Oslo Business School
Working Paper 6/2017

OSLO AND AKERSHUS
UNIVERSITY COLLEGE
OF APPLIED SCIENCES

# Integration and Competition for Innovations in Science-based Industries

Tapas Kundu and Seongwuk Moon

# Integration and Competition for Innovations in Science-based Industries

Tapas Kundu[*,†] Seongwuk Moon[‡]

September 21, 2017

**Abstract**

We develop a model to understand how competition for innovation affects the organization of research activity and property-rights allocation in science-based industries. We consider a vertical production process with a division of labour between research and commercialization. We analyze firms' incentive for integration in the presence of upstream competition for innovation. Integration adversely affects an integrated firm's R&D investment and creates positive externality for the independent firms. For a sufficiently strong externality, a semi-integrated structure appears in equilibrium. The model can thus explain the coexistence of integrated and independent research firms and conforms to the evidence of R&D competition in science-based industries. Interestingly, a non-integrated arrangement can sometime appear in equilibrium even though a semi-integrated arrangement has higher innovation probability and aggregate industry payoff. This is because those who gain from integration cannot commit to compensate the losing parties at the contracting stage. We analyze the effects of resource constraints and inter-customer licensing on the industry structure and their implications for the competition for innovation.

JEL Code: L22; O31; O32

Keywords: R&D contest; Innovation, Vertical integration; Science-based Industry.

[*]Oslo Business School, Oslo Akershus University College of Applied Sciences. Email: tapas.kundu@hioa.no.
[†]School of Business and Economics, UiT the Arctic University of Norway.
[‡]Sogang University Graduate School of Management of Technology. Email: seongwuk@sogang.ac.kr.

# 1 Introduction

Competition for innovation is essential to the growth of a science-based industry.[1] The starting point of our paper is the observation that the organization of research activity and the allocation of ownership rights affect incentives in a competition for innovation. The division of labour between research and commercialization creates a natural vertical structure in the production process of a science-based industry. We see a complex picture of how firms draw their boundaries along this vertical structure. Consider, for example, the case of the biotech industry. At one end of the spectrum, there are large pharmaceutical companies that maintain in-house laboratories, pursue scientific research to discover and identify drug candidates, conduct clinical trials in multiple phases to further develop and commercialize their innovations, and finally, compete in the product market. At the other end, there are small entrepreneurial biotech firms that specialize in preclinical R&D, operate only in the early stages of a drug discovery, and are funded through venture capital and private equity. Following discoveries, these biotech firms interact with other specialized firms in a market for technology for further development and commercialization of their innovations. In between the two ends, multiple types of R&D contracts exist in forms of collaboration, alliance, and partnership among research laboratories, universities, biotech firms, and pharmaceutical companies (Arora et al. 2004, Gans et al. 2008).[2] These contracts make way for the successful transfer of knowledge and rights from those who do R&D to those who commercialize. The transfer of technology through licensing occurs at various stages of a drug development process. While most licensing deals typically take place in the discovery and early development stages, the frequency of late-stage licensing deals has increased over the years (Cartwright 2013, Grabowski and Kyle 2014).[3]

The organization of innovation research and allocation of property rights across various types of firms operating in both upstream and downstream sections of the market have implications for the industry. These industry features are part of the institutional arrangements under which all participants of a typical science-based industry operate. While these arrangements affect a firm's incentives to innovate in a competition for innovation, they are also influenced by how firms compete to innovate. The close interaction between competition for innovation and the institutional arrangements lead us to the following research questions.

[1] Pisano (2010) describes a "science-based" business as the one that "attempts not only to use existing science but also to advance scientific knowledge and capture the value of the knowledge it creates." We also adhere to the same definition in this paper and focus on industries in which the final product comes through a process that involves research for creating new knowledge and subsequent investment for commercializing the value of the knowledge. Examples, among others, include biotechnology, nanotechnology, chemical and semiconductor industries.

[2] The pharmaceutical industry has made an extensive use of the market for technology in the last two decades. The number of high-valuation alliances and partnerships has increased several folds in this period (Grabowski and Kyle 2014).

[3] Reduced financial constraints encourage biotech firms to be more entrepreneurial and give them incentives for late-stage contracting (Grabowski and Kyle 2014). Gans et al. (2008) finds evidence of frictions in the market for technology in forms of high uncertainty associated with a patent's scope and challenges in transferring tacit knowledge. Such frictions can discourage pharmaceutical firms from early-stage licensing.

First, how does competition for innovation affect the organization of innovation research and property-rights allocation at an industry level? Second, how do these institutional arrangements in turn influence competition for innovation? Finally, what sorts of institutional arrangements do we see in a competitive market equilibrium, and how does the equilibrium arrangement fare in terms of economic efficiency? While the existing literature acknowledges the effects of various institutional arrangements such as property-rights allocation and the possibility of technology transfer on innovation incentives (see, for example, Aghion and Tirole 1994, Arora et al. 2004), our point of departure is that we study the effect of competition for innovation on firms' incentives to innovate in a framework with endogenous organization of innovation research, firm boundaries, and property-rights allocations.

To answer these research questions, we develop a simple model of competition for innovation that closely resembles the innovation-generating process in a typical science-based industry. We consider a vertical production process with a division of labour between innovation research and commercialization. Innovation is an intermediate good that is produced out of innovation research and is used as an input in the commercialization process. We refer to firms that specialize in commercialization as customers and firms that specialize in innovation research as research units. Our basic model considers a two-tier duopoly setting.[4] The innovation-generating process unfolds in three stages. In the first, customers decide whether to integrate with research units by owning property rights of their research output. In the second, the research units make non-verifiable investments (which we call 'effort' in the model) in research and compete for innovation. Finally, if an innovation is realized, then customers bargain to acquire a license for commercialization.

We characterize the competitive equilibrium of the game and show that two types of R&D arrangements can arise in equilibrium. In one of these two forms, which we refer to as no integration, all research units compete for innovation while retaining the property rights of their research output. The successful firm licenses its technology to the customers in the post-innovation stage. In the other form, which we refer to as semi integration, one of the two research units (some but not all research units in a framework with more than two firms) sells the ownership rights of its research output to a potential customer before it competes for innovation with the other firm that retains the ownership rights of its output. A semi-integration arrangement in equilibrium is interesting for two reasons. First, starting with a symmetric framework, we find in equilibrium simultaneous allocation of property rights of innovation in the upstream and downstream sections of the market. Second, some of the customers in the downstream market can acquire innovation from two sources: An external source occurs when a research unit retains the property rights of its output and succeeds in making an innovation. An internal source occurs when a customer owns the property rights of the successful research unit's output.

[4]The model can be easily extended to a setting with more than two players in both upstream and downstream markets. We find the results are robust to such an extension.

Evidence from science-based industries supports both of these features. For instance, in 2002, the top ten largest pharmaceutical companies conducted the majority of their development projects in-house while buying 47 per cent of their development candidates from external sources such as biotech firms and universities (Pisano 2006). Biotech start-ups and university lab spin-offs often compete intensively for novel science-based technologies that can be subsequently commercialized by pharmaceutical corporations.[5] Large pharmaceutical firms also establish internal R&D units to develop novel technologies and compete with independent research firms.[6]

Technological competition between research-focused firms and firms doing both research and commercialization is also common in other innovative industries (Gans and Stern 2003, Norbäck and Persson 2009). Other findings from our analytical model indicate that with a fixed number of research units, a semi-integration arrangement can also have higher innovation probability than a no-integration arrangement. In some cases, no integration occurs at equilibrium even though a semi-integration arrangement can generate higher aggregate payoffs.

To establish these results, we develop the model with three key features. The first feature is related to the concept of integration. In line with the property-rights approach in the organization literature (see, for example, Grossman and Hart 1988, Hart and Moore 1988, Aghion and Tirole 1994), we consider integration based on allocation of ownership rights of the research output before firms engage in innovation research. More specifically, an integrated research unit transfers the property rights of a forthcoming innovation to the corresponding integrated customer before it makes any non-verifiable investment in innovation research and before it competes for innovation. Consequently, integration occurs before an innovation is realized. In contrast, an independent research unit transfers rights after it succeeds in making an innovation but before the commercialization process.[7]

The concept of integration has certain implications for our analysis. For example, we deal with two types of market—one at the pre-innovation stage and the other at the post-innovation stage. The contrast between these two markets is similar to the difference between a "market for innovation" involving transaction of intellectual property for the creation of new technology and a "market for technology" involving transactions for the use and diffusion of existing technology (Arora et al. 2001).[8] Furthermore, a market for innovation in our framework includes

[5]University-based research teams (Harvard and UCSF) and biotech startup (Genentech) raced intensively to find how to express human insulin in bacteria in the beginning of biotech industry (Stern 1995).

[6]For instance, in developing HIV medicines based on integrase inhibitors that block HIV integrase, large pharmaceutical corporation Merck and biotech companies such as ViiV Healthcare and Gilead competed with their own drugs. Specifically, in this integrase inhibitor drug class, Merck & Co has developed raltegravir (RAL) while ViiV Healthcare and Gilead have developed dolutegravir (DTG) and elvitegravir (EVG) respectively (AIDS info, FDA-Approved HIV Medicines). Examining 4,057 pharmaceutical projects by forty largest pharmaceutical companies, Guedj (2005) showed that the novelty of drugs from in-house R&D was not statistically different from drugs obtained from other sources.

[7]Our notion of integration here is close to the concept of backward integration in the vertical integration literature (Lafontaine and Slade 2007). In backward integration, the manufacturers decide whether to "make or buy" the input.

[8]The United States Department of Justice in its Antitrust Guidelines for the Licensing of Intellectual Property

all transactions that involve transfer of ownership rights of forthcoming innovations from the innovator to the commercializing entity. Therefore, an integrated environment in our framework refers to a wide range of situations including in-house laboratories of pharmaceutical companies, acquisition of research firms by pharmaceutical companies, and R&D contracts between a research firm and a commercializing firm in which the research firm transfers the rights of its potential research outputs before they are realized. Thus, an early-stage licensing deal in which a biotech firm grants licenses after identifying a molecule with potential application—but before conducting research to further study and develop the drug candidate—can also be an example of an integrated arrangement.

The second key feature of our model deals is related to how we model uncertainty in the innovation-generating process. We consider two types of uncertainty. First, we assume that making a successful innovation is a stochastic event. Although a firm's chance of making an innovation increases with its research investment, success is not guaranteed. We present this uncertainty through a probabilistic relationship between the investment in innovation research and the time of delivery of an innovation. In addition, we assume the absence of information regarding the exact nature of innovation at the pre-innovation stage. An innovation is not well defined until it is realized. Therefore, parties cannot contract for delivery of a specific innovation. To address the lack of information, we consider an incomplete-contract framework similar to the framework considered in Aghion and Tirole (1994). A contract in a market for innovation describes only the allocation of property rights of a forthcoming innovation against a possible transfer fee from the licensee to the licensor. In contrast, the contracted license fee will depend on the exact valuation of innovation by the licensees in a market for technology that exists at the post-innovation stage.

The third feature is related to the form of competition. We model competition in the form of an innovation contest. The underlying assumption here is that there is a temporary monopoly rent for the first innovator—the winner of the innovation contest.[9] The assumption of a monopoly rent is not new in the innovation literature, and it makes a contest an ideal framework for modeling competition.[10] The contest framework brings our model close to the models studied in the literature on innovation tournaments (Schmidt 2008). In these models, a firm often has a double incentive for investing in R&D. Specifically, an investment in R&D increases a firm's

(U.S. Department of Justice 2017) also makes a similar distinction between a "market for technology" and a "market for research and development" based on the differences between (a) transaction of assets comprising of "intellectual property that is licensed ... and its close substitutes" and (b) transaction of assets comprising of "research and development related to the identification of a commercializable product, or directed to particular new or improved goods or processes, and the close substitutes for that research and development."

[9]We do not explicitly model the product-market competition as our focus remains on the competition for making an innovation as an intermediate good. The assumption of a monopoly rent in the product market makes our analysis simple and tractable. One can however get equivalent results in a model of innovation for a cost-reducing technology that can foster a producer's competitiveness in a product-market competition with multiple producers.

[10]See Scotchmer (2004) for a discussion on the roles of patent and protection in fostering innovation. See Konrad (2007) for a comprehensive analysis of contest frameworks.

chance of winning the contest and the reward from winning. In our model, research units make non-verifiable investments in research to increase their chances of winning the contest. An investment in research also increases the chance of making a successful innovation. Thus, we consider an innovation contest a productive contest.

These features lead us to our main findings. Incomplete contract and non-verifiability of research investment together imply that property-rights allocation is the key source of incentive in our model. Because we treat allocation of property rights among customers and research units as endogenous, three forms of industry competition are feasible: competitions under full integration, semi integration, and no integration. We show that only competition under no integration or competition under semi integration can arise in equilibrium. Integration reduces an integrated research unit's motivation to make a non-verifiable investment. Thus, an integrated customer is adversely affected because the integrated research unit exerts low effort. However, there is a positive externality of integration on the rent-seeking effort level of an independent research unit in an innovation contest. An increase in the non-integrated research unit's effort level also increases the payoff for all customers when the aggregate innovation probability is higher in semi integration than with no integration. If the increase in the customer's payoff is sufficiently high, then an integrated customer can compensate for its loss from the reduced effort of its own integrated research unit. Thus, a semi-integration arrangement survives in equilibrium.

The theoretical implications of this finding are interesting from multiple perspectives. First, it explains the existence of an integrated arrangement in a framework in which integration has a direct dampening effect on non-verifiable research investment. While this loss of incentive to exert innovative effort can indeed be a disadvantage for integration, we can explain the integration between a customer and their supplier via the presence of other suppliers sharing a common environment. In our model, the positive externality of an integration arrangement on the independent firm's investment drives the possibility of integration. Second, integration occurs in the absence of other known driving forces including uncertainty and credit constraints in the upstream market. A research unit's decision to integrate is not to avoid the uncertainty of innovation because all agents are assumed to be risk neutral. Credit constraints are often cited as the possible reason behind integration (Grabowski and Kyle 2014). We find a seemingly opposite effect in resource constraints on the possibility of integration because it reduces the extent of positive externalities arising from integration. An independent research unit, if resource-constrained, cannot expand its effort to the fullest extent while competing with an integrated research unit in an innovation contest. Therefore, the possibility of semi integration in equilibrium typically reduces with the extent of resource constraints in the upstream market.

All equilibrium structures can be socially inefficient. A semi-integration arrangement leads to an asymmetric distribution of industry-wide R&D costs, which are not socially desirable with a convex cost function. The efficiency of a no-integration arrangement can, however, be ambiguous. On one hand, rent-seeking incentives in an innovation contest typically leads to over-investment of R&D effort. On the other hand, the possibility of bargaining with customers at the post-innovation stage reduces incentives for R&D investment. These two effects collectively determine the optimal effort level in a no-integration arrangement. We show that the first effect dominates the second effect if a potential innovation has a high expected value or if the customers have low bargaining power. In these cases, no integration produces socially wasteful investment. Welfare comparisons between the two possible equilibrium arrangements shows if semi integration occurs in equilibrium, it also generates higher aggregate payoff than no integration. The converse, however, is not true. No integration can arise in equilibrium even when a semi-integration arrangement generates higher aggregate industry payoffs. This is because in our model those who benefit from an integration cannot necessarily commit to compensate those who lose in any credible way.

## 1.1 Related Literature

Our paper relates to several strands of literature. This paper contributes to the literature on the effects of integration and competition on innovation incentives (Aghion and Tirole 1994, Brocas 2003, Chen and Sappington 2010, Liu 2016). Aghion and Tirole (1994) explain how the allocation of property rights can affect R&D investment in industries with innovative product market. In their framework, investments in both upstream and downstream markets are necessary for innovation. While we share the role of property rights in providing incentive, our focus is on the competition for innovation as an intermediate good and how the contest-like competition affects investment in the upstream market. Chen and Sappington (2010) consider the role of the product-market competition on the incentives for innovation in the upstream market. However, we consider the production market competition in a reduced form with an assumption that a successful innovation gives the downstream commercializing firm a temporary monopoly power. In a similar framework, Brocas (2003) studies incentives for integration and its implications on R&D investments. Unlike our model, upstream firms can simultaneously innovate and use substitutable technologies and the focus is on the effect of switching costs on the R&D investment. In Liu (2016), integration has a positive coordination effect that boosts payoffs of the integrated units. The decision to integrate critically depends on the relative relevance of investment in the upstream and downstream markets. On the other hand, we consider a direct negative effect of integration on the integrated firms. Our work complements these studies in its attention to the role of integration on innovation incentives while sharing a common vertical production process with a division of labour between research and commercialization. However, we differ in how we model the competition for innovation and for providing an explanation for integration based on its positive externality on non-integrated firms.

The innovation literature also focused on the question of coexistence of research-focused firms and firms specializing in both research and commercialization in the context of science-based industries. Their interrelationship has often been examined from the perspective of either collaboration between small and large firms (Acs and Audretch 1988, Baumol 2010) or competition between entrant and incumbent firms (Gans and Stern 2000, Norbäck and Persson 2009). Our study explains the coexistence in an otherwise symmetric framework of competition. We recognize a new role of large corporations in competition for innovation. These corporations generate positive externality to independent research-focused firms' R&D efforts. Because of the positive externality, semi integration can generate a larger surplus than no integration. Our results support the coexistence of startups and large corporations but from a different perspective than the previous literature.

Our model also shares common features with models studied in the literature on vertical integration and foreclosure that began with the seminal works of Ordover et al. (1990) and Salinger (1988).[11] The primary concern in this literature has been the strategic use of foreclosure through integration to alter the market power and price- and quantity-setting abilities of firms in competition. These models typically associate integration with potential supply constraints for the independent downstream customers (Salinger 1988, Bolton and Whinston 1993, Chen 2001) or demand constraints for the independent upstream suppliers (Stefanadis 1997). Thus, integration provides the integrated customer and supplier with rent-seeking opportunities. We differ from these models in one critical aspect—we do not impose any restriction due to integration on demand and supply of the non-integrated firms. We allow both inter-customer trading as well as trading between an integrated customer and an independent innovator. A firm's bargaining power in a transaction at the post-innovation market for technology solely depends on the distribution of the customers' valuations for an innovation, which is independent of the industry structure. Thus, strategic foreclosure is not a reason for integration in our set up.

We also make a theoretical contribution to the contest literature (Konrad 2007). The contest models often consider specific success functions such as the Tullock success function due to their axiomatic foundations (Tullock 1980, Skaperdas 1996). The contest success function in our model has a game-theoretic foundation. It is derived from an underlying game in which innovation is an uncertain event and players strategically exert effort strategically. We also show that the conditional contest-success probability given there is an innovation, coincides with the Tullock success function for a suitable choice of innovation probability. To our advantage, the derived contest success function is multiplicatively separable in efforts. This makes the derivation of marginal effects of effort on contest-success probabilities and payoffs easy and tractable. The framework is particularly useful in modeling contests in which the efforts are productive and the value of the contest prize is uncertain.

[11] Rey and Tirole (2007) provide a nice summary of this literature.

# 2 The model

We consider a game with four players - Two upstream research units, $RU_1$ and $RU_2$, and two downstream customers, $C_1$ and $C_2$. The research units perform research necessary to realize an innovation. The customers can only commercialize an innovation. The game proceeds in three stages − Pre-innovation contracting of ownership rights, an innovation contest and post-innovation bargaining.

## 2.1 Pre-innovation contracting in a market for innovation

We consider an incomplete contract framework, similar to the one considered in Aghion and Tirole (1994).[12] The exact nature of innovation is unknown at the contracting stage and so the value of an innovation is not contractible. A research unit's effort is also not contractible. A contract can specify only the allocation of ownership rights of any forthcoming innovation.

In stage 1, $C_1$ and $C_2$ simultaneously offer prices $p_1$ and $p_2$. The two research units observe prices and decide whether or not to sell ownership rights of any forthcoming innovation. If a research unit sells the right, it receives no further reward when an actual innovation is realized. We call this case a case of integration, and refer to the corresponding customer-research unit pair as integrated. We assume that a customer (or a research unit) can be integrated only with one research unit (or one customer). If a customer or a research unit is not integrated, we refer to it as independent.

As customers offer prices simultaneously, we must specify a matching mechanism by which a research unit is matched with a customer in an integration arrangement. Consider a price profile $(p_1, p_2)$. First, suppose that both research units are willing to integrate at the highest offered price. We then randomly select one research unit and match it with the customer offering the highest price. If the other research unit is willing to sell the rights at the second highest price, it is matched with the other customer. Next, suppose that no research unit is willing to integrate at the highest offered price. Then, there will be no integration. Finally, suppose that only one research unit is willing to integrate at the highest offered price. We then match the willing research unit with the customer offering the highest price. The other research unit remains independent. If two customers offer the same price, we randomly select one customer as the one offering the highest price, and follow the above matching procedure.[13]

[12] In comparison to Aghion and Tirole (1994), in our model the bargaining power lies with the customers. In this sense, the model is more aligned with the notion of backward integration than forward integration.

[13] We assume that the customer offering the second highest price does not renegotiate its price offer after one of the two research units is integrated with the customer offering the highest price. We make this assumption to keep our analysis simple. The assumption, however, does not affect our results in any significant way. This is because in our model, when the customer offering the highest price gets integrated, the other customer does not gain any additional advantage in dealing with the independent research unit as an independent research unit will always have an option to sell its innovation to an integrated customer in the post-innovation stage.

### 2.2 Innovation contest

Next, consider the stage in which research units engage in research competition. We normalize the minimum effort to zero. The probability of a research unit making an innovation for a given effort level $e \in [0, 1]$ is given by an increasing function $q(e) \in [0, 1]$. The cost of effort is given by an increasing function $c(e)$ with $c(0) = 0$. Additional assumptions will be needed to support the first-order approach in various scenarios. We defer discussions of those to Section 3.2.

We model research competition in the form of an innovation contest. Consider time in an interval $[0, 1]$. At the beginning of time, both research units simultaneously incur effort cost. Effort cost is sunk and it cannot be altered once the contest begins. Effort is non-verifiable and therefore non-contractible in our model. We interpret effort as applications of researchers' knowledge and skill that are not easily measurable or verifiable and can only be driven by output-related incentive. Assuming that effort is invested only at the beginning of the contest makes the analysis simple and tractable. A research unit wins the contest if it comes up with an innovation ahead of its competitor in the fixed time interval $[0, 1]$. Let $x_i$ denote the time that $RU_i$ takes to make an innovation. We assume that $x_i$ follows a uniform distribution over the time interval $\left[0, \frac{1}{q(e_i)}\right]$, so that the probability that $RU_i$ makes an innovation within the time interval $[0, 1]$ is exactly $q(e_i)$. If no research unit innovates within the time interval $[0, 1]$, the contest ends at time 1 with no innovation. Otherwise, the contest ends at the time when a research unit comes up with an innovation ahead of its competitor.

For a given effort profile $\underline{e} = (e_1, e_2)$ such that $e_i$ denotes $RU_i$'s effort level, $RU_i$'s probability of winning the innovation contest is $\pi_i(\underline{e}) = Pr\left[x_i = min\left\{x_1, x_2\right\} \leq 1\right]$ where $x_1 \sim Uniform\left[0, \frac{1}{q(e_1)}\right]$, and $x_2 \sim Uniform\left[0, \frac{1}{q(e_2)}\right]$. We can rewrite these winning probabilities in the following simpler forms.

$$\pi_1(\underline{e}) = Pr\left[x_1 = min\left\{x_1, x_2\right\} \leq 1\right] = \int_0^1 q(e_1)\left(1 - tq(e_2)\right)dt = q(e_1)\left(1 - \frac{q(e_2)}{2}\right), \tag{1}$$

and,

$$\pi_2(\underline{e}) = q(e_2)\left(1 - \frac{q(e_1)}{2}\right). \tag{2}$$

Note that $\pi_1(e_1, e_2) = \pi_2(e_2, e_1)$. The sum of these winning probabilities is the probability of realizing an innovation. We denote the innovation probability for a given effort profile $\underline{e}$ by $\pi_{inv}(\underline{e})$. Therefore, $\pi_{inv}(\underline{e}) = 1 - (1 - q(e_1))(1 - q(e_2))$.

### 2.3 Post-innovation bargaining in a market for technology

The game moves to the post-innovation bargaining stage when an innovation contest ends with a successful innovation. If an independent research unit wins the contest, it has the ownership right of the innovation and it can bargain with customers over a licensing fee. If an integrated research unit wins the contest, the corresponding integrated customer has the ownership right of the innovation and can either commercialize the innovation or can bargain with the other customer over a licensing fee. Let $v_i$ denote $C_i$'s value of a successful innovation. We assume that $v_i$ follows a symmetric distribution around its expected value $\overline{v}$ and $v_1$ and $v_2$ are independently distributed. Let $v_{\max} = \max\{v_1, v_2\}$ and $v_{\min} = \min\{v_1, v_2\}$.

We model the bargaining game in reduced form. For simplicity, we consider symmetric Nash-bargaining payoff. Specifically, when a seller (either an independent research unit or an integrated customer) with a reservation value of the innovation $r_s$ trades with a buyer (a customer) with an innovation value $r_b$, we assume the additional value $(r_b - r_s)$ will be equally split between the seller and the buyer. Therefore, payoffs of the buyer and the seller are $\frac{r_b - r_s}{2}$ and $\frac{r_b + r_s}{2}$ respectively.[14] When an integrated customer sells the commercialization right to the other customer (such a possibility may arise if the corresponding integrated research unit wins the innovation contest), the integrated customer's reservation value is its own valuation of the innovation. On the other hand, when an independent research unit sells the commercialization right to one customer, its reservation value will be the innovation value realized by the other customer.

## 2.4 Payoffs and solution concept

We assume that all players are risk neutral.

The ex post payoff of $RU_i$ is given by

$$U_i^{RU} = \begin{cases} p - c(e_i) & \text{if } RU_i \text{ is integrated} \\ \frac{v_{max}+v_{min}}{2} - c(e_i) & \text{if } RU_i \text{ is independent and wins the contest} \\ -c(e_i) & \text{if } RU_i \text{ is independent and does not win the contest} \end{cases},$$

where $p$ is the price at which $RU_i$ sells the ownership right of any forthcoming innovation and $e_i$ is the effort level of $RU_i$ .

The ex post payoff of $C_i$ is

$$U_i^{C} = \begin{cases} \frac{v_i - v_{min}}{2} & \text{if } C_i \text{ is not integrated} \\ \frac{v_{max}+v_i}{2} - p & \text{if } C_i \text{ is integrated with some } RU_j \text{ that wins the contest} \\ \frac{v_i - v_{min}}{2} - p & \text{if } C_i \text{ is integrated with some } RU_j \text{ that does not win the contest} \end{cases},$$

where $p$ is the price at which $C_i$ buys the ownership right of any forthcoming innovation and $v_i$ is $C_i$'s realized innovation value. We consider the subgame perfect Nash Equilibrium in pure strategies as the solution concept. We focus only on pure strategies due to their analytical

[14] The assumption of equal split of the additional rent is common in the innovation literature (see Aghion and Tirole 1994).

tractability.

We make some simplifying assumptions for analytical tractability. For instance, we consider fixed transfer fees in exchange of the property rights in the market for innovation. We do not consider non-linear contracts, which are not uncommon in practice. For example, research units may receive a bonus if an innovation is commercialized. Such an incentive can positively affect R&D effort. However, as long as the bonus level differs from the ex post bargaining payoff of an independent research unit, there will be a difference in the effort levels chosen by two types of firms. We are interested in studying the impact of this difference, which can be analyzed with more tractability in our simple setting.

We also assume that an integrated customer has little control over its integrated research unit's choice of effort at the innovation contest. There are several reasons for this assumption. First, we interpret effort as applications of researchers' knowledge and skill. It may not be perfectly measurable given the uncertain nature of the innovation process. We also assume zero effort is associated with a positive success probability to reflect that an integrated customer can possibly control the routinized activities. Besides, our concept of integration is broad and covers various R&D arrangements. Thus, we intend to include contract research where a research firm can operate on its own even after selling the rights of its research output in some research projects. In this case, it is expected that the research unit decide its own R&D investment.

# 3 Equilibrium analysis

We solve the game by backward induction.

## 3.1 Post-innovation bargaining

We will study the expected payoff of the four players at the beginning of the post-innovation stage. At this point, it is worthwhile to introduce two simplifying notations. As $v_i$ s are independent and symmetrically distributed around the expected value $\overline{v}$, it can be easily shown that $E\left(\frac{v_{max}+v_{min}}{2}\right)=\overline{v}$.[15] We denote $E\left(\frac{v_{max}-v_{min}}{2}\right)$ by $\underline{v}$.[16] Together, we can write

$$E(v_{max})=\overline{v}+\underline{v},\ E(v_{min})=\overline{v}-\underline{v}.$$

$\overline{v}$ and $\underline{v}$ are important parameters in our model. While $\overline{v}$ measures an innovation's expected value, $\underline{v}$ measures the bargaining power of customers in the bargaining game. A customer's

[15]The distribution function and the density function of $v_{max}$ are given by $G(x)=F^2(x)$and $g(x)=2f(x)F(x)$ where $F$ and $f$ denote the distribution function and density function of $v_i$ respectively. And, the distribution function and the density function of $v_{min}$ are given by $H(x)=1-(1-F(x))^2$ and $h(x)=2f(x)(1-F(x))$. Therefore,

$$E\left(\frac{v_{max}+v_{min}}{2}\right)=\frac{1}{2}\int 2xf(x)(F(x)+1-F(x))\,dx=\int xf(x)\,dx=\overline{v}.$$

[16]Precisely, $\underline{v}=E\left(\frac{v_{max}-v_{min}}{2}\right)=\int xf(x)(2F(x)-1)\,dx$.

bargaining power is low when valuations have high positive correlation. In this case, $v_{max}$ is likely to be close to $v_{min}$, or equivalently, $\underline{v}$ is close to zero.

The innovation contest can lead to three different cases: (i) an independent research unit wins the innovation contest, (ii) an integrated research unit wins the innovation contest, and (iii) the innovation contest results in no successful innovation.

First, consider that an independent research unit wins the innovation contest. The winning research unit bargains with the customer who has the maximum valuation and its reservation value is the second highest valuation. As there are only two customers, the winning research unit's expected payoff is given by $E\left(\frac{v_{max}+v_{min}}{2}\right)=\bar{v}$. The losing research unit has zero payoff at this stage. A customer has an expected payoff of $\frac{1}{2}E\left(\frac{v_{max}-v_{min}}{2}\right)=\frac{\underline{v}}{2}$.

Second, consider that an integrated research unit wins the innovation contest. The corresponding integrated customer, who owns the innovation, can either commercialize the innovation or bargain with the other customer if the other customer has higher valuation. The expected payoff of the winning customer is therefore $E\left(\frac{v_{max}+v_i}{2}\right)=\overline{v}+\frac{\underline{v}}{2}$. The expected payoff of the independent customer is $E\left(\frac{v_i-v_{min}}{2}\right)=\frac{\underline{v}}{2}$. Each of the two research units will have zero expected payoff.

It is worth noting that the aggregate expected payoff of all the four players is the same as the expected value of $v_{max}$. By allowing inter-customer trading of license, we rule out the possibility of an ex post inefficient situation in which an innovation is not commercialized at the maximum possible value.

Finally, if an innovation contest leads to no successful innovation, the bargaining game is trivially resolved with each player having zero expected payoff at the post-innovation stage.

### 3.2 Innovation contest

We now solve for the optimal effort levels in the innovation contest. We assume that the success function $q(e)$ and the cost function $c(e)$ are such that there exists a unique solution of the payoff maximization problem and the solution lies in the open interval $(0,1)$. Our first assumption below is sufficient (though not necessary) to ensure that we can find a unique solution in various scenarios by solving the first-order condition. Formally, we assume:

**Assumption 1.** $\overline{v}q(e)-c(e)$ is strictly concave in $e$.

Assumption 1 is typically satisfied as long as the cost function $c(e)$ is sufficiently convex compared to the success function $q(e)$. Our second assumption is sufficient to ensure that the solution of the payoff-maximization problem in various scenarios lies in the open interval $(0,1)$. Formally, we assume:

**Assumption 2.** $\overline{v}q'(1)-c'(1)<0<\frac{\underline{v}}{2}q'(0)-c'(0)$.

In the remainder of our paper, Assumptions 1 and 2 hold true unless explicitly stated.[17]

We can have three different industry structures at the beginning of an innovation contest: (i) both research units are integrated, (ii) one of the research unit integrates while the other does not, and (iii) no research unit integrates. We call these three structures as *full integration* (FI), *semi integration* (SI) and *no integration (NI)* respectively.

First, consider the case of full integration. As the integrated research units get zero payoff at the post-innovation stage, they exert no effort. By (1) and (2), the winning probabilities of the two research units are identical and it is given by $q(0)\left(1-\frac{q(0)}{2}\right)$.

Next, consider the case of semi integration. At this stage, without loss of generality, we assume that $RU_2$ is integrated to $C_2$, and $RU_1$ remains independent. $RU_2$ therefore exerts no effort as it gets zero payoff at the post-innovation stage. $RU_1$'s expected payoff, given an effort level $e_1$, is $\overline{v}q(e_1)\left(1-\frac{q(0)}{2}\right)-c(e_1)$. The optimal effort of $RU_1$, denoted by $e^{SI}$, satisfies the following first-order condition:

$$\overline{v}\left(1-\frac{q(0)}{2}\right)q'\left(e^{SI}\right)-c'\left(e^{SI}\right) = 0. \tag{3}$$

Finally, consider the case of no integration. In a symmetric Nash equilibrium, the optimal effort levels of both research units, denoted by $e^{NI}$, solves the following condition:

$$e^{NI} = \underset{e\in[0,1]}{argmax}\ \overline{v}q(e)\left(1-\frac{q\left(e^{NI}\right)}{2}\right)-c(e).$$

From the first-order condition, we get that $e^{NI}$ satisfies

$$\overline{v}\left(1-\frac{q\left(e^{NI}\right)}{2}\right)q'\left(e^{NI}\right)-c'\left(e^{NI}\right) = 0. \tag{4}$$

By differentiating (3) and (4) and applying Assumption 1, we find that $e^{SI}$ and $e^{NI}$ are increasing in $\overline{v}$. Further, a comparison of the effort levels shows that $e^{SI} > e^{NI} > 0$. Although integration dampens the integrated research unit's incentive to exert effort, it creates a positive externality to the independent research unit's effort level. The following lemma formally proves this observation.

**Lemma 1.** $e^{SI} > e^{NI} > 0$.

*Proof.* First note that $e^{NI} > 0$ by Assumption 2. Denote $\left(1-\frac{q(0)}{2}\right)$ and $\left(1-\frac{q\left(e^{NI}\right)}{2}\right)$ by $A$ and $B$ respectively. We have $A > B$ as $e^{NI} > 0$. Note that $e^{NI}$ solves $\overline{v}Bq'\left(e^{NI}\right)-c'\left(e^{NI}\right)=0$.

[17] Assumptions 1 and 2 do not affect our results in any significant way. If we relax Assumption 1, we will have to deal with multiple solutions and subsequently, with an equilibrium selection problem. If we relax Assumption 2, we will have boundary solution, which makes the solution insensitive to changes in the parameter values to some extent.

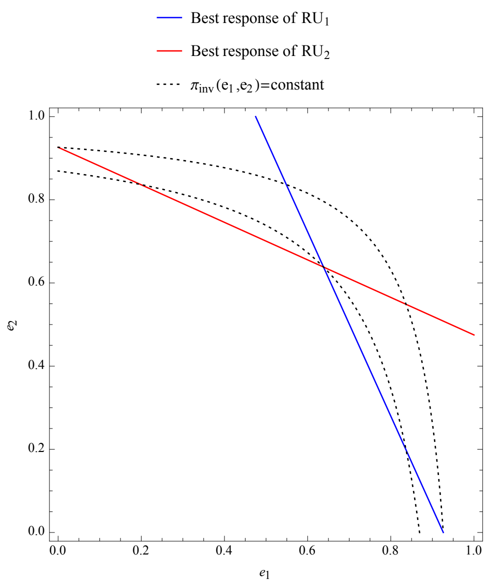


Figure 1: Response functions ($\alpha = 0.8$)

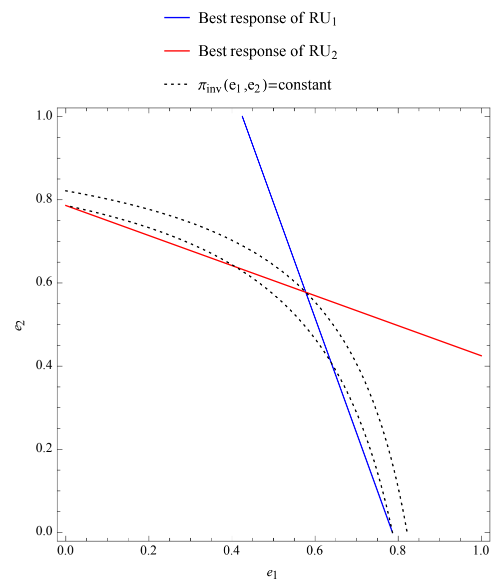


Figure 2: Response functions ($\alpha = 0.4$)

As $A > B$ and $q'\left(e^{NI}\right) > 0$, we must have $\overline{v}Aq'\left(e^{NI}\right) - c'\left(e^{NI}\right) > 0$. Further note that $e^{SI}$ solves $\overline{v}Aq'\left(e^{SI}\right) - c'\left(e^{SI}\right) = 0$. By Assumption 1, $\overline{v}Aq\left(e\right) - c\left(e\right)$ is also strictly concave and therefore, we must have $e^{NI} < e^{SI}$. □

Below we provide an example with a specific form of linear success function $q\left(e\right)$.

**Example 1.** Consider the following binary distribution of customer valuation: $v_i$ can be take two values, 3 or 1, each with 0.5 probability. Therefore, $\overline{v} = 2$ and $\underline{v} = 0.5$. Let $c\left(e\right) = e^2$ and $q\left(e\right) = \frac{1-\alpha}{4} + \frac{3+\alpha}{4}e$ for $\alpha \in [0,1]$. In Figure 1, we consider $\alpha = 0.8$. The two straight lines plot the best-response functions of the two research units in the $\left(e_1, e_2\right)$ space (the flatter one corresponds to $RU_2$'s best response for a given choice of $e_1$). The response functions intersect each other at the optimal effort level of an independent research unit in no integration ($e^{NI} = 0.638$). The point of intersection of the response function of $RU_2$ (the response function of $RU_1$) and the vertical axis (the horizontal axis) is the optimal effort level of an independent research unit in semi integration ($e^{SI} = 0.926$). The dotted curves present the choices of $e_1$ and $e_2$ at which the innovation probability $\pi_{inv}\left(e_1, e_2\right)$ is constant. For $\alpha = 0.8$, we have $\pi_{inv}\left(e^{NI}, e^{NI}\right) = 0.882$ and $\pi_{inv}\left(e^{SI}, 0\right) = 0.933$. In Figure 2, we consider $\alpha = 0.4$ and we plot the response functions. For $\alpha = 0.4$, we have $e^{NI} = 0.578$, $e^{SI} = 0.786$, $\pi_{inv}\left(e^{NI}, e^{NI}\right) = 0.871$ and $\pi_{inv}\left(e^{SI}, 0\right) = 0.846$. Unlike in Figure 1, in this case, we have $\pi_{inv}\left(e^{SI}, 0\right) < \pi_{inv}\left(e^{NI}, e^{NI}\right)$.

As illustrated in Example 1, the effect of integration on the innovation probability can be ambiguous. We denote the innovation probabilities, computed at the optimal effort profile, in cases of full integration, semi integration and no integration by $\pi^{FI}_{inv}$, $\pi^{SI}_{inv}$ and $\pi^{NI}_{inv}$ respectively.

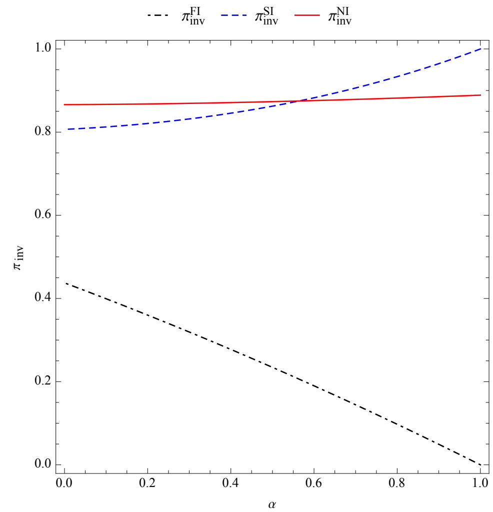


Figure 3: Innovation probability

We have

$$
\begin{aligned}
\pi_{inv}^{FI} &= \pi_{inv}(0,0) = 1-(1-q(0))^2, \\
\pi_{inv}^{SI} &= \pi_{inv}\left(e^{SI},0\right) = 1-(1-q(0))\left(1-q\left(e^{SI}\right)\right), \\
\pi_{inv}^{NI} &= \pi_{inv}\left(e^{NI},e^{NI}\right) = 1-\left(1-q\left(e^{NI}\right)\right)^2.
\end{aligned} \tag{5}
$$

We find that the innovation probability is the least in case of full integration. The comparison between $\pi_{inv}^{SI}$ and $\pi_{inv}^{NI}$ is however ambiguous. Define

$$\Delta_{inv} := \pi_{inv}^{SI} - \pi_{inv}^{NI}.$$

**Lemma 2.** *$\Delta_{inv} \geq 0$ if and only if $(1-q(0))\left(1-q\left(e^{SI}\right)\right) \leq \left(1-q\left(e^{NI}\right)\right)^2$. Further, $\pi_{inv}^{FI} \leq \min\left\{\pi_{inv}^{SI}, \pi_{inv}^{NI}\right\}$.*

The proof follows by comparing $\pi_{inv}^{FI}$, $\pi_{inv}^{SI}$ and $\pi_{inv}^{NI}$ and by the fact that $e^{SI} > e^{NI} > 0$. The above two lemmata give us an important insight to the effect of contest on innovative effort. In semi integration, an independent firm faces weak competition form the integrated firm. However, weak competition can sometime boost R&D investment of the independent firm and results in higher aggregate innovation probability.

In the following example, we compare the innovation probabilities for a particular class of linear success function.

**Example 2.** We continue with the same parameter specification considered in Example 1. We assume that $q(e) = \frac{1-\alpha}{4} + \frac{3+\alpha}{4}e$ for $\alpha \in [0,1]$. Figure 3 plots the innovation probabilities $\pi_{inv}^{FI}$, $\pi_{inv}^{SI}$ and $\pi_{inv}^{NI}$ as functions of $\alpha$. An independent research unit's effort is always higher in semi integration. For high values of $\alpha$, the innovation probability is higher in semi integration than in no integration.

## 3.3 Pre-innovation contracting

At the pre-innovation contracting stage, customers simultaneously offer prices to buy ownership rights. After observing a price profile $(p_1, p_2)$, the research units decide whether or not to integrate. Depending on their integration decisions, three different industry structures may arise in equilibrium. Table 3.3 presents payoffs of the research units and the customers in different structures. Without loss of generality, we consider that $RU_2$ and $C_2$ are integrated in semi integration. As inter-customer trading is allowed in the post-innovation stage and as customers' valuations are independently drawn, both customers would have an expected payoff of at least $\frac{\underline{v}}{2}$ times the innovation probability. However, the integrated customer pays a price to integrate and it additionally receives a payoff from integration, which equals $\overline{v}$ times the probability of the integrated research units winning the contest. In the following subsections, we analyze equilibrium possibilities in various structures.

Table 3.3: Payoff in different market structures

| | Full integration | | No integration | | Semi integration | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 1 | 2 | 1 (independent) | 2 (integrated) |
| $RU$ | $p$ | $p$ | $(\overline{v}/2)\,\pi_{inv}^{NI}$ $-c\left(e^{NI}\right)$ | $(\overline{v}/2)\,\pi_{inv}^{NI}$ $-c\left(e^{NI}\right)$ | $\overline{v}\pi_1\left(e^{SI},0\right)$ $-c\left(e^{SI}\right)$ | $p$ |
| $C$ | $(\overline{v}/2)\,\pi_{inv}^{FI}$ $+(\underline{v}/2)\,\pi_{inv}^{FI}$ $-p$ | $(\overline{v}/2)\,\pi_{inv}^{FI}$ $+(\underline{v}/2)\,\pi_{inv}^{FI}$ $-p$ | $(\underline{v}/2)\,\pi_{inv}^{NI}$ | $(\underline{v}/2)\,\pi_{inv}^{NI}$ | $(\underline{v}/2)\,\pi_{inv}^{SI}$ | $\overline{v}\pi_2\left(e^{SI},0\right)+$ $(\underline{v}/2)\,\pi_{inv}^{SI}$ $-p$ |

### 3.3.1 Equilibrium with full integration

In an equilibrium with full integration, both customers must offer the same price. It is because the customer offering the higher price will otherwise have a strict incentive to decrease the offered price without affecting its chance to integrate. We denote the common price by $p$. In full integration, a research unit has an incentive to integrate if $p$ is above its opportunity cost of integration, which is given by $\overline{v}\pi_1\left(e^{SI},0\right)-c\left(e^{SI}\right)$. On the other hand, the customer is only willing to offer a price $p$ below its relative benefit from integration (derived in the proof of Lemma 3), which is given by $(\overline{v}/2)\,\pi_{inv}^{FI}+\frac{\underline{v}}{2}\pi_{inv}^{FI}-\frac{\underline{v}}{2}\pi_{inv}^{SI}$. Therefore, an equilibrium with full integration exists for some price $p$ if and only if

$$\overline{v}\pi_1\left(e^{SI},0\right)-c\left(e^{SI}\right) \quad\leq\quad \frac{\overline{v}}{2}\pi_{inv}^{FI}-\frac{\underline{v}}{2}\left(\pi_{inv}^{SI}-\pi_{inv}^{FI}\right) \tag{6}$$

In this case, the optimal price $p$ coincides with the lower bound of (6) as prices are offered before integration decisions are made. The following lemma documents the finding.

**Lemma 3.** *An equilibrium with full integration exists if and only if condition (6) holds true.*

A formal proof is given in the appendix. The following proposition shows that the condition required for the existence of an equilibrium with full integration is not satisfied for any parameter values.

**Proposition 1.** *There is no competitive equilibrium with full integration.*

*Proof.* The inequality (6) can be rewritten as

$$\frac{\underline{v}}{2}\left(\pi_{inv}^{SI}-\pi_{inv}^{FI}\right)\leq\frac{\overline{v}}{2}\pi_{inv}^{FI}-\left(\overline{v}\pi_{1}\left(e^{SI},0\right)-c\left(e^{SI}\right)\right).$$

The left hand side is always positive as $\pi_{inv}^{SI}>\pi_{inv}^{FI}$. But the right hand side is always negative as $\overline{v}\pi_{1}\left(e^{SI},0\right)-c\left(e^{SI}\right)>\overline{v}\pi_{1}\left(0,0\right)-c\left(0\right)=\frac{\overline{v}}{2}\pi_{inv}^{FI}$. Hence, (6) cannot be satisfied. □

The mechanism behind this result is as follows. Note that the contracted price is simply a transfer between a research unit and a customer. Therefore, in any equilibrium, a customer-research unit pair must be able to maximize the joint payoff. In full integration, a customer-research unit pair gets a joint payoff of $\frac{\overline{v}+\underline{v}}{2}\pi_{inv}^{FI}$. They can however deviate to a semi-integration arrangement, which provides this pair a joint payoff of $\overline{v}\pi_{1}\left(e^{SI},0\right)-c\left(e^{SI}\right)+\frac{\underline{v}}{2}\pi_{inv}^{SI}$. The joint payoff from deviation is always higher than the joint payoff in full integration.

### 3.3.2 Equilibrium with no integration

In a no-integration equilibrium, both research units are not willing to integrate at the maximum offered price. A research unit prefers not to integrate if the maximum price is less than its payoff in no integration, which is $\frac{\overline{v}}{2}\pi_{inv}^{NI}-c\left(e^{NI}\right)$. On the other hand, a customer is willing to offer a price only up to its benefit from integration given the other customer is not integrated. A customer's benefit from integration in this case (derived in the proof of Lemma 4) is $\overline{v}\pi_{2}\left(e^{SI},0\right)+\frac{\underline{v}}{2}\pi_{inv}^{SI}-\frac{\underline{v}}{2}\pi_{inv}^{NI}$. Therefore if an equilibrium with no integration exists, the following must be true:

$$\begin{aligned}\overline{v}\pi_{2}\left(e^{SI},0\right)+\frac{\underline{v}}{2}\pi_{inv}^{SI}-\frac{\underline{v}}{2}\pi_{inv}^{NI} &\leq \frac{\overline{v}}{2}\pi_{inv}^{NI}-c\left(e^{NI}\right)\\ \Leftrightarrow\Delta_{inv} &\leq \frac{2\eta}{\underline{v}},\end{aligned}\tag{7}$$

where

$$\eta=\overline{v}\pi_{2}\left(e^{NI},e^{NI}\right)-c\left(e^{NI}\right)-\overline{v}\pi_{2}\left(e^{SI},0\right).$$

The parameter $\eta$ measures the difference between an independent research unit's payoff in a no-integration equilibrium and an integrated research unit's payoff in a semi-integration equilibrium.

It is easy to see that $\eta$ is always positive and increasing in $\overline{v}$.[18] The condition (7) will be violated only if $\pi_{inv}^{SI}$ is sufficiently greater than $\pi_{inv}^{NI}$. The following lemma shows that condition (7) is indeed a necessary and sufficient condition to have an equilibrium with no integration. A formal proof is given in the appendix.

**Lemma 4.** *An equilibrium with no integration exists if and only if condition (7) holds true.*

### 3.3.3 Equilibrium with semi integration

Without loss of generality, assume that in a typical semi-integration arrangement $RU_2$ is integrated to $C_2$ and $RU_1$ and $C_1$ are not integrated. If an equilibrium with semi integration exists, then $p_2$ must be above $RU_2$'s opportunity cost of integration, which is $\frac{\overline{v}}{2}\pi_{inv}^{NI} - c\left(e^{NI}\right)$. Similarly, $p_2$ must be below $C_2$'s benefit from integration in semi integration, which is $\overline{v}\pi_2\left(e^{SI},0\right) + \frac{\underline{v}}{2}\pi_{inv}^{SI} - \frac{\underline{v}}{2}\pi_{inv}^{NI}$. Thus, in order to sustain semi integration in equilibrium, we must have

$$\begin{aligned} \overline{v}\pi_2\left(e^{SI},0\right) + \frac{\underline{v}}{2}\pi_{inv}^{SI} - \frac{\underline{v}}{2}\pi_{inv}^{NI} &\geq \frac{\overline{v}}{2}\pi_{inv}^{NI} - c\left(e^{NI}\right) \\ \Leftrightarrow \Delta_{inv} &\geq \frac{2\eta}{\underline{v}}. \end{aligned} \tag{8}$$

The following lemma shows that (8) is also a necessary and sufficient condition to have an equilibrium with no integration. In such an equilibrium, we will have $p_2 = \frac{\overline{v}}{2}\pi_{inv}^{NI} - c\left(e^{NI}\right)$ and the optimal response of $C_1$ would be to offer a price strictly below $p_2$. In response, $RU_2$ integrates with $C_2$ while $RU_1$ remains independent. A formal proof of the lemma is given in the appendix.

**Lemma 5.** *An equilibrium with no integration exists if condition (8) holds true.*

The following proposition characterizes all the competitive equilibria in pure strategies.

**Proposition 2.** *There always exists a competitive equilibrium. The equilibrium exhibits semi integration if and only if (8) holds true. Otherwise, we have no integration in equilibrium.*

The proof directly follows from the preceding discussion, and is therefore skipped.

The proposition shows that semi integration is likely to occur if the innovation probability is sufficiently higher in semi integration than in no integration. To understand the mechanism behind this result, think of a market structure with no integration to begin with. A research unit (assume $RU_2$) will be willing to integrate if the price is as high as its payoff under no integration. $RU_2$'s decision to integrate, however, increases $RU_1$'s rent-seeking motivation and subsequently, $RU_1$'s effort level in contest. An increase in $RU_1$'s effort increases not only $RU_1$'s

[18] We have $\overline{v}\pi_2\left(e^{NI},e^{NI}\right) - c\left(e^{NI}\right) \geq \overline{v}\pi_2\left(e^{NI},0\right) \geq \overline{v}\pi_2\left(e^{SI},0\right)$, where the first inequality follows from the fact that $e^{NI}$ is $RU_2$'s best response given that $RU_1$ exerts $e^{NI}$ levels of effort and the second inequality follows from the fact that $e^{SI} > e^{NI}$. Application of the envelope theorem shows that $\overline{v}\pi_2\left(e^{NI},e^{NI}\right) - c\left(e^{NI}\right)$ is increasing in $\overline{v}$. As $\pi_2\left(e^{SI},0\right)$ decreases with $\overline{v}$, $-\overline{v}\pi_2\left(e^{SI},0\right)$ is increasing in $\overline{v}$. Together, we find that $\eta$ is increasing in $\overline{v}$.

expected payoff, but also all customers' expected payoff, if the aggregate innovation probability is higher in semi integration than in no integration. If the increase in innovation probability is sufficiently high, the customer $C_2$ can improve its payoff from integration, even after paying $RU_2$ its asking price.[19] We therefore see semi integration in our framework because integration creates positive externality on the independent research unit's effort in an innovation contest and it subsequently benefits all customers by increasing the aggregate innovation probability.

The effect of the (expected) innovation value on the existence condition (8) is not necessarily unidirectional. This is because the difference in innovation probability, $\Delta_{inv}$, can move in both direction with a change in $\overline{v}$ (as both $e^{SI}$ and $e^{NI}$ increase with $\overline{v}$). The effect of the parameter $\underline{v}$ is however straightforward. For high values of $\underline{v}$, semi integration is less likely to arise. Recall that $\underline{v}$ reflects the customer's bargaining power. For a very specialized innovation, which only one customer can possibly commercialize with high surplus, $\underline{v}$ will be high. Therefore, semi integration is less likely to occur in such a case.

Although our basic model considers a duopoly setting in both upstream and downstream markets, the model can be easily extended to a setting with more than two firms in each market. While increasing the number of firms does not bring any new tradeoffs in our model, we find our results are robust to such an extension. In a framework with $m$ firms in the upstream market and $n$ firms in the downstream market, industry structure with various degrees of semi-integration is possible. In a semi-integration arrangement with $k \in \{0, 1, 2, ..., \min\{m, n\}\}$ integration, an independent research unit's optimal effort $e^{SI}$ solves

$$\overline{v}q'\left(e^{SI}\right)\left(1-\frac{q\left(0\right)}{2}\right)^{k}\left(1-\frac{q\left(e^{SI}\right)}{2}\right)^{m-k-1}-c'\left(e^{SI}\right)=0. \tag{9}$$

We find that in a general framework, a full integration arrangement cannot arise. The intuition is similar to what we find in the duopoly setting. In full-integration, there is no independent research unit who can benefit from the positive externality of integration. The total surplus is too small to compensate the loss that any integrating customer incurs from the low effort of its own integrated research unit. The model predicts a unique equilibrium in which we see either no integration or a semi-integration in which some but not all research units integrate.

# 4 Discussion

## 4.1 Efficiency

Because of the inter-customer licensing, the customer with the maximum valuation will always commercialize the innovation in our model. We can so write the social value of an innovation at

[19] Note that the customer also gets a premium from integration, which is given by $\overline{v}\pi_2\left(e^{SI}, 0\right)$. But the premium is never sufficient to compensate the research unit's opportunity cost of integration.

an effort profile $(e_1, e_2)$ as

$$W(e_1, e_2) = E(v_{max})\, \pi_{inv}(e_1, e_2) - c(e_1) - c(e_2).$$

Let $W^* := \max_{(e_1,e_2)\in[0,1]^2} W(e_1, e_2)$. As utility can be transferred among players at no cost in our model (through contracted prices between research units and customers and through licensing fees between customers), an outcome is efficient only if the aggregate payoff is $W^*$.[20] For tractability, we assume that the social value of innovation is concave in efforts. Formally, we assume:

**Assumption 3.** $W(\underline{e})$ is strictly concave in $\underline{e}$.

Assumption 3 is satisfied if the cost function $c(e)$ is sufficiently convex. From the first-order condition, we can uniquely characterize the symmetric, value-maximizing effort profile $(e^w, e^w)$, in which $e^w$ satisfies the following condition:

$$(\overline{v} + \underline{v})(1 - q(e^w))\, q'(e^w) - c'(e^w) \quad = \quad 0. \tag{10}$$

Assumption 2 implies that $e^w > 0$, and so full integration cannot be an efficient outcome. Because of the convexity of the cost function, semi integration is also not an efficient outcome as the asymmetric distribution of effort costs reduces social value of an innovation. A no-integration outcome, on the other hand, has a symmetric distribution of costs. It can however result in more or less supply of effort compared to the socially optimal effort level. The following lemma shows that a no-integration outcome can be socially wasteful if the success probability at the optimal effort level in no integration is sufficiently high.

**Lemma 6.** *$e^w \leq e^{NI}$ if and only if $\frac{2\underline{v}}{\overline{v}+2\underline{v}} \leq q\left(e^{NI}\right)$.*

*Proof.* In the Appendix. □

Two opposite forces drive the above result. On the one hand, rent-seeking incentives can lead to excess efforts.[21] On the other hand, ex-post bargaining with customers reduces incentive to exert effort. It is easy to see that $e^{NI}$ is increasing in $\overline{v}$ and it does not depend on $\underline{v}$. On the other hand, $\frac{2\underline{v}}{\overline{v}+2\underline{v}}$ is decreasing in $\overline{v}$ and increasing in $\underline{v}$. Together, we can conclude that research units put more than socially optimal effort level in a no-integration outcome if $\overline{v}$ is sufficiently high or if $\underline{v}$ is sufficiently low.

**Corollary 1.** *$e^w \leq e^{NI}$ if and only if $\overline{v}$ is sufficiently high or if $\underline{v}$ is sufficiently low.*

[20] In other words, for any outcome (let us call it $A$) with aggregate payoff less than $W^*$, we can always construct another payoff profile, supported by some effort profile yielding higher social value of innovation, that Pareto dominates the payoff profile associated with $A$.

[21] With non-productive contest, rent-seeking incentives always lead to socially wasteful effort. Chung (1996) considers a special type of productive contest, in which contest prize is a function of aggregate efforts, and shows that players exert more than the socially optimal effort in equilibrium.

The model thus predicts that if an innovation is highly valuable or if customers' valuations are positively correlated, a no-integration arrangement is likely to be socially wasteful. Thus, for large-scale innovations and for less-specialized innovations (so that multiple customers can commercialize the innovation), no integration generates socially wasteful effort.

While all arrangements, namely full integration, semi integration and no integration, can be potentially inefficient, below we make welfare comparison among them. In particular, we study whether an equilibrium arrangement can be dominated by another arrangement in terms of social value of innovation. The social value of an innovation, computed at the optimal effort profile in the innovation contest, in cases of full integration, semi integration and no integration are respectively given by

$$\begin{aligned} W^{FI} &:= & W(0,0) &= & (\overline{v}+\underline{v})\,\pi_{inv}^{FI} \\ W^{SI} &:= & W\left(e^{SI},0\right) &= & (\overline{v}+\underline{v})\,\pi_{inv}^{SI}-c\left(e^{SI}\right) \\ W^{NI} &:= & W\left(e^{NI},e^{NI}\right) &= & (\overline{v}+\underline{v})\,\pi_{inv}^{NI}-2c\left(e^{NI}\right). \end{aligned} \tag{11}$$

To compare different structures in terms of the social value of an innovation, we introduce a notion of inefficiency here. We call an industry structure *inefficient* if there exists an alternative structure with higher social value of an innovation, computed at the optimal effort profile in the innovation contest.

It can be shown that the full integration structure is always inefficient as the social value of an innovation in full integration is dominated by the social value of an innovation in semi integration. The comparison between the cases of semi integration and no integration is ambiguous. In particular,

$$W^{SI}\le W^{NI} \quad\Leftrightarrow\quad \Delta_{inv}\le\frac{c\left(e^{SI}\right)-2c\left(e^{NI}\right)}{\overline{v}+\underline{v}}. \tag{12}$$

Recall that the innovation is commercialized at the maximum customer valuation, $v_{\max}$, which has an expected value of $\overline{v}+\underline{v}$. The right hand side expression in (12) therefore measures the difference in total effort between the two cases of semi integration and no integration, per unit of the expected value of innovation. As the condition in (12) differs from the equilibrium characterizing condition in (8), it is obvious that the competitive equilibrium may not necessarily be efficient. The following proposition shows that a no-integration equilibrium can be inefficient while a semi-integration equilibrium cannot be inefficient.

**Proposition 3.** *A semi-integration equilibrium is not inefficient. In contrast, a no-integration equilibrium, can be inefficient if the following is true*

$$\frac{\left(c\left(e^{SI}\right)-2c\left(e^{NI}\right)\right)}{\overline{v}+\underline{v}}\le\Delta_{inv}\le\frac{2\eta}{\underline{v}}.$$

*Proof.* In the Appendix. □

An important insight from the efficiency analysis is that even when a no-integration arrangement is observed in equilibrium, it does not necessarily yields high aggregate industry payoff than semi integration. Why do we see inefficient no-integration arrangement in equilibrium? A particular arrangement can be sustained in equilibrium as long as each pair of a research unit and a customer cannot make themselves better off by deviating to an alternate arrangement. Consider a situation in which no integration is observed in equilibrium. It implies that in this situation if an independent pair of research unit and customer (call it pair $A$) decides to integrate, the joint payoff will be less. However, such a move can have a positive externality on the effort level of the other independent research unit in an innovation contest. Thereby, the integration decision by pair $A$ may increase the joint payoff of the other pair (call it pair $B$) in a semi-integration arrangement compared to what pair $B$ is currently getting in the no-integration equilibrium. If the gain in pair $B$'s joint payoff exceeds the loss in pair $A$'s joint payoff, the no-integration arrangement is inefficient. However, as pair $A$ is not compensated for the loss it makes from integration, we can still observe no integration in equilibrium. Customers end up restricting themselves from offering higher price as the independent research unit cannot commit to compensate the integrated customer at the contracting stage.

### 4.2 Resource constraints

So far we did not assume any potential resource constraints for the customers and the research units. Such constraints can have implications for our analysis. To illustrate these implications, below we model the credit constraints in a very simple manner.

Consider first the possibility that research units face borrowing constraint. Suppose that a research unit can borrow up to $c(L) > 0$. We are interested to see how various values of $L$ would effect the equilibrium condition. If $L \geq e^{SI}$, the constraint has no impact on the equilibrium arrangement. If $e^{NI} \leq L < e^{SI}$, then the effort profile in no integration is unaffected; effort of an independent research unit in semi integration reduces to $L$.[22] Such reduction can adversely affect the possibility of semi integration in equilibrium. If, for example, $RU_2$ and $C_2$ are integrated in semi integration, the claimed price for integration must lie above the $RU_2$'s opportunity cost of integration and below $C_2$'s benefit from integration. In this case, $RU_2$'s opportunity cost of integration is $\overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$ and $C_2$'s benefit is $\overline{v}\pi_2(L, 0) + \frac{v}{2}(\pi_1(L, 0) + \pi_2(L, 0)) - \frac{v}{2}\left(\pi_1\left(e^{NI}, e^{NI}\right) + \pi_2\left(e^{NI}, e^{NI}\right)\right)$. After rearranging terms, we get the following condition to sustain semi integration in equilibrium:

$$\overline{v}\pi_2(L, 0) + \frac{v}{2}(\pi_1(L, 0) + \pi_2(L, 0)) \quad \geq \quad \frac{v}{2}\pi_{inv}^{NI} + \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right). \tag{13}$$

[22] An independent research unit's payoff in semi integration is strictly increasing in the range $[0, L]$ for any $L < e^{SI}$.

At $L = e^{SI}$, the condition (13) coincides with the existence condition of a semi-integration equilibrium (8). Further, it can be shown that the expression in the left hand side of (13) is increasing in $L$ and the condition is not satisfied at $L = e^{NI}$. Therefore, a borrowing constraint can adversely affect the possibility of semi integration in equilibrium as $L$ decreases and when $e^{NI} \leq L < e^{SI}$.

As $L$ further decreases from $e^{NI}$ (consider values between 0 and $e^{NI}$), effort of an independent research unit in both semi integration and no integration will reduce to $L$. The corresponding equilibrium condition for existence of semi integration changes to the following:

$$\overline{v}\pi_2(L,0) + \frac{\underline{v}}{2}(\pi_1(L,0) + \pi_2(L,0)) \geq \frac{\underline{v}}{2}(\pi_1(L,L) + \pi_2(L,L)) + \overline{v}\pi_2(L,L) - c(L). \qquad (14)$$

The above condition in (14) is not satisfied for any value of $0 \leq L < e^{NI}$. In this case, only no-integration equilibrium survives. It is worth noting that we will still not see full integration in equilibrium for any $L > 0$. The condition for existence of full integration in equilibrium is

$$\overline{v}\pi_1(L,0) - c(L) \leq \frac{\overline{v}}{2}\pi_{inv}^{FI} - \frac{\underline{v}}{2}\left((\pi_1(L,0) + \pi_2(L,0)) - \pi_{inv}^{FI}\right), \qquad (15)$$

which is not satisfied for any $L > 0$. Borrowing constraints for research units thus increase the possibility of no integration relative to semi integration in equilibrium. To see the intuition behind it, note that semi integration exists as the integration decision by one research unit create positive externality on the independent research unit's effort level. A borrowing constraint dampens the impact of such positive externality.

A resource constraint on the customer's side can be introduced in a similar way, such that there is an exogenous upper bound on the price that it can offer at the pre-innovation contracting stage. The resource constraint does not affect the possibility of no integration in any adverse way as the customers do not pay any price upfront in a no-integration arrangement. It can however, adversely affect possibility of semi integration in equilibrium if the customer cannot arrange funds as high as the research unit's reservation price, which is $\frac{\overline{v}}{2}\pi_{inv}^{NI} - c\left(e^{NI}\right)$. In such a situation, we will see no integration in equilibrium.

We must interpret our findings from the above analysis with caution. Our model assumes no fixed cost to participate in the innovation contest. If we allow such fixed cost and if research units are heavily resource-constrained, then only full integration can be observed. However, in absence of any fixed cost, we find that the adverse effects of resource constraints in the upstream markets are more prominent on a semi-integrated arrangement than on a no-integration arrangement.

### 4.3 Inter-customer trading

In our basic model, customers can trade the ownership right of an innovation between them at the post-innovation stage. Inter-customer licensing is not uncommon in practice (Arora et al. 2004). However, to find its effect in our model, we here investigate the model outcome in absence of inter-customer trading.

The possibility of inter-customer trading does not have a direct impact on a research unit's incentive to put effort. It can however change the customers' expected payoffs at the pre-innovation contracting stage. To see this, consider the event in which an integrated research unit (assume for example, $RU_2$) wins the innovation contest and the corresponding integrated customer (assume for example, $C_2$) has low valuation of the innovation. The other customer (in this case, $C_1$) receives zero payoff in absence of inter-customer trading, even if its value of the innovation may be high.

Table 4.3: Payoff in absence of inter-customer trading

| | Full integration | | No integration | | Semi integration | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 1 | 2 | 1 | 2 |
| $RU$ | $p$ | $p$ | $(\overline{v}/2)\,\pi_{inv}^{NI}$ $-c\left(e^{NI}\right)$ | $(\overline{v}/2)\,\pi_{inv}^{NI}$ $-c\left(e^{NI}\right)$ | $\overline{v}\pi_1\left(e^{SI},0\right)$ $-c\left(e^{SI}\right)$ | $p$ |
| $C$ | $(\overline{v}/2)\,\pi_{inv}^{FI}$ $-p$ | $(\overline{v}/2)\,\pi_{inv}^{FI}$ $-p$ | $(\underline{v}/2)\,\pi_{inv}^{NI}$ | $(\underline{v}/2)\,\pi_{inv}^{NI}$ | $(\underline{v}/2)\,\pi_1\left(e^{SI},0\right)$ | $\overline{v}\pi_2\left(e^{SI},0\right)+$ $(\underline{v}/2)\,\pi_1\left(e^{SI},0\right)$ $-p$ |

In Table 4.3, we present the expected payoffs of research units and customers in different structures. The condition to have full integration in equilibrium is the following:

$$(\underline{v}/2)\,\pi_1\left(e^{SI},0\right) \leq (\overline{v}/2)\,\pi_{inv}^{FI} - \left(\overline{v}\pi_1\left(e^{SI},0\right) - c\left(e^{SI}\right)\right). \tag{16}$$

The condition is never satisfied for any parameter values, implying that an equilibrium with full integration does not exist, even in absence of inter-customer trading. Comparing the research unit's opportunity cost of integration with the customer's relative benefit from integration, one can derive the condition that uniquely determine whether we observe semi integration or no integration in equilibrium. The condition is as follows:

$$\frac{\underline{v}}{2}\left(\pi_1\left(e^{SI},0\right) - \pi_{inv}^{NI}\right) \geq \overline{v}\pi_2\left(e^{NI},e^{NI}\right) - c\left(e^{NI}\right) - \overline{v}\pi_2\left(e^{SI},0\right). \tag{17}$$

If (17) is satisfied, we observe semi integration in equilibrium; Otherwise, no integration is observed in equilibrium. Comparing (17) with (8), we find that if inter-customer trading is ruled out, semi integration is less likely to be sustained in equilibrium.

### 4.4 Innovation contest

#### 4.4.1 Contest success function

Contest success function (CSF) is an important part of modeling a contest. Because of strong axiomatic foundation, many contest models assume CSF of the following additive form:

$$p_i\left(e_1,\ldots,e_n\right)=\begin{cases}\frac{f(e_i)}{\sum_{j=i}^{n} f(e_j)} & \text{if } \max\left\{f\left(e_1\right),\ldots,f\left(e_n\right)\right\}>0\\ \frac{1}{n} & \text{otherwise,}\end{cases} \tag{18}$$

where $f$ is a positive, increasing function. Skaperdas (1996) shows that any CSF with the following five axiomatic properties − imperfect discrimination, monotonicity, anonymity, consistency and independence − must be of the additive form (18).[23] In addition, Skaperdas (1996) shows that the above axioms together with the assumption of homogeneity of degree zero generate Tullock CSF (Tullock 1980), which is the most commonly used CSF in the rent-seeking contest literature:

$$p_i\left(e_1,\ldots,e_n\right)=\begin{cases}\frac{e_i^r}{\sum_{j=i}^{n} e_j^r} & \text{if } \max\left\{e_1,\ldots,e_n\right\}>0\\ \frac{1}{n} & \text{otherwise .}\end{cases}. \tag{19}$$

We derive a research unit's winning probability from an underlying environment, in which we treat innovation as a probabilistic event. A player's winning probability therefore composes of two factors - probability of making an innovation and probability of winning the contest. In an innovation contest, an independent $RU_i$ chooses $e_i$ to maximize payoff, $\overline{v}\pi_i\left(\underline{e}\right)-c\left(e_i\right)$, where $\pi_i\left(\underline{e}\right)$ is $RU_i$'s winning probability. We can rewrite the payoff as follows:

$$\overline{v}\cdot\pi_i\left(\underline{e}\right)-c\left(e_i\right)=\left(\overline{v}\cdot\pi_{inv}\left(\underline{e}\right)\right)\frac{\pi_i\left(\underline{e}\right)}{\pi_{inv}\left(\underline{e}\right)}-c\left(e_i\right), \tag{20}$$

where $\overline{v}\pi_{inv}\left(\underline{e}\right)$ is the expected value of an innovation and $\frac{\pi_i(\underline{e})}{\pi_{inv}(\underline{e})}$ is $RU_i$'s contest-success probability given an innovation is realized. The contest-success probability $\frac{\pi_i(\underline{e})}{\pi_{inv}(\underline{e})}$ satisfies all the five desired axiomatic properties, and it can therefore be expressed in an additive form (18):

$$\begin{aligned}\frac{\pi_i\left(e_i,e_j\right)}{\pi_{inv}\left(e_i,e_j\right)} &=\frac{q\left(e_i\right)\left(1-\frac{q(e_j)}{2}\right)}{q\left(e_i\right)\left(1-\frac{q(e_j)}{2}\right)+q\left(e_j\right)\left(1-\frac{q(e_i)}{2}\right)}\\ &=\frac{f\left(e_i\right)}{\sum_{j=i}^{n} f\left(e_j\right)}\end{aligned}$$

where $f\left(e_i\right)=\frac{q(e_i)}{\left(1-\frac{q(e_i)}{2}\right)}$. With suitable choice of $q\left(e_i\right)$ (for example, consider $q\left(e_i\right)=\frac{2e_i^r}{e_i^r+2}$), the contest-success probability coincides with the Tullock CSF (19). Thus, we generalize the

[23] Clark and Riis (1998) extend the result to non-anonymous CSF.

Tullock contest framework in modeling contest with uncertain prize and the derived winning probability has a game-theoretic foundation. An advantage of our model is that the winning probability $\pi_i(e_i, e_j) = q(e_i)\left(1 - \frac{q(e_j)}{2}\right)$ is multiplicatively separable, which makes it easy to derive the marginal effects of a player's effort on the winning probability and payoff.

### 4.4.2 Productive contest

An innovation contest is a productive contest. In a non-productive contest, a player's effort contributes only to her success in winning the contest. In contrast, in a productive contest, a player's effort increases both the expected prize value and her success probability. Therefore, a player's incentive to exert effort differs between a productive contest and a non-productive contest. This observation has implication on our findings regarding existence of semi-integration equilibrium.

Semi integration occurs in equilibrium only if the innovation probability is sufficiently higher in semi integration than in no integration. Lemma 2 shows that the condition $e^{SI} > e^{NI}$ is a sufficient condition to have higher innovation probability in semi integration. In a two-player contest, this condition refers to a situation in which a player's effort level given the other player puts no effort, is more than the equilibrium effort. Below we show that a productive contest is essential to satisfy the condition $e^{SI} > e^{NI}$. Specifically, a contest with fixed prize value and CSF in additive form, cannot satisfy $e^{SI} > e^{NI}$.

To see this, consider a general two-player productive contest framework, in which the payoff to player $i \in \{1, 2\}$ is given by

$$v(e_1, e_2)\, p_i(e_1, e_2) - c(e_i),$$

where $v(e_1, e_2)$ is the prize value and increasing in $e_i$, $p_i(e_1, e_2)$ is $i$'s contest-success probability and $c(e_i)$ is $i$'s cost of effort. We assume that $p_i(e_1, e_2)$ can be written in additive form: $p_i(e_1, e_2) = \frac{f(e_i)}{f(e_1)+f(e_2)}$, $i \in \{1, 2\}$ and $f$ is an increasing function. For simplicity, we assume a symmetric framework, in which two players have the same value function $v$ and the same cost function $c$. We further assume that the payoff is strictly concave in $e_i$ so that we follow the first-order approach. Let $b(e)$ denote the best response of player $i$, given the other player's effort $e$. We denote the symmetric equilibrium effort by $e^*$, and therefore, $b(e^*) = e^*$. We are interested to find out the condition for $b(0) > e^*$.

**Lemma 7.** *A necessary condition for* $e^* < b(0)$ *is* $\frac{\partial v(e^*,0)}{\partial e_1} \cdot p_1(e^*, 0) - \frac{\partial v(e^*,e^*)}{\partial e_1} \cdot p_1(e^*, e^*) > 0$.

*Proof.* In the Appendix. □

For a non-productive contest, $\frac{\partial v(e_1,e_2)}{\partial e_1} = 0$ at any $(e_1, e_2)$. Hence, a non-productive contest can never satisfy the necessary condition stated in Lemma 7. Example 3 discusses two Tullock-contest models, one with productive effort and the other with non-productive effort. The purpose

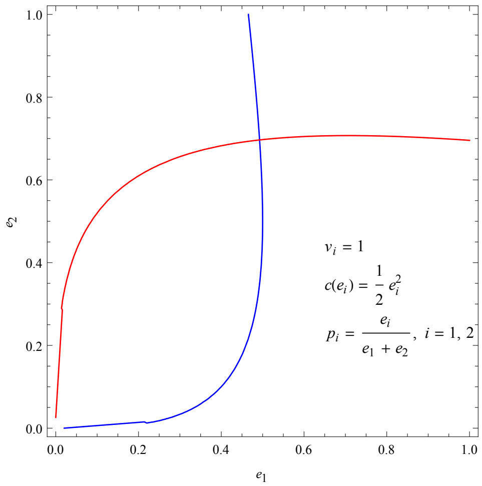


Figure 4: Best-response curves in a non-productive Tullock contest

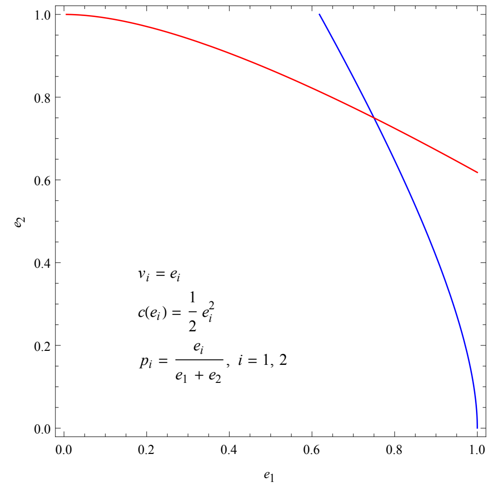


Figure 5: Best-response curves in a productive Tullock contest

of this example is to illustrate how shapes of the best-response curves differ between the two types of contest. Similar to the example of Tullock contest with productive effort, the innovation contest in our model generates decreasing best-response functions.

**Example 3.** Consider a two-player contest with the Tullock CSF (19) with $r = 1$. Let $c(e) = \frac{1}{2}e^2$. Figure 4 plots the best-response curves in the case of non-productive effort with a fixed prize value (set at 1). The best-response curves are concave and are increasing at effort levels close to zero. Figure 5 plots the best-response curves when the prize value is given by $v_i = e_i$. The response curves in this case are decreasing.

The patent race models are also related to the productive-contest models. In the classic model of patent race as pioneered by Loury (1979) and Dasgupta and Stiglitz (1980) (referred as LDS model hereafter), multiple firms compete for a patent. The patent has a fixed value and the probability of making a discovery in the infinite time horizon is always one. Time is costly in the sense that an early discovery is better than a late discovery. The winner's payoff decreases with expected time of discovery. An implication of positive time-discount factor is that if a firm expands its effort (keeping others effort at a fixed level), it increases not only its chance of winning the patent, but also its payoff from winning as the expected time of discovery reduces. Baye and Hoppe (2003) show that LDS models are strategically equivalent to Tullock rent seeking contest model with fixed prize value when the time-discount factor approaches zero. It implies that the response curves can be increasing at effort levels close to zero when the discount factor approaches zero. However, if the discount factor is close to zero, we are effectively dealing with a situation when players are contesting for a prize with a deterministic value. It can be shown that if the discount factor is sufficiently higher than zero, the LDS models can also exhibit decreasing response curves – when one firm reduces effort, the other firm responds by increasing effort.

# 5 Conclusion

In this paper, we develop a simple model to study the competition for innovation on industry structure and innovation incentive. The model shows, under certain conditions, the coexistence of integrated and independent R&D arrangements that appear in equilibrium even though integration can have a negative effect on the integrated firms' R&D investment. The results follow from the positive externality of integration on other independent firms' incentive for innovation. An important lesson from the efficiency analysis is that a no-integration arrangement can arise in equilibrium even when a semi-integrated arrangement can generate higher innovation probability and aggregate industry payoff. The inefficient no-integration equilibrium arises because those who benefit from integration cannot commit to compensating the losing parties in any credible way. Though we draw many of our illustrative examples from the biotech and pharmaceutical industries, our results should be applicable to any industry in which the final products need research-based inputs, and a successful innovation can give the innovator a significant rent in competition. Examples include nanotechnology, chemical, and semiconductor industries.

Our findings have important implications for policymaking. First, mergers and acquisitions are often matters of concern for their potential depressing effects on consumer welfare. In science-based industries, the question is even more complex because the industry structure can also affect innovation frequency and long-term growth. Our study highlights a specific externality of integration that can impact the industry's innovation frequency. Competition policies should consider this effect when evaluating the role of integration. Second, in our model, the motivation for innovation comes from the underlying competition to grab the reward of being the first innovator. However, the capacity to innovate depends on institutional factors and constraints. In fact, resource constraints in the upstream market can severely limit the effect of positive externality generated from integration. The findings emphasize the importance of credit availability to the entrepreneurial research-focused firms in boosting their efforts in the competition for innovation.

We make some simplifying assumptions to keep our analysis tractable. For example, we assume a frictionless market for technology, which is not always observed in practice (Gans et al. 2008). Frictions in the market for technology, in forms of asymmetric information, weak patent laws, and complexity in defining the scope of patents can also adversely affect the bargaining power of the entrepreneurial firms operating in the upstream market. In our framework, such a reduction in bargaining power will also limit the effect of positive externality of integration. Thus, policies should strengthen the market for technology. Our current study does not address many other issues including the complementarity of R&D investment in innovation research or the multi-stage innovation process. These are equally important for the growth of science-based industries. We leave these questions for future studies.

# Appendix

Our analysis focuses on the pure strategies only. At the pre-innovation contracting stage, $C_i$ offers a price $p_i \in [0, \infty)$. Let $\mathbf{p} = (p_1, p_2)$ denote a price profile. At the pre-innovation contracting stage, $RU_i$ decides whether to integrate with a customer. Its integration strategy is given by a tuple $\mathbf{int}_i = \left(int_i^f, int_i^s\right)$. The first component $int_i^f(\mathbf{p})$ is $RU_i$'s integration decision at $p_{\max}$, the maximum price of the price profile $\mathbf{p}$. The second component $int_i^s(\mathbf{p})$ is $RU_i$'s integration decision at $p_{\min}$, the second highest price of the price profile $\mathbf{p}$, given that the other research unit is integrated with the customer offering the highest price $p_{\max}$. For simplicity, we assume that $int_i^f(\mathbf{p})$ and $int_i^s(\mathbf{p})$ take binary values, 0 and 1, such that the value 1 corresponds to a decision to integrate. Let $\mathbf{int} = (\mathbf{int}_1, \mathbf{int}_2)$ denote a profile of integration strategies. The research units simultaneously decide the effort level in the innovation contest. $RU_i$'s effort strategy is to choose $e_i \in [0, 1]$, given a price-integration strategy profile $(\mathbf{p}, \mathbf{int})$. A pure strategy of $RU_i$ is given by $\sigma_i = (\mathbf{int}_i, e_i)$. Below we present the proofs that are omitted in the main text.

**Proof of Lemma 3:**

*Proof.* If an equilibrium with full integration exists, then it must be the case that the research units are willing to integrate at both prices $p_1$ and $p_2$. It is easy to see that in this case, both customers will offer the same price in equilibrium, as otherwise the customer offering the higher price can increase her payoff by decreasing price. We denote the common price by $p$. As $RU_2$ integrates at $p$ when $RU_1$ is already integrated, we must have

$$p \geq \overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right). \tag{21}$$

$C_2$'s expected payoff in this equilibrium is $\left(\overline{v} + \frac{\underline{v}}{2}\right)\pi_2(0,0) + \frac{\underline{v}}{2}\pi_1(0,0) - p$. The first component is $C_2$'s expected payoff when $RU_2$ wins the contest times the probability that $RU_2$ wins the contest. Recall that (from our discussion in section 3.1) the expected payoff of the customer integrated with the winning research unit is $E\left(\frac{v_{\max}+v_i}{2}\right) = \overline{v} + \frac{\underline{v}}{2}$. The second component is $C_2$'s expected payoff when $RU_1$ wins the contest times the probability that $RU_1$ wins the contest. After simplifying, we can rewrite $C_2$'s expected payoff as $\overline{v}\pi_2(0,0) + \frac{\underline{v}}{2}\pi_{inv}^{FI} - p$.

On the other hand, if $C_2$ deviates by lowering its price, its expected payoff will be $\frac{\underline{v}}{2}\left(\pi_1\left(0, e^{SI}\right) + \pi_2\left(0, e^{SI}\right)\right) = \frac{\underline{v}}{2}\pi_{inv}^{SI}$. Comparing the above expressions, the no deviation condition for $C_2$ is given by

$$p \;\leq\; \overline{v}\pi_2(0,0) - \frac{\underline{v}}{2}\left(\pi_{inv}^{SI} - \pi_{inv}^{FI}\right). \tag{22}$$

From (21) and (22), we see that a necessary condition to have an equilibrium with full integration is that

$$\overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right) \;\leq\; \overline{v}\pi_2(0,0) - \frac{\underline{v}}{2}\left(\pi_{inv}^{SI} - \pi_{inv}^{FI}\right). \tag{23}$$

The above condition is also a sufficient condition to have an equilibrium with full integration. To see this, we construct an equilibrium as follows. Let us denote $\overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right)$ by $A$ and $\overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$ by $B$. We have $B \leq A$ as $\overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right) \geq \overline{v}\pi_2\left(0, e^{NI}\right) - c\left(e^{NI}\right) \geq \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$. Consider the following strategies. $C_i$ chooses $p_i = A$. The integration strategies $\left(\left(int_1^f(\mathbf{p}), int_1^s(\mathbf{p})\right), \left(int_2^f(\mathbf{p}), int_2^s(\mathbf{p})\right)\right)$ by the research units are as follows:

$$\begin{aligned}
int_1^f(\mathbf{p}) &= \begin{cases} 1 & \text{if } p_{\max} \geq A \\ 0 & \text{otherwise} \end{cases} \\
int_2^f(\mathbf{p}) &= \begin{cases} 1 & \text{if } p_{\max} \geq B \\ 0 & \text{otherwise} \end{cases} \\
int_1^s(\mathbf{p}) = int_2^s(\mathbf{p}) &= \begin{cases} 1 & \text{if } p_{\min} \geq A \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{24}$$

And $RU_i$'s effort strategy $e_i$ is as follows:

$$e_i = \begin{cases} 0 & \text{if } RU_i \text{ is integrated} \\ e^{NI} & \text{if both research units are not integrated} \\ e^{SI} & \text{otherwise} \end{cases} \tag{25}$$

**Claim 1**: The integration strategies and the effort strategies given by (24) and (25) are Nash equilibrium strategies in the subgame induced by the price profile $\mathbf{p}$.

We have already shown in section 3.2 that that $e_i$s are the Nash equilibrium effort strategies in the innovation contest. We will now show that for a given price profile $\mathbf{p}$, $\mathbf{int}_1 = \left(int_1^f(\mathbf{p}), int_1^s(\mathbf{p})\right)$ is $RU_1$'s optimal integration strategy given $RU_2$ follows $\mathbf{int}_2 = \left(int_2^f(\mathbf{p}), int_2^s(\mathbf{p})\right)$ and vice versa. Note that $RU_i$'s expected

payoff from no integration when the other research unit is integrated is given by $A$. Therefore, it prefers to integrate if and only if $p_{\min} \geq A$. We next show that $int_1^f$ is $RU_1$'s best response against $RU_2$'s first stage integration strategy $int_2^f$ and vice versa. To see this, note that for any price profile with $p_{\max} \geq A$, both research unit's dominant strategy is to integrate, as a research unit's maximum payoff from non-integration can never exceed $A$ in any situation. Similarly, for any profile with $p_{\max} < B$, both research unit's dominant strategy is not to integrate, as it can always a payoff as high as $B$ by non-integration. Finally, if $p_{\max}$ lies in the interval $[B, A]$, and if one of the research unit integrates, the other research unit's optimal strategy is not to integrate and vice versa. This completes the proof of Claim 1.

Further, note that the customers by offering $p_i = A$, can induce both firms to integrate, and given condition (23), none of the customer can improve the payoff by lowering its offered price when the other customer offers a price equal to $A$. Hence the above strategies constitute a subgame perfect Nash equilibrium. These strategies will lead to an outcome of full integration as both research unit integrate at $p_1 = p_2 = A$. $\square$

**Proof of Lemma 4:**

*Proof.* If an equilibrium with no integration exists, then it must be the case that both research units are not willing to integrate at the maximum price. Assume that the research units face a price profile $(p_1, p_2)$. We compare $RU_2$'s payoff from integration and that from no integration. When $RU_1$ does not integrate, $RU_2$'s payoff from integrating is $max\{p_1, p_2\}$ and from not integrating is $\overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$. If an equilibrium with no integration exists, we must have

$$max\{p_1, p_2\} \leq \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right). \tag{26}$$

We next consider the customers' incentive to offer low prices. Suppose that $C_1$ offers a price $p_1 \leq \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$. If $C_2$ also offers a price $p_2 \leq \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$, its expected payoff is $\frac{\underline{v}}{2}\pi_{inv}\left(e^{NI}, e^{NI}\right) = \frac{\underline{v}}{2}\pi_{inv}^{NI}$. If $C_2$ deviates by increasing its price above (weakly) $\overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$, then one of the two research units (without loss of generality, assume $RU_2$) will choose to accept the offer. In such a case, $C_2$'s expected payoff will be given by $\left(\overline{v} + \frac{\underline{v}}{2}\right)\pi_2\left(e^{SI}, 0\right) + \frac{\underline{v}}{2}\pi_1\left(e^{SI}, 0\right) - p_2 = \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{\underline{v}}{2}\pi_{inv}^{SI} - p_2$. Comparing the above expressions, the no deviation condition for $C_2$ is given by

$$p_2 \quad \geq \quad \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{\underline{v}}{2}\pi_{inv}^{SI} - \frac{\underline{v}}{2}\pi_{inv}^{NI}. \tag{27}$$

Hence, a necessary condition to have an equilibrium with no integration is

$$\begin{aligned} \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{\underline{v}}{2}\pi_{inv}^{SI} - \frac{\underline{v}}{2}\pi_{inv}^{NI} \quad &\leq \quad \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right) \\ \Leftrightarrow \Delta_{inv} \quad &\leq \quad \frac{2\eta}{\underline{v}}. \end{aligned} \tag{28}$$

The above condition is also a sufficient condition to have an equilibrium with full integration. To see this, assume that condition (28) holds true and we consider the following strategies. $C_i$ chooses $p_i = 0$. As before, we denote $\overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right)$ by $A$ and $\overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$ by $B$. And, we consider the integration strategies $\left(\left(int_1^f(\mathbf{p}), int_1^s(\mathbf{p})\right), \left(int_2^f(\mathbf{p}), int_2^s(\mathbf{p})\right)\right)$ and the effort strategies given by (24) and (25) respectively.

As shown in the proof of lemma 3 (see Claim 1 in the proof), the integration strategies and the effort strategies are the Nash equilibrium strategies in the subgame induced by the price profile $\mathbf{p}$. We will have to show that $p_1 = 0$ and $p_2 = 0$ are Nash equilibrium price strategies by the customer. To see this, let us suppose that $C_1$ offers $p_1 = 0$. By increasing $p_2 \geq B$, $C_2$ can get a payoff of $\overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{\underline{v}}{2}\pi_{inv}^{SI} - p_2$, which can never be higher than its current payoff from no integration $\frac{\underline{v}}{2}\pi_{inv}^{NI}$ as the condition (28) holds true. Hence, the above mentioned strategies are indeed Nash equilibrium strategies. In this equilibrium, no research unit integrates. $\square$

**Proof of Lemma 5:**

*Proof.* Without loss of generality, we assume that $RU_2$ is integrated to $C_2$, and $RU_1$ and $C_1$ are not integrated in semi integration. If an equilibrium with semi integration exists, then it must be the case that $RU_1$ does not integrate at $p_{\min}$. Notice that the payoff of a non integrated research unit is $A = \overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right)$ when the other research unit is integrated. Hence, we must have $p_{\min} \leq A$. However, there are two possibilities in which we can see semi integration. First, both research units are willing to integrate at $p_{\max}$, and second, $RU_2$ integrates at $p_{\max}$ but $RU_1$ does not integrate at $p_{\max}$. We assume that $p_{\min} \leq A$ and analyze the two cases below.

Case 1: Both research units are willing to integrate at $p_{\max}$. When $RU_2$ is integrated, $RU_1$ gets $\frac{1}{2}p_{\max} + \frac{1}{2}A$ by integrating ($RU_1$ is matched with the customer offering $p_{\max}$ with $\frac{1}{2}$ probability) and it gets $A$ by not integrating. Hence, in this case we must have $p_{\max} \geq A$.

Case 2: $RU_2$ is willing to integrate at $p_{\max}$, but $RU_1$ is not. Comparing $RU_1$'s payoff from integration and no integration (when $RU_2$ is integrated), we see that $p_{\max} \leq A$. Similarly, comparing $RU_2$'s payoff from integration and no integration (when $RU_1$ is not integrated) we see that $p_{\max} \geq B = \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$. Hence, in this case we must have $p_{\max} \in [B, A]$.

Next, we look at the customers' optimal price responses. For given $p_1$, we consider the optimal response of $RU_2$.

If $p_1 < B$, $C_2$ gets $\frac{v}{2}\pi_{inv}^{NI}$ by offering $p_2 < B$. And, if it offers $p_2 \geq B$, one of the research unit integrates while the other is not. Therefore, by offering $p_2 \geq B$, $C_2$ gets $\overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - p_2$, which is decreasing in $p_2$. Hence, when $p_1 < B$, the optimal response of $C_2$ is $B$ if $B \leq \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - \frac{v}{2}\pi_{inv}^{NI}$, and any $p_2 < B$ if $B > \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - \frac{v}{2}\pi_{inv}^{NI}$.

If $p_1 \in [B, A]$, $C_2$ gets $\frac{v}{2}\pi_{inv}^{SI}$ by offering $p_2 < p_1$ (as only one research unit integrates with $C_1$ in that case). By offering $p_2 \geq p_1$, $C_2$ gets $\overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - p_2$, which is always less than $\frac{v}{2}\pi_{inv}^{SI}$ for all $p_2 \geq B$. This is because $\overline{v}\pi_2\left(e^{SI}, 0\right) \leq \overline{v}\pi_2\left(e^{NI}, 0\right) \leq \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right) = B$. Hence, when $p_1 \in [B, A]$, the optimal response of $C_2$ is any $p_2 < p_1$.

Finally, if $p_1 > A$, $C_2$ gets $\frac{v}{2}\pi_{inv}^{SI}$ by offering $p_2 < A$ (as only one research unit integrates with $C_1$ in that case). By offering $p_2 \geq A$, $C_2$ gets $\overline{v}\pi_2(0, 0) + \frac{v}{2}\pi_{inv}^{FI} - p_2$, which is always less than $\frac{v}{2}\pi_{inv}^{SI}$ for all $p_2 \geq A$. Hence, when $p_1 > A$, the optimal response of $C_2$ is any $p_2 < A$.

The optimal response of $C_1$ for a given price $p_2$ would also be symmetric. It is evident in no circumstances, any customer would offer a price as high as $A$. Thus the case 1 depicted above, in which $p_{\max} \geq A$, will never be realized in equilibrium. Therefore, if we see semi integration in equilibrium, it must be that case 2 holds true, in which we have $p_{\max} \in [B, A]$. From the optimal response functions, we see that such a possibility can occur only if $B \leq \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - \frac{v}{2}\pi_{inv}^{NI}$, in which case $p_{\max} = B$ and $p_{\min} < p_{\max}$. Hence a necessary condition to have an equilibrium with semi integration is

$$\begin{aligned}\overline{v}\pi_2\left(e^{SI},0\right)+\frac{v}{2}\pi_{inv}^{SI}-\frac{v}{2}\pi_{inv}^{NI} &\geq \overline{v}\pi_2\left(e^{NI},e^{NI}\right)-c\left(e^{NI}\right)\\ \Leftrightarrow \Delta_{inv} &\geq \frac{2\eta}{\underline{v}}.\end{aligned}\tag{29}$$

The above condition is also a sufficient condition to have an equilibrium with full integration. To see this, assume that condition (29) holds true and we denote $\overline{v}\pi_2\left(0,e^{SI}\right)-c\left(e^{SI}\right)$ by $A$ and $\overline{v}\pi_2\left(e^{NI},e^{NI}\right)-c\left(e^{NI}\right)$ by $B$. Consider the following strategies. $C_1$ chooses $p_1=0$ and $C_2$ chooses $p_2=B$. And, we consider the integration strategies $\left(\left(int_1^f(\mathbf{p}),int_1^s(\mathbf{p})\right),\left(int_2^f(\mathbf{p}),int_2^s(\mathbf{p})\right)\right)$ and the effort strategies given by (24) and (25) respectively.

As shown in the proof of lemma 3 (see Claim 1 in the proof), the integration strategies and the effort strategies are the Nash equilibrium strategies in the subgame induced by the price profile $\mathbf{p}$. From our derivation of the optimal response functions above, we see that $p_1=0$ and $p_2=B$ are Nash equilibrium price strategies when the condition (29) holds true. Hence, the above mentioned strategies are indeed Nash equilibrium strategies. In this equilibrium, $RU_2$ integrates with $C_2$ while $RU_1$ and $C_1$ are not integrated. □

**Proof of Lemma 7:**

*Proof.* From the first-order condition (and as payoff is concave in $e_i$),

$$e^*<b(0)\Leftrightarrow c'(e^*)<\frac{\partial v(e^*,0)}{\partial e_1}\cdot p_1(e^*,0)+v(e^*,0)\cdot\frac{\partial p_1(e^*,0)}{\partial e_1}.$$

We replace $c'(e^*)$ by $\frac{\partial v(e^*,e^*)}{\partial e_1}\cdot p_1(e^*,e^*)+v(e^*,e^*)\cdot\frac{\partial p_1(e^*,e^*)}{\partial e_1}$, which follows from the condition that characterizes the equilibrium value $e^*$. Therefore, a necessary and sufficient condition for $e^*<b(0)$ is that

$$v(e^*,e^*)\cdot\frac{\partial p_1(e^*,e^*)}{\partial e_1}-v(e^*,0)\cdot\frac{\partial p_1(e^*,0)}{\partial e_1}<\frac{\partial v(e^*,0)}{\partial e_1}\cdot p_1(e^*,0)-\frac{\partial v(e^*,e^*)}{\partial e_1}\cdot p_1(e^*,e^*).\tag{30}$$

Given the additive form of $p_i(e_1,e_2)$, we have

$$\frac{\partial p_1(e_1,e_2)}{\partial e_1}=\frac{f'(e_1)f(e_2)}{\left(f(e_1)+f(e_2)\right)^2},$$

and

$$\frac{\partial^2 p_1(e_1,e_2)}{\partial e_2\partial e_1}=\frac{f'(e_1)f'(e_2)\left(f(e_1)+f(e_2)\right)\left(f(e_1)-f(e_2)\right)}{\left(f(e_1)+f(e_2)\right)^4}.$$

Therefore, $\frac{\partial^2 p_1(e_1,e_2)}{\partial e_2\partial e_1}>0$ if $f(e_2)<f(e_1)$, or equivalently, $e_2<e_1$. We thus have $\frac{\partial p_1(e^*,e_2)}{\partial e_1}>0$ for all $e_2<e^*$. In particular, $\frac{\partial p_1(e^*,0)}{\partial e_1}<\frac{\partial p_1(e^*,e^*)}{\partial e_1}$. Further, $v(e^*,0)\leq v(e^*,e^*)$ as $v$ is increasing in $e_2$. Together, we get that $v(e^*,e^*)\cdot\frac{\partial p_1(e^*,e^*)}{\partial e_1}-v(e^*,0)\cdot\frac{\partial p_1(e^*,0)}{\partial e_1}>0$. From (30), we get a necessary condition for $e^* < b(0)$ is

$$\frac{\partial v(e^*, 0)}{\partial e_1} \cdot p_1(e^*, 0) - \frac{\partial v(e^*, e^*)}{\partial e_1} \cdot p_1(e^*, e^*) > 0.$$

□

**Proof of Lemma 6:**

*Proof.* Note that $e^{NI}$ satisfies (4) and $e^w$ satisfies (10). Denote $(\overline{v} + \underline{v})\left(1 - q\left(e^{NI}\right)\right)$ and $\overline{v}\left(1 - \frac{q\left(e^{NI}\right)}{2}\right)$ by $A$ and $B$ respectively. A direct comparison of $A$ and $B$ shows that $A \leq B$ if and only if $\frac{2\underline{v}}{\overline{v}+2\underline{v}} \leq q\left(e^{NI}\right)$. By (4), $B = \frac{c'\left(e^{NI}\right)}{q'\left(e^{NI}\right)}$. Therefore,

$$\begin{aligned} A \leq B \quad &\Leftrightarrow \quad (\overline{v} + \underline{v})\left(1 - q\left(e^{NI}\right)\right) \leq \frac{c'\left(e^{NI}\right)}{q'\left(e^{NI}\right)} \\ &\Leftrightarrow \quad (\overline{v} + \underline{v})\left(1 - q\left(e^{NI}\right)\right) q'\left(e^{NI}\right) - c' \leq 0 \\ &\Leftrightarrow \quad e^w \leq e^{NI} \text{ (by Assumption 3).} \end{aligned}$$

□

**Proof of Proposition 3:**

*Proof.* From (7), (8) and (12), we see that a no-integration equilibrium is inefficient if

$$\frac{\left(c\left(e^{SI}\right) - 2c\left(e^{NI}\right)\right)}{\overline{v} + \underline{v}} \leq \Delta_{inv} \leq \frac{2\eta}{\underline{v}}, \tag{31}$$

and a semi-integration equilibrium is inefficient if

$$\frac{2\eta}{\underline{v}} \leq \Delta_{inv} \leq \frac{\left(c\left(e^{SI}\right) - 2c\left(e^{NI}\right)\right)}{\overline{v} + \underline{v}}. \tag{32}$$

It can be shown with example that (31) is not vacuous. Below we show that (32) cannot hold true. In particular, we will show whenever $W^{SI} \leq W^{NI}$, or equivalently, the right-side inequality in (32) holds true, we cannot observe semi-integration arrangement in equilibrium, or equivalently, the left-side inequality cannot hold true. To see this, it is useful to start with the following claim:

$$2\overline{v}\pi_2\left(e^{SI}, 0\right) \leq \pi_{inv}^{SI} - c\left(e^{SI}\right). \tag{33}$$

The claimed relationship in (33) can be derived as follows:

$$
\begin{aligned}
\overline{v}\pi_2\left(e^{SI},0\right) \leq \overline{v}\pi_2\left(0,0\right) = \overline{v}\pi_1\left(0,0\right) &\leq \overline{v}\pi_1\left(e^{SI},0\right) - c\left(e^{SI}\right), \\
\Leftrightarrow \overline{v}\pi_2\left(e^{SI},0\right) + \overline{v}\pi_2\left(e^{SI},0\right) &\leq \overline{v}\pi_2\left(e^{SI},0\right) + \overline{v}\pi_1\left(e^{SI},0\right) - c\left(e^{SI}\right), \\
\Leftrightarrow 2\overline{v}\pi_2\left(e^{SI},0\right) &\leq \overline{v}\pi_{inv}^{SI} - c\left(e^{SI}\right).
\end{aligned}
$$

We rewrite (33) as $2\overline{v}\pi_2\left(e^{SI},0\right) + \underline{v}\pi_{inv}^{SI} \leq \overline{v}\pi_{inv}^{SI} + \underline{v}\pi_{inv}^{SI} - c\left(e^{SI}\right) = W^{SI}$. If a semi-integration arrangement is inefficient, then $W^{SI} \leq W^{NI}$, and therefore

$$
2\overline{v}\pi_2\left(e^{SI},0\right) + \underline{v}\pi_{inv}^{SI} \leq W^{NI} = \left(\overline{v} + \underline{v}\right)\pi_{inv}^{NI} - 2c\left(e^{NI}\right).
$$

After rearranging terms, we get

$$
\begin{aligned}
\underline{v}\pi_{inv}^{SI} - \underline{v}\pi_{inv}^{NI} &\leq \overline{v}\pi_{inv}^{NI} - 2c\left(e^{NI}\right) - 2\overline{v}\pi_2\left(e^{SI},0\right), \\
\Leftrightarrow \frac{\underline{v}}{2}\left(\pi_{inv}^{SI} - \pi_{inv}^{NI}\right) &\leq \overline{v}\pi_2\left(e^{NI},e^{NI}\right) - 2c\left(e^{NI}\right) - 2\overline{v}\pi_2\left(e^{SI},0\right), \\
\Leftrightarrow \frac{\underline{v}}{2}\Delta_{inv} &\leq \eta,
\end{aligned}
$$

which implies that we cannot observe semi integration in equilibrium.