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Time-varying covariance structures
A DCC-GARCH approach to testing the CAPM

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Abstract

This master's thesis examines the implications of applying time-varying covariance structures between four major asset classes in the US economy within the framework of the conditional Capital Asset Pricing Model (CAPM). The DCC-GARCH-in-mean model is employed to estimate the time-varying covariance structures. To construct the market portfolio, weights based on market values are utilized and updated for each time period. The findings reveal significant evidence of time-varying risk premia and risk exposures (beta), thereby supporting the notion of a time-varying covariance structure in the CAPM. However, we also find that intercepts in CAPM equations are significant, in contradiction to the CAPM. Consequently, it is suggested that while there is a need to account for time-varying estimation of market risk, solely relying on this factor may not adequately explain the variations in expected returns. Hence, the results imply the inclusion of additional factors to enhance the model's explanatory power.

Preface

This thesis concludes our master's degree in economics and business administration at Oslo Metropolitan University. We're grateful to have had this opportunity to extend our knowledge of asset pricing and gain a deeper understanding of volatility modelling. The work of this thesis has felt meaningful, and academically stimulating.

First and foremost, we are sincerely grateful to our supervisor, Associate Professor Johann Reindl, for all inspiration, patience, and theoretical support during this project. We would also like to thank Gard Ålrust for providing access to Bloomberg, which was crucial for our market portfolio calculation. Lastly, we would like to extend a heartfelt *thank you* to Julie for her amazingness.

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1 Introduction

Modeling of asset volatility has gained significant attention in financial research. Volatility is a measure of the risk, but since it is not directly observable, we need models to estimate it. Assets react to new information in the market in different ways, but we know that some assets are more interlinked and tend to react similarly when exposed to shocks. In asset pricing, it is important to understand the volatility dynamics, and it is therefore helpful to analyze their co-movement, or covariance.

The Capital Asset Pricing Model (CAPM) has been a fundamental part of asset pricing since it was introduced by Sharpe (1964) and Lintner (1965). The model attempts to explain the complexity of financial markets with a simple and practical framework. It assumes that there is a constant, linear relationship between asset returns and market risk. Extensive research and empirical tests have been performed of the model's validity, and although empirical tests yield mixed results, it provides an accessible approach to understanding the relationship between the individual asset and the overall market.

The strong assumptions, and consequently lack of empirical evidence in support, has led to several efforts to extend and improve the model. One of the more regarded extensions of the CAPM were formulated by Fama and French (1992) with the *three-factor model*. In addition to market risk exposure, they included firm characteristics in explaining returns. Engle (1982) and Bollerslev (1986) introduced the ARCH and GARCH – framework, respectively, which allowed for the estimation of time-varying variance. Later, they join forces and show how the multivariate GARCH model is estimated and apply it to empirical data. They find that covariances between asset returns are not constant over time, indicating that the assumption of constant covariances in CAPM may not hold. This led to the introduction of the *conditional CAPM*, in which the covariance structures are allowed to be time-varying (Bollerslev et.al, 1988).

This thesis attempts to contribute to the existing research by updating the study by Bollerslev et al., with certain extensions in the data and methodology. Their title, *A Capital*

Asset Pricing Model with Time-varying Covariances, is fitting for the work presented here. We compare findings for a new sample period. The econometric model applied to the data is within the GARCH framework, with an extension called DCC-GARCH which allows for the study of time varying covariance structures, and how these affect returns. To test the CAPM, conditional covariance structures are included in augmentations of the mean equation. This is applied to monthly return series for four asset classes. We construct a value-weighted market portfolio which is updated for each time period. The sample period spans 30 years. Our research question is:

"Does time-varying covariance structures improve the CAPM?"

The first part of our analysis centers around the estimation of the risk aversion coefficient and the market risk premium. Consistent with the literature by Cuthbertson and Nitzsche (2005) and Bali and Engle (2010), we estimate the risk aversion coefficient to be 5.32. Additionally, we analyze the relationship between the risk aversion coefficient and the market variance, demonstrating how changes in risk aversion impact the premium awarded to investors. We observe that during periods of high market volatility, the market risk premium increases, while in relatively calm periods, it remains below 0.5%.

The second part of our analysis deals with testable implications of the CAPM. We find significant evidence of time-variation in risk premia and risk exposures, in support of the CAPM with time-varying covariance structure. However, we also find that intercepts in CAPM equations are significant, in contradiction to the CAPM. Our results suggest that even though it seems that there should be a time-varying component included, this alone is not sufficient for the model to adequately explain the variations in expected returns. The results suggest that other factors should be included to improve the model.

The thesis is organized in the following way: Section 2 gives an overview of the theoretical framework, and our hypotheses. Section 3 describes the methodology and empirical application. Section 4 describes the data and its properties. In section 5 we present the findings and test results. Section 6 concludes with suggestions on continued research.

2 Theoretical Framework

2.1 Capital Asset Pricing Model

The traditional capital asset pricing model (CAPM) was introduced by Sharpe (1964) and Lintner (1965), and expands on the mean-variance optimization suggested by Markowitz (1952). According to CAPM, in equilibrium, the expected return of an asset above the risk-free interest rate is directly related to the expected return of the market. It explains how the expected excess return of an asset is a function of the co-movement of that asset with the market (β_i) and the market risk premium ($E[r_M] - r_f$). In the traditional CAPM the expected excess return of an asset i is given by:

$$E[r_i] - r_f = \beta_i\big(E[r_M] - r_f\big) \tag{1}$$

There are a few assumptions to this model. Firstly, all investors have the same perception of what makes up the efficient one-period ahead portfolio. Secondly, all investors share the same expectations on the return properties, such as means and covariances. And lastly, the market is efficient: there exists no transaction costs or taxes, and investors have access to borrowing at the risk-free rate (Sharpe, 1964).

While the CAPM provides a framework on how the expected excess return on the asset is connected to the market, it does not *explain* the market return. According to Cuthbertson and Nitzsche, the expected excess return on the market, or the market risk premium, can be expressed as a product of the market risk aversion (δ) and the market variance (σ_M^2) (2005, p. 137). This can be interpreted as the level of compensation an investor will require to hold the market portfolio:

$$E[r_M] - r_f = \delta\sigma_M^2 \tag{2}$$

Inserting (2) in (1) gives:

$$E[r_i] - r_f = \beta_i\delta\sigma_M^2 \tag{3}$$

It follows then, that in equilibrium an investor will require a premium for taking on risk equal to the variance of the market times the prevailing market risk aversion. The CAPM assumes that investors have some level of risk aversion ($\delta > 0$), which means that from (3) an increase in market risk will result in a higher expected return on the asset, ceteris paribus. In financial markets we distinguish between systematic and idiosyncratic risk. The latter is associated with an individual asset, while the former is non-diversifiable, and thus inherent in the marketplace. The risk-aversion would tell us how much compensation we require for that systematic risk. It can thus be approached as the aggregate relative risk aversion in the overall economy (Ng, 1991, p. 1508). It is not directly measurable, but it is common to *infer* a risk aversion coefficient based on observed financial data. According to Cuthbertson and Nitzsche plausible ranges for the risk aversion coefficient is between 3 and 6 (2005, p. 659). This is consistent with Chou (1988) and Bali and Engle (2010) who found it to reach an estimated value around 4.5, while work by Poterba and Summers (1986) estimates it lower, at 3.5. Since the risk coefficient is an inferred measure, the accuracy of the estimated coefficient is sensitive to measurement error and is therefore best used as a tool for the theoretical understanding of investor behavior.

The beta estimate in (3) is used to identify the degree to which an asset is exposed to the systematic risk, and at what level an investor should be compensated to hold that risk. Therefore, according to CAPM a reward is given only on account of how much an asset contributes to the overall portfolio risk. The beta can then be defined as the covariance between the return on an asset and the market portfolio, times the *inverse* of the variance of the market return (Cuthbertson & Nitzsche, 2005, p. 118):

$$\beta_i = \frac{cov(r_{i,M})}{\sigma_M^2} \tag{4}$$

2.2 Time-varying covariances and the CAPM

As we can see from (4), the covariance estimate is used in identifying the beta. We know however that the covariance matrix of returns is time-varying. The intuition seems reasonable, that an asset's covariance with the market changes in certain periods. Then, this implies that the betas will be time-varying, and we can express the time-varying CAPM:

$$E_{t-1}[r_{i,t}] - r_{f,t} = \beta_{i,t}\delta\sigma^2_{M,t} \quad (5)$$

Equation (5) follows from (3), taking into account that the covariance matrix is time-varying. Similarly, the market risk premium would also be expressed conditionally:

$$E_{t-1}[r_{M,t}] - r_{f,\,t} = \delta\sigma^2_{M,t} \quad (6)$$

Equation (6) follows from (2), taking into account that the market risk premium is time-varying. Due to the weak empirical evidence of the traditional CAPM's ability to price assets, this is a common approach for modelling variances (Nieto et al., 2014, p. 14). Allowing for time-variation in the betas is supported by works like Fama and French (1997, p. 175) and Ferson and Harvey (1999, p. 1334), where they show how time-varying betas can help explain the expected returns. Lewellen and Nagel, however, argue that even though they found that the betas were time-varying, the conditional time-varying beta was not enough to explain the anomalies in the CAPM framework (2006, p. 311).

Robert Engle formulated the Autoregressive Conditional Heteroskedasticity (ARCH) model in 1982. Initially applied for modelling inflation volatility in the UK, the model is now widely used in financial and macroeconomic time series econometrics. The ARCH recognizes the tendency of current variance to be a function of past errors. It assumes a time-varying variance equation, in contrast to the conventional time series modelling of the time, which operated under the assumption of constant unconditional variance (Engle R. , 1982).

Tim Bollerslev further proposed a generalization of the ARCH model in 1986. Whereas the ARCH process only includes past error terms, the GARCH also includes the lagged *conditional variance* term in the current variance equation. A large movement in an asset price that happened recently will therefore increase the current variance equation. The intuition behind this is that increased price movements in assets tend to affect the volatility in the following periods. Thus, the GARCH model assumes that the conditional variance follows an autoregressive process, and by modelling this variance, it seeks to capture the persistence and clustering of the time-varying volatility (1986).

In their 1988-paper *A Capital Asset Pricing Model with Time-Varying Covariances*, Bollerslev et al. test the CAPM by allowing the covariances to be time-varying. They use the GARCH framework to estimate the conditional covariances to see if time-varying estimates would improve the precision of the pricing model. The overall market consist of an indefinite number of assets, but it can be tested empirically by taking the most important asset classes, as shown by Bollerslev et. al and later by Cuthbertson and Nitzsche (2005, p. 670).

Therefore, we can specify a multivariate GARCH model to perform empirical tests of (5) and (6) to test the asset pricing model. Bollerslev et al. use three assets, while we expand with an additional asset class, i.e., $i = 4$.

$$r_{M,t} = w_{1,t-1}r_{1,t} + w_{2,t-1}r_{2,t} + w_{3,t-1}r_{3,t} + w_{4,t-1}r_{4,t}$$
$$E_{t-1}\left[r_{M,t}\right] = \sum_{i=1}^{4} w_{i,t-1}\, E_{t-1}\left[r_{i,t}\right] \tag{7}$$

From (7), the expected excess return on the market can be defined as the weighted average of the individual asset returns. The market's return variance is then given at:

$$\sigma_{M,t}^2 = var_{t-1}(w_{1,t-1}r_{1,t} + w_{2,t-1}r_{2,t} + w_{3,t-1}r_{3,t} + w_{4,t-1}r_{4,t})$$
$$= \sum_{i=1}^{4}\sum_{j=1}^{4} cov_{t-1}(r_{i,t}, r_{j,t})\, w_{i,t-1}w_{j,t-1} \tag{8}$$

The above expressions will be used in our modelling of the market variance. As for empirical testing of the CAPM we modify expression (5):

$$
\begin{aligned}
E_{t-1}[r_{i,t}] - r_{f,t} &= \delta \beta_{i,t} \sigma_{M,t}^2 \\
&= \delta \frac{cov_{t-1}(r_{i,M,t})}{\sigma_{M,t}^2} \sigma_{M,t}^2 \\
&= \delta cov_{t-1}(r_{i,M,t})
\end{aligned} \tag{9}
$$

To extract $cov_{t-1}(r_{i,M,t})$ we make use of $r_{M,t}$ from (7). This can be exemplified for $i = 1$:

$$
\begin{aligned}
cov_{t-1}(r_{1,t}, r_{M,t}) &= cov_{t-1}(r_{1,t}, (w_{1,t-1} r_{1,t} + w_{2,t-1} r_{2,t} + w_{3,t-1} r_{3,t} + w_{4,t-1} r_{4,t})) \\
cov_{t-1}(r_{1,t}, r_{M,t}) &= w_{1,t-1} var_{t-1}(r_{1,t}) + w_{2,t-1} cov_{t-1}(r_{1,t}, r_{2,t}) + w_{3,t-1} cov_{t-1}(r_{1,t}, r_{3,t}) + w_{4,t-1} cov_{t-1}(r_{1,t}, r_{4,t})
\end{aligned} \tag{10}
$$

where the last line is derived by applying the bilinearity property of covariance calculation. Finally, inserting (10) in (9) gives:

$$
\begin{aligned}
E_{t-1}[r_{1,t}] - r_{f,t} &= \delta \left(w_{1,t-1} var_{t-1}(r_{1,t}) + w_{2,t-1} cov_{t-1}(r_{1,t}, r_{2,t}) + w_{3,t-1} cov_{t-1}(r_{1,t}, r_{3,t}) + w_{4,t-1} cov_{t-1}(r_{1,t}, r_{4,t}) \right) + \varepsilon_{1,t} \\
&\vdots \\
E_{t-1}[r_{4,t}] - r_{f,t} &= \delta \left(w_{1,t-1} cov_{t-1}(r_{1,t}, r_{4,t}) + w_{2,t-1} cov_{t-1}(r_{2,t}, r_{4,t}) + w_{3,t-1} cov_{t-1}(r_{3,t}, r_{4,t}) + w_{4,t-1} var_{t-1}(r_{4,t}) \right) + \varepsilon_{4,t}
\end{aligned} \tag{11}
$$

Going forward, and in the spirit of Bollerslev et al. (1988), we express the CAPM relationship of many assets in the form of vector and matrix. The covariance matrix ($\boldsymbol{H_t}$) times the previous period value weights ($\boldsymbol{w_{t-1}}$) gives $\boldsymbol{H_t w_{t-1}}$. Then the CAPM requires the following for expected excess return of an asset:

$$
\begin{aligned}
\boldsymbol{\mu_t} &= \delta \begin{bmatrix} \sigma_{1,t}^2 & \sigma_{12,t} & \sigma_{13,t} & \sigma_{14,t} \\ \sigma_{12,t} & \sigma_{2,t}^2 & \sigma_{23,t} & \sigma_{24,t} \\ \sigma_{13,t} & \sigma_{23,t} & \sigma_{3,t}^2 & \sigma_{34,t} \\ \sigma_{14,t} & \sigma_{24,t} & \sigma_{34,t} & \sigma_{4,t}^2 \end{bmatrix} \begin{bmatrix} w_{1,t-1} \\ w_{2,t-1} \\ w_{3,t-1} \\ w_{4,t-1} \end{bmatrix} \\
&= \delta \boldsymbol{H_t w_{t-1}}
\end{aligned} \tag{12}
$$

where δ is still the average market risk aversion coefficient and $\sigma_{ij,t}$ is simply the same as $cov_{t-1}(r_{i,t}, r_{j,t})$: the covariances of *excess returns* at time *t*, given the information at *t-1*.

The conditional variance of the market return is given by:

$$\sigma_{M,t}^2 = \begin{bmatrix} w_{1,t-1} & w_{2,t-1} & w_{3,t-1} & w_{4,t-1} \end{bmatrix} \begin{bmatrix} \sigma_{1,t}^2 & \sigma_{12,t} & \sigma_{13,t} & \sigma_{14,t} \\ \sigma_{12,t} & \sigma_{2,t}^2 & \sigma_{23,t} & \sigma_{24,t} \\ \sigma_{13,t} & \sigma_{23,t} & \sigma_{3,t}^2 & \sigma_{34,t} \\ \sigma_{14,t} & \sigma_{24,t} & \sigma_{34,t} & \sigma_{4,t}^2 \end{bmatrix} \begin{bmatrix} w_{1,t-1} \\ w_{2,t-1} \\ w_{3,t-1} \\ w_{4,t-1} \end{bmatrix} \qquad \textbf{(13)}$$

$$= \boldsymbol{w}_{t-1}' \boldsymbol{H}_t \boldsymbol{w}_{t-1}$$

and the conditional mean is then derived at:

$$\mu_{M_t} = \begin{bmatrix} w_{1,t-1} & w_{2,t-1} & w_{3,t-1} & w_{4,t-1} \end{bmatrix} \begin{bmatrix} \mu_{1,t} \\ \mu_{2,t} \\ \mu_{3,t} \\ \mu_{4,t} \end{bmatrix} \qquad \textbf{(14)}$$

$$= \boldsymbol{w}_{t-1}' \boldsymbol{\mu}_t$$

By inserting (12 **)** in (14) the market risk premium can be expressed:

$$\mu_{M_t} = \delta \sigma_{M,t}^2 \qquad \textbf{(15)}$$

2.3 The Hypotheses

Testable implications of the time-varying covariance structure for expected returns in (12) are summarized here and performed in section *5.6 Tests of the CAPM*:

T.1 – There are no systematic effects of non-β terms. i.e. – $\alpha_i = 0$

T.2 – The time-varying risk premium term does not improve the model i.e. – $\delta = 0$

T.3 – Time-varying risk premium does improve the model, compared to a specification that includes a static β. i. e. – $\delta = 0$

T.4 – No other source of risk than the market – i.e., GARCH-in-mean coefficient $\delta = 0$.

3 Methodological Design

3.1 DCC-GARCH

An extension of a multivariate GARCH model called Dynamic Conditional Correlation (DCC-) GARCH is applied in this thesis. It belongs to the class of models where the conditional covariance matrix can be decomposed into conditional standard deviations and correlations (Engle R. , 2002). Instead of modelling individual volatility expressions, we will fit the joint properties of the assets in a multivariate model that will recognize the co-moving nature of these assets. As we proceed with our model, we will extract properties such as covariance and correlation structures and apply them in an empirical framework. Before we derive the multivariate models, we will describe the univariate GARCH model. In doing so, notations will be familiar when we proceed. Bollerslev (1986) define the univariate GARCH(1,1) model:

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} \tag{16}$$

With mean:

$$r_t = \mu + \varepsilon_t \tag{17}$$

$$\varepsilon_t = h_t^{1/2} z_t \tag{18}$$

Where r_t is the return of an asset and μ the value of the unconditional mean. ε_t is the error term which follows a so-called *white noise* process with a time-varying conditional distribution. It is distributed normally with a mean zero and time-varying variance, i.e., $\sim N(0, h_t)$. In (16) the conditional variance h_t is a function of past squared returns ε_{t-1}, and previous period conditional variance, h_{t-1}. From (18), we observe how the conditional variance h_t is absorbed into the disturbance term in (17) and represents misspecification (Engle et al., 1987).

ω, α and β are model parameters that will be estimated. In the following these will be referred to as intercept, *ARCH* and *GARCH* – coefficients, respectively. In order to ensure a positive variance, the parameters must satisfy $\omega > 0, \alpha \geq 1, \beta \geq 0$, and a stable GARCH(1,1)-model requires $(\alpha + \beta) < 1$ (Bollerslev, 1986):

$$\omega = (1 - \alpha - \beta)V_L \tag{19}$$

Rearranging (19), the unconditional variance, or the *long run variance* (V_L), can be obtained. In a stable GARCH-model the effect h_{t-1} on h_t will eventually die, and the model will revert to its mean – the long run variance V_L. $(\alpha + \beta)$ is a measure of persistence in conditional variance. When the coefficients are close to 1, it follows that any past shock in volatility will be highly persistent in the current variance estimation. In the case of $(\alpha + \beta) \geq 1$, the *un*conditional variance is not defined, and "we have a non-stationary (explosive) series in the conditional variance" (Cuthbertson & Nitzsche, 2005, p. 659).

There are several reasons why we would want to extend from a univariate model to a model framework that includes more variables. Firstly, we know that financial assets covary, and the pricing of such a portfolio will then be affected by the nature of this covariance. Secondly, correlations within a portfolio are not constant over the sample periods. If volatility is modelled for assets individually, we will overlook the dynamic processes that *run in the background*. With the transition into multivariate GARCH, we aim at capturing these structures more accurately.

The DCC-GARCH-model enables us to estimate the time-variation that exist in covariance- and correlation structures. It can be expanded for many assets, reducing the computational burden of matrix estimations without substantially increasing the parameters (Engle R. , 2002). Another important benefit is that it includes the immediate disturbance and lagged correlation structure in the estimation of the conditional correlation. By including the immediate disturbance, the model adjusts for the heteroskedasticity which eliminates the bias of underlying volatility processes in the error terms. This improves the accuracy of the conditional correlation estimates (Celik, 2012). Engle (2002) defines the conditional covariance matrix**:**

$$H_t = D_t R_t D_t \quad (20)$$

Where $\boldsymbol{D_t}$ is a diagonal matrix of conditional standard deviations obtained by estimating a separate GARCH model for each return series:

$$\boldsymbol{D_t} = \sqrt{\omega + \alpha\varepsilon_{t-1}\varepsilon'_{t-1} + \beta D^2_{t-1}} \quad (21)$$

and $\boldsymbol{R_t}$ is the adjusted correlation matrix:

$$\boldsymbol{R_t} = Q_t^{*-1} Q_t Q_t^{*-1} \quad (22)$$

$\boldsymbol{R_t}$ is where the DCC-model differs from a constant multivariate model (CCC-GARCH) because it estimates a new matrix for each period t. To estimate $\boldsymbol{R_t}$ we first need to estimate the dynamic correlation matrix, $\boldsymbol{Q_t}$:

$$\boldsymbol{Q_t} = (1 - \lambda_1 - \lambda_2)\boldsymbol{R} + \lambda_1 \boldsymbol{\varepsilon_{t-1}\varepsilon'_{t-1}} + \lambda_2 \boldsymbol{Q_{t-1}} \quad (23)$$

where $\mathbf{R}$ is the weighted average of the unconditional covariance matrix, i.e., the matrix which the covariances revert to in the long run. The parameters in $\mathbf{R}$ can be approached as quasicorrelations (StataCorp. 2023). It is estimated directly, as opposed to V_L in (19), which is obtained through the estimated intercept ω. Next, the model is adjusted for the immediate disturbance in the residuals to the correlation ($\boldsymbol{\varepsilon_{t-1}\varepsilon'_{t-1}}$), with model coefficient λ_1. The last term represents the persistence of correlation, with $\boldsymbol{Q_{t-1}}$ being the previous period correlation matrix, and λ_2 the persistence parameter. Q_t^* in (22) is the diagonal matrix of the squared elements of Q_t.

Thus, λ_1 and λ_2 can be said to measure the short- and long run volatility impact. Although these have similar properties to the ARCH α and GARCH β coefficients, a great computational benefit of the DCC-GARCH is that the number of model parameters in (23) will not increase – regardless of how many time series are included in the model. Engle describes that it has “the flexibility of univariate GARCH but not the complexity of conventional multivariate GARCH” (2002, p. 339).

With the estimation of (21) - (23) complete, we arrive back at $\boldsymbol{H_t}$ in (20), where we can retrieve a conditional covariance matrix for every time period in our sample. In a later section we will describe the data in more detail, but of now we would like to visualize the conditional covariance matrix of returns of four broad asset classes, which make up the overall market portfolio: *treasury bills* and *treasury bonds*, *equity*, and *corporate bonds*:

$$\boldsymbol{H_t} = \begin{bmatrix} \sigma^2_{bills,t} & \sigma_{bills,bonds,t} & \sigma_{bills,equity,t} & \sigma_{bills,cbonds,t} \\ \sigma_{bonds,bills,t} & \sigma^2_{bonds,t} & \sigma_{bonds,equity,t} & \sigma_{bonds,cbonds,t} \\ \sigma_{equity,bills,t} & \sigma_{equity,bonds,t} & \sigma^2_{equity,t} & \sigma_{equity,cbonds,t} \\ \sigma_{cbonds,bills,t} & \sigma_{cbonds,bonds,t} & \sigma_{cbonds,equity,t} & \sigma^2_{cbonds,t} \end{bmatrix} \quad \textbf{(24)}$$

To optimize our model, we estimate the log likelihood function for every period and summarize for all observations:

$$logL = \sum_{t=1}^{T} -\frac{1}{2}(n \ln 2\pi + \ln(\det H_t) + \varepsilon_t H_t^{-1} \varepsilon_t^T) \quad \textbf{(25)}$$

Next, the expression in (25) is maximized with the built-in optimization tool in MS Excel, by allowing for these parameters to be changed: The starting values for model intercepts, ARCH and GARCH coefficients, starting correlations for each of the assets, along with the DCC-coefficients. Lastly, we specify a mean equation that includes the (value weighted) conditional covariance matrix and the market risk aversion, by inserting (12) in (17) this gives:

$$\boldsymbol{r_t} = \boldsymbol{\mu} + \delta \boldsymbol{H_t} \boldsymbol{w_{t-1}} + \boldsymbol{\varepsilon_t} \quad \textbf{(26)}$$

This is what is called the GARCH-in-mean, or GARCH-M, as applied in Bollerslev et al. (1988). Equation (26) is the empirical implementation of the theoretical results derived in (12).

The DCC-GARCH-M specification is not supported by the standard econometric packages available and would require a tailor-made approach that we are unable to develop. We estimate the model in MS Excel, adapting a template provided by NEDL (2021) to fit our empirical specification. Because we cannot calculate the numerical derivation of the log-likelihood function in MS Excel, we are not able to obtain the standard errors for the coefficients, which is important in determining the reliability of the model. However, we are able to test the joint significance of two competing models using the likelihood-ratio test. Furthermore, some hypotheses involve model specifications that are possible to implement in statistical software, and we will use them when applicable.

In the appendix, we include a comparison of the DCC-GARCH-M model estimated in MS Excel against a DCC-GARCH model estimated in statistical software. The estimates from the latter show highly significant α and β coefficients for all assets except treasury bonds, where the β is insignificant. This indicates that there exists volatility persistence structures in the data that is suited for modelling. Furthermore, similarities in the log likelihood estimates suggests that the MS Excel-model is correctly specified.

3.2 Market portfolio

The crucial estimation in our model is the market variance, from which many of the other elements are derived. In addition to the covariance matrix, the weights of the assets are needed to complete the estimation of the market variance. The calculation of the percentage market weights ($\mathrm{w_i}$) can calculated from market values of the asset classes (MV_i):

$$w_i = \sum_{i=1}^{N} \left(\frac{MV_i}{\sum_{i=1}^{N} MV_i} \right) \quad \textbf{(27)}$$

Finally, with the weights calculated from expression (27), we can estimate the market variance from (13), and by allowing for a *risk aversion* parameter to be estimated in the optimization, we can also derive the market risk premium from (15).

We're pleased to have obtained the appropriate data for market values. In their paper on the global market portfolio Doeswijk et.al (2014) express their frustration of the nearly impossible task in obtaining market values of the underlying assets in an index. The choice of assets is inspired by Bollerslev et al. who construct an overall market portfolio on similar assets (1988, p. 121). Although the asset classes are the same, it is worth noting that we do have somewhat differing definitions of bills and bonds. They applied the 6-month treasury bills and 20-year Treasury Bond. As for stocks they used the NYSE value-weighted equity returns. For treasury bills we have used an index with maturities below 1 year, and for treasury bonds we have used an index with maturities of 10 years and above. To proxy equity we have used the S&P 500 Total Return Composite. While different, we believe that by expanding the definition of short- and long-term bonds and including an additional asset in the corporate bond index, we approach a more complete representation of the investment universe available. This is backed by Evans (1994) who finds that he can account for more of the variation in excess returns when corporate bonds are added to the market portfolio and Doeswijk et.al points to the increasing importance of including assets other than just equity and treasury bonds (2019, p. 522). We recognize that a fully reflective market portfolio is close to impossible to attain, as is claimed by Roll (1977). It may never completely reflect the true market portfolio, but we believe that our proxy of the overall market has captured a big chunk of the investing possibilities available to the investor.

To summarize, the overall investing universe is therefore a combination of short-term treasury bills, long term treasury bonds, corporate bonds, together with equities.

4 Data description

Return data is calculated from indices for US Treasury bills, US Treasury bonds, US Corporate bonds and the S&P500 Composite. All series include total returns. There are a total of 372 observations in the sample period from 01/1992 until 12/2022. See appendix for details on the indices.

Discrete returns are directly calculated from the indices. The *monthly return* for period t is calculated as: $r_t = \frac{Index\ Price_t}{Index\ Price_{t-1}} - 1$, where t refers to the last trading day of the month.

Periods of recession are included as shaded areas in all figures throughout the thesis. This is done to highlight periods where we would expect heightened volatility, and otherwise shifts in a trend or estimate that breaks with the near past. In the sample data, three recessions occur: 04-11/2001, 01/2008-06/2009 and 03-04/2020. Recession data is collected from the *National Bureau of Economic Research* and is US-specific (Federal Reserve Bank of St. Louis, 2023). In the following, these periods will be referred to as the *20xx – recession.* See source data for definition of a recession, as this is outside the scope of this thesis.

In Table 1, descriptive statistics are shown for *excess returns*. The most volatile asset, in terms of standard deviation, is unsurprisingly *equity* with a monthly standard deviation of 0.0428, and excess mean of 0.67 %. In contrast, *bills* observe a standard deviation of 0.0006 and mean of 0.02%. All asset classes, except for *bonds*, observed lowest returns during the recession of 2008 with *equity* -16.9 %, *cbonds* -7.9 % and *bills* -0.2 %. *Bonds* had the lowest return of -9.0 % in July 2004. All assets had its highest return in the recession of 2008, except for *equity* which had its highest return in the recession of 2020.

Variable	N	Mean	SD	Min	Max	Skewness	Kurtosis
bills	372	.00025	.00063	-.0017	.00465	1.842	11.79
bonds	372	.00336	.03037	-.09013	.1227	.1959	4.241
equity	372	.00671	.04277	-.1688	.1282	-.5933	4.097
cbonds	372	.00269	.01668	-.07919	.06796	-.6546	6.778

Table 1 - Descriptive statistics

With kurtosis above 3, all assets show evidence of fat tails, with corporate bonds being the highest of the riskier assets. Unsurprisingly, treasury bills show a large positive skew and high kurtosis, albeit the magnitudes of these are low. The normality test confirms that the data is not normally distributed, which is common for financial return series. However, we proceed under the assumption of normally distributed returns.

Skewness and kurtosis tests for normality

Variable	Obs	Pr(skewness)	Pr(kurtosis)	Joint test Adj chi2(2)	Prob>chi2
bills	372	0.0000	0.0000	131.73	0.0000
bonds	372	0.1192	0.0006	12.53	0.0019
equity	372	0.0000	0.0015	24.34	0.0000
cbonds	372	0.0000	0.0000	48.20	0.0000

Table 2 - Skewness and kurtosis tests for normality

Figure 1 shows the relative weights of the assets in the sample period. If we adjust for the fact that we have included another asset in our investment universe, the market weights of Bollerslev et al. (1988) looks quite similar: From a visual inspection, the market portfolio of Bollerslev et al. consists of equity fluctuating around 70 – 85% and T-bill and T-bonds 5 – 15% each.

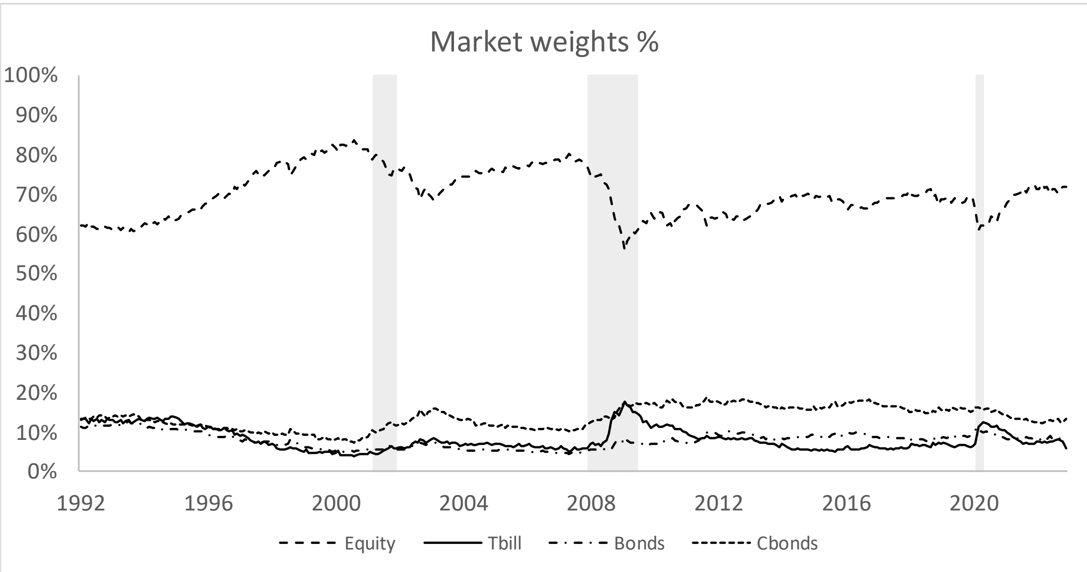

Figure 1 - Market weights

The total market capitalization of our constructed portfolio is 44 578.7 billion USD at the end of the sample period – 12/2022. This is a reduction of 20% compared to the end of 2021, but an increase of nearly 267% from the crisis of 2008. In Table 3 some periods of interests are highlighted to quantify asset values in detail. We have included the start and end sample period of 01/1992 and 12/2022. In between we have included time stamps of periods before and after a big shift in market weights, notably the ones around the year 2000 and 2008.

	01.1992	01.2000	03.2003	12.2006	06.2009	12.2022
Panel A - market values in Billion USD						
T-Bill	584.3	684.8	928.0	923.4	2 019.1	2 588.4
T-Bonds	513.8	779.9	818.7	764.0	970.3	3 920.4
Equity	2 788.2	11 739.7	7 819.4	12 729.1	8 044.9	32 132.9
C-Bonds	605.4	1 173.5	1 799.9	1 719.5	2 305.6	5 937.0
Total	4 491.7	14 377.9	11 366.0	16 136.0	13 339.9	44 578.7
Panel B - market values in %						
T-Bill	13.0 %	4.8 %	8.2 %	5.7 %	15.1 %	5.8 %
T-Bonds	11.4 %	5.4 %	7.2 %	4.7 %	7.3 %	8.8 %
Equity	62.1 %	81.7 %	68.8 %	78.9 %	60.3 %	72.1 %
C-Bonds	13.5 %	8.2 %	15.8 %	10.7 %	17.3 %	13.3 %
Total	100.0 %	100.0 %	100.0 %	100.0 %	100.0 %	100.0 %

Table 3 - Selected market weights and values

Lastly, a visualization of the market value development over the sample period is presented in Figure 2.

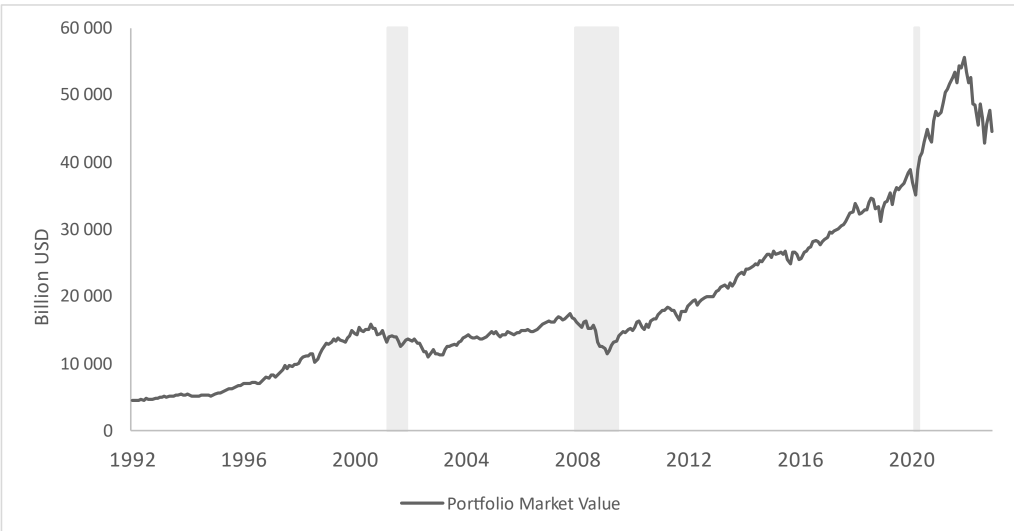

Figure 2 - Market values

5 Findings

In this section we begin with the estimate of the risk aversion coefficient and the estimated market risk premium. Then we will introduce the model estimates and analyze the time-varying covariances with the application in betas and asset risk premia. Next, we will present the estimates of the dynamic correlations of the assets and the market. Lastly, we will present the test results for the hypotheses presented in 2.3.

5.1 Market risk premium

From equation (15) we see that the market risk premium is composed of the conditional market variance, and a constant risk aversion coefficient. We have estimated the risk aversion coefficient to 5.32. This is consistent with the estimates presented in Cuthbertson and Nitzsche (2005) and Bali and Engle (2010). Considering this, our estimate of the risk aversion coefficient seems reasonable. Ideally, we would have liked to include a time-varying coefficient for market risk aversion, but for simplicity, we model it as a static coefficient throughout our sample period.

The estimated market risk premium is shown in Figure 3. It is plotted against the market variance to show how the risk aversion coefficient proportionally affects the premium awarded to an investor. The beginning of our sample period follows a period of high market volatility in the late 1980s, with the market risk premium coming down and stabilizing in the beginning of the 1990s. Similarly, in periods of relative calm, i.e., between 2004 and 2008 it is below 0.5%. During periods of high degrees of market uncertainty, we observe increased levels, with monthly premium > 1%, as evident on multiple occasions in our sample period.

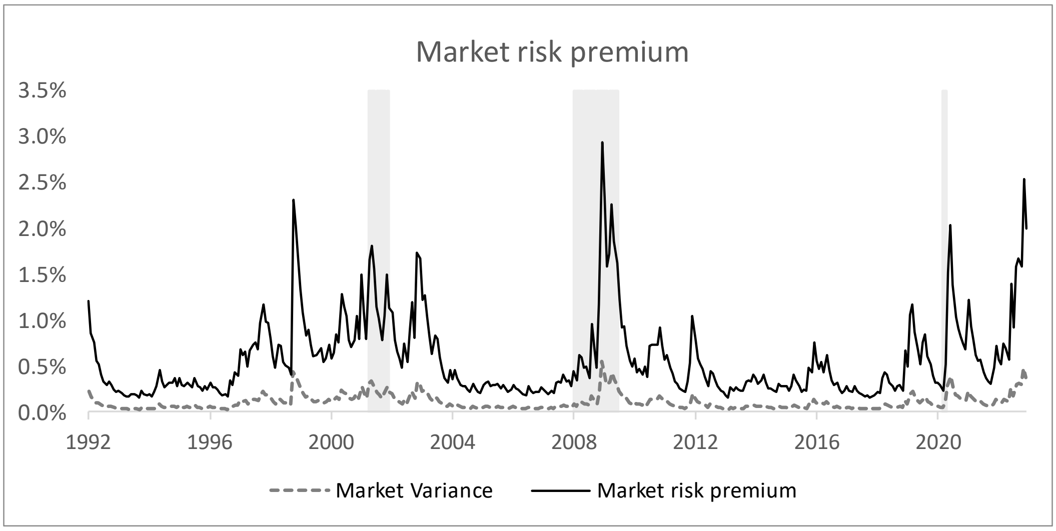

Figure 3 - Market risk premium

5.2 Model estimates

Table 4 presents the model estimates. There are distinct differences in the conditional variance persistence among the assets. The model estimates for *equity* and *bills* shows a high level of persistence with $(\alpha + \beta)$ close to 1. For corporate bonds, however, the estimate is much lower with $(\alpha + \beta)$ estimated at 0.66. Similarly, *treasury bonds* exhibits some of the same characteristics as *corporate bonds*, with a relatively low $(\alpha + \beta)$ at 0.62. We also note that the estimated intercept ω is higher for these assets.

Bolleslev et. al. (1988) estimate a similar variance persistence for bills (0.911) and bonds (0.629). However, for *equity* they estimate a lower persistence (0.547), which is substantially different from the estimate presented here.

Model estimates	**Tbill**	**Bonds**	**Equity**	**Cbonds**
Mean equation:				
$risk\ aversion, \hat{\delta}$ **5.32**				
Variance equation:				
$intercept, \hat{\omega}$	0.0000	0.0005	0.0001	0.0002
$arch, \hat{\alpha}$	0.293	0.241	0.278	0.479
$garch, \hat{\beta}$	0.700	0.380	0.701	0.184
$(\hat{\alpha} + \hat{\beta})$	0.993	0.621	0.979	0.663
$unconditional\ variance, V_L\ (see\ eq. (19))$	0.0000	0.0014	0.0051	0.0006
$unconditional\ volatility, \sqrt{V_L}\ (see\ eq. (19))$	0.0018	0.0368	0.0713	0.0249

DCC parameters:	
Starting correlation in R	
$\hat{\rho}_{tbill,\,tbonds}$	0.146
$\hat{\rho}_{tbill,equity}$	0.113
$\hat{\rho}_{tbonds,equity}$	0.292
$\hat{\rho}_{tbill,cbonds}$	0.132
$\hat{\rho}_{tbonds,cbonds}$	0.971
$\hat{\rho}_{equity,cbonds}$	0.383
Dynamic correlations	
$\hat{\lambda}_1$	0.101
$\hat{\lambda}_2$	0.882
$\log \boldsymbol{likelihood}$	**5002.20**

Table 4 - Baseline model estimates

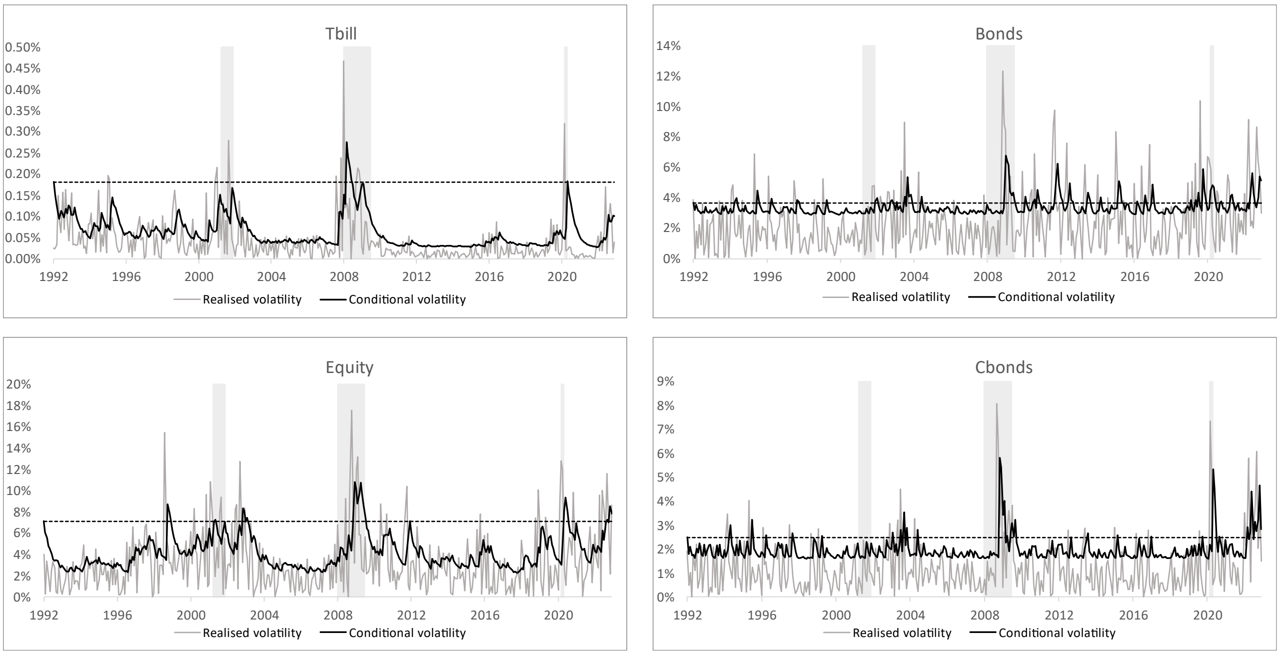

Figure 4 - Volatility estimates

Figure 4 shows the estimated conditional volatility plotted against the realised volatility for all four assets. The conditional volatility is specified using the diagonal conditional standard deviation from (21) with model estimates from Table 4. The realised volatility is simply expressed as the absolute value of the residuals. The subplots in Figure 4 also include the long run volatility, expressed as the root V_L in (19). From a visual inspection, the estimated conditional volatility seem to be capturing the fluctuations in realised volatility quite well. The GARCH β in *Tbill* seems to be a bit high, as the decay rate of the estimate is a bit slow, perhaps most evident during the 2020-recession. In *Equity* it seems that the estimate is a bit slow in reacting to shocks, always lagging behind. Here we also observe how model starting values might affect the estimate, evident by the spread in the beginning of the sample period. As mentioned earlier in the discussion about the application of statistical software, it is worth noting that the GARCH β estimate for treasury bonds is statistically insignificant.

5.3 Dynamic betas

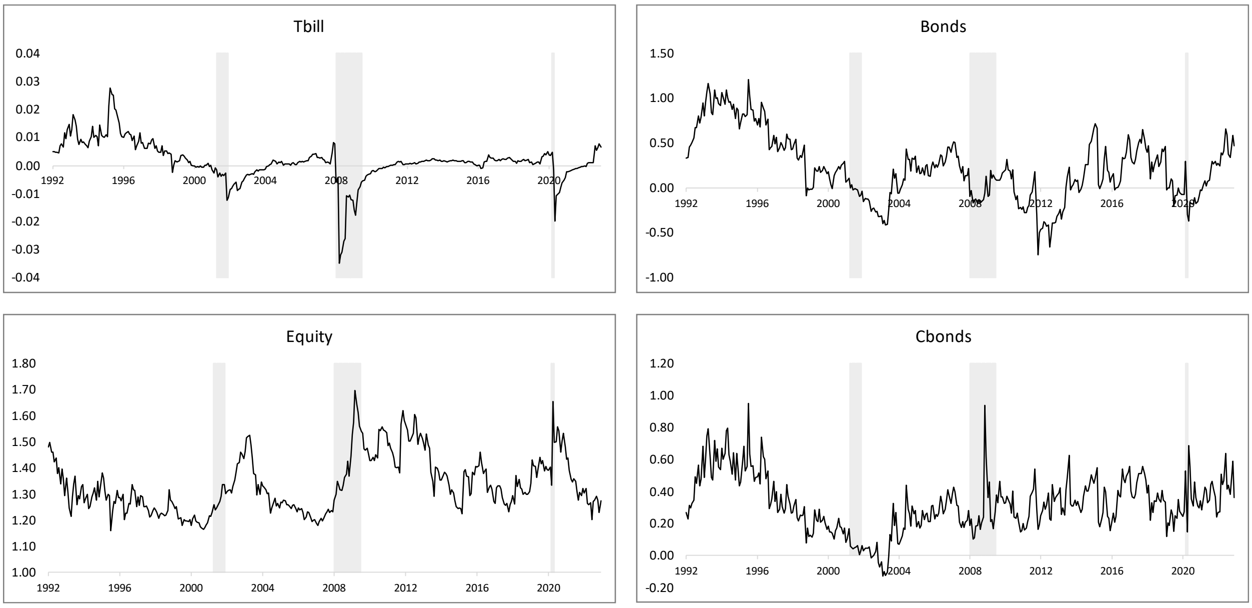

Figure 5 - Beta estimates

The estimates in Figure 5 follow from equation (4), considering that the covariance matrix is time-varying. The estimated beta for treasury bills shows it relatively stable around zero throughout the sample period. It does, however, cross below zero during recession periods. This is consistent with treasury bills being viewed as a safer asset to hold during crises, as Cheema et. al. (2022) find when analyzing safe haven assets. Furthermore, as the return series includes treasury bills with maturities below one year, this will include, and have similarities to, the risk-free rate (which is proxied by the 3-month bill). It is then not surprising that the estimated risk exposure to the market is close to zero, with the implication that the investor is not awarded a premium in holding this asset class. This is consistent with the estimation of Bollerslev et al. (1988), who's beta ranges between zero and 0.05 in the calmer periods. In the recession periods we observe a slight negative beta, which is driven by a negative change in covariance between the asset and the market, and obviously an increase in the market variance.

The beta estimate for treasury bonds shows substantial movement in our sample period. It increases sharply in the beginning, before it steadily declines until it reaches -0.4 in May of 2003. It further increases in the calm period between the two first recessions, before decreasing during the start of the recession of 2008. Similar to treasury bills, the estimated bonds beta turns negative in recession periods, suggesting that it is considered a safer asset to hold in these periods, evident by the negative co-movement of returns with the market. With increased market risk, investors typically seek defensive assets that are less exposed to the market, here expressed by a lower beta. This is exemplified in the second half of 2011 when insecurity caused by the *debt ceiling crisis* affect the beta estimate for treasury bonds – decreasing from 0.18 to negative 0.74 in a couple of months. This was driven by a negative covariance with the market, which was increasingly volatile, resulting in a sudden drop in estimated beta. Moreover, in the event of an actual default on its debt by the United States, the impact would extend beyond treasury bonds and bills, and result in an increase in risk premia. If the risk-free rate used for discounting is no longer risk-free, it would significantly influence the pricing of all assets.

The beta estimates for corporate bonds exhibit a similar pattern to that of treasury bonds up until the 2008 recession. From here a shift happens, and the two asset classes *break free* from each other. Firstly, we observe how the corporate bonds beta spikes towards the end of 2008, with a peak of 0.94, before returning to 0.36 in Jan 2009. Meanwhile the treasury bonds beta is negative in both time periods. Secondly, we note how treasury bonds fluctuates a lot more than corporate bonds in the period after the 2008 recession – with periods of negative betas both in 2013-2014 and during the 2020-recession. In comparison, corporate bonds ranges between positive 0.2 and 0.6 with a high of 0.69 in May 2020. This indicates that corporate bonds are more exposed to the systematic risk in the market.
The 2020-high is lower than the 2008-high. This finding is consistent with Cheema et al. (2022), who observed that corporate bonds played a relatively weak role as a safe haven asset during the 2008 recession. However, during the 2020 recession, corporate bonds exhibited stronger safe haven characteristics.

The estimated beta of equity is well above 1 for the entire sample period, and ranges between 1.2 and 1.7. Due to equity's large portion in the overall market portfolio, it contributes strongly to the estimated market variance in (13). This shows that the risk contribution of equity has a larger impact on the overall market volatility than the other three assets**.** Furthermore, its inherent risky nature compared to the safer assets in the market portfolio *inflates* the beta, in the sense that we would expect it to be closer to one. However, adjusting for the fact that we have included another safe asset in the market portfolio, and the different economic climates for interest rates and bonds, the estimated betas might not be too different from their findings of Bollerslev et al. (1988)

In conclusion, after reaching its highest value of 1.69 in Mar 2009, the equity beta continues to stay high (over 1.5 most of the time up until 2013), indicating that the equity market is the leading risk contributor, with the implication that investors demand a higher premium to hold equity. Compared to the other assets we find that treasury bills and bonds move in the opposite direction, whereas corporate bonds stay relatively stable during this period.

5.4 Asset risk premia

The estimated risk premia are shown in Figure 6. Note that it shows monthly returns. The specification of the expected excess return follows from CAPM in (9), where the individual excess return is a product of the covariance with the market and the estimated coefficient for risk aversion. Time variation in asset risk premia can be due to changes in an asset's perceived risk profile, i.e., to what extent it is exposed to the systematic risk, through the covariance of returns. The following discussion will show similarities to section 5.3, since return covariances with the market is the main input in both.

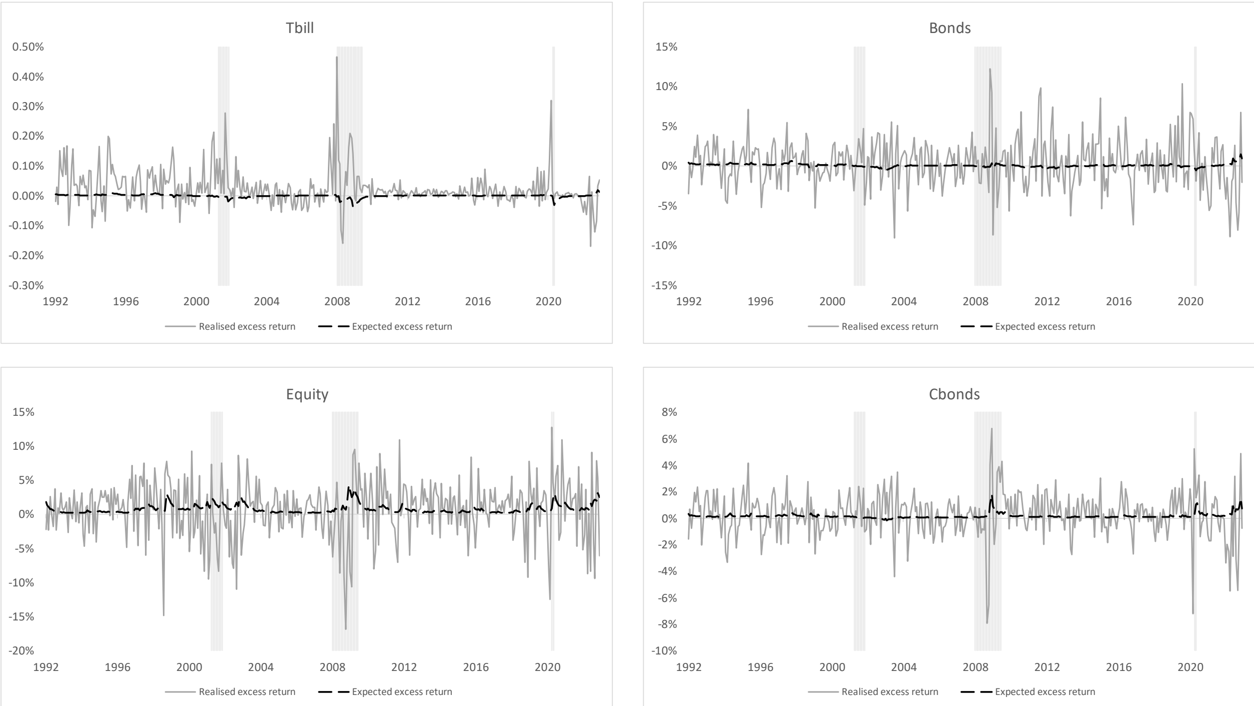

Figure 6 - Asset risk premia

The risk premium for treasury bills is stable around zero for the entire sample period, however turning briefly negative in the recession – periods. This is consistent with the findings of Bollerslev et al. (1988), adjusting for frequency of observations (they report quarterly risk premia) and specification with intercepts in the expected return. They find it to be quite stable throughout their sample period, except for an increase in risk premium in the highly volatile, high interest regime in the beginning of the 1980s - where it seems likely that an investor would be awarded for holding the asset class. In contrast, the risk-free rate has not exceeded 5% per annum in any of the recession periods in our sample.

The risk premium of treasury bonds fluctuates for the most part in the range 0 – 1%, however becoming slightly negative in the recession periods, implying little return covariance with the market outside these periods. For equity the estimated risk premium fluctuates quite a lot during the sample period, with highs up to 4.25% in end of 2008, and the lowest of 0.2% in October 2017. The levels closely follow the movement in the market variance, as per previous discussion. Note that the expected excess return of equity is non-negative. Equity is considered the riskier asset in the portfolio, and for investors to justify taking on additional risk, the premium associated with it needs to be positive. Otherwise, the investor would rather hold a less risky asset. Adjusting for the intercept in the findings of Bollerslev et. al (1988), the risk premium of equity is similarly non-negative. The risk premium for corporate bonds is quite low and stable up until 2008. During the 2008 and the 2020-recession, however, it increases substantially - (1.81%) and (1.05%), respectively.

Estimated risk premia towards the end of the sample period show an increase for all assets. From Figure 5 it is evident that individual covariances with the market increases more than the market variance. This is true for all assets, except equity, where we observe a reduction in the beta, as the relative strength of its covariance with the market is smaller than the overall market variance.

5.5 Dynamic correlations

From the components of the DCC-model we can retrieve time-varying correlations coefficients, $\frac{cov_{t-1}(r_{i,M,t})}{\sigma_{i,t}\sqrt{\sigma_{M,t}^2}}$, where the conditional standard deviation, $\sigma_{i,t}$, is obtained from the $\boldsymbol{D_t}$ matrix in (21). Figure 7 shows the correlations of the four assets with the market. We observe how the correlation between equity and the market is close to 1 for the entire sample period. As previously discussed, this is due to equity's large share in the constructed market portfolio, which results in a strong co-movement between equity and the market. In the following, we will not devote any further attention to this asset's correlation with the market.

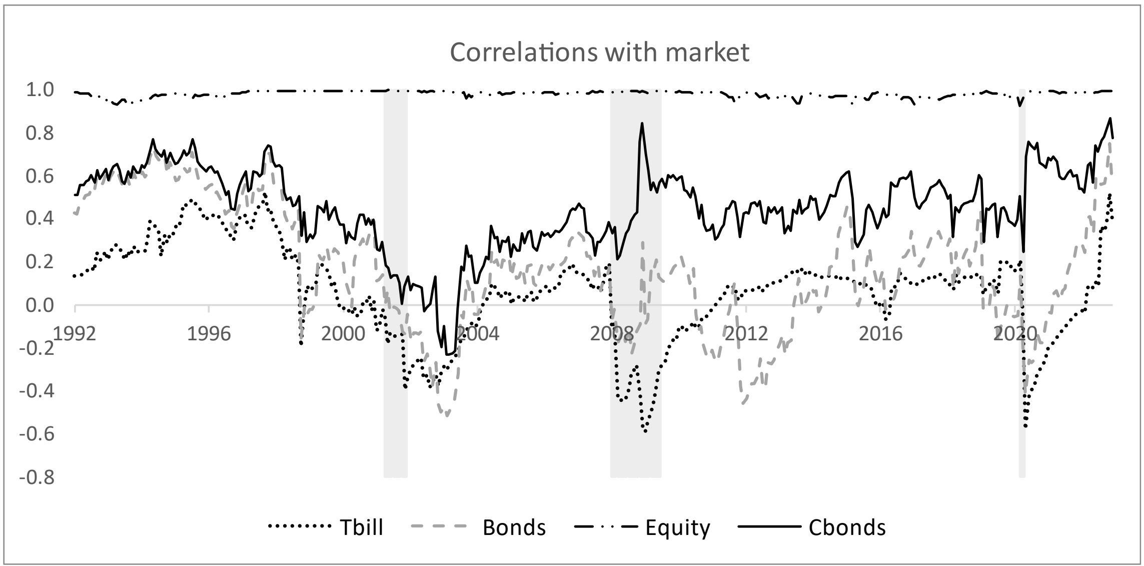

Figure 7 - Comparison time-varying correlations

From the beginning of the sample period until the recession of 2008 the assets are similarly correlated with the market, but in 2008 they deviate. The correlation coefficient of corporate bonds increases and stabilizes in the range of 0.40 and 0.60 until the recession of 2020 where we observe an even higher correlation with the market. This could be due to a rapid growth in the corporate bond market, having tripled in market value from 2 to 6 trillion USD between the financial crisis of 2008 and the pandemic – and therefore making up a larger share of the constructed market portfolio, as is evident from Figure 1. The correlation of treasury bonds and the market ranges between -0.4 and 0.4 from the beginning of the 2008- recession and onwards. We do, however, observe a large negative shift in correlations at the end of 2011, as highlighted in section 5.3. Treasury bills is weakly correlated with the market around zero, with some negative shifts in the recession periods. This is consistent with Doeswijk et al. (2019) who also finds that the return correlations between government securities and the market decreases in “down markets”.

In our data we observe time variation in the correlations between the assets and the market. Albeit somewhat stable in calmer periods, when exposed to volatility shocks, the correlation with the market changes drastically. Most notably perhaps in 2008 and 2020.
In the period after the 2020 – recession we observe a pattern with similarities to the mid-1990s. Towards the end of our sample period, a *correlation clustering* seems to be emerging among the assets. Negatively correlated after the shock of the pandemic, Bonds and Bills increases to levels not seen since the calmer period of the mid-90s. The correlation of corporate bonds is positively affected by the pandemic but remains high and increasing. Ultimately, these assets are kept within a small range with a relatively *high* correlation with the market. Changes in interest rates might help explain the similar pattern we see for the two periods: The clustering of the 90s occurs after an increase in interest rates. The same is the case for the period after the 2020 recession when interest rates are increased quickly.

Another period of interest is seen in the second half of 1998 when a sudden drop in the stock market occurs. This increase in volatility is evident from the *equity* sub plot in Figure 4, along with the model's ability of seemingly capturing the dynamics of the volatility. This drop in equity drastically affects the estimated correlations between treasury bonds and *equity* with a negative change of approx. 52 percentage points from Sept to Oct. Similar for Bills, the correlations with the equity decreases by approx. 37 percentage points. Simultaneously, we observe how the estimated correlation between Treasury Bonds and Bill changes, with an increase of approx. 14 percentage points. The dynamics of the correlation between corporate and treasury bonds is also of some interest. Before this event, the correlation between these assets had not been below 0.95. After the shock to *equity*, we observe how the correlation decreases by 14 percentage points and does not reach historical levels until five years later. While the market for corporate bonds was not affected by the shock, it seems that investors might be more willing to accept a smaller risk premium in choosing a safer asset – treasury bonds or bills. This is also evident from the negative asset risk premium estimated in this period.

5.6 Tests of the CAPM

To test the validity of our model we perform likelihood-ratio tests for different augmentations of the model. The likelihood-ratio test evaluates the goodness-of-fit between two *competing* models. It estimates the statistical significance of an unrestricted model against a model where there is a restriction to one or more of the parameters. If the restriction holds, i.e., is supported by the data, there would not be much change in the residuals and the likelihood-ratio would be small (Cuthbertson & Nitzsche, 2005, p. 508). In this case we would not be able to reject the null hypotheses, implying that the restricted model is already a good fit for the data. We can therefore indirectly test the asset pricing model by including terms and assess the test results against theory. All tests were done by maximum likelihood estimations, and then comparing log likelihood values in a likelihood ratio test, where L_2 is the unrestricted model and L_1 is the restricted:

$$LR = 2(log\ L_2 - log\ L_1) \sim \chi^2(k) \qquad (28)$$

The likelihood ratio test stat is asymptotically chi-square distributed with k degrees of freedom, which is equal to the difference in the number of parameters between the restricted and unrestricted model (Wilks, 1938).

Below, in Table 5, different model specifications are estimated to test the CAPM.
Model 1 is the baseline model introduced previously. We then extend the model with intercepts and different properties of the market risk premium and betas.
Mean equations are specified in the relevant sub sections of the tests.

	Model estimates	Model 1	Model 2	Model 3	Model 4	Model 5
Mean equation	$constant, \hat{\mu}$					
	T-Bill	N/A	0.0002	0.0002	0.0001	0.0001
	T-Bonds	N/A	0.0010	0.0014	0.0004	-0.0001
	Equity	N/A	0.0038	0.0063	0.0041	0.0042
	C-Bonds	N/A	0.0004	0.0007	-0.0013	-0.0016
	$constant, \hat{\beta}$					
	T-Bill	N/A	N/A	N/A	0.02	0.01
	T-Bonds	N/A	N/A	N/A	0.45	0.06
	Equity	N/A	N/A	N/A	1.27	-0.59
	C-Bonds	N/A	N/A	N/A	0.93	0.25
	$risk\ aversion, \hat{\delta}$	5.32	3.30	N/A	3.20	7.27
Variance equation	$intercept\ \hat{\omega}$					
	T-Bill	2.23E-08	1.74E-08	1.63E-08	1.77E-08	2.01E-08
	T-Bonds	5.14E-04	5.03E-04	5.03E-04	5.25E-04	4.77E-04
	Equity	1.06E-04	1.08E-04	9.74E-05	9.18E-05	9.72E-05
	C-Bonds	2.09E-04	1.90E-04	1.84E-04	1.67E-04	1.67E-04
	$arch, \hat{\alpha}$					
	T-Bill	0.293	0.265	0.260	0.258	0.269
	T-Bonds	0.241	0.232	0.212	0.234	0.257
	Equity	0.278	0.304	0.289	0.280	0.285
	C-Bonds	0.479	0.417	0.358	0.414	0.476
	$garch, \hat{\beta}$					
	T-Bill	0.700	0.725	0.732	0.731	0.718
	T-Bonds	0.380	0.391	0.399	0.374	0.403
	Equity	0.701	0.680	0.698	0.710	0.707
	C-Bonds	0.184	0.243	0.278	0.331	0.302
	$unconditional\ variance, V_L\ (see\ eq. (19))$					
	T-Bill	0.0000	0.0000	0.0000	0.0000	0.0000
	T-Bonds	0.0014	0.0013	0.0013	0.0013	0.0014
	Equity	0.0051	0.0072	0.0073	0.0099	0.0122
	C-Bonds	0.0006	0.0006	0.0005	0.0007	0.0008
DCC paramaters	$\boldsymbol{Starting\ correlation\ in\ R}$					
	$\hat{\rho}_{tbill,\ tbonds}$	0.146	0.024	0.029	-0.128	-0.089
	$\hat{\rho}_{tbill,equity}$	0.113	-0.024	-0.023	0.001	0.009
	$\hat{\rho}_{tbonds,equity}$	0.292	0.203	0.159	0.347	0.390
	$\hat{\rho}_{tbill,cbonds}$	0.132	0.022	0.021	-0.072	-0.035
	$\hat{\rho}_{tbonds,cbonds}$	0.971	0.967	0.965	0.984	0.986
	$\hat{\rho}_{equity,cbonds}$	0.383	0.331	0.298	0.413	0.452
	$\boldsymbol{Dynamic\ correlations}$					
	$\hat{\lambda}_1$	0.101	0.108	0.108	0.107	0.109
	$\hat{\lambda}_2$	0.882	0.875	0.875	0.881	0.879
	$\log likelihhood$	5002.20	5033.62	5031.88	5039.48	5044.13

Table 5 - DCC-GARCH model estimates

5.6.1 T.1 - intercept α

The expected excess return, as formulated by the CAPM, does not include other measures of risks, other than that of the risk on the portfolio. The investor is not rewarded for idiosyncratic risk, i.e. asset-specific intercept terms should not exist. One of the earliest empirical tests of the validity of the sole risk measure in the CAPM is done by Fama and MacBeth (1973). They test the hypothesis that there are *no systematic effects of non-β risk*. They find that they are not able to reject this, implying that there is no other measure of risk than the portfolio risk affecting average returns. It seems natural then to perform a test in the same spirit as above. Our baseline model with mean equation for the expected excess return without the intercept α_i is tested against an unrestricted model where intercepts are included.

Restricted model: $E_{t-1}[r_{i,t}] - r_{f,\,t} = \beta_{i,\,t}(E_{t-1}[r_{M,t}] - r_{f,\,t}) + \epsilon_{i,t}$

Unrestricted model: $E_{t-1}[r_{i,t}] - r_{f,\,t} = \alpha_i + \beta_{i,\,t}(E_{t-1}[r_{M,t}] - r_{f,\,t}) + \epsilon_{i,t}$

We are testing if the included $\alpha_i = 0, \quad i = 1,2,3,4$

5 033.62	$log\ L_2\ of\ Model\ 2\ - unrestricted\ model$
5 002.20	$log\ L_1\ of\ Model\ 1\ - restricted\ model$
62.85	$2(log\ L_2 - log\ L_1) \sim \chi^2(4)$
0.000	$P - value$

The test result shows a significant difference in the unrestricted model, which includes an intercept in the expected return. Therefore, we reject the implication of the CAPM that the intercept term α for all i is zero. This rejection of the CAPM, by the fact that the intercepts are significant, could indicate that our modelling of the market portfolio is not complete, or lend support to questioning the empirical relevance of the theoretical model. As it stands, the *restricted model* is not able to explain the average excess return only through the specification of beta times the risk premium component. However, this result is consistent with Bollerslev et al. (1988), who found significant negative intercepts. They argued that these negative intercepts were a consequence of differences in capital gains taxes on long-term assets, which would incentivize investors to hold onto the assets even during periods of negative returns. In *Model 2 – unrestricted model,* the estimated intercepts are low and positive (see parameter $\hat{\mu}$ in Table 5). It is worth noting that in the first half of our sample period, there was a continuous reduction in capital gains taxes on long-term assets, but in 2013, taxes increased. Consequently, this increase would *reduce* the incentives for investors to hold onto the assets. In the context of the CAPM, one of the key assumptions of the model is that effective markets have no differences in tax treatment among assets. Ultimately, without standard errors of the intercepts, it becomes challenging to express confidence in these estimates.

5.6.2 T.2 - Excluding time-varying risk premia

In the second test we dig deeper into the time-varying risk premium component of our model. Does it at all contribute? In section 3.1 we describe how the return series, when run in a statistical software, show significant ARCH and GARCH – coefficients, at least for three of the assets. This suggests that there are multivariate GARCH structures present in the time series. But do these GARCH structures affect the mean itself?

We are testing a nested model with only an intercept against an unrestricted model containing the time-varying risk premium term.

Restricted model: $E_{t-1}\left[r_{i,t}\right] - r_{f,\,t} = \alpha_i \ + \epsilon_{i,t}$

Unrestricted model: $E_{t-1}\left[r_{i,t}\right] - r_{f,\,t} = \alpha_i + \beta_{i,\,t}\left(E_{t-1}\left[r_{M,t}\right] - r_{f,\,t}\right) + \epsilon_{i,t}$

We are testing if the risk premium term $\beta_{i,\,t}\left(E_{t-1}\left[r_{M,t}\right] - r_{f,\,t}\right) = 0,\ \text{for}\ \ i = 1, 2, 3, 4$, which is equivalent of testing $\delta = 0$ in (12).

5 033.62	$log\ L_2\ of\ Model\ 2\ - unrestricted\ model$
5 031.88	$log\ L_1\ of\ Model\ 3 - restricted\ model$
3.48	$2(log\ L_2 - log\ L_1) \sim \chi^2(1)$
0.062	$P - value$

The unrestricted model is an improvement of the restricted and is statistically significant at the 10% level. We can therefore reject the null hypothesis of the time-varying risk premium term being equal to zero. This indicates that the time-varying risk premium component should be included in the modelling of the excess returns. I.e., the multivariate GARCH structures that exist in the data affect the mean. This result give support to the theory of the conditional CAPM, and is consistent with the findings of Bollerslev et al. (1988) and Fama and French (1997, p. 175).

5.6.3 T.3 - Static β

Next, we would like to continue testing the nature of the risk premium term. Is perhaps the best fit for our data the use of a constant beta? To test this, we specify the nested model with a constant beta, and from there expand the model with an included time-varying beta term. The intuition behind this being that the unrestricted model, if found significant in the likelihood ratio test, will reject the notion that time-varying beta term is zero. Simply put, a rejection of the null hypothesis would give support to the presence of time-varying structures in the data.

Restricted model:

$$E_{t-1}[r_{i,t}] - r_{f,\,t} = \alpha_i + \beta_i(E_{t-1}[r_{M,t}] - r_{f,\,t}) + \epsilon_{i,t}$$

Unrestricted model:

$$E_{t-1}[r_{i,t}] - r_{f,\,t} = \alpha_i + \beta_i(E_{t-1}[r_{M,t}] - r_{f,\,t}) + \beta_{i,\,t}(E_{t-1}[r_{M,t}] - r_{f,\,t}) + \epsilon_{i,t}$$

We are testing if the risk premium term $\beta_{i,\,t}(E_{t-1}[r_{M,t}] - r_{f,\,t}) = 0, \;\; i = 1, 2, 3, 4$, which is equivalent of testing $\delta = 0$ in (12).

5 044.13	$log\,L_2\ of\ Model\ 5\ - unrestricted\ model$
5 039.48	$log\,L_1\ of\ Model\ 4 - restricted\ model$
9.30	$2(log\,L_2 - log\,L_1) \sim \chi^2(1)$
0.002	$P - value$

The likelihood-ratio test stat is presented above. The unrestricted model improves the nested model significantly, and we can further conclude that the time-varying beta term improves the model. This indicates that there is time-varying risk premium component in our modelling of the excess returns. It’s interesting to note that the parameter values for the static betas change drastically. Most notably, in the coefficient for equity, where it changes from 1.31 to a negative beta of 0.59. In lieu of our test results above, we would expect the static beta coefficients to be insignificant had we been able to retrieve the standard errors. Without these, we must settle with the joint result of the overall model improving with the inclusion of a time-varying risk component.

5.6.4 T.4 - GARCH-in-mean

The GARCH-in-mean specification should only hold for the market portfolio, and not for the individual asset classes. To test (6), we have implemented univariate GARCH-in-mean regressions as specified in (26). These are done in statistical software, and we're able to retrieve the standard errors for the estimates. The results are shown in Table 6. *Market* is the estimated return series for the market risk premium.

The significant GARCH-in-mean δ coefficient on the mean is as expected since the market risk premium is a function of its own variance. We note that the coefficient is very large compared to the values described in section 2.1. Furthermore, even though we observe highly significant α and β coefficients, the sum $(\alpha + \beta) > 1$. In this case, the long run variance V_L is not defined and the model does not work for the intended purpose. We tried different model specifications (various lags of α and β, no intercept in mean), but did not see an improvement. GARCH-in-mean δ coefficient remained high in all specifications.

The included intercept in the mean is statistically significant. From theory the intercept should not have a significant effect on the mean. But perhaps due to a sample size not large enough to even out the average this is the case. We observe an average of 0.58% in the estimated return series for the market risk premium.

As an additional test, we would like to test the CAPM in relations to the GARCH-in-mean coefficient on the assets. We use the same specification in (26) to test (9).
The results show that intercepts and GARCH-M coefficients for Equity and Corporate Bonds are statistically insignificant, lending support to theory presented about CAPM: There should neither be an effect from an intercept nor its own variance in the expected excess return.

The test results for Bonds differ from the others. We note that the GARCH β is slightly negative and highly insignificant. For the GARCH model to work all parameters should be non-negative. The negative coefficients violate this constraint, and it makes little sense interpreting the significant GARCH-M coefficient. *Tbill* is not included as convergence was not achieved for the optimization - most likely because of the small variations in the excess returns.

	Model estimates	**Market**	**Bonds**	**Equity**	**Cbonds**
Mean equation	$intercept, \hat{\mu}$	0.0026	0.0126	0.0055	-0.0001
		(0.000)***	(0.004)***	(0.003)	(0.002)
	$GARCH-M, \hat{\delta}$	104.233	-9.679	1.785	10.982
		(12.958)***	(4.743)**	(2.245)	(8.277)
Variance equation	$intercept, \hat{\omega}$	1.47E-08	6.61E-04	7.63E-05	5.26E-05
		(1.48E-08)	(1.47E-04)***	(4.28E-05)*	(2.88E-05)*
	$arch, \hat{\alpha}$	0.462	0.282	0.204	0.132
		(0.048)***	(0.077)***	(0.050)***	(0.052)**
	$garch, \hat{\beta}$	0.755	-0.012	0.774	0.668
		(0.019)***	(0.153)	(0.049)***	(0.158)***
	$unconditional\ variance, V_L\ (see\ eq.(19))$	*not defined*	0.001	0.003	0.000

Table 6 - Univariate GARCH-in-mean estimates

*** Significance level of 1%
** Significance level of 5%
* Significance level of 10%

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6 Conclusion

The estimated market risk aversion coefficient is 5.32, which is in line with previous studies. The DCC-model estimates reveal differences in variance persistence among assets, with corporate and treasury bonds exhibiting lower persistence compared to equity and bills.

The analysis of the time-variation in our data reveals several key findings. Firstly, the dynamic betas indicate that treasury bills and bonds maintain stability and serve as safer assets during recessions, while corporate bonds show increased exposure to systematic risk during recession periods. Equity consistently exhibits a beta above 1, reflecting its large relative share in the constructed market portfolio. Secondly, estimated asset risk premia show stable risk premiums for treasury bills, fluctuating premiums for treasury bonds, and a low and stable premium for corporate bonds, which increases substantially during recessions. Overall, correlations between equity and the market remain consistently close to one, corporate bonds exhibit an increasing correlation with the market, and treasury bonds and bills show varying correlations with occasional negative shifts during recessions.

To examine our research question, *"Does time-varying covariance structures improve the CAPM?",* we have formulated hypotheses that we tested using likelihood ratio tests or statistical software. The test results are summarized here:

- In **T.2** we investigate the contribution of time-varying risk premia in the model. The results indicate that the risk premium term is significantly different from zero, supporting the inclusion of the time-varying risk premium component in the modeling of excess returns.
- In **T.3** we test the use of static versus time-varying risk premium. The results reveal that the model with time-varying risk premium term significantly improves the model compared to the specification with a static risk measure. This further supports the presence of time-varying risk premium component in the modeling of excess returns.
- In **T.4** we perform a split test of GARCH-in-mean. First on the market risk premium and then on the CAPM. In the test on the market risk premium, we find a significant GARCH-in-mean coefficient, supporting the theoretical expectation. However, the presence of a significant constant term and ($\alpha + \beta$) >1 suggest potential instability in

the variance specification. Then, in the test on the CAPM the results show that when constants and its own variance is added to the mean it is statistically insignificant, which we expect from the CAPM.

- In **T.1** we examine the inclusion of intercepts in the mean. The results show that the model specification with intercepts is significantly better, and thus rejecting the CAPM. This suggests the presence of unaccounted risk measures in the model, and questions the completeness of the market proxy.

Further research should consider expanding the market portfolio by including a larger number of assets, and intangible assets like human capital. We would also suggest that further research allow for a time-varying measure of market risk aversion, this would extend the dynamics of the model.

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Appendix

Data

Bloomberg Short Treasury Total Return Index LT12TRUU and Bloomberg index US Long Treasury Total Return LUTLTRUU are used to calculate the returns series of bills and bonds respectively. To calculate the returns for corporate bonds we have used the index Bloomberg US Corporate Total Return Value LUACTRUU. The equity returns are calculated from the S&P500COMP index, retrieved from Refinitiv Datastream.

The US government publishes a monthly statement with aggregated market value of all outstanding debt. We got in contact with Tyler Atkinson in the Dallas Fed who kindly provided us with broken down market value data on bills, notes, and bonds. Data set on market value of stocks are retrieved for the S&P500COMP index in Refinitiv Datastream, and the market value of the corporate bonds index is retrieved from Bloomberg.

As a proxy for the risk-free rate of return we have used the 1-month US T-bill. The return series is collected from Kenneth French's website. It seems to be the industry standard – we have noted that the NBIM also use this as a proxy for the risk-free rate. See GIPS report 2021.

Model comparison

As a contribution to the reliability of our MS Excel calculations we include a comparison between the model estimated in MS Excel with a model estimated using statistical software (STATA). The model output from STATA is specified as a DCC-GARCH(1,1).

The difference between the models is the *GARCH-in-mean* specification used in Excel. It allows for testing of the conditional CAPM in (9) by estimating a risk aversion coefficient which is multiplied with $cov_{t-1}(r_{i,M,t})$. This term is specified in the mean equation and the residuals are adjusted for this expected return. This specification of the mean is not available in STATA, only for the univariate model (which is applied in T.4).

Although the specifications are not the same, we note that the log likelihood estimates are similar (5002.20 vs 5003.13). We have tested different starting values, including the STATA-output as starting values for our model to check if the excel optimization is "stuck" at a local maximum. Considering how close the log likelihoods are, it seems like the Excel-implementation is not too far off from the estimation that happens in STATA, even considering the difference between the two model specifications. We know however, that STATA treats the starting values differently than we do in the Excel implementation. In t=1 we set $\boldsymbol{Q}_t = \boldsymbol{R}$, and let t=2 be the first period that estimates $\boldsymbol{Q}_t$ according to (23). Similarly, In t=1 we set $\boldsymbol{D}_t = \sqrt{\boldsymbol{V_L}}$, and let t=2 be the first period that estimates $\boldsymbol{D}_t$ according to (21). For the immediate disturbance term, $\boldsymbol{\epsilon_{t-1}\epsilon'_{t-1}}$, in t=1 this is set equal to zero. STATA, in comparison, uses various econometric techniques in estimating more accurate starting values. We refer to the STATA manual listed in references for details.

Most of the coefficients in the STATA output are highly significant. A few similarities of the coefficients can be observed, where high correlation between treasury and corporate bonds is present in both outputs. The DCC parameters are similarly small and close to 0.1 for both λ_1 (0.101 and 0.091), and high for λ_2 (0.882 and 0.805). However, there are many differences of the coefficients. The Excel output for α and β coefficients of corporate bonds are 0.479 and 0.184, while for STATA they are 0.213 and 0.443. These values provide contradicting explanations of the persistence characteristics for corporate bonds. Another notable difference is the starting correlation between treasury bonds and equity, where the

Excel model estimates it to be 0.292, and STATA to negative 0.072. Considering that all except one of α and β coefficients are highly significant, the STATA output indicate that the data is well suited for volatility persistence estimation.

STATA does not allow constraints other than equalities / inequalities, which we would need to constrain $(\alpha_{tbill} + \beta_{tbill}) < 1$, as the sum of these coefficients exceed 1. In the Excel implementation we have not applied any constraints to the parameters.

Model estimates - Excel	Tbill	Bonds	Equity	Cbonds
Mean equation:				
$risk\ aversion, \hat{\delta}$ **5.32**				
Variance equation:				
$intercept, \hat{\omega}$	0.0000	0.0005	0.0001	0.0002
$arch, \hat{\alpha}$	0.293	0.241	0.278	0.479
$garch, \hat{\beta}$	0.700	0.380	0.701	0.184
$(\hat{\alpha}+\hat{\beta})$	0.993	0.621	0.979	0.663
$unconditional\ variance, V_L\ (see\ eq.(19))$	0.0000	0.0014	0.0051	0.0006
$unconditional\ volatility, \sqrt{V_L}\ (see\ eq.(19))$	0.0018	0.0368	0.0713	0.0249

DCC parameters:	
Starting correlation in R	
$\hat{\rho}_{tbill,\ tbonds}$	0.146
$\hat{\rho}_{tbill,equity}$	0.113
$\hat{\rho}_{tbonds,equity}$	0.292
$\hat{\rho}_{tbill,cbonds}$	0.132
$\hat{\rho}_{tbonds,cbonds}$	0.971
$\hat{\rho}_{equity,cbonds}$	0.383
Dynamic correlations	
$\hat{\lambda}_1$	0.101
$\hat{\lambda}_2$	0.882
$\log likelihood$	**5002.20**

Model estimates - STATA	Tbill	Bonds	Equity	Cbonds
Variance equation:				
$intercept, \hat{\omega}$	0.0000	0.0005	0.0001	0.0001
	(0.0000)***	(0.0001)***	(0.0000) **	(0.0000)***
$arch, \hat{\alpha}$	0.450	0.328	0.152	0.213
	(0.091)***	(0.082)***	(0.035)***	(0.068)***
$garch, \hat{\beta}$	0.641	0.166	0.807	0.443
	(0.048)***	(0.159)	(0.044)***	(0.144)***
$(\hat{\alpha}+\hat{\beta})$	1.091	0.494	0.959	0.656
$unconditional\ variance, V_L\ (see\ eq.(19))$	*not defined*	0.0010	0.0020	0.0003
$unconditional\ volatility, \sqrt{V_L}\ (see\ eq.(19))$	*not defined*	0.0322	0.0451	0.0176

DCC parameters:	
Starting correlation in R	
$\hat{\rho}_{tbill,\ tbonds}$	0.394
	(0.072)***
$\hat{\rho}_{tbill,equity}$	0.061
	(0.089)
$\hat{\rho}_{tbonds,equity}$	-0.072
	(0.086)
$\hat{\rho}_{tbill,cbonds}$	0.303
	(0.078)***
$\hat{\rho}_{tbonds,cbonds}$	0.791
	(0.045)***
$\hat{\rho}_{equity,cbonds}$	0.215
	(0.084) **
Dynamic correlations	
$\hat{\lambda}_1$	0.091
	(0.018)***
$\hat{\lambda}_2$	0.805
	(0.032)***
$\log likelihood$	**5003.13**

*** Significance level of 1%
** Significance level of 5%
* Significance level of 10%

```
. mgarch dcc (Tbill Bonds Stocks Cbonds =, noconstant), arch(1) garch(1)
```