years

CIR			
A	99,9991	**E**	8,7784
		L	91,2207
	99,9991		99,9991
SCR base case		-1,7784	
Stress scenario interest rates			
A	101,6220	**E**	8,7741
		L	92,8478
	101,6220		101,6220
SCR stress case		0,0043	
Stress scenario stocks			
A	92,8023	**E**	6,5781
		L	86,2243
	92,8023		92,8023
SCR stress case		2,2004	
Total SCR		2,2025	
Required capital		0,4241	

Figure 6-12 Results CIR 3 m Nibor M 10 year



6.2.4 3 month T Bill - time period 10 years monthly

We have examined whether the use of a different rate interest rate for calibration would be reflected in the results.

Table 6-15: Parameters 3 m T-Bill 10 year

T BILL 3MTH (2004-2014)	Vasicek		CIR	
	Monthly	Annual	Monthly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,052927	0,183344	0,052927	0,183344
Std. Dev interest rate	0,003254	0,011434	0,018908	0,069861
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,028457	-0,341488	-0,123454	-1,481442

Table 6-16 Results Vasicek 3 m T-Bill M 10 year

Vasicek

A	100,5748	E	6,5859
		L	93,9888
	100,5748		100,5748

SCR base case	0,4141

Stress scenario interest rates

A	105,2038	E	6,2639
		L	98,9399
	105,2038		105,2038

SCR stress case	0,3220

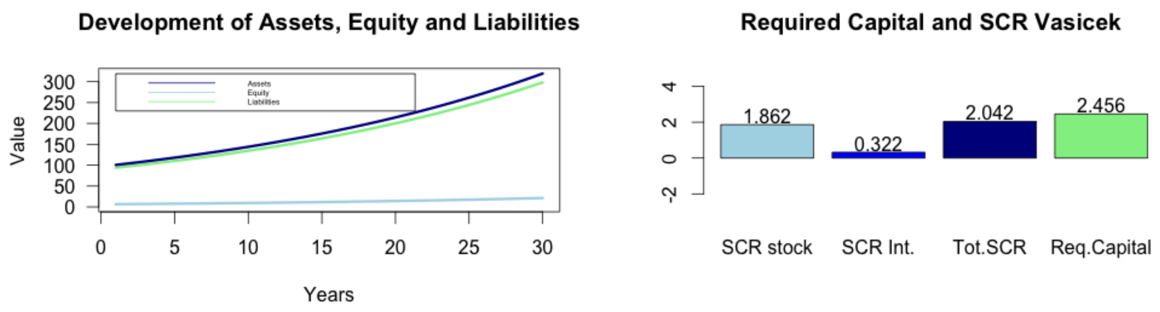
Stress scenario stocks

A	93,3378	E	4,7240
		L	88,6138
	93,3378		93,3378

SCR stress case	1,8620

Total SCR	2,0421
Required capita	2,4562

Figure 6-13 Results Vasicek 3 m T-Bill 10 year



Appendix B 8 shows the stress scenarios development. Using a monthly 3 month T-Bill compared to the standard base case has greatest impact on shareholders and required capital. Value of equity increases with 10.11%, resulting in a reduced required capital with 18.69 %. The difference in standard deviation for interest rate is not big enough to explain the differences, but the kappa is 9% higher with T-bills. A faster mean reverting process can, as elaborated before explain some of the differences. There is also a possibility that the differences originate from coincidences.

Table 6-17 Results CIR 3 m T-Bill 10 year

CIR

A	99,8652	**E**	8,4211
		L	91,4440
	99,8652		99,8651

SCR base case	-1,4211

Stress scenario interest rates

A	101,1084	**E**	8,4040
		L	92,7044
	101,1084		101,1084

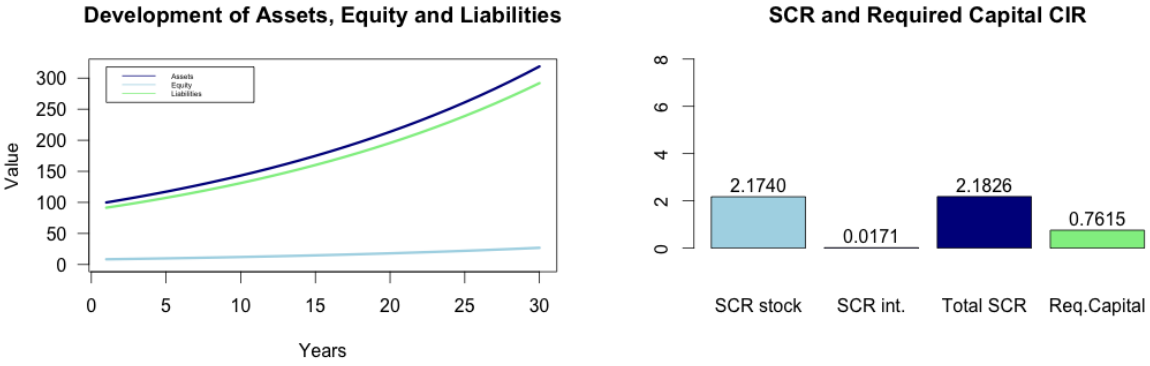
SCR stress case	0,0171

Stress scenario stocks

A	92,6776	**E**	6,2471
		L	86,4305
	92,6776		92,6776

SCR stress case	2,1740
Total VaR	2,1826

Figure 6-14 Results CIR 3 m T-Bill M 10 year



Compared with the standard base case, the parameters calibrated from the monthly 3 month T-Bill, produced larger kappa and standard deviation. From the higher standard deviation we would expect larger differences in the values. This was not the case with the results shown in table 6.16. The reason for this is partially due to the high kappa. A kappa over one causes the interest rate to use less than one time step to drift back to the long-term mean. Thereby the effect of high variance in interest rates is reduced. Figure in Appendix B 9 show the stress scenarios development.

6.2.5 3 month Libor - time period 10 years

We have examined whether the use of a different rate interest rate for calibration would be reflected in the results. Parameters used in the analysis with monthly 3 month Libor for a 10 years time period are shown in table 6.17.

Table 6-18: Parameters 3 m Libor M 10 years

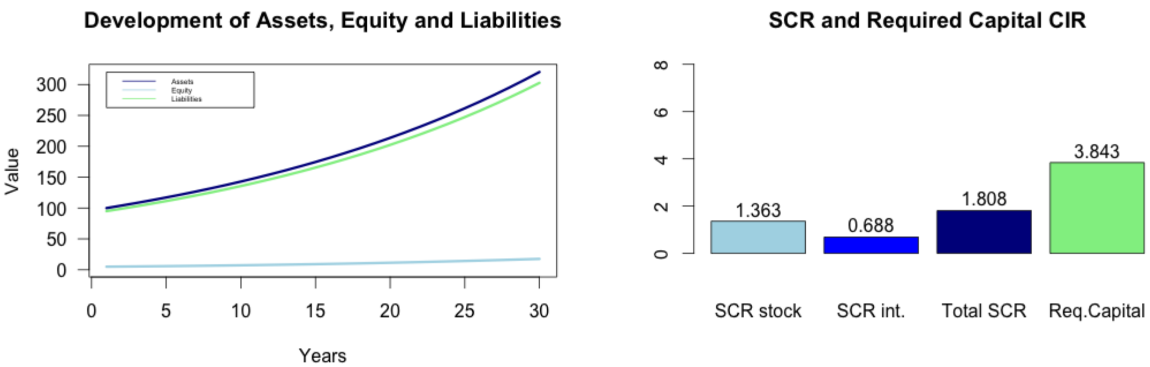
LIBOR 3MTH (2004-2014)	Vasicek		CIR	
	Monthly	Annual	Monthly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,052927	0,183344	0,052927	0,183344
Std. Dev interest rate	0,002656	0,009228	0,015612	0,054426
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,006183	-0,074192	-0,012590	-0,151081

The final balance sheet for Vasicek becomes slightly errors specified, since it is not discounted correctly back to the start value of 100. There is a decrease in the parameters calibrated for the 3 month Libor, compared to the 3 month Nibor in the base case. This applies to both the standard deviation for interest rates and kappa. Liabilities have a value increase, while value of equity decreases compared to the base case. We refrain to comment the result from Vasicek here because of the questionable discounted value, the results can be seen in the Appendix B 10 and Appendix B 11. Figure in Appendix B 12 show the stress scenarios development. The focus will rather be on results from CIR.

Table 6-19 Results CIR 3 m Libor M 10 years

	CIR		
A	99,8830	**E**	4,9652
		L	94,9178
	99,8830		99,8830
SCR base case	2,0348		
	Stress scenario interest rates		
A	106,8842	**E**	4,2770
		L	102,6072
	106,8842		106,8842
SCR stress case	0,6882		
	Stress scenario stocks		
A	92,6959	**E**	3,6019
		L	89,0940
	92,6959		92,6959
SCR stress case	1,3633		
Total SCR	1,8084		
Required capital	3,8433		

Figure 6-15 Results CIR 3 m Libor M 10 years



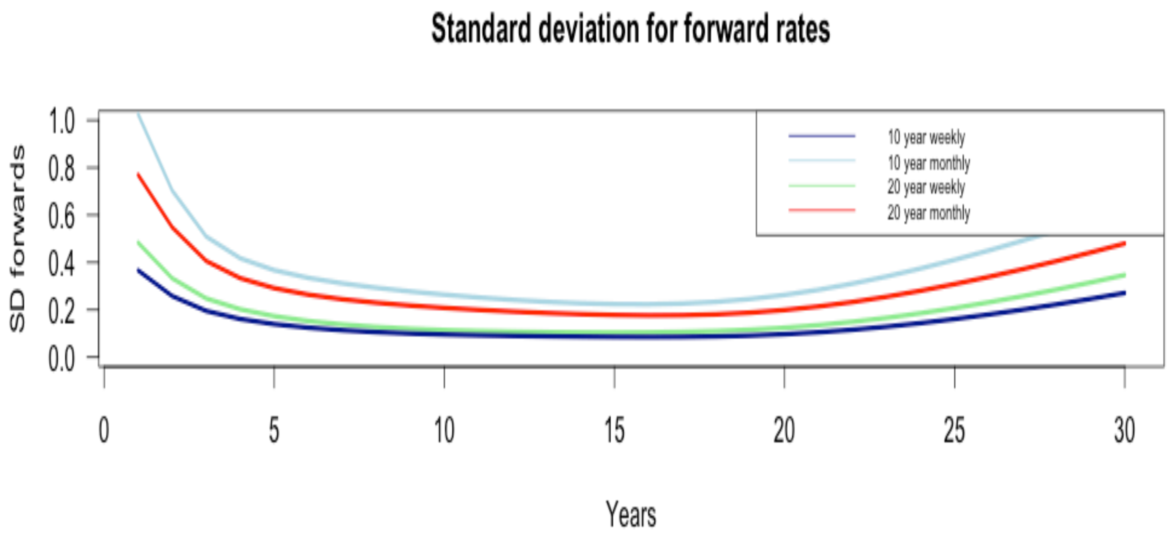
Kappa has a much lower value compared to the kappa for the base case. Results from the other analyzes with CIR did not leave a large effect for the stress scenario for interest rates. The 3 months Libor shows a larger change than other analysis have done with this model. The effect translates into the value of liabilities, which have a large increase. Equity has smaller values compared to the base case, it is especially the value of equity in the stress scenario for stocks that experience a decline.

In CIR the effect of using Libor instead of Nibor causes a major difference in value of equity, a reduction of 42.56% compared to the value of equity in the base case. 3 month Libor shows less risk in the portfolio in that SCR has decreased. The required capital is now 5.77 times higher than before. Figure in Appendix B 13 show the stress scenarios development.

6.3 Libor Market Model

Under we will present the results when we use a different time period and frequency for the forward rates. To compare results with the LMM 10 year monthly forward rates we use 10 years weekly, 20 years monthly and weekly forwards. The evolvement of the different standard deviations linked to the data sets are presented under in Figure 6-16

Figure 6-16: Standard deviations forward rates



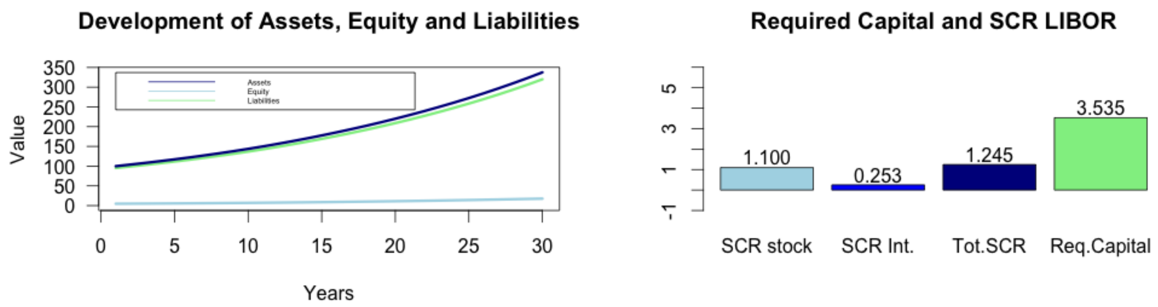
The monthly standard deviations, both for 10 and 20 years time period are noticeably higher than the weekly standard deviations. This will naturally expose the monthly based simulations with more risk, and should thereby increase the requirements for capital in these situations. It is interesting to see how the return distributions cause the equities to differ much more, relatively speaking, then liabilities. A general comment for all tree results is that the LMM model discount all the simulated assets correctly back to the start value of 100, within an acceptable margin of error.

6.3.1 LMM 10 year weekly

Table 6-20 Results LMM weekly 10 year

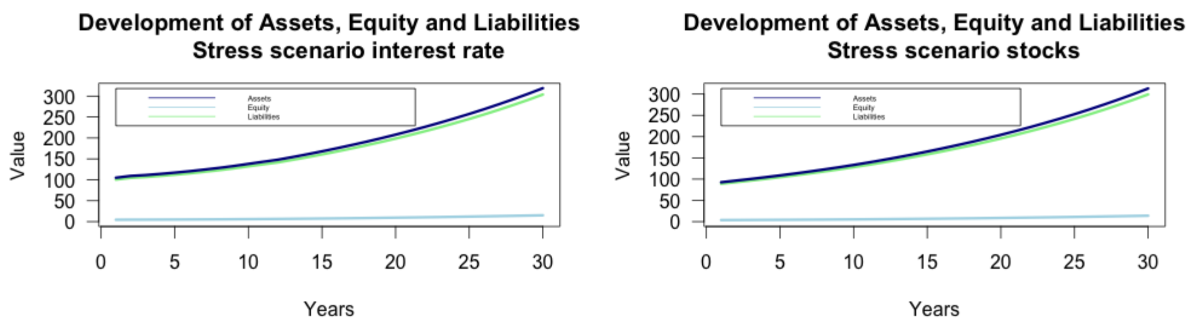
LMM			
A	99,8481	**E**	4,7107
		L	95,1375
	99,8481		99,8481
SCR base case			2,2893
Stress scenario interest rates			
A	104,9054	**E**	4,4578
		L	100,4477
	104,9054		104,9055
SCR stress case			0,2529
Stress scenario stocks			
A	92,6544	**E**	3,6111
		L	89,0433
	92,6544		92,6544
SCR stress case			1,0996
Total SCR			1,2455
Required capital			3,5348

Figure 6-17 Results LMM weekly 10 year



Analysis with weekly data instead of monthly data shows more realistic results. We avoids the steep rise in forward - and spot rates, see figure 6.28 compared to figure 6.5. The main difference is that value of equity is increased from the standard base case, with 67.66%. In addition is SCR higher for the weekly data. The life insurance company is required to have a higher level of capital to cover the quantifiable risk. It is remarkable that the SCR is increasing just by switching the calibration from monthly data to weekly data.

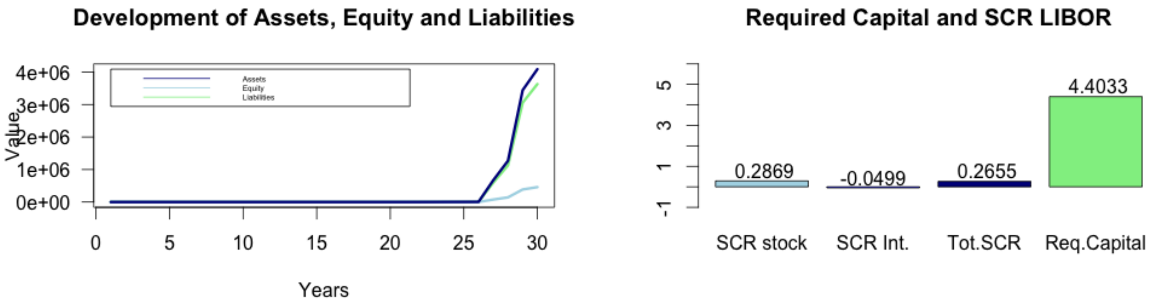
Figure 6-18 Stress scenario LMM weekly 10 year



6.3.2 LMM 20 year monthly

Changing the time period for the monthly forward rates gives little impact on the results in the LMM. These results can be seen in the Figure 6-19 and Figure 6-20. The justification of the enormous spot rates is explained above in the standard base case for LMM.

Figure 6-19 Results LMM monthly 20 year

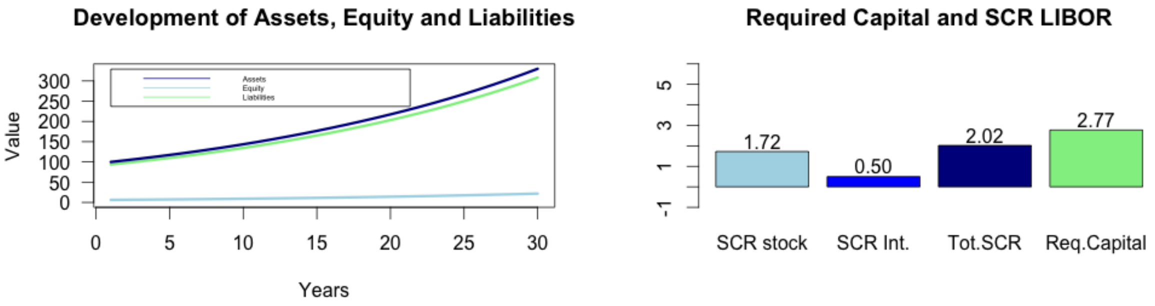


6.3.3 LMM 20 year weekly

Table 6-21 Results LMM weekly 20 year

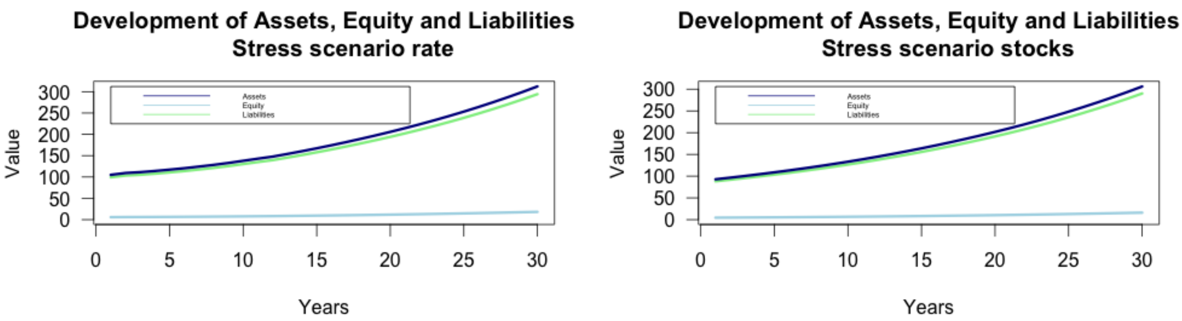
LMM			
A	99,9281	**E**	6,2490
		L	93,6790
	99,9281		99,9281
SCR base case			0,7510
Stress scenario interest rates			
A	104,9942	**E**	5,7491
		L	99,2452
	104,9942		104,9942
SCR stress case			0.4999443
Stress scenario stocks			
A	92,7272	**E**	4,5260
		L	88,2012
	92,7272		92,7272
SCR stress case			1,7230
Total SCR			2,0199
Required capital			2,7709

Figure 6-20 Results LMM weekly 20 year



Results from the analysis with the weekly data for a 20 years time period have a final balance sheet similar to the final balance sheet for the weekly 10 years data. Differences are evident when comparing to the standard base case for the LMM. The weekly data get a higher value of equity, which is an advantage for the shareholders. Calibration of the LMM with this dataset is not favorable for the policyholders. The results we get with this data series is more similar to the results we get from the standard base case with Vasicek.

Figure 6-21 Results stress scenario LMM weekly 20 year



Summary LMM

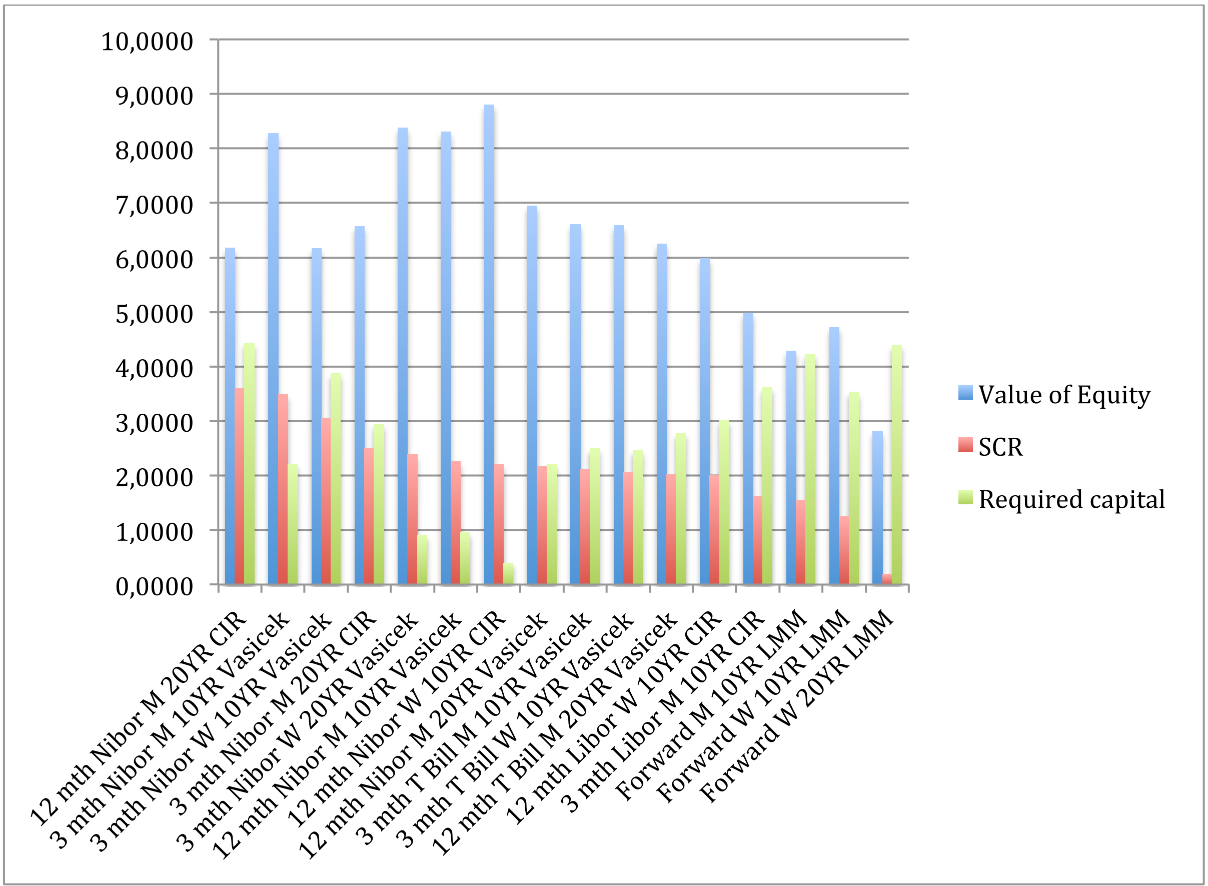
The general remarks we observe from LMM is how the large standard deviations in the monthly forward rates influence the shareholder value negatively compared when calibrating with weekly data. Because of the higher associated risk with monthly data, we see how the required capital is reduced when using weekly data, were the risk is smaller. This is a reasonably development since the higher incorporated risk should make the insurance companies to hold more capital additional to the book equity for enduring unfavorably events.

6.4 Summary of analysis and results

Table 6-22 Summary of results

Model	Interes rate	Data Frequency	Time period	Value of Equity	SCR	Required capital
CIR	12 mth Libor	monthly	20YR	6,1735	3,5987	4,4252
Vasicek	3 mth Nibor	weekly	20YR	8,2780	3,4913	2,2133
CIR	12 mth Libor	weekly	10YR	6,1715	3,0423	3,8707
Vasicek	3 mth Nibor	weekly	10YR	6,5683	2,5060	2,9377
CIR	12 mth Nibor	weekly	20YR	8,3830	2,3886	0,9056
CIR	3 mth Nibor	monthly	20YR	8,3089	2,2630	0,9541
CIR	12 mth Nibor	weekly	10YR	8,8062	2,2041	0,3980
Vasicek	3 mth T Bill	weekly	10YR	6,9501	2,1612	2,2112
Vasicek	12 mth Nibor	monthly	10YR	6,6114	2,1084	2,4970
Vasicek	3 mth T Bill	monthly	10YR	6,5859	2,0521	2,4562
LMM	Forward	weekly	20YR	6,2490	2,0199	2,7709
Vasicek	3 mth Nibor	monthly	10YR	5,9811	2,0026	3,0214
Vasicek	12 mth T Bill	monthly	20YR	4,9864	1,6135	3,6174
Vasicek	12 mth Nibor	monthly	20YR	4,2894	1,5495	4,2354
LMM	Forward	weekly	10YR	4,7197	1,2455	3,5348
LMM	Forward	monthly	10YR	2,8096	0,1924	4,3828

Figure 6-22 Summary of results



Table

Figure 6-22 and figure Table 6-22 shows some selected results, it shows the value of equity for the base case, SCR and required capital. The table is sorted by ranking the highest SCR.

Norwegian life insurance companies have to fulfill the guaranteed interest return each year. Liabilities in a life insurance company have basically long term to maturity, but the guaranteed interest return leads to a short-term focus in the investment management. A significantly part of the life insurance company`s assets are invested in securities where value and return is dependent on the interest rate level.

Results from Vasicek and LMM shows that a high value of equity leads to a high SCR, while the required capital is low. And reverse, a low value of equity gives a low SCR and a high required capital. Results of the CIR do not provide an equally clear trend. Analysis from CIR with Nibor, both time periods, frequencies and 3 – and 12 month, gives results with an increased value of equity, same time as the required capital has a low value.

It is Vasicek with the 3 month Nibor, 20 years time period and on a weekly basis, which gives the largest SCR. The LMM with monthly data for a 10 years time period produces the SCR with the lowest value.

It is consistently almost for all results from Vasicek and LMM that the value of equity is decreased, while it is increased for CIR. Analysis with LMM gives the smallest values for SCR. Weekly data seems to produce both higher values of equity and SCR compared to monthly.

The table shows that we get varying results depending on the interest rate used, the model we use, as well as the frequency and time period for the data.

6.4.1 Other results

In addition, we run analysis for remaining data series consisting of various interest rates for time periods of 10 years and 20 years on a monthly – and weekly basis.

Results from these analyses can be seen in the appendix. We have chosen to exclude some series because of inconstancy in the results, these are: Libor 3 month weekly 10 and 20 years, Libor 12 month 10 year weekly and Libor 3 month 20 year monthly.

Included are:
Appendix D 1 12 m Libor monthly 10Y
Appendix D 2 12 m T-Bill monthly 10Y
Appendix D 3 12 m Nibor monthly 20Y
Appendix D 4 12 m T-Bill monthly 20Y
Appendix D 5 3 m Libor monthly 20Y
Appendix D 6 3 m T-Bill monthly 20Y
Appendix D 7 12 m Libor monthly 20Y
Appendix D 8 12 m Nibor weekly 20 Y
Appendix D 9 3 m T-Bill 10Y
Appendix D 10 12 m T-Bill 10Y
Appendix D 11 3 m T-Bill weekly 10Y
Appendix D 12 12 m T-Bill weekly 20Y

7 Scenarios

In this chapter we look at different scenarios for the base case, where we change some of the assumptions and parameters in the model.

We want to see how the value distribution will evolve when the risk of the portfolio change. Changing the asset allocation, respectively with an increase and decrease in the stock ratio, changes the risk of the portfolio.

We have scenarios where the interest rate in Vasicek and CIR, and the forward rate for LMM, increase to 6 % and decrease to 3 %.

The proposal from the Norwegian FSA of reducing the guaranteed interest rate from 2.5% to 2.0 % for new contracts was declined. It would be interesting to see what effect the proposal would have had on the value of our insurance model, if adopted. We will also run a scenario where the guarantee increases to 3 %. One must keep in mind that we make a huge assumption that the change applies to all contracts, not just the new ones.

The duration of 6 we use for bonds is probably high. If we assume the duration is reduced to 4, it could have a major impact on value distribution, especially because of the high bond ratio in our base case.

Parameters used in these analyses are the parameters used in the base case, see Table 6-1 and Table 13-1 for the LMM.

7.1 Increase in stock allocation to 25 %

In this scenario we wanted to increase the risk of the portfolio. We did this by changing the stock allocation to 25 %, by adjusting the bond allocation down to 70%.

Table 7-1 Results Vasicek stock ratio 25%

Vasicek			
A	100,8255	E	5,314336
		L	95,51117
	100,8255		100,825506

SCR base case	1,685664

Stress scenario interest rates			
A	105,7617	E	5,033727
		L	100,728
	105,7617		105,761727

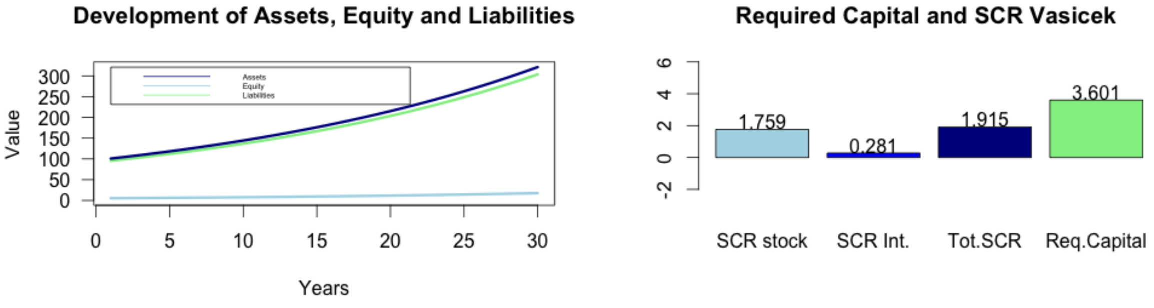
SCR stress case	0,2806094

Stress scenario stocks			
A	89,48968	E	3,555053
		L	85,93463
	89,48968		89,489683

SCR stress case	1,759283

Total SCR	1,915069
Required capital	3,600733

Figure 7-1 Results Vasicek stock ratio 25%

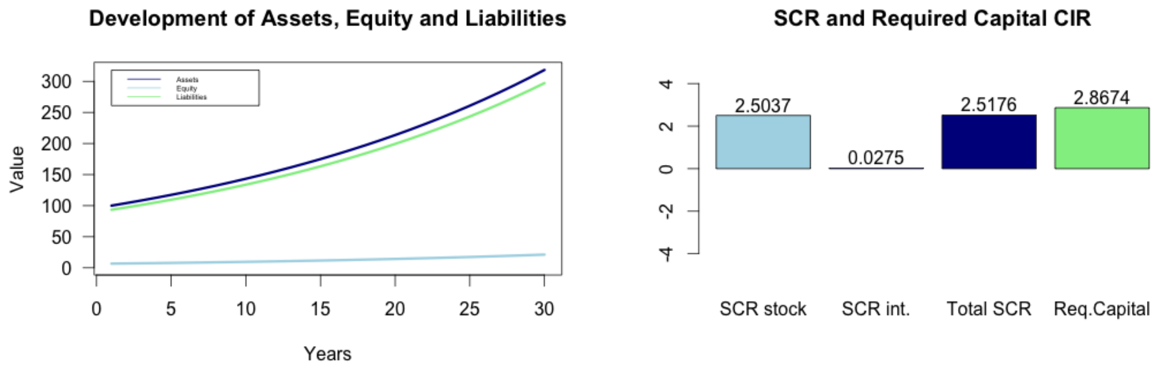


Results from stress scenarios for Vasicek are in the Appendix C 1.

Table 7-2 Results CIR stock ratio 25 %

	CIR		
A	99,99258	**E**	6,650111
		L	93,34247
	99,99258		99,992581
SCR base case		0,349889	
	Stress scenario interest rates		
A	101,8368	**E**	6,622622
		L	95,21417
	101,8368		101,836792
SCR stress case		0,027489	
	Stress scenario stocks		
A	88,74739	**E**	4,14641
		L	84,60098
	88,74739		88,74739
SCR stress case		2,503701	
Total SCR		2,517558	
Required capital		2,867447	

Figure 7-2 Result CIR stock ratio 25 %



Results from stress scenarios CIR stocks are in the Appendix C 2

Table 7-3 Result LMM stock ratio 25 %

LMM			
A	100,6072	**E**	3,0073
		L	97,5999
	100,6072		100,6072
SCR base case			3,9927
Stress scenario interest rates			
A	104,9654	**E**	3,0756
		L	101,8898
	104,9654		104,9654
SCR stress case			-0,0683
Stress scenario stocks			
A	89,2883	**E**	2,6118
		L	86,6764
	89,2883		89,2883
SCR stress case			0,3955
Total SCR			0,3661
Required capital			4,3588

Figure 7-3 Results LMM stock ratio 25 %

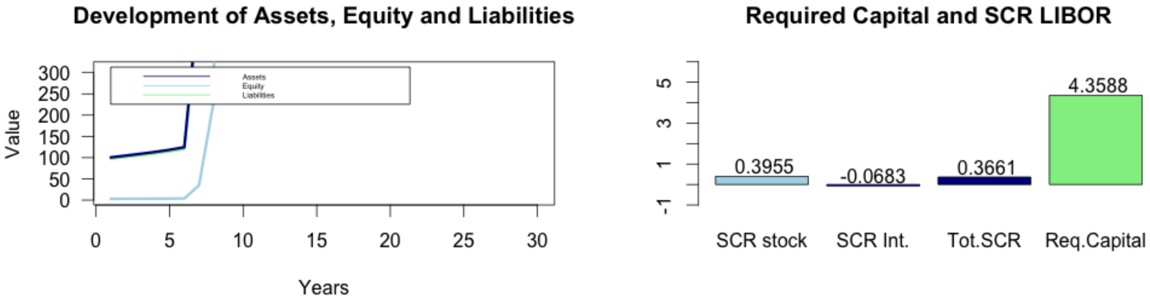
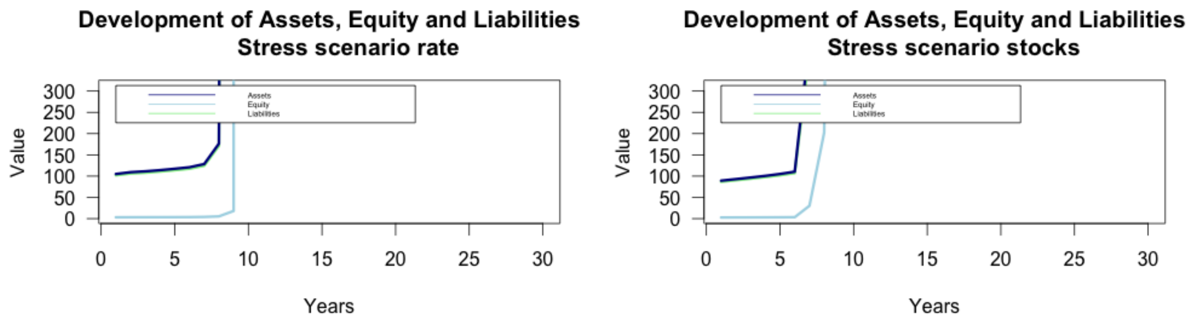


Figure 7-4 Results LMM Stress scenario stock ratio 25%



Results from these analysis shows that both Vasicek and CIR gets a decrease in the value of equity. This gives an increase in the required capital compared to the standard base cases for Vasicek and CIR, the life insurance company is more in need of in addition to book equity. LMM do not get an increase in the required capital compared to the standard base case. One reason for this may be that the value of equity is low, both in the standard base case and in this scenario.

When the stock allocation is adjusted up, Vasicek gives a total SCR that has a lower value compared to the standard base case. We thought initially that since an increase in stock allocation would increase risk in the portfolio, that total SCR would also increase. Both CIR and LMM have an increase in SCR.

The stress scenarios for interest rates do not have as much an effect compared to the standard base cases. The final balance has decreased for all three models, the explanation for this is the new bond allocation. Bond prices increases in value when interest rates falls, and since we have less bonds in the portfolio the final balance are adversely affected. The greatest change is for the stress scenarios for stocks. A fall in stock prices has greater consequences when there are a higher proportion of stocks. With less bonds in the portfolio it will not get as great positive impact of a fall in interest rates, while with more stocks there will be a greater negative impact during a fall in stock prices.

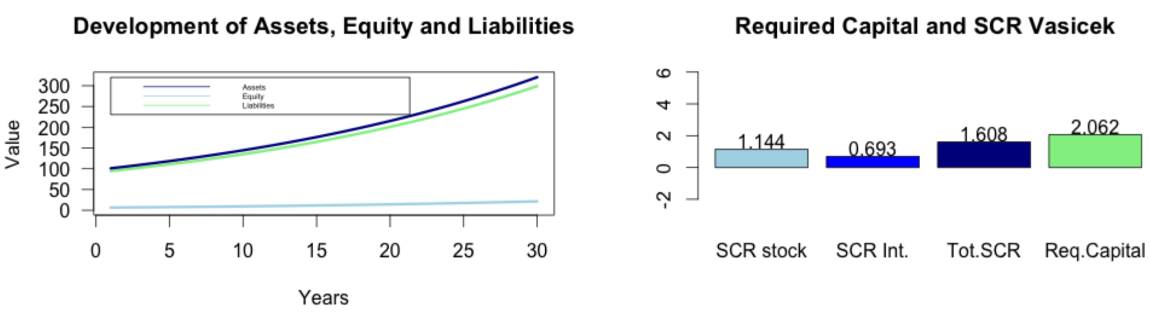
7.2 Decrease in stock allocation to 10 %

In this scenario, we have lowered the stock allocation to 10%.

Table 7-4 Results Vasicek stock ratio 10 %

Vasicek			
A	101,0213	**E**	6,5454
		L	94,4759
	101,0213		101,0213
SCR base case		0,4546	
Stress scenario interest rates			
A	107,0260	**E**	5,8519
		L	101,1742
	107,0260		107,0261
SCR stress case		0,6935	
Stress scenario stocks			
A	96,4790	**E**	5,4010
		L	91,0780
	96,4790		96,4790
SCR stress case		1,1444	
Total SCR		1,6075	
Required capital		2,0622	

Figure 7-5 Results Vasicek stock ratio 10 %



Results from stress scenarios for Vasicek stock 10 % are in the Appendix C 3.

Table 7-5 Result CIR stock ratio 10 %

CIR			
A	99,9934	**E**	9,7692
		L	90,2241
	99,9934		99,9934

SCR base case	-2,7692

Stress scenario interest rates			
A	102,2337	**E**	9,8986
		L	92,3341
	102,2337		102,2327

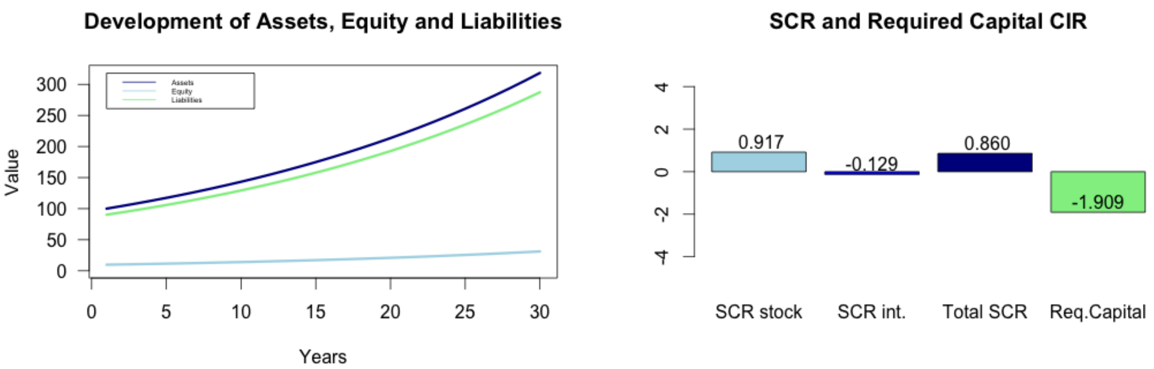
SCR stress case	-0,1294

Stress scenario stocks			
A	95,4957	**E**	8,8519
		L	86,6438
	95,4957		95,4957

SCR stress case	0,9173

Total SCR	0,8600
Required capital	-1,9093

Figure 7-6 Results CIR stock ratio 10 %



Results from stress scenarios for CIR stock ratio 10% are in the Appendix C 4.

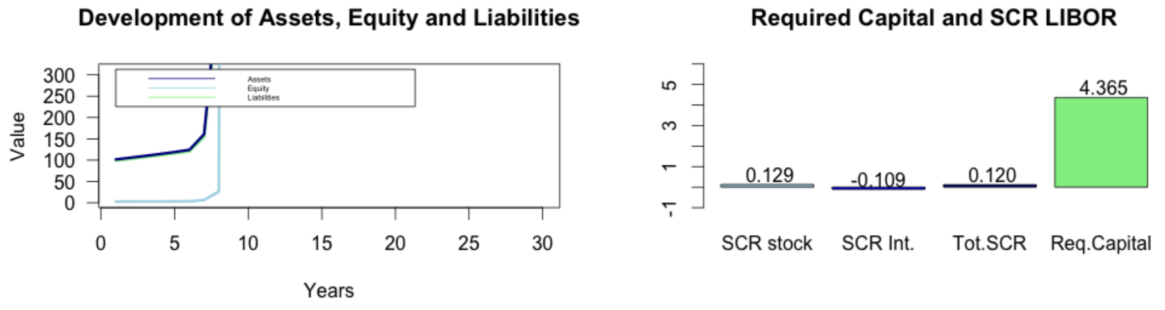
Table 7-6 Results LMM stock ratio 10 %

LMM			
A	101,6786	E	2,7555
		L	98,9231
	101,6786		101,6786
SCR base case			4,2445

Stress scenario interest rates			
A	106,8392	E	2,8646
		L	103,9746
	106,8392		106,8392
SCR stress case			-0,1091

Stress scenario stocks			
A	97,1019	E	2,6266
		L	94,4754
	97,1019		97,1019
SCR stress case			0,1289
Total SCR			0,1203
Required capital			4,3648

Figure 7-7 Results LMM stock ratio 10 %



Results from stress scenarios for the LMM with a stock ratio of 10 % are in the Appendix C 5.

The values are compared with the standard base case. When reducing the portfolio risk we expect the required capital to be reduced and market value of both of equity and liabilities to increase. There is also reasons to expect that the SCR contribution from stressing stocks should be less than with the standard base case.

The change in required capital compared with the standard base case has different effect in all three models. Vasicek reduces the required capital by 31.7 %, and LMM has no significant change. With the CIR model the required capital becomes negative, and is reduced compared with the standard base case 4.76 times. Intuitively it is a bit wrong that the required capital can be negative, but it is originating from how it is calculated, and that the market value of liabilities is higher than book value. The market value of equity is increased respectively by 9.43 % and 12.57 % when the model is calibrated with Vasicek and CIR. Surprisingly a reduction with -1.9 is observed from the LMM. The values of liabilities only have minimal changes.

After reducing the stock ratio to 10 %, the SCR originating from the stress case for stocks is reduced substantially for all models compared with the standard base case. This is naturally because the effect of a fall in stock prices will have less impact with 10 % compared to 16 % stock ratio.

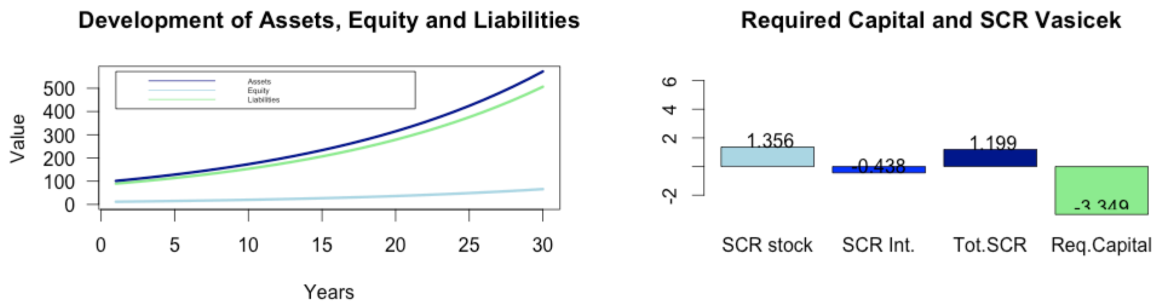
7.3 Increase in interest rate to 6 %

Further we have increased the start rate and the long term mean, theta in Vasicek and CIR to 6 %. The equilibrium rate is set to 6 % for the forwards in the LMM.

Table 7-7 Results Vasicek interest rate and theta 6 %

Vasicek			
A	101,1453	E	11,5478
		L	89,5974
	101,1453		101,1453
SCR base cas	-4,5478		
Stress scenario interest rates			
A	109,8562	E	11,9857
		L	97,8705
	109,8562		109,8562
SCR stress ca	-0,4379		
Stress scenario stocks			
A	93,8681	E	10,1919
		L	83,6762
	93,8681		93,8681
SCR stress ca	1,3559		
Total SCR	1,1986		
Required cap	-3,3493		

Figure 7-8 Results Vasicek interest rate and theta 6 %

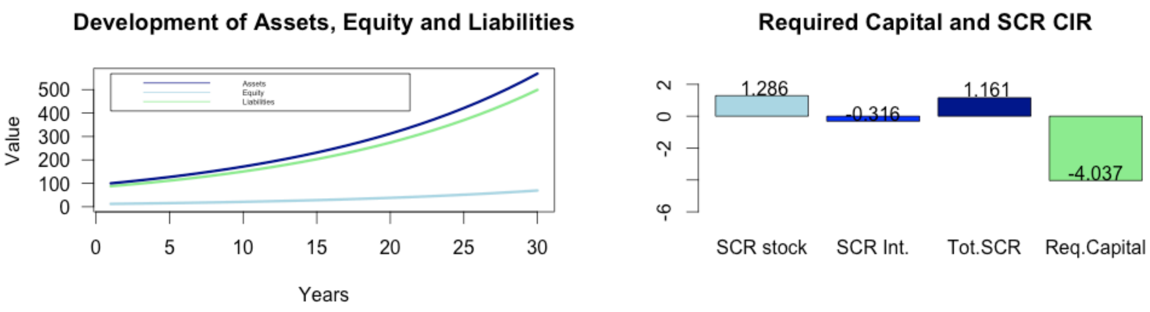


Results from stress scenarios for Vasicek with a 6 % interest rate are in the Appendix C 6

Table 7-8 Results CIR Interest rate and theta 6 %

	CIR		
A	99,9922	**E**	12,1980
		L	87,7942
	99,9922		99,9922
SCR base cas		-5,1980	
	Stress scenario interest rates		
A	103,1340	**E**	12,5136
		L	90,6203
	103,1340		103,1339
SCR stress ca		-0,3157	
	Stress scenario stocks		
A	92,7957	**E**	10,9116
		L	81,8840
	92,7957		92,7957
SCR stress ca		1,2863	
Total VaR		1,1611	
SCR		-4,0368	

Figure 7-9 Results CIR interest rate and theta 6 %

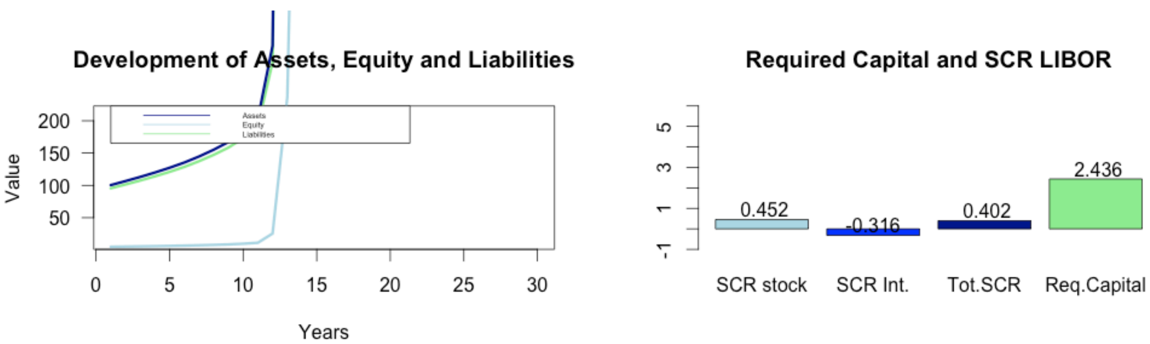


Results from stress scenarios for CIR with a 6 % interest rate are in the Appendix C 7.

Table 7-9 Results LMM forward rate 6 %

	LMM		
A	100,4469	**E**	4,9652
		L	95,4817
	100,4469		100,4468
SCR base case		-4,9652	
	Stress scenario interest rates		
A	110,2994	**E**	5,2812
		L	105,0182
	110,2994		110,2994
SCR stress case		-0,3161	
	Stress scenario stocks		
A	93,2126	**E**	4,5133
		L	88,6992
	93,2126		93,2125
SCR stress case		0,4519	
Total SCR		0,4016	
Required capital		2,4364	

Figure 7-10 Results LMM forward rate 6 %



Stress scenario figures are in Appendix C 8.

We have increased the start rate in Vasicek and CIR to 6 %, the long-term mean is also increased to 6 %. When the interest rate increases it is normal to expect the bond value to decrease. With our bond ratio of 79 %, we would expect a rather large decrease in the asset value. The increase in interest rates should have a positive effect on the value of equity since it is easier to fulfill the guaranteed interest return to policyholders without increasing the risk.

The relationship between stock value and interest rate changes are to some extent a little contra intuitive. If the interest rate goes down, this usually reflects less risk in the market and should increase the stock prices. From another point of view, government bond rates are usually sat down to boost the economy because the market is in a recessionary phase. The correlation we use in the standard base case between MSCI world and short rates for respectively Vasicek and CIR are -0.077 and -0.0466.

When r is increased from 4 % to 6 % we get results with a large increase in the value of equity compared to the standard base case for both Vasicek and CIR. This is also true for the stress scenarios for interest rates and stocks. Liabilities decrease in value. There is a large reaction in the stress scenario for interest rate. The final balance sheet has a large increase compared to both the standard base case and the base case. Stress scenarios for stocks have a similar final balance sheet as the standard base case, but the difference is that the value of equity has increased. There are a decrease in both the SCR and required capital. Required capital is negative for both Vasicek and CIR. When the value of equity has increased to this high level, it is not as large need for in addition to book value equity.

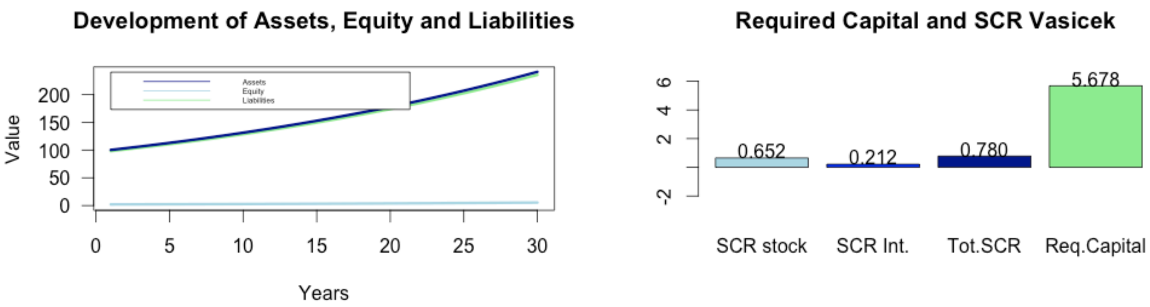
7.4 Decrease in interest rate to 3 %

We have decreased the start rate and theta in Vasicek and CIR to 3 %, in the LMM the equilibrium rate is set to 3 % for the forward.

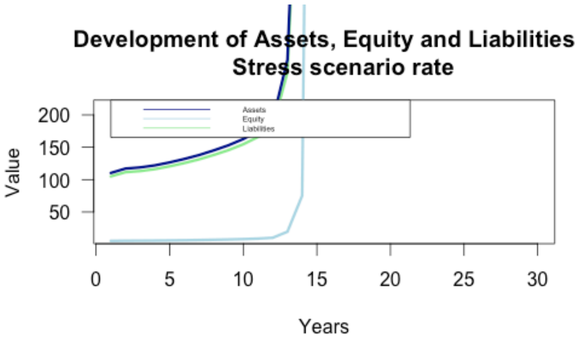
Table 7-10 Results Vasicek interest rate 3 %

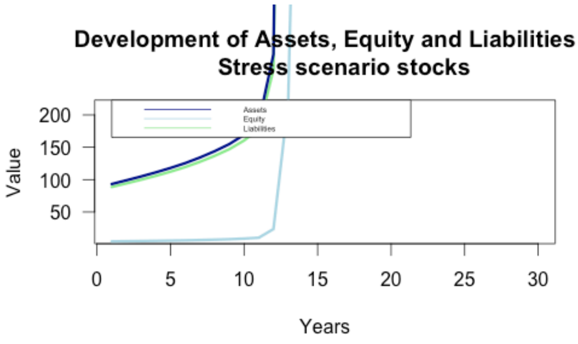
	Vasicek		
A	100,1567	**E**	2,1014
		L	98,0553
	100,1567		100,1567
SCR base case	4,8986		
	Stress scenario interest rates		
A	104,0723	**E**	1,8899
		L	102,1824
	104,0723		104,0723
SCR stress case	0,2116		
	Stress scenario stocks		
A	92,9503	**E**	1,4492
		L	91,5012
	92,9503		92,9503
SCR stress case	0,6523		
Total SCR	0,7799		
Required capital	5,6784		

Figure 7-11 Results Vasicek interest rate 3 %



Appendix C 8 Stress scenario LMM Forward 6 %





Appendix C 9. Include stress scenarios for Vasicek with a 3 % interest rate.

Appendix C 8 Stress scenario LMM Forward 6 %

Table 7-11 Results CIR interest rate 3 %

CIR

A	99,9871	**E**	4,3268
		L	95,6603
	99,9871		99,9871

SCR base cas	2,6732

Stress scenario interest rates

A	101,5434	**E**	4,2300
		L	97,3134
	101,5434		101,5434

SCR stress ca	1,8418

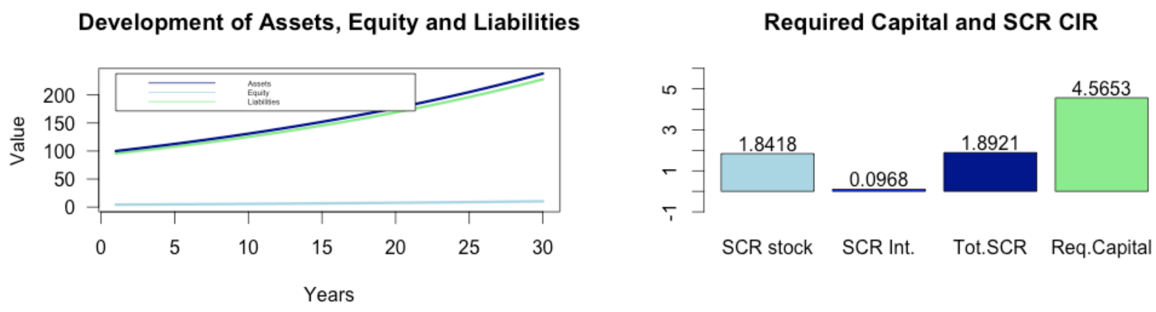
Stress scenario stocks

A	92,7910	**E**	2,4850
		L	90,3060
	92,7910		92,7910

SCR stress ca	1,8418

Total SCR	1,8921
Required caı	4,5653

Figure 7-12 Results CIR Interest rate 3 %

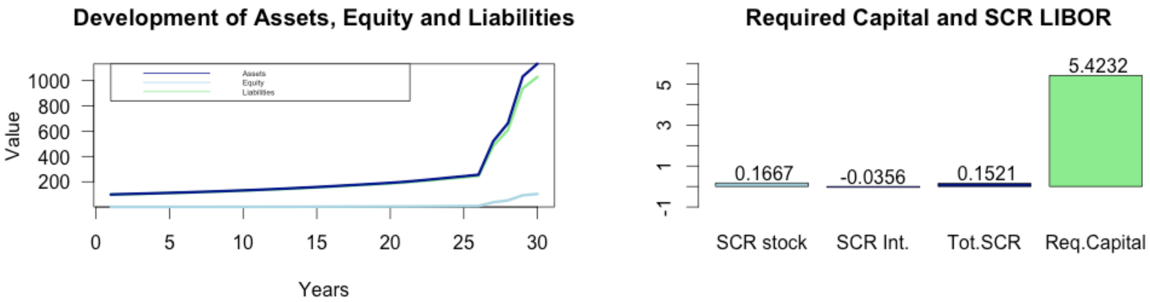


Results from stress scenarios for CIR with a 3 % interest rate are in the Appendix C 10.

Table 7-12: Results LMM forward rate 3%

	LMM		
A	100,0427	**E**	1,7289
		L	98,3138
	100,0427		100,0427
SCR base case		-1,7289	
	Stress scenario interest rates		
A	104,9072	**E**	1,7646
		L	103,1426
	104,9072		104,9072
SCR stress case		-0,03560492	
	Stress scenario stocks		
A	92,8353	**E**	1,5622
		L	91,2731
	92,8353		92,8353
SCR stress case		0,1667	
Total SCR		1,8921	
Required capital		5,4232	

Figure 7-13 Results LMM Forward rate 3%



Stress scenario figure for LMM are in Appendix C 11.

We have lowered the interest rate and long-term mean to 3 %. When the interest rate is at 3 %, only 0.5% over the guaranteed interest return, we expect that the shareholders will experience the largest loss in value. Policyholders will naturally receive their guaranteed return and therefore we do not expect a significantly movement in liabilities from the standard base case.

All three models have a decrease in the value of equity compared to the standard base cases, Vasicek and CIR are most affected. While the value of equity in LMM decreases slightly, but the value of equity was also low in the standard base case. The value of liabilities are increased because of the low interest rate, bond values are increasing. The final balance sheet for stress scenario for interest in Vasicek is slightly affected when the interest rates are lowered, but the value of equity has a major decrease. This also applies for CIR. Value of equity is unaffected in the stress scenario for LMM. Stress scenarios for stocks give the same final balance sheet as the standard base cases. Required capital is increasing for all three models, there is a greater need for in addition to book value equity when the interest rate is low. Total SCR is decreasing in value for both Vasicek and CIR, while it is increasing for LMM.

7.5 Increase in guaranteed interest return to 3 %

We have set the guaranteed interest return up to 3 %.

Table 7-13 Results Vasicek g= 3%

Vasicek			
A	100,9412	E	3,956568
		L	96,98459
	100,9412		100,941158

SCR base case	3,043432

Stress scenario interest rates			
A	106,518	E	3,414705
		L	103,1033
	106,518		106,518005

SCR stress case	0,5418627

Stress scenario stocks			
A	93,67865	E	2,684978
		L	90,99367
	93,67865		93,678648

SCR stress case	1,271589

Total SCR	1,612322
Required capital	4,655754

Figure 7-14 Results Vasicek g= 3%

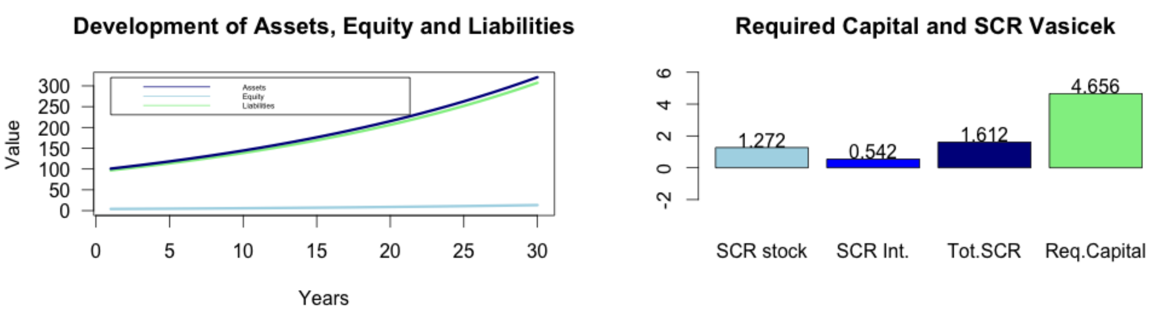


Table 7-14 Results CIR g= 3%

CIR			
A	99,99227	**E**	6,677688
		L	93,31458
	99,99227		99,992268

SCR base case	0,322312

Stress scenario interest rates			
A	102,0735	**E**	6,562786
		L	95,51074
	102,0735		102,073526

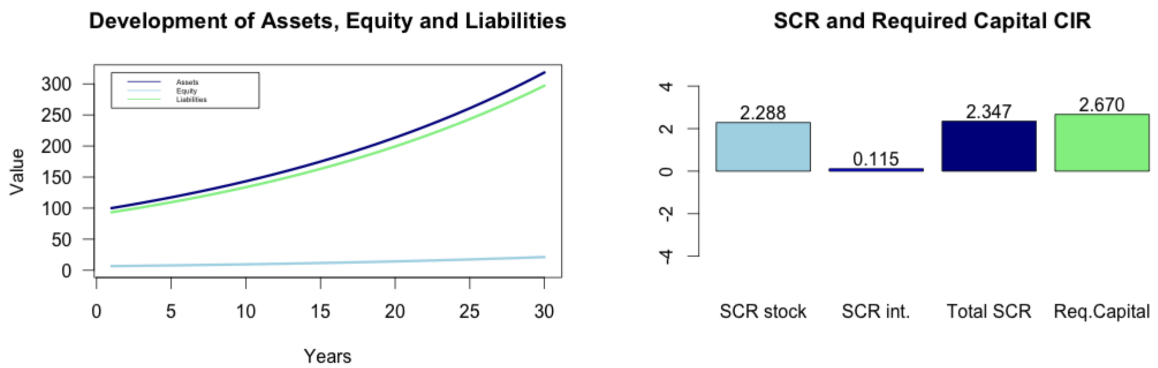
SCR stress case	0,114901

Stress scenario stocks			
A	92,79579	**E**	4,389886
		L	88,40591
	92,79579		92,795796

SCR stress case	2,287801

Total SCR	2,347362
Required capital	2,669675

Figure 7-15 Results CIR g = 3%

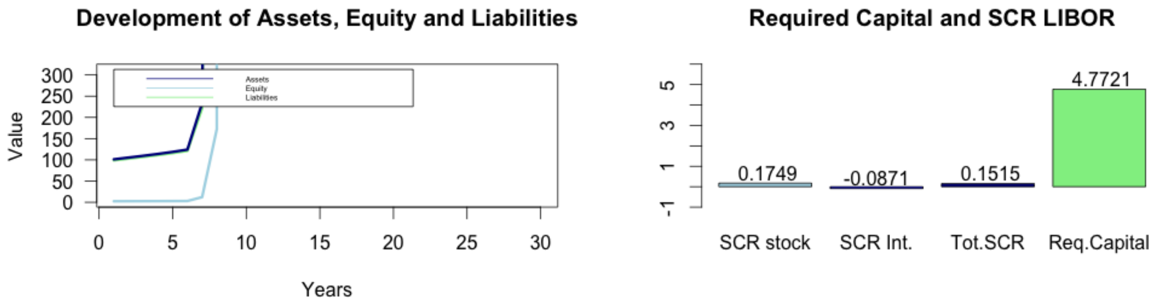


Results from stress scenarios for Vasicek and CIR with g = 3 % are in Appendix C 12 and Appendix C 13.

Table 7-15 Results LMM g=3%

LMM			
A	101,2352	**E**	2,3794
		L	98,8558
	101,2352		101,2352
SCR base case			4,6206
Stress scenario interest rates			
A	106,0747	**E**	2,4665
		L	103,6082
	106,0747		106,0747
SCR stress case			-0,0871
Stress scenario stocks			
A	93,9450	**E**	2,2045
		L	91,7405
	0,0000		0,0000
SCR stress case			0,1749
Total SCR			0,1515
Required capital			4,7721

Figure 7-16 Results LMM g = 3%



Results from stress scenarios for LMM with g = 3% are in the Appendix C 14.

When we change the guaranteed interest return, g, to 3 %, we see that the value of equity is decreased and the value of liabilities have increased, in comparison to both the standard base cases and the scenarios where g is 2 %. This is consistent regardless of which model we look at. An increase in the guaranteed interest return for the contracts will lead to an increase in the value of the liabilities and the policyholders. The required capital for all the models has also increased compared to the standard base cases and the scenarios where g is 2 %. When the value of equity is declined there is a need for in addition to book equity. Vasicek has a lower total SCR compared to the standard base case and the scenarios where g is 2 %, this is also the case for the LMM. For CIR the total SCR is higher than the total SCR for the standard base case and the scenarios where g is 2 %.

The stress scenarios for interest rates and stocks do not show abnormal results compared with the stress scenarios for other analysis we have looked at.

7.6 Decrease in guaranteed interest return to 2 %

We have set the guaranteed interest return down to 2 %.

Table 7-16 Results Vasicek g= 2%

	Vasicek		
A	100,9412	**E**	7,413704
		L	93,52746
	100,9412		100,941164

SCR base case	-0,413704

	Stress scenario interest rates		
A	106,518	**E**	7,101333
		L	99,41664
	106,518		106,517973

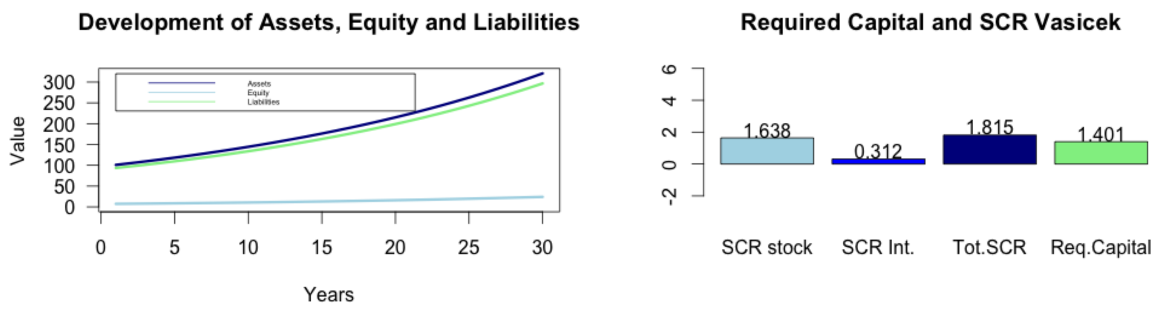
SCR stress case	0,3123703

	Stress scenario stocks		
A	93,67865	**E**	5,775229
		L	87,90342
	93,67865		93,678649

SCR stress case	1,638475

Total SCR	1,814934
Required capital	1,40123

Figure 7-17 Results Vasicek g = 2%

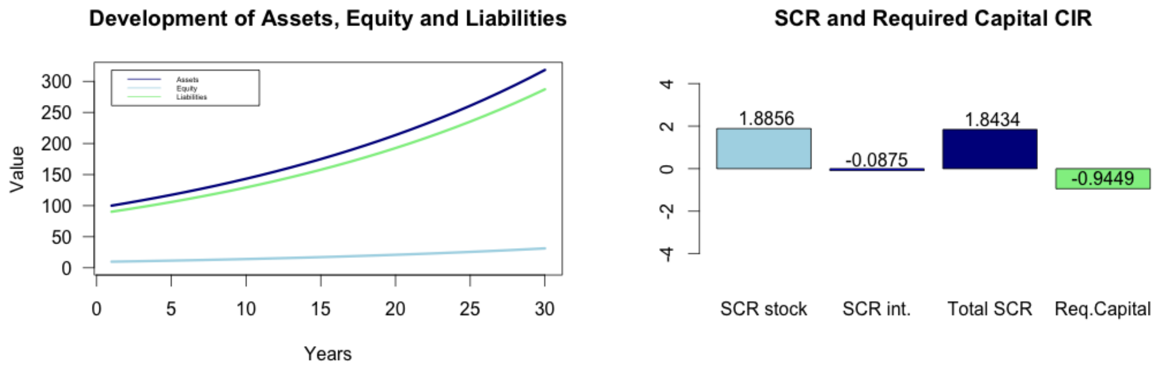


Results from stress scenarios for Vasicek with g= 2 % are in the Appendix C 15.

Table 7-17 Results CIR g= 2%

	CIR		
A	99,99227	**E**	9,788285
		L	90,20398
	99,99227		99,992265
SCR base case		-2,788285	
	Stress scenario interest rates		
A	102,0735	**E**	9,875829
		L	92,19769
	102,0735		102,073519
SCR stress case		-0,087544	
	Stress scenario stocks		
A	92,79579	**E**	7,902664
		L	84,89313
	92,79579		92,795794
SCR stress case		1,885621	
Total SCR		1,843408	
Required capital		-0,9448766	

Figure 7-18 Results CIR g= 2%

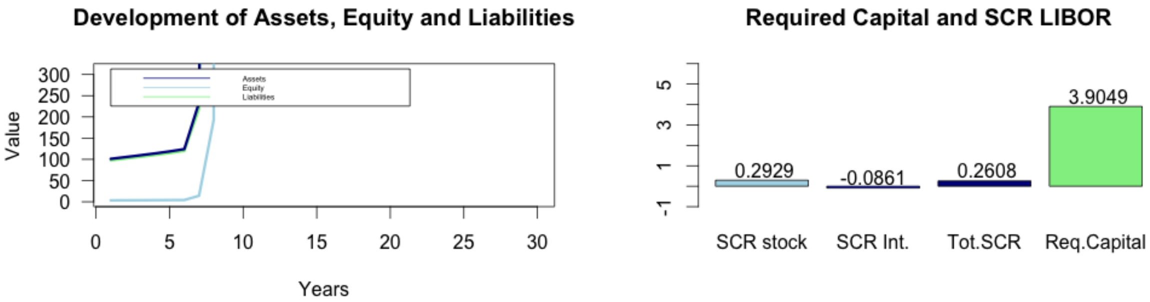


Results from stress scenarios for CIR with g= 2 % are in the Appendix C 16.

Table 7-18 Results LMM g=2%

LMM			
A	101,2352	**E**	3,3559
		L	97,8793
	101,2352		101,2352
SCR base case			3,6441
Stress scenario interest rates			
A	106,0747	**E**	3,4420
		L	102,6327
	106,0747		106,0747
SCR stress case			-0,0861
Stress scenario stocks			
A	93,9450	**E**	3,0629
		L	90,8821
	93,9450		93,9450
SCR stress case			0,2929
Total SCR			0,2608
Required capital			3,9049

Figure 7-19 Results LMM g=2%



Results from stress scenarios for LMM with g=2% are in the Appendix C 17.

Results from the analysis where the guaranteed interest return is 2 % shows the opposite results compared to the previous scenarios where g is 3 %. Value of equity is increased for all scenarios compared to the standard base cases and scenarios where g is 3 % for all models. By reducing the guaranteed interest return, the value of liabilities is also reduced. The life insurance company needs to cover the guaranteed interest return of 2 %, this is clearly much easier than having to cover a guaranteed interest return of 3 %. In this situation it is more likely that the value of equity will increase, which is the case here.

Vasicek has a total SCR that is higher compared to the scenarios where g is 3 %, but it is smaller compared to the SCR for the standard base case. SCR for the LMM has an increased value compared to the standard base case and the scenarios where g is 3 %. For CIR the value of required capital is smaller than SCR, which indicates that the market value of liabilities is higher than book value.

Required capital has decreased for all the models compared to both standard base case and the scenarios where g is 3 %. The value of equity has, as mentioned above, increased in value and therefore it is not the same need in addition to equity as is the case when g is 3%.

The stress scenarios produce similar results as for the standard base cases.

7.7 Lower duration, 4

Bond duration is changed to 4 in this scenario.

Table 7-19 Results Vasicek Duration 4

Vasicek			
A	100,7254	E	7,078872
		L	93,64657
	100,7254		100,725442

SCR base case	-0,078872

Stress scenario interest rates			
A	105,0369	E	6,380227
		L	98,65667
	105,0369		105,036897

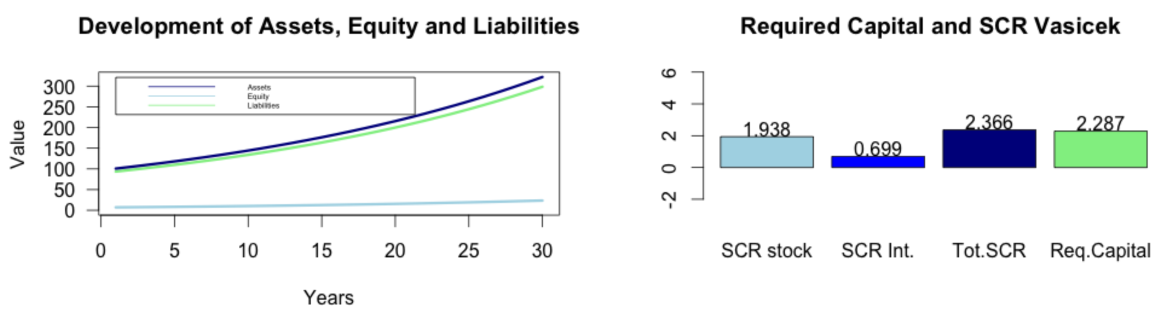
SCR stress case	0,6986453

Stress scenario stocks			
A	93,47788	E	5,141143
		L	88,33674
	93,47788		93,477883

SCR stress case	1,937729

TotalSCR	2,365731
Required capital	2,286859

Figure 7-20 Results Vasicek Duration 4

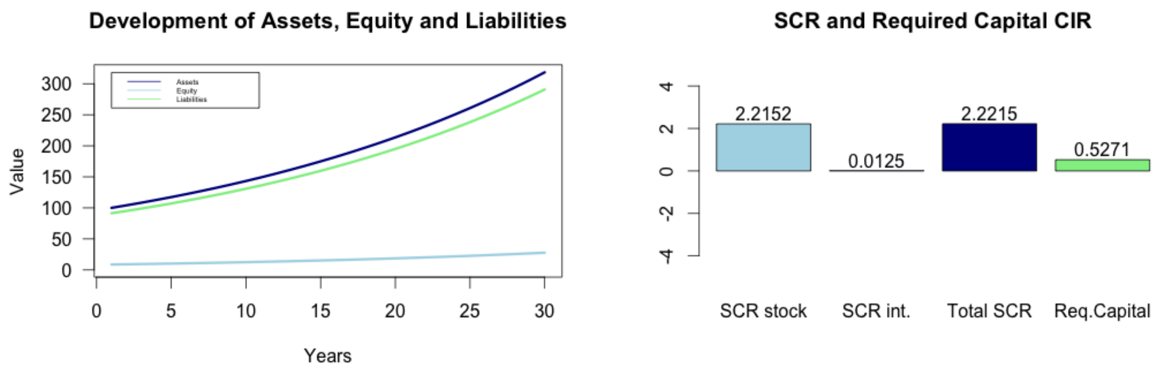


Results from stress scenarios for Vasicek with duration 4 are in the Appendix C 18.

Table 7-20 Results CIR Duration = 4

	CIR		
A	99,99207	**E**	8,694446
		L	91,29763
	99,99207		99,992076
SCR base case		-1,694446	
	Stress scenario interest rates		
A	101,9911	**E**	8,681918
		L	93,30915
	101,9911		101,991068
SCR stress case		0,012528	
	Stress scenario stocks		
A	92,79561	**E**	6,479211
		L	86,31639
	92,79561		92,795601
SCR stress case		2,215235	
TotalSCR		2,221526	
Required capital		0,5270795	

Figure 7-21 Results CIR Duration = 4

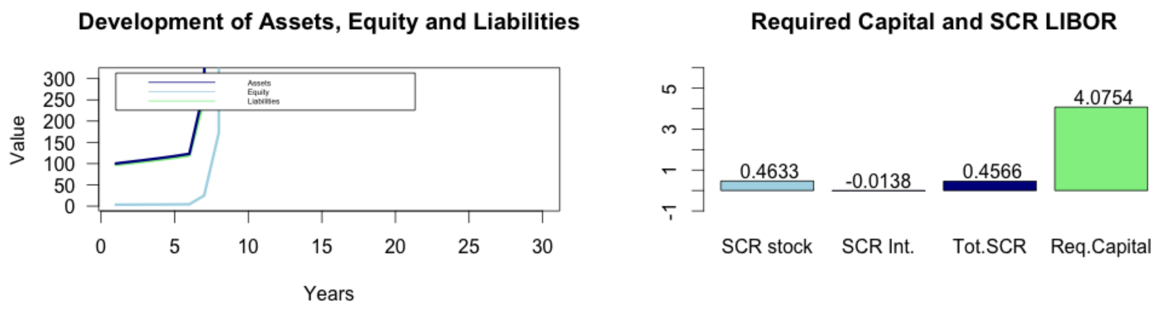


Results from stress scenarios for CIR with duration 4 are in the Appendix C 19.

Table 7-21 Results LMM Duration = 4

LMM			
A	100,3026	**E**	3,3811
		L	96,9215
	100,3026		100,3026
SCR base case			3,6189
Stress scenario interest rates			
A	105,1656	**E**	3,3950
		L	101,7706
	105,1656		105,1656
SCR stress case			-0,0138
Stress scenario stocks			
A	93,0807	**E**	2,9178
		L	90,1629
	93,0807		93,0807
SCR stress case			0,4633
Total SCR			0,4566
Required capital			4,0754

Figure 7-22 Results LMM Duration = 4



Results from stress scenarios for LMM with duration 4 are in the Appendix C 20.

General, for a zero coupon bond a lower duration reduces the risk of the bond, since it takes shorter time to receive back the face value. So when the duration goes down we would expect the value to go up.

Value of equity is increasing for all scenarios for all models, while the value of liabilities is decreasing. The changes are greatest for Vasicek, while CIR is changing minimal in value. Both Vasicek and CIR have a higher market value than book value for liabilities, this leads to a required capital smaller than the total SCR. Required capital is reduced for the three models compared to the standard base cases. Results from the stress scenarios, both interest rates and stocks, gives no major differences from the standard base cases.

Summary

By changing the parameters in the models we get results for different scenarios, some results were as expected, while some were surprising. There is interesting to see how single factors can affect the value, in some cases with a significantly large impact. The greatest differences appeared in the scenarios for different stock allocations and the scenarios with adjusted interest rate and long-term mean, where we saw distinct changes, especially in the stress scenarios, for both interest rates and stocks. By changing the guaranteed interest return we got major differences in the value of equity and the value of liabilities. Ideally, we should have run more analysis with other values of g to see what results that would have given.

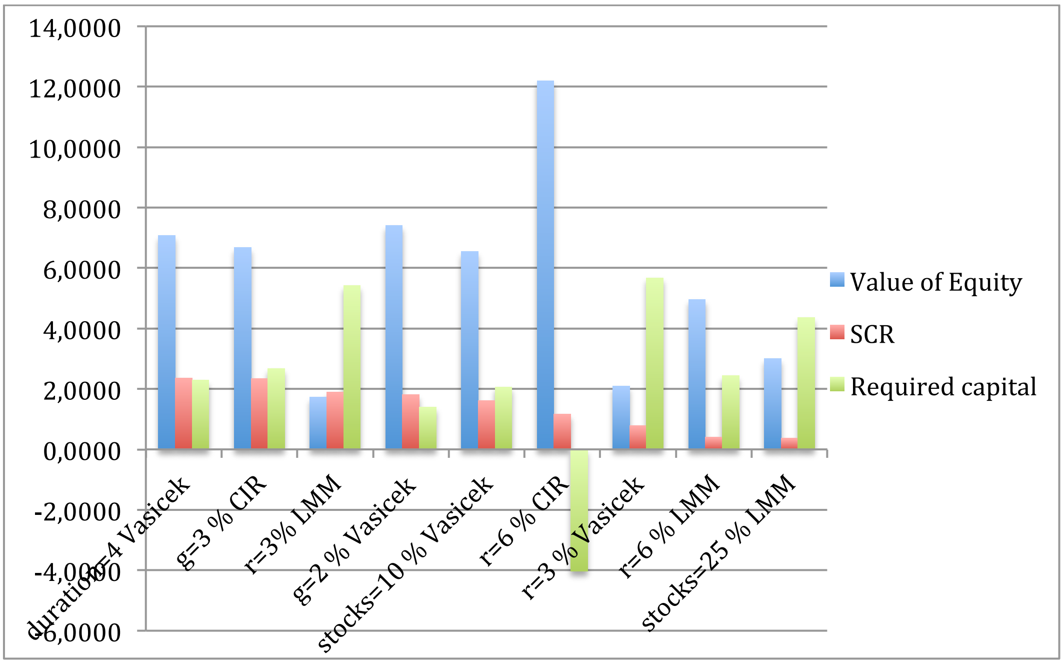
A recurring phenomenon for CIR compared with Vasicek, was the higher mean reversion parameter. It causes the effect from changing single parameters to quickly pass, because it was faster pulled back to the long term mean. The large difference in the kappa parameter is questionable, and in the worst case may weaken the results.

Some selected results are shown in table Table 7-22 and figure Figure 7-23. The table is sorted by ranking the highest SCR.

Table 7-22 Summary table scenarios

Model	Parameter	Value of Equity	SCR	Required capital
Vasicek	duration=4	7,0789	2,3657	2,2869
CIR	g=3 %	6,6777	2,3474	2,6697
LMM	r=3%	1,7289	1,8921	5,4232
Vasicek	g=2 %	7,4137	1,8149	1,4012
Vasicek	stocks=10 %	6,5454	1,6075	2,0622
CIR	r=6%	12,1980	1,1611	-4,0368
Vasicek	r=3%	2,1014	0,7799	5,6784
LMM	r=6%	4,9652	0,4016	2,4364
LMM	stocks=25 %	3,0073	0,3661	4,3588

Figure 7-23 Summary table scenarios



When we adjust the duration down to 4, Vasicek gets the highest value for SCR compared to other scenarios the value of equity is also increased. LMM has a low value of SCR if we change the stock allocation to 25 %, but can see from the required capital that there is need for in addition to book value equity. The greatest change in value of equity is for CIR when we set the interest rate and speed of mean reversion to 6 %, required capital is negative in this case. All three models reacts mostly the same way when we change a parameter, an exception is CIR when stock allocation is 25 %, where total SCR and required capital is increase, and it is the opposite for Vasicek and LMM. There is however a difference in how much change it is.

8 Conclusion

Our results show that both choice of model and calibration affects the capital requirements. It was interesting and disturbing to see after calibrating the three models with a variety of datasets, that we got divergent results. At the risk of drawing a too firm conclusion, the results indicates that life insurance companies can get the capital requirements they want by using a model and calibration that fits their needs. Since life insurance companies and regulators to an extent have different incentives, they will argue for and against the use of different models and calibrations. Internal models must be approved prior to use, and there are already restrictions on the models. These restrictions should if possible be even stricter.

Especially, by using the model Cox, Ingersoll and Ross, we observe a higher market value of equity than compared to the other models. This should generally give lower requirements for capital in addition to book equity. We see that the speed of mean reversion, kappa, is calibrated with a high value for CIR compared to Vasicek. Which results in small changes in value of equity and in the whole balance sheet in the stress scenario for interest rates, compared to the base case.

If the proposal from the Norwegian FAS, of decreasing the guaranteed interest rate had been approved, this would have increased the shareholder value at the expense of the policyholder. By changing single parameters, like adjusting the guaranteed interest down, all the models show an apparently similar reaction. When changing the time series used in calibrations, the results were more divergent. This may indicate that life insurance companies have incentives to get their own internal model approved.

Potentially policy implications for life insurance companies and the Financial Supervisory Authority in conjunction with Solvency II should contain clear restrictions on how to calculate capital requirements. It should be established strict guidelines for which models that are allowed, and how they are calibrated.

All conclusions are based on the condition that the models are correctly programmed and calibrated.

Weaknesses in the report

For the Libor market model we should ideally have held the standard deviation constant from year 16, where it starts to increase. Instead of instant US forward rates we should have used swap rates. The formula for bond pricing is inconsistent with the formula for calculating the spot rates from forward rates. The formula we use for spot rates is a discrete time, and bond pricing is on a continuous time. We only run 30 000 simulations for LMM, due to machine capacity, this is unfortunate, but should not affect the results significantly.

Our results from Vasicek and some times CIR does not get discounted correctly back to the start value. This probably ordinates from a misspecification in the simulations code.

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10 Appendix A: Symbols

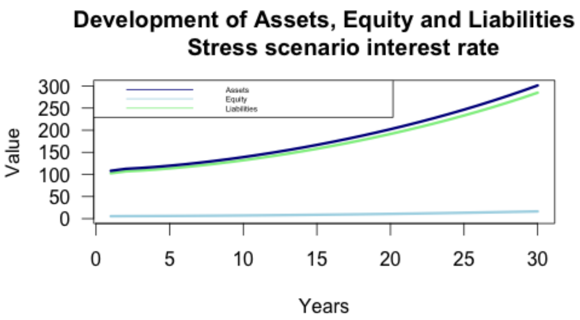
Symbols

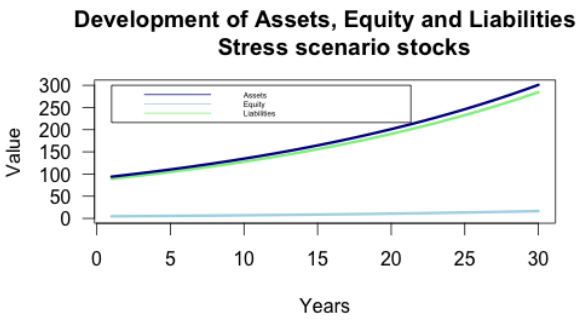
A_t	Assets at time t
ASR	Additional statutory reserve
BSCR	Basic solvency capital requirement
E_t	Equity at time t
g	Guaranteed return (in % − terms)
k	Kappa
L_t	Liabilities at time t
MVAR	Market value adjustment reserve
N	Number of simulations
R_0	Start rate
r_t	Interest rate at time t
$R_{A,t}$	Return before allocaton $=R_p$
r_t^b	Bond return
$R_{k,t}$	Return after allocation to MVAR
$r_{k,t}$	Profit after allocation to MVAR
$R_{P,t}$	Portfolio return at time t
r_p	Portfolio return
$R_{S,t}$	Shareholder return at time t
r_t^s	Stock return
r_t^{sr}	Short rate return
S	Stock price
$SCR_{Interest\ rate}$	Solvency capital requirement for interest rates
SCR_{Stocks}	Solvency capital requirement for stocks
SCR_{Total}	Total solvency capital requirement
Std	Standard deviation portfolio
T	Time
t	Bonus for shareholders
VaR	Value at risk
W_L	Profit sharing to customer fund, after ASR

θ	Theta

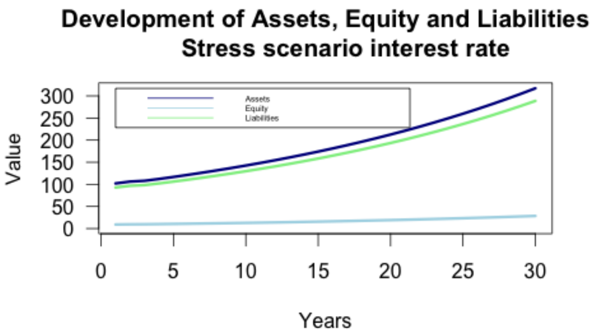
11 Appendix B

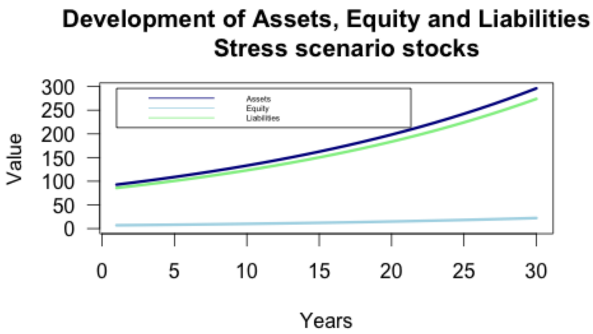
Appendix B 1 Stress scenario Vasicek 3mNibor W



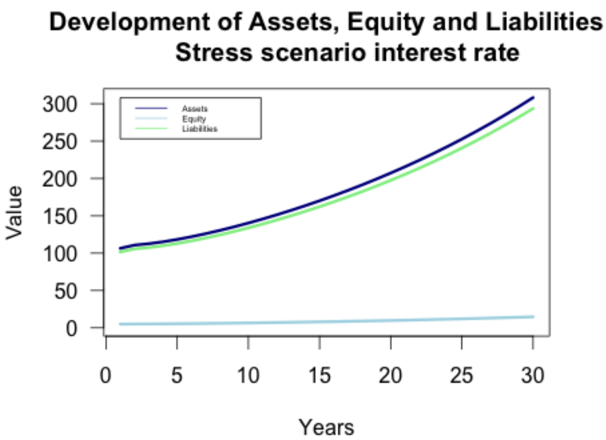


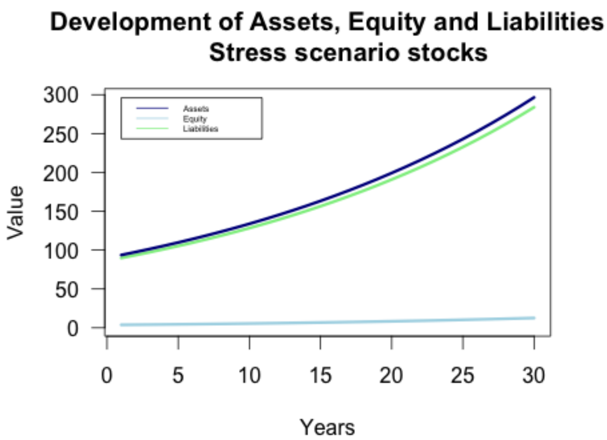
Appendix B 2 Stress scenario CIR 3m Nibor W



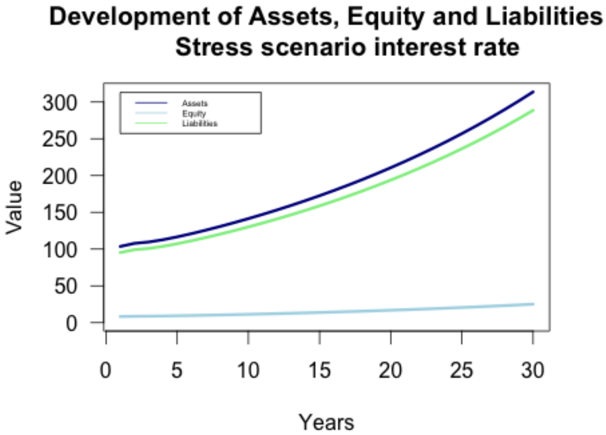


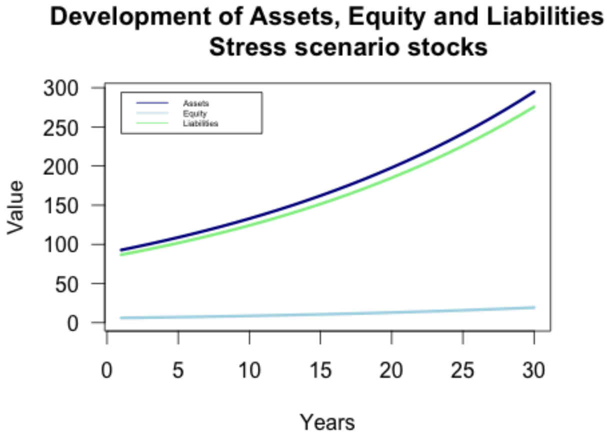
Appendix B 3 Stress scenario Vasicek 3mNibor M20Y





Appendix B 4 Stress scenario CIR 3m Nibor M 20Y





Appendix B 5 Results CIR 12 m Nibor M 10 year

CIR

A	99,99914	**E**	8,778432
		L	91,2207
	99,99914		99,999132

E.value	-1,778432

Stress scenario interest rates

A	101,622	**E**	8,774126
		L	92,84784
	101,622		101,621966

VaR stress case	0,004306054

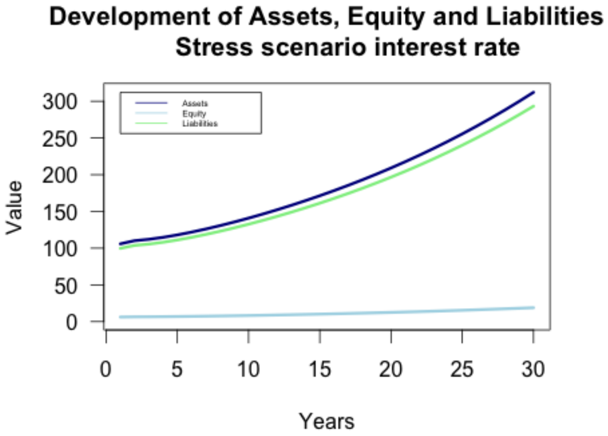
Stress scenario stocks

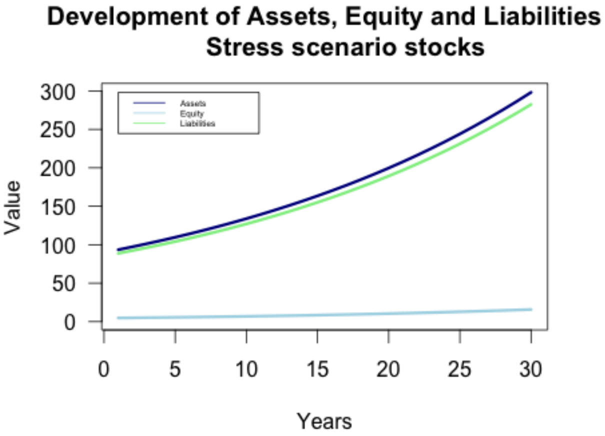
A	92,80234	**E**	6,578056
		L	86,22428
	92,80234		92,802336

VaR stress case	2,200376

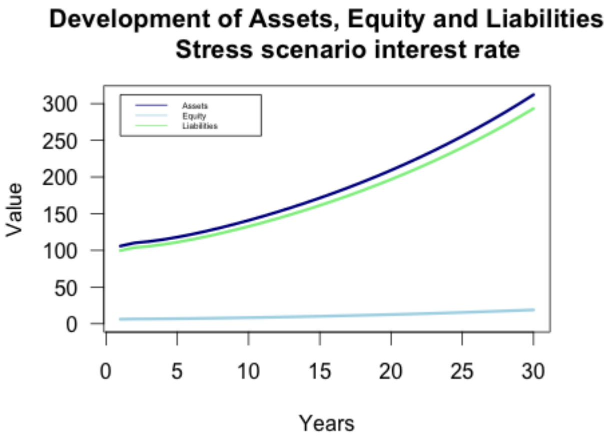
Total VaR	2,202533
SCR	0,4241004

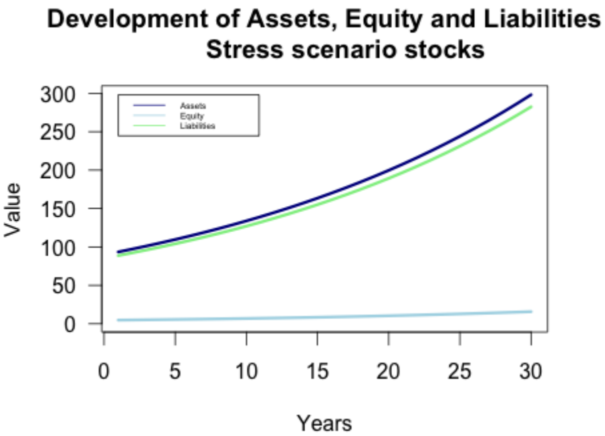
Appendix B 6 Stress scenario Vasicek 12 m Nibor 10Y



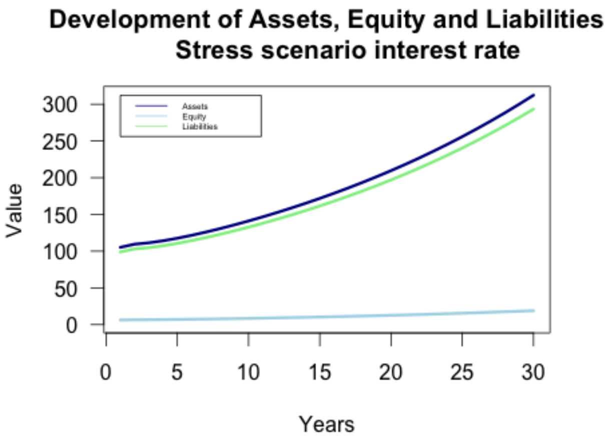


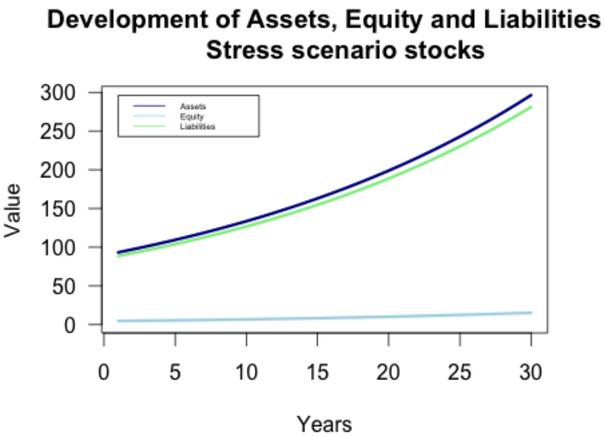
Appendix B 7 Stress scenario CIR 12 m Nibor 10Y



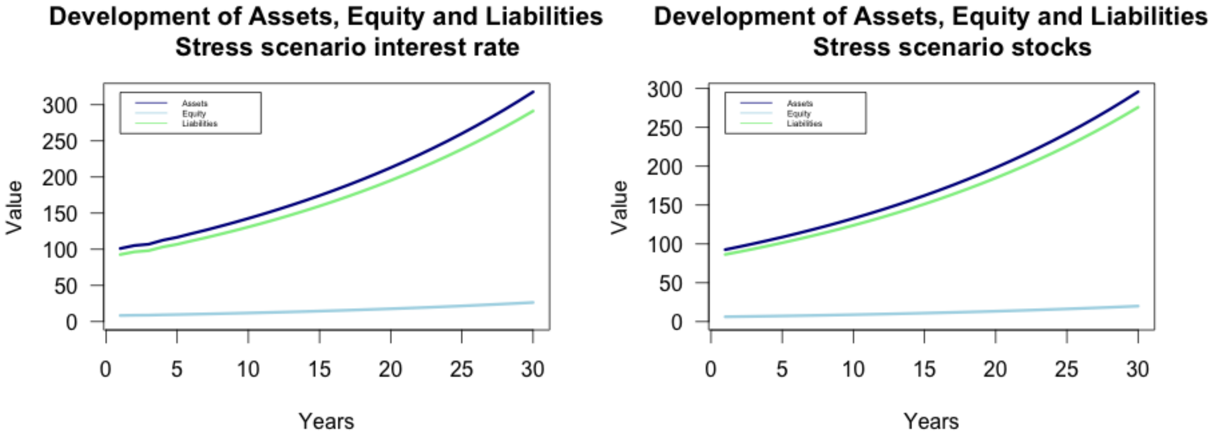


Appendix B 8 Stress scenario Vasicek 3 m T-BILL 10Y





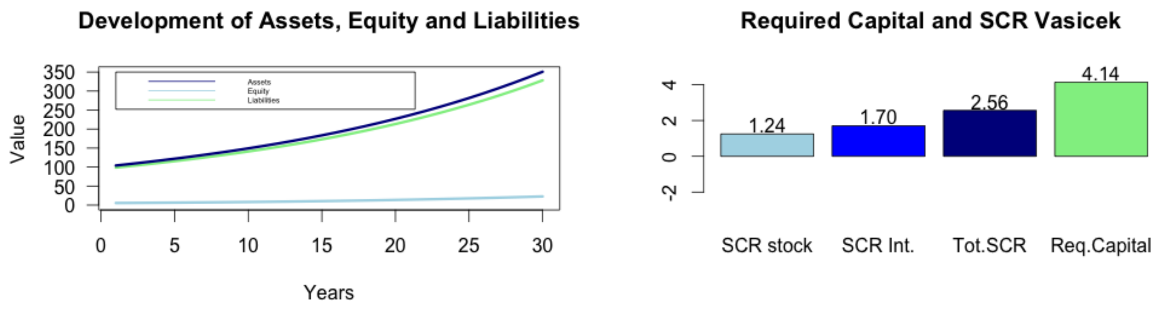
Appendix B 9 Stress Scenario CIR 3 m T-Bill 10 Y



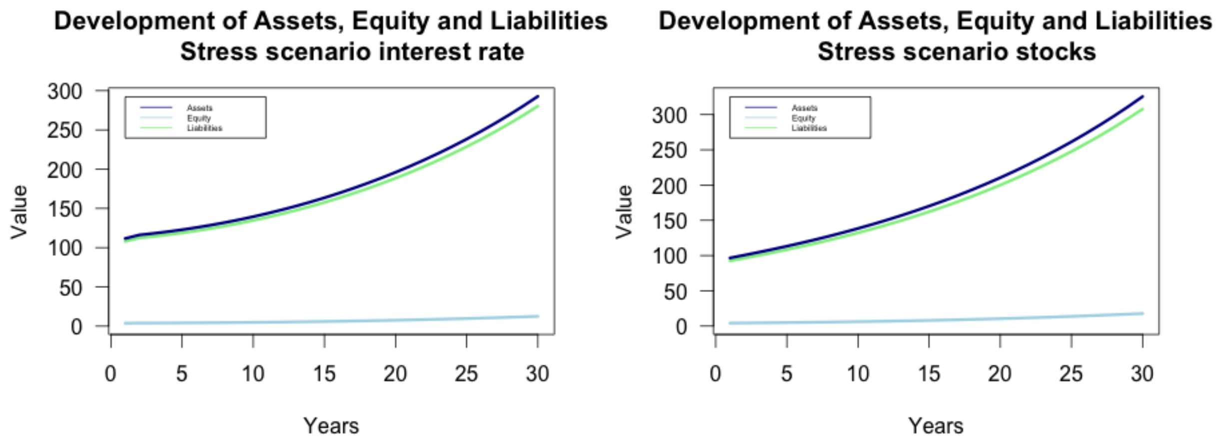
Appendix B 10 Results Vasicek 3 m Libor 10 year

	Vasicek		
A	104,0662	**E**	5,4229
		L	98,6433
	104,0662		104,0662
SCR base case		1,5771	
	Stress scenario interest rates		
A	111,6459	**E**	3,7184
		L	107,9274
	111,6459		111,6458
SCR stress case		1,7044	
	Stress scenario stocks		
A	96,5910	**E**	4,1788
		L	92,4122
	96,5910		96,5910
SCR stress case		1,2441	
Total SCR		2,5638	
Required capital		4,1410	

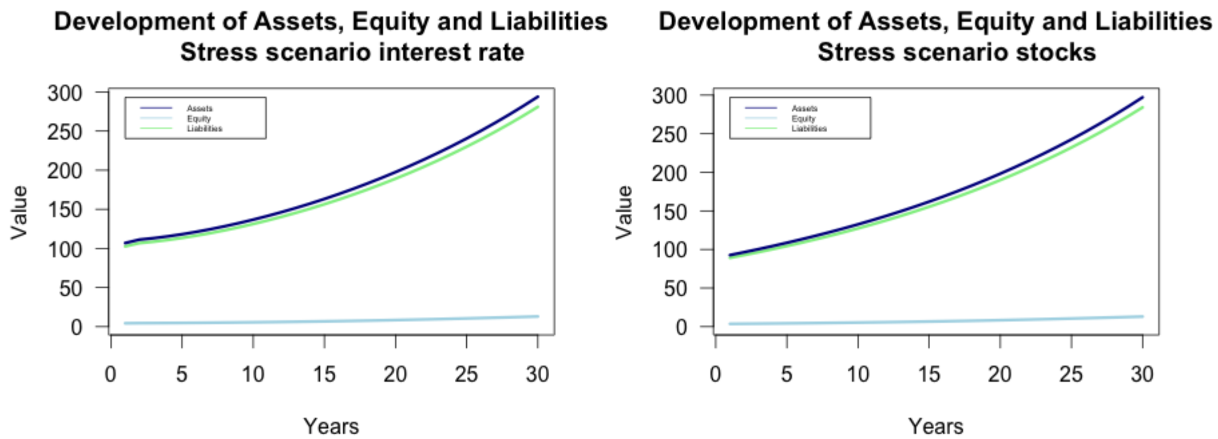
Appendix B 11 Results Vasicek 3 m Libor 10 year



Appendix B 12Stress scenario Vasicek 3m Libor 10 Y



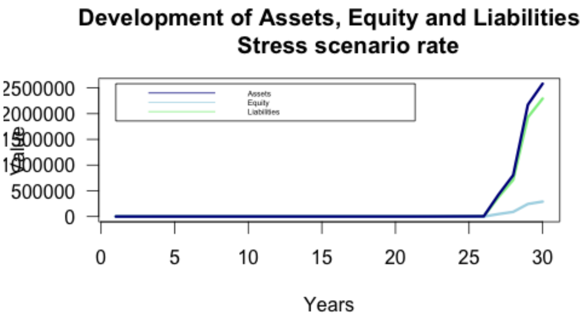
Appendix B 13 Stress scenario CIR 3m Libor 10Y

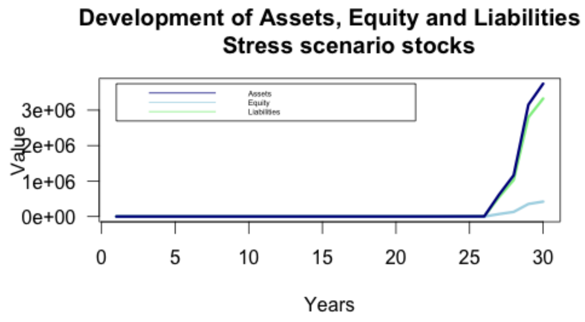


Appendix B 14 Results LMM 20Y M

LMM			
A	100,0887	E	2,8623
		L	97,2265
	100,0887		100,0887
SCR base case			4,1377
Stress scenario interest rates			
A	105,0517	E	2,9121
		L	102,1396
	105,0517		105,0517
SCR stress case			-0.0498872
Stress scenario stocks			
A	92,8789	E	2,5753
		L	90,3036
	92,8789		92,8789
SCR stress case			0,2869
Total SCR			0,2655
Required capital			4,4033

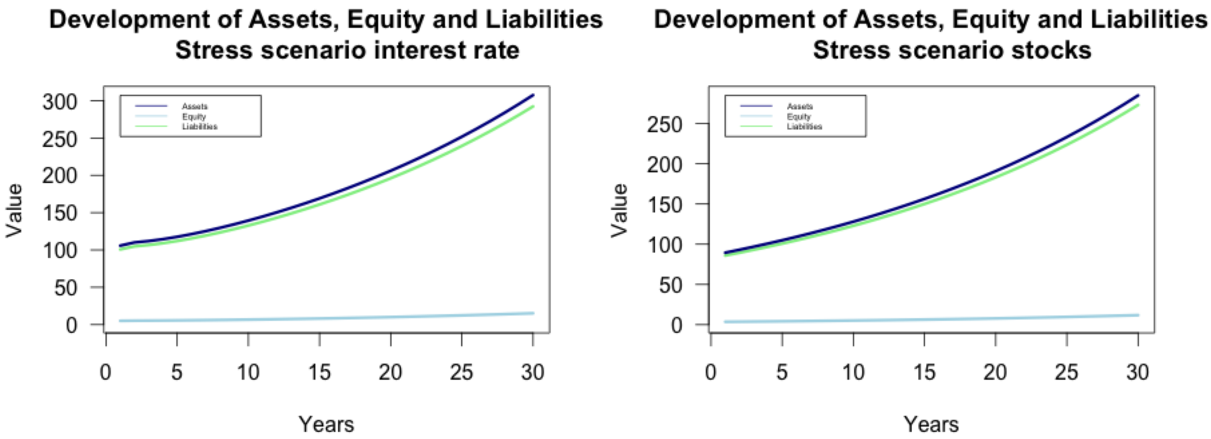
Appendix B 15 Results Stress scenario LMM 20Y M



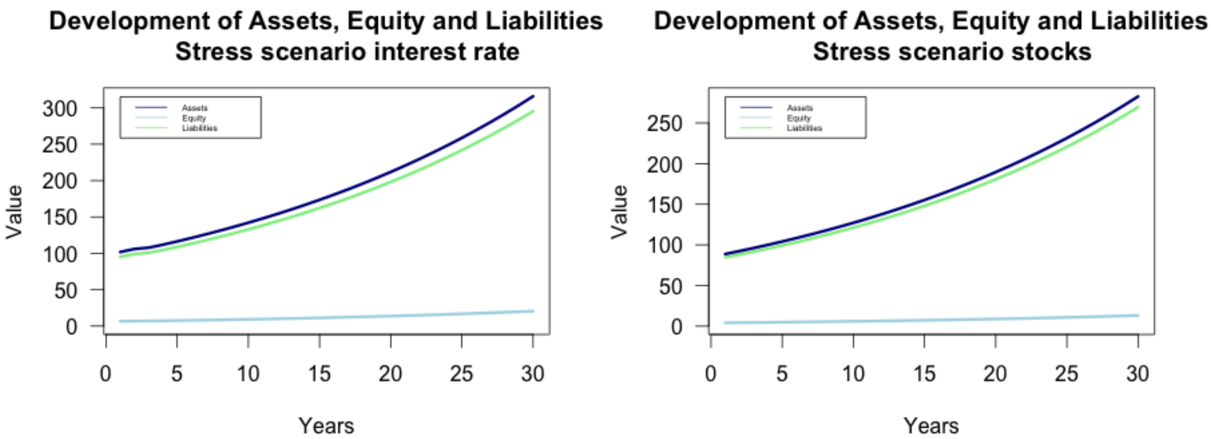


12 Appendix C

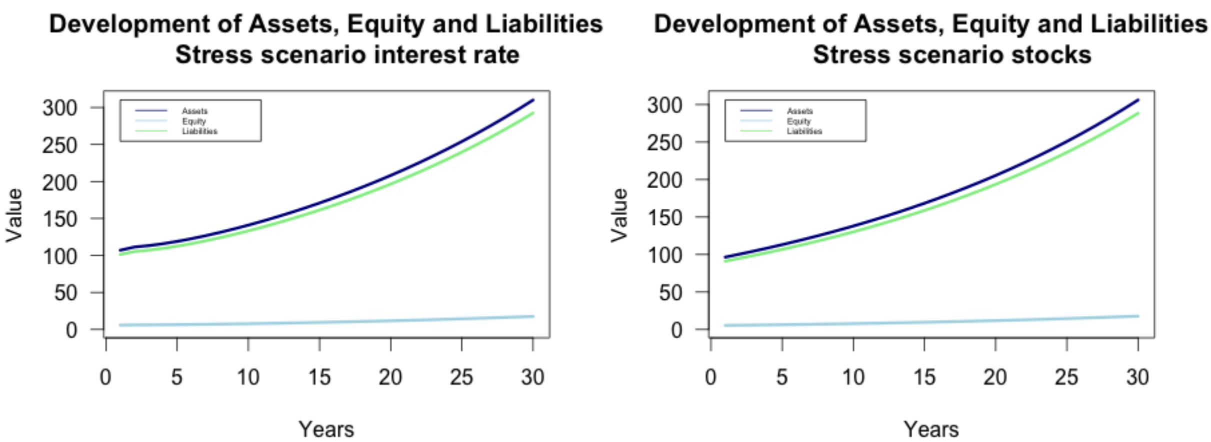
Appendix C 1 Stress scenario Vasicek stock ratio 25 %



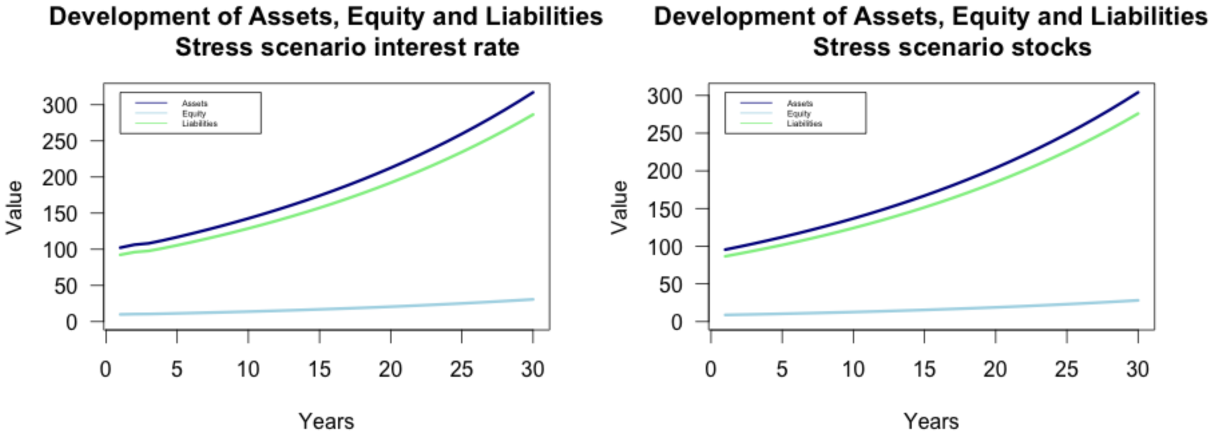
Appendix C 2 Stress scenario CIR stock ratio 25 %



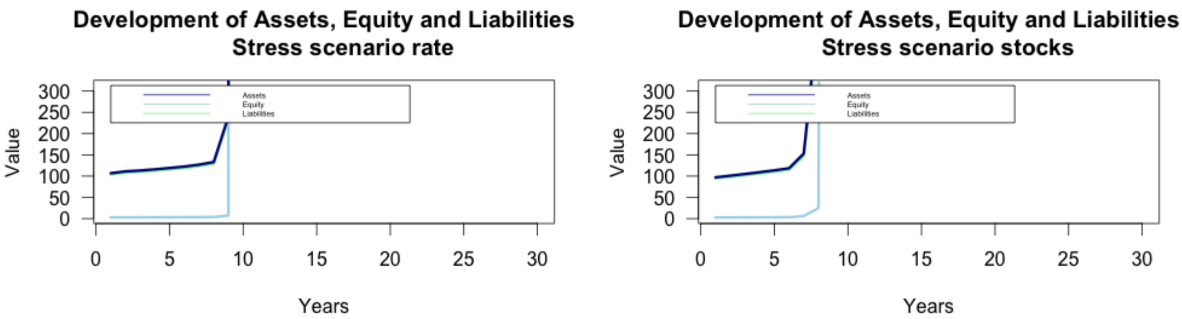
Appendix C 3 Stress scenario Vasicek 10 %



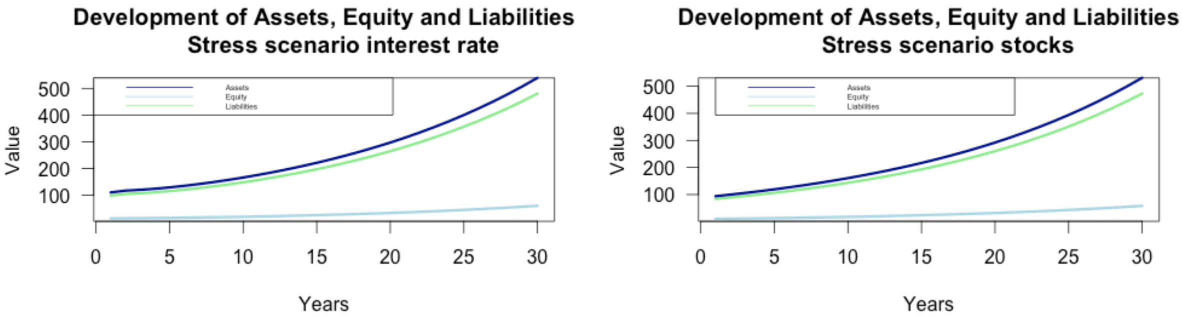
Appendix C 4 Stress scenario CIR stock ratio 10 %



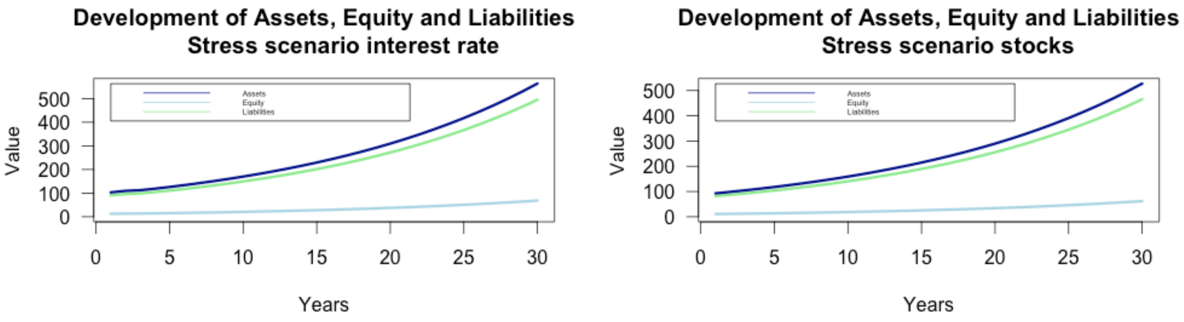
Appendix C 5 Stress scenario LMM stock ratio 10 %



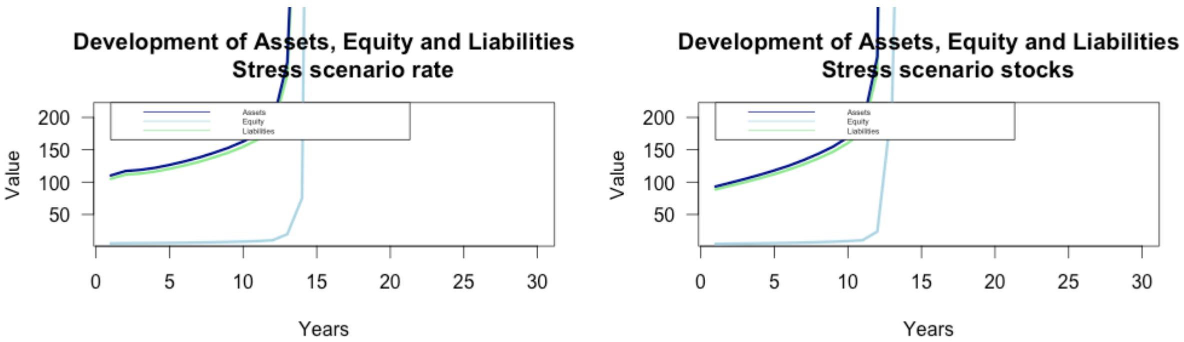
Appendix C 6 Stress scenario Vasicek r= 6% and theta= 6%



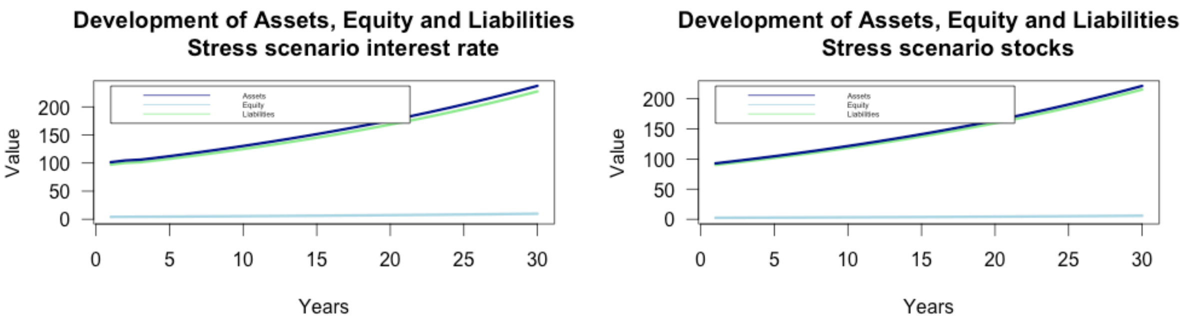
Appendix C 7 Stress scenario CIR r= 6% and theta =6%



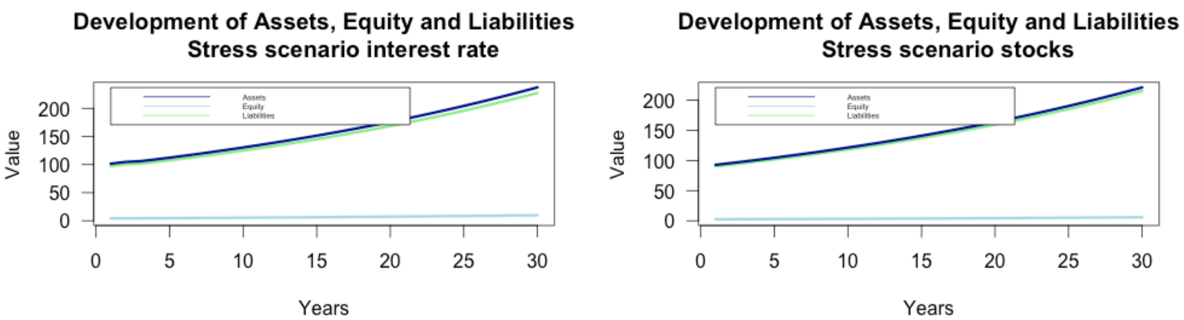
Appendix C 8 Stress scenario LMM Forward 6 %



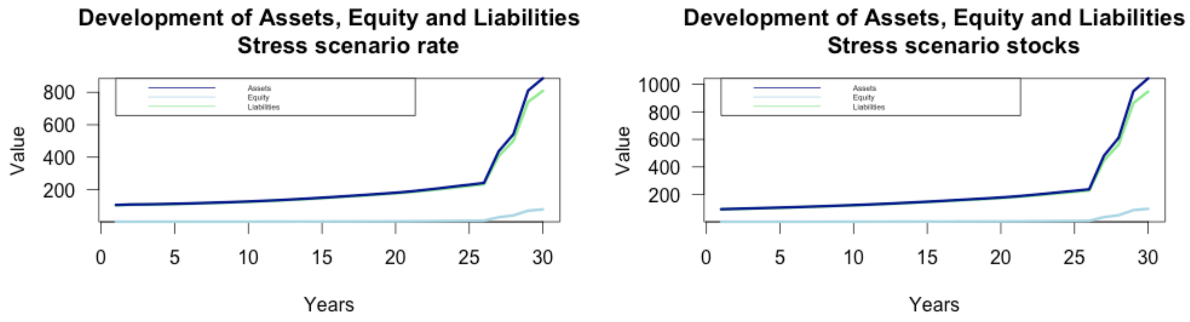
Appendix C 9 Stress scenario vasicke r=3% and theta=3%



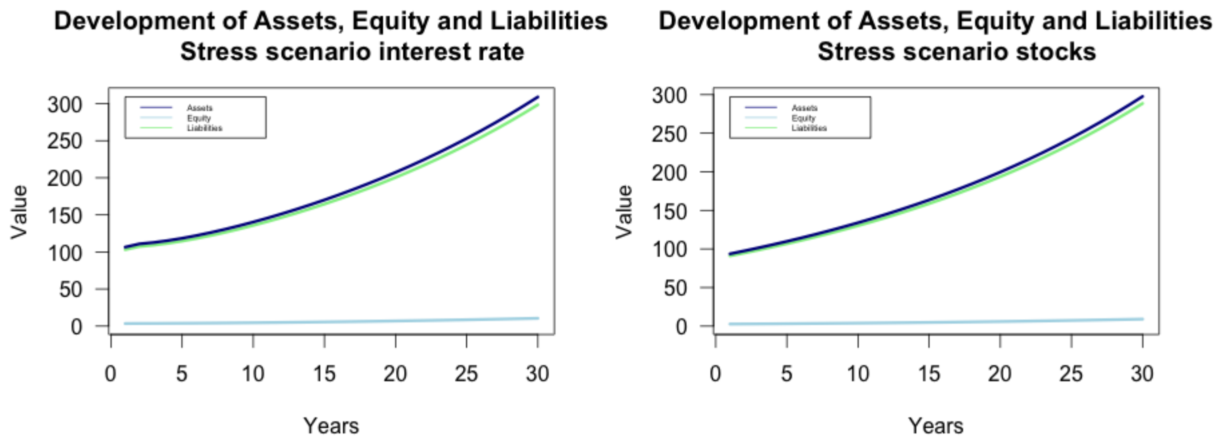
Appendix C 10 Stress scenario CIR r=3% and theta=3%



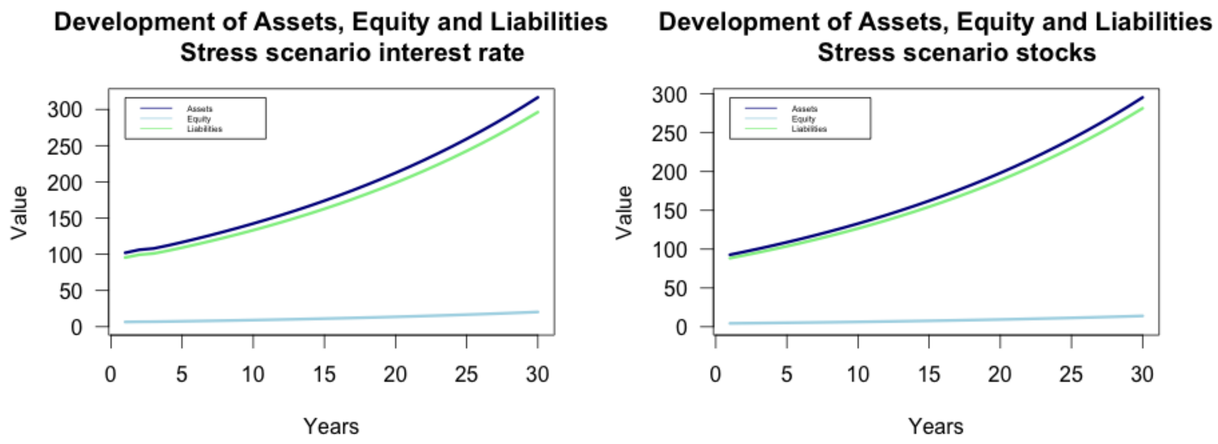
Appendix C 11 Stress scenario LMM forward 3 %



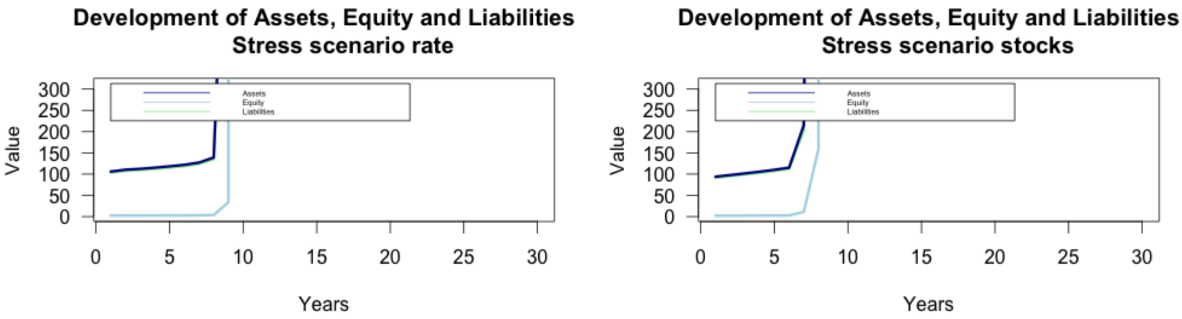
Appendix C 12 Stress scenario Vasicek g=3%



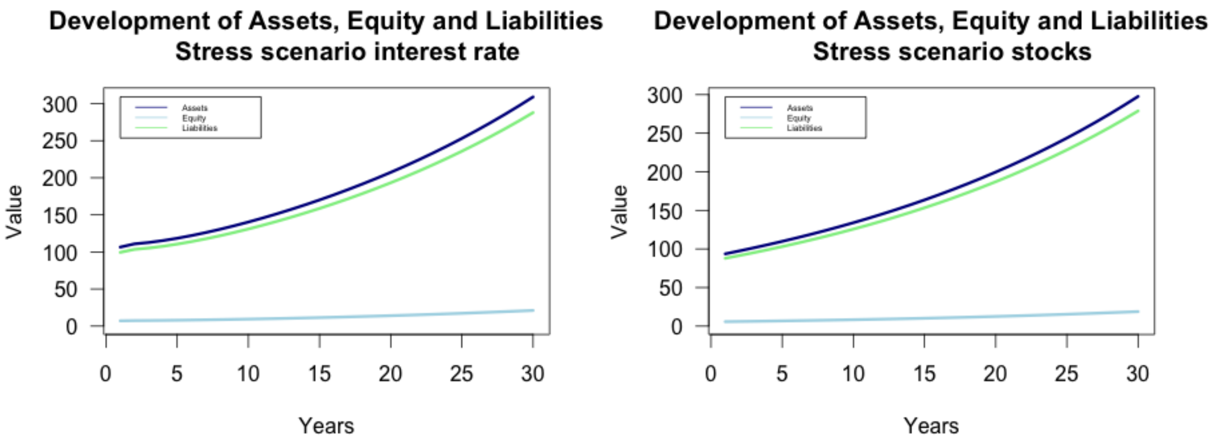
Appendix C 13 Stress scenario CIR g=3%



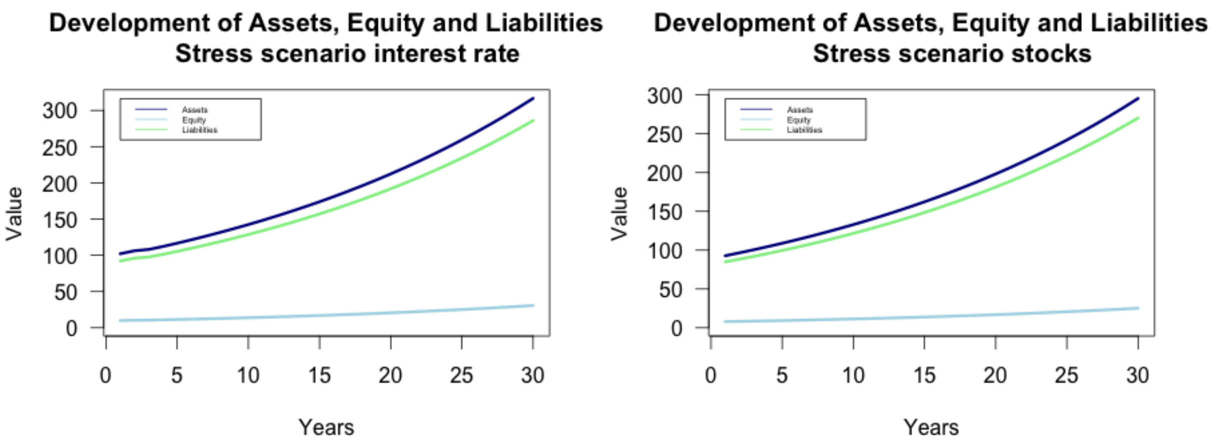
Appendix C 14 Stress scenario LMM 3%



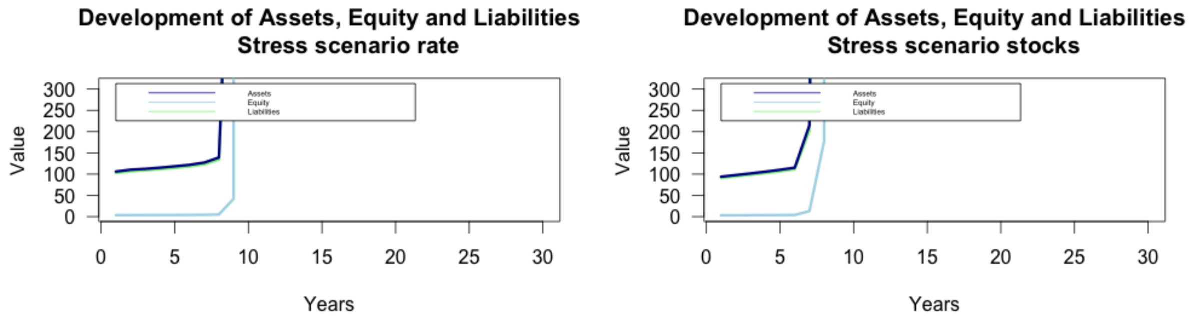
Appendix C 15 Stress scenario Vasicek g=2%



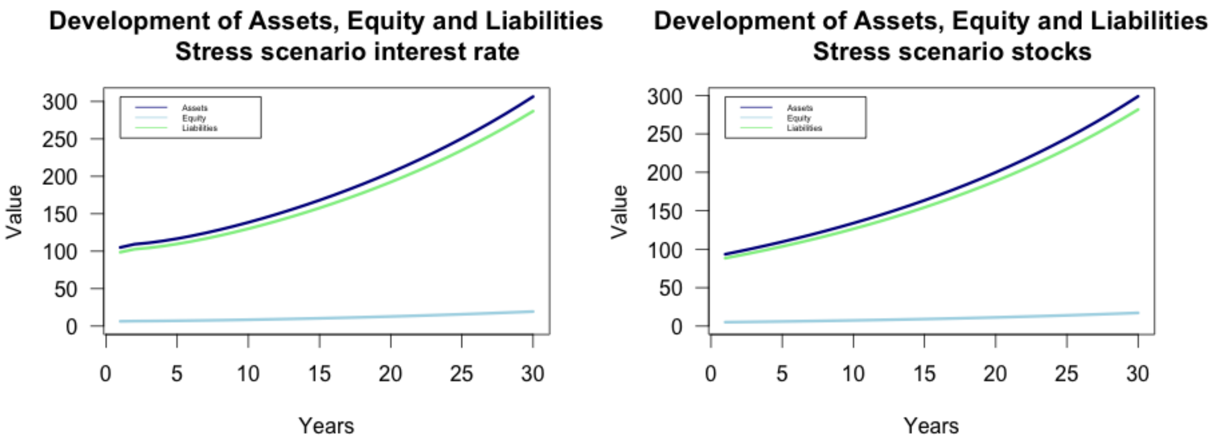
Appendix C 16 Stress scenario CIR g=2%



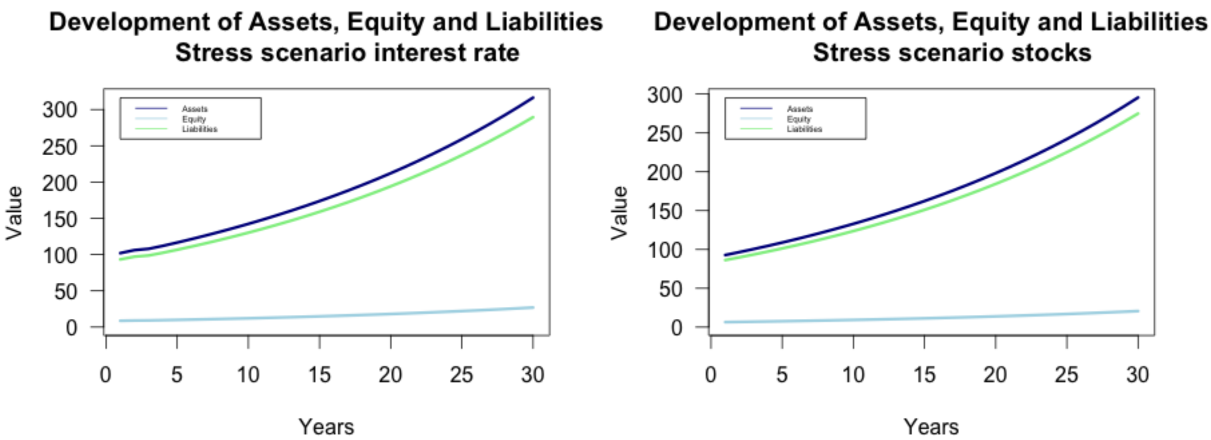
Appendix C 17 Stress scenario LMM g=2%



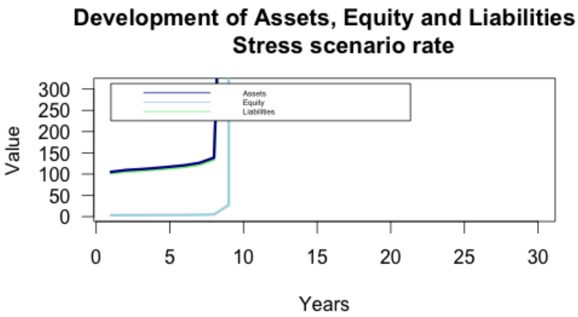
Appendix C 18 Stress scenario Vasicek duration 4

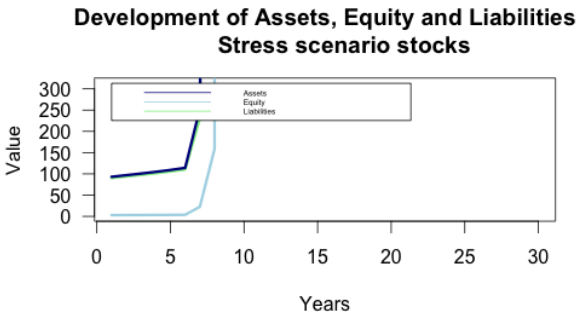


Appendix C 19 Stress scenario CIR duration 4



Appendix C 20 Stress scenario LMM duration 4





13 Appendix D

Appendix D 1 12 m Libor monthly 10Y

LIBOR 12MTH (2004-2014)	Vasicek Monthly	Vasicek Annual	CIR Monthly	CIR Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,052927	0,183344	0,052927	0,183344
Std. Dev rate	0,002291	0,007951	0,013146	0,045569
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,004062	-0,048750	-0,001368	-0,016418

Vasicek				**CIR**			
A	105,7050	**E**	6,1499	**A**	99,5583	**E**	3,7863
		L	99,5552			**L**	95,7720
	105,7050		105,7050		99,5583		99,5583
SCR base case	0,8501			**SCR base case**	3,2137		
Stress scenario interest rates				***Stress scenario interest rates***			
A	113,6859	**E**	3,6199	**A**	109,0009	**E**	1,5099
		L	110,0660			**L**	107,4910
	113,6859		113,6859		109,0009		109,0009
SCR stress case	2,5300			**SCR stress case**	2,2764		
Stress scenario stocks				***Stress scenario stocks***			
A	98,1160	**E**	4,7566	**A**	92,3945	**E**	3,0041
		L	93,3594			**L**	89,3904
	98,1160		98,1160		92,3945		92,3945
SCR stress case	1,3933			**SCR stress case**	0,7822		
Total SCR	3,4449			**Total SCR**	2,7522		
Required capital	4,2950			**Required capital**	5,9659		

Appendix D 2 12 m T-Bill monthly 10Y

T BILL 12MTH (1994-2014)	Vasicek Monthly	Vasicek Annual	CIR Monthly	CIR Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,052927	0,183344	0,052927	0,183344
Std. Dev rate	0,002931	0,010261	0,018956	0,069634
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,020986	-0,251834	-0,112892	-1,354710

Vasicek				CIR			
A	100,9189	**E**	6,4655	**A**	99,9223	**E**	8,4891
		L	94,4655			**L**	91,4362
	100,9189		100,9309		99,9223		99,9253
SCR base case	0,5345			**SCR base case**	-1,4891		
Stress scenario interest rates				***Stress scenario interest rates***			
A	106,4985	**E**	5,8933	**A**	101,2843	**E**	8,4780
		L	100,6053			**L**	92,8063
	106,4985		106,4986		101,2843		101,2843
SCR stress case	0,5601			**SCR stress case**	0,0081		
Stress scenario stocks				***Stress scenario stocks***			
A	93,6583	**E**	4,6413	**A**	92,7308	**E**	6,3111
		L	89,0170			**L**	86,4196
	93,6583		93,6583		92,7308		92,7308
SCR stress case	1,8121			**SCR stress case**	2,1750		
Total SCR	0,3863			**Total SCR**	2,1791		
Required capital	4,1664			**Required capital**	0,6930		

Appendix D 3 12 m Nibor monthly 20Y

NIBOR 12MTH (1994-2014)	Vasicek Monthly	Vasicek Annual	CIR Monthly	CIR Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,047876	0,165849	0,047876	0,165849
Std. Dev rate	0,003315	0,011559	0,014980	0,053156
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,013087	-0,157045	-0,047342	-0,568109

Vasicek				CIR			
A	101,6720	E	4,2894	A	100,0034	E	8,5832
		L	97,3826			L	91,4211
	101,6720		101,6720		100,0034		100,0043
SCR base case		2,7106		SCR base case		-1,5832	

Stress scenario interest rates				*Stress scenario interest rates*			
A	108,2344	E	3,5951	A	103,1238	E	8,5760
		L	104,6392			L	94,5478
	108,2344		108,2343		103,1238		103,1238
SCR stress case		0,6943		SCR stress case		0,0072	

Stress scenario stocks				*Stress scenario stocks*			
A	94,4496	E	3,2332	A	92,8065	E	6,3319
		L	91,2165			L	86,4746
	94,4496		94,4496		92,8065		92,8065
SCR stress case		1,0809		SCR stress case		2,2513	
Total SCR		1,5495		Total SCR		2,2549	
Required capital		4,2354		Required capital		0,6717	

Appendix D 4 12 m T-Bill monthly 20Y

T BILL 12MTH (1994-2014)	Vasicek		CIR	
	Monthly	Annual	Monthly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,047876	0,165849	0,047876	0,165849
Std. Dev rate	0,004235	0,014867	0,019292	0,070623
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,026431	-0,317167	-0,106449	-1,277385

Vasicek				CIR			
A	100,5122	E	4,9864	A	99,9645	E	8,8424
		L	95,5258			L	91,1222
	100,5122		100,5122		99,9645		99,9645
SCR base case		2,0136		SCR base case		-1,8424	

Stress scenario interest rates				Stress scenario interest rates			
A	105,2002	E	4,6761	A	101,4086	E	8,8516
		L	100,5241			L	92,5570
	105,2002		105,2002		101,4086		101,4086
SCR stress case		0,3103		SCR stress case		-0,0093	

Stress scenario stocks				Stress scenario stocks			
A	93,2928	E	3,5603	A	92,7637	E	6,6279
		L	89,7325			L	86,1358
	93,2928		93,2928		92,7637		92,7637
SCR stress case		0,3103		SCR stress case		2,2144	
Total SCR		1,6135		Total SCR		2,2098	
Required capital		3,6174		Required capital		0,3675	

Appendix D 5 3 m Libor monthly 20Y

LIBOR 3MTH (1994-2014)	Vasicek		CIR	
	Monthly	Annual	Monthly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,047876	0,165849	0,047876	0,165849
Std. Dev rate	0,002366	0,008205	0,014638	0,050584
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,001939	-0,023271	0,004921	0,059055

Vasicek				CIR			
A	115,3305	E	7,1858	A	99,7496	E	3,9092
		L	108,1447			L	95,8404
	115,3305		115,3305		99,7496		99,7496
SCR base case	-0,1858			SCR base case	3,0908		

Stress scenario interest rates				*Stress scenario interest rates*			
A	122,8929	E	3,8373	A	108,5113	E	2,3988
		L	119,0556			L	106,1125
	122,8929		122,8929		108,5113		108,5113
SCR stress case	3,3485			SCR stress case	1,5103		

Stress scenario stocks				*Stress scenario stocks*			
A	107,0313	E	5,7581	A	92,5723	E	2,9753
		L	101,2731			L	89,5970
	107,0313		107,0312		92,5723		92,5723
SCR stress case	1,3978			SCR stress case	0,9339		
Total SCR	4,2245			Total SCR	2,1363		
Required capital	4,0686			Required capital	5,2271		

Appendix D 6 3 m T-Bill monthly 20Y

T BILL 3MTH (1994-2014)	Vasicek		CIR	
	Monthly	Annual	Monthly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,047876	0,165849	0,047876	0,165849
Std. Dev rate	0,003477	0,012345	0,022983	0,089273
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,048406	-0,580873	-0,212107	-2,545281

	Vasicek				CIR		
A	100,1264	E	8,059125	A	80,9486	E	5,14553
		L	92,06729			L	75,80307
	100,1264		100,126415		80,9486		80,9486
SCR base case		-1,059125		SCR base case		1,85447	

	Stress scenario interest rates				*Stress scenario interest rates*		
A	103,2525	E	7,999566	A	80,62924	E	5,148871
		L	95,25294			L	75,48037
	103,2525		103,252506		80,62924		80,629241
SCR stress case		0,05955905		SCR stress case		-0,003341	

	Stress scenario stocks				*Stress scenario stocks*		
A	92,97167	E	5,875238	A	75,11747	E	3,635825
		L	87,09643			L	71,48165
	92,97167		92,971668		75,11747		75,117475
SCR stress case		2,207719		SCR stress case		1,509706	
Total SCR		2,238093		Total SCR		1,508038	
Required capital		1,155136		Required capital		3,362508	

Appendix D 7 12 m Libor monthly 20Y

LIBOR 12MTH (1994-2014)	Vasicek		CIR	
	Monthly	Annual	Monthly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,047876	0,165849	0,047876	0,165849
Std. Dev rate	0,002509	0,008706	0,013092	0,045461
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,003566	-0,042788	-0,0048065	-0,0576782

Vasicek				CIR			
A	117,2702	E	7,277687	A	99,8444	E	6,1735
		L	109,9925			L	93,6709
	117,2702		117,270187		99,8444		99,8444
SCR base case		-0,277687		SCR base case		0,8265	
Stress scenario interest rates				*Stress scenario interest rates*			
A	124,6732	E	4,11509	A	105,6679	E	3,6143
		L	120,5581			L	102,0536
	124,6732		124,67319		105,6679		99,8444
SCR stress case		3,162597		SCR stress case		2,5592	
Stress scenario stocks				*Stress scenario stocks*			
A	108,8324	E	5,902675	A	92,6601	E	4,6179
		L	102,9297			L	88,0422
	108,8324		108,832375		92,6601		92,6601
SCR stress case		1,354303		SCR stress case		1,5556	
Total SCR		4,014881		Total SCR		3,5987	
Required capital		3,757903		Required capital		4,4252	

Appendix D 8 12 m Nibor weekly 20 Y

NIBOR 12MTH (1994-2014)	Vasicek Weekly	Vasicek Annual	CIR Weekly	CIR Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,023431	0,168961	0,023431	0,168961
Std. Dev rate	0,001398	0,004850	0,006189	0,044797
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,002191	-0,113920	-0,007485	-0,389229

Vasicek			
A	101,3831	E	6,821612
		L	94,56145
	101,3831		101,383062
SCR base case			6,821612
Stress scenario interest rates			
A	108,3826	E	5,615472
		L	102,7671
	108,3826		108,382572
SCR stress case			1,20614
Stress scenario stocks			
A	94,0271	E	4,987285
		L	89,03982
	94,0271		94,027105
SCR stress case			1,844527
Total SCR			2,661167
Required capital			2,829356

CIR			
A	100,0032	E	8,8062
		L	91,1970
	100,0032		100,0032
SCR base case			8,8062
Stress scenario interest rates			
A	101,8783	E	8,7992
		L	93,0791
	101,8783		101,8783
SCR stress case			0,0071
Stress scenario stocks			
A	92,8073	E	6,6056
		L	86,2017
	92,8073		92,8073
SCR stress case			2,2006
Total SCR			2,2041
Required capital			0,3979

Appendix D 9 3 m T-Bill 10Y

T BILL 3MTH (2004-2014)	Vasicek		CIR	
	Weekly	Annual	Weekly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,026122	0,188369	0,026122	0,188369
Std. Dev rate	0,001420	0,010270	0,007809	0,057035
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,005630	-0,292747	-0,025249	-1,312936

Vasicek			
A	100,6746	E	6,950081
		L	93,72454
	100,6746		100,674621
SCR base case			0,049919
Stress scenario interest rates			
A	105,8018	E	6,522339
		L	99,27947
	105,8018		105,801809
SCR stress case			0,4277415
Stress scenario stocks			
A	93,43269	E	5,034687
		L	88,398
	93,43269		93,432687
SCR stress case			1,915394
Total SCR			2,161247
Required capital			2,211166

CIR			
A	99,9821	E	8,678038
		L	91,30406
	99,9821		99,982098
SCR base case			-1,678038
Stress scenario interest rates			
A	101,3834	E	8,663408
		L	92,71995
	101,3834		101,383358
SCR stress case			0,01463053
Stress scenario stocks			
A	92,78793	E	6,500694
		L	86,28723
	92,78793		92,787924
SCR stress case			2,177344
Total SCR			2,184696
Required capital			0,5066581

Appendix D 10 12 m T-Bill 10Y

T BILL 12MTH (2004-2014)	Vasicek		CIR	
	Weekly	Annual	Weekly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,026122	0,188369	0,026122	0,188369
Std. Dev rate	0,001389	0,010041	0,007909	0,057799
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,004880	-0,253745	-0,026436	-1,374664

Vasicek			
A	100,8877	E	6,740455
		L	94,14725
	100,8877		100,887705
SCR base case			0,259545
Stress scenario interest rates			
A	106,4547	E	6,188023
		L	100,2667
	106,4547		106,454723
SCR stress case			0,5524324
Stress scenario stocks			
A	93,63167	E	4,881656
		L	88,75002
	93,63167		93,631676
SCR stress case			0,5524324
Total SCR			2,187962
Required capital			2,447507

CIR			
A	99,98906	E	8,660964
		L	91,3281
	99,98906		99,989064
SCR base case			-1,660964
Stress scenario interest rates			
A	101,3283	E	8,642555
		L	92,68578
	101,3283		101,328335
SCR stress case			0,01840922
Stress scenario stocks			
A	92,79456	E	6,487046
		L	86,30752
	92,79456		92,794566
SCR stress case			2,173918
Total SCR			2,183181
Required capital			0,5222171

Appendix D 11 3 m T-Bill weekly 10Y

T BILL 3MTH (1994-2014)	Vasicek		CIR	
	Weekly	Annual	Weekly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,023431	0,168961	0,023431	0,168961
Std. Dev rate	0,001427	0,004969	0,008253	0,060914
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,010303	-0,535762	-0,045923	-2,387977

Vasicek			
A	100,0333	E	9,411297
		L	90,62199
	100,0333		100,033287
SCR base case			-2,411297
Stress scenario interest rates			
A	103,3373	E	9,413085
		L	93,92422
	103,3373		103,337305
SCR stress case			-0,0017879
Stress scenario stocks			
A	92,83498	E	7,214785
		L	85,6202
	92,83498		92,834985
SCR stress case			2,196512
Total SCR			2,195619
Required capital			-0,2156782

CIR			
A	86,89487	E	5,394205
		L	81,50067
	86,89487		86,894875
SCR base case			1,605795
Stress scenario interest rates			
A	86,18926	E	5,256352
		L	80,93291
	86,18926		86,189262
SCR stress case			0,1378535
Stress scenario stocks			
A	80,64177	E	3,822658
		L	76,81911
	80,64177		80,641768
SCR stress case			1,571547
Total SCR			1,644812
Required capital			3,250607

Appendix D 12 12 m T-Bill weekly 20Y

T BILL 12MTH (1994-2014)	Vasicek		CIR	
	Weekly	Annual	Weekly	Annual
Start rate R0	0,040000	0,040000	0,040000	0,040000
Std. Dev stocks	0,023431	0,168961	0,023431	0,168961
Std. Dev rate	0,002012	0,006997	0,008088	0,059624
Mean reversion, theta	0,040000	0,040000	0,040000	0,040000
Speed of mean reversion, kappa	-0,007399	-0,384729	-0,043401	-2,256848

Vasicek

A	100,2089	**E**	8,960944
		L	91,24793
	100,2089		100,208874
SCR base case			-1,960944

Stress scenario interest rates

A	104,5072	**E**	8,830264
		L	95,6769
	104,5072		104,507164
SCR stress case			0,1306803

Stress scenario stocks

A	92,99869	**E**	6,744734
		L	86,25395
	92,99869		92,998684
SCR stress case			2,21621
Total SCR			2,284355
Required capital			0,3234111

CIR

A	91,6958	**E**	5,782016
		L	85,91378
	91,6958		91,695796
SCR base case			1,217984

Stress scenario interest rates

A	91,41765	**E**	5,598205
		L	85,81945
	91,41765		91,417655
SCR stress case			0,1838112

Stress scenario stocks

A	85,09727	**E**	4,102593
		L	80,99468
	85,09727		85,097273
SCR stress case			1,679423
Total SCR			1,778467
Required capital			2,996451