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Prospective primary teachers' efficacy to teach mathematics: measuring efficacy beliefs and identifying the factors that influence them

Aisling Leavy[1] · Annette Hessen Bjerke[2] · Mairéad Hourigan[1]

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Abstract
Prospective teachers' mathematics efficacy beliefs affect school placement experiences and influence later teacher behaviour and subsequent student outcomes. These efficacy beliefs are open to change during initial teacher education and become more resistant to change once the teacher enters the workforce. Therefore, it is important to measure mathematics teacher efficacy early in initial teacher education and identify low efficacy beliefs. This study measured the mathematics teaching efficacy beliefs of 402 prospective primary teachers. Arising from Rasch analysis of the efficacy scores of these teachers, 22 participants who displayed a range of mathematics teaching efficacy beliefs were interviewed to gain insights into factors affecting these beliefs. Participants communicated greater confidence in tasks requiring procedural (tell-items) than conceptual knowledge (explain-items). Both low and high efficacy participants reported traditional experiences of mathematics teaching and learning as students. However, high efficacy participants reported mastery of mathematics within these traditional contexts, whereas their low efficacy peers did not report mastery. Furthermore, all participants reported how memories of their school mathematics experiences informed their level of efficacy when thinking about teaching mathematics, thus revealing the powerful influence of mastery experiences.

Keywords Mathematics teaching efficacy · Prospective primary teacher · Self-efficacy beliefs · Teacher efficacy beliefs · Mastery · Rasch analysis

1 Introduction

Almost half a century ago, Albert Bandura (1977) suggested that people's behaviours are strongly influenced by their beliefs about their capabilities to engage in behaviours that bring about specific outcomes. These *self-efficacy beliefs*, situated within his broader

✉ Aisling Leavy
aisling.leavy@mic.ul.ie

1 Department of STEM Education, Mary Immaculate College - University of Limerick, South Circular Road, Limerick, Ireland

2 Department of Primary and Secondary Teacher Education, Faculty of Education and International Studies, Oslo Metropolitan University, Pilestredet, Oslo, Norway

socio-cognitive theory, reflect a person's belief in their ability to succeed. Consequently, they contribute strongly to the goals people make, the efforts they invest in executing a plan of action to achieve those goals, how they respond when obstacles are encountered, and their general wellbeing as they engage with everyday life tasks. The influence of self-efficacy beliefs extends beyond individuals' personal lives and impacts their professional lives (Bandura, 1997)—this is particularly true for teachers. The last several decades have seen growing acknowledgment and appreciation of the critical role played by teacher efficacy beliefs in teacher development.

2 Teacher efficacy beliefs

Teachers' efficacy beliefs, which are teachers' beliefs in their own ability to support student learning and achievement, have received extensive consideration in the teacher education literature. In outlining his theory of self-efficacy, Bandura (1977) distinguishes between efficacy expectations and outcome expectations. A similar delineation is present in the research literature on teacher efficacy beliefs which examines personal teaching efficacy and teaching outcome expectancy. Personal teaching efficacy, the focus of this research study and described in detail below, is the teacher's personal belief about their capability to execute a task, in this case, to teach effectively. Teaching outcome expectancy, developed from Bandura's (1977) outcome expectancy belief that "a person's estimate that a given behaviour will lead to a certain outcome" (p.193), is the teacher's judgment about the outcomes of performing a task, in this case, the beliefs about the positive learning outcomes for students arising from effective teaching.

Teacher efficacy beliefs are an extension of Bandura's (1977) efficacy expectations which are described as "the conviction that one can successfully execute the behaviour required to produce the outcomes" (p. 193). Specifically, teacher efficacy beliefs have been described as "a simple idea with significant implications" (Tschannen-Moran & Woolfolk Hoy, 2001, p. 783). These implications extend into many realms of teacher behaviour and subsequent student outcomes. Essentially, teachers are more likely to engage in tasks in which they perceive themselves competent (Pajares, 1996). Consequently, when faced with obstacles, teachers with greater efficacy beliefs are more likely to persist in their efforts to meet teaching goals (Gibson & Dembo, 1984). These teachers possess more positive attitudes and enthusiasm about teaching and have higher confidence in their teaching abilities (Guskey, 1988). They are open to new ideas and more likely to experiment in the classroom (Depaepe & König, 2018; Raudenbush et al., 1992), set high expectations for students and work longer with students experiencing difficulties (Gibson & Dembo, 1984), adopt innovative instructional strategies, and demonstrate a positive disposition to curriculum innovation and educational reform (Gabriele & Joram, 2007; Ghaith & Yaghi, 1997; Guskey, 1988). In the first year of teaching, when efficacy beliefs about mathematics teaching have been found to decline (Charalambous et al., 2008), teachers with higher teacher efficacy beliefs demonstrate greater satisfaction in teaching and experience less stress (Tschannen-Moran et al., 1998). Moreover, these efficacy beliefs positively impact teacher motivation (Midgley et al., 1989) and student achievement (Bruce & Ross, 2008; Enochs et al., 2000). In contrast, teachers with low teaching efficacy beliefs are more likely to demonstrate higher stress levels (Gresham, 2008; Tschannen-Moran et al., 1998), are less likely to affect student learning and outcomes, and are less likely to stay in the field of education (Glickman & Tamashiro, 1982; Tschannen-Moran et al., 1998).

In the context of teacher education, a particularly relevant distinction is that efficacy beliefs pertain to self-perception of competence rather than the actual level of competence. In particular, how prospective teachers estimate their abilities may have consequences for how they approach teaching and the effort they exert during school placement experiences. Bandura (1997, p. 35) emphasised this point in his statement, "A capability is only as good as its execution. The self-assurance with which people approach and manage difficult tasks determines whether they make good or poor use of their capabilities. Insidious self-doubts can easily overrule the best of skills" (Bandura, 1997, p. 35). Consequently, it is important to examine the factors influencing self-efficacy development, particularly teacher efficacy beliefs.

Four main factors influence efficacy beliefs: performance accomplishments (based on mastery experiences), vicarious experiences, social persuasion, and physiological and emotional states—the most influential being *mastery experiences* (Bandura, 1977). The central role of mastery experiences is emphasised by Bandura when he states that "persistence in activities that are subjectively threatening but in fact relatively safe produces, through experiences of mastery, further enhancement of self-efficacy and corresponding reductions in defensive behavior" (p. 191). These expectations of mastery affect both initiation and coping behaviours (Bandura, 1977). High expectations of mastery influence whether people will try to cope in certain situations or perhaps even avoid these situations, thus influencing the choice of activity or behavioural setting. Once engaged in an activity or setting, expectations of mastery can affect coping efforts, the degree of persistence, and the amount of effort invested when faced with obstacles. Thus, the experience of mastery arising from effective performance of a task increases self-efficacy, whereas failure weakens self-efficacy (Bandura, 1977; Tschannen-Moran et al., 1998). The influence of mastery experiences on teacher engagement (Han et al., 2016), teaching efficacy beliefs (Wilson et al., 2020), and mathematics teaching efficacy in particular (Middleton et al., 2004; Phelps, 2010; Usher, 2009), has been reported in several studies. School placement experiences, wherein prospective teachers experiment with teaching mathematics, develop a sense of competence and greater efficacy (Charalambous et al., 2008; Hoy & Spero, 2005). Similarly, the development of mastery experiences for early career teachers supports efficacy development (Mulholland & Wallace, 2001). *Vicarious experiences* are secondary experiences provided by social models. Seeing someone like themselves succeed through persistent efforts can strengthen efficacy beliefs in mathematics teaching (Bandura, 1977; Phelps, 2010; Usher, 2009). Charalambous et al. (2008) found that modelling and verbal feedback provided to prospective teachers by mentors, alongside interactions with tutors, peers, and pupils as part of school placements, positively influence teacher efficacy beliefs. The latter verbal feedback and support, termed *social persuasion*, can strengthen mathematics teaching self-efficacy beliefs (Phelps, 2010; Usher, 2009) and is most effective when it comes from a trustworthy and credible persuader; for example, feedback from students enhances teacher efficacy (Charalambous et al., 2008; Mulholland & Wallace, 2001). However, Bandura (1995) suggests that it is easier to undermine self-efficacy beliefs through negative feedback, which can, in turn, lead to avoidance of similar or more challenging situations. Finally, a person's own *physical and emotional states*, such as stress and fatigue, can add to feelings of mastery or incompetence and, in turn, influence efficacy beliefs.

2.1 Measuring teacher efficacy beliefs

Several instruments have been developed to assess general teacher efficacy beliefs beginning with items created by the RAND organisation (Armor et al., 1976), then a two-factor teacher efficacy scale focusing on personal and general teaching efficacy (Gibson & Dembo, 1984), a 30-item scale assessment of instructional efficacy developed by Bandura (1997), and the 24-item Teachers' Sense of Efficacy Scale (TSES) developed by Tschannen-Moran and Woolfolk Hoy (2001). Such efforts to measure teacher efficacy beliefs have focused predominantly on in-service rather than prospective teachers (Charalambous et al., 2008) and have tended to neglect domain-specific measures such as the teaching of mathematics (Charalambous et al., 2008; Klassen et al., 2011). Indeed, a systematic review of teacher efficacy research from 1998 to 2009 carried out by Klassen et al. (2011) identified three directions for further study: the need for more qualitative approaches to explore teacher efficacy, the need for more significant insights into the sources that support the development of teacher efficacy, and attention to domain-specific insights from subject areas such as science, mathematics, and reading.

3 Efficacy beliefs about teaching and learning mathematics

3.1 Efficacy beliefs about learning mathematics

There is a plethora of research examining the role of self-efficacy in the learning of mathematics, much of it examining problem posing and problem solving. Almost three decades ago, Pajares and Miller (1994) found that efficacy was a stronger predictor of problem solving than gender, mathematics concept, and other factors. Recent studies of school children suggest a positive relationship between problem-posing self-efficacy and the originality of problems posed (Limin et al., 2013) and performance in problem construction (Philippou et al., 2001). Given the important role of efficacy beliefs, identifying interventions and experiences that promote positive efficacy is important. The benefits of exposure to teaching methods that prompt multiple solutions when problem solving have been demonstrated for low efficacy students (Schukajlow et al., 2019). Similarly, carefully designed experiences in mathematics content courses for prospective teachers, such as classroom discourse and instructor support during mathematics problem solving, were shown to promote mastery goals (Morrone et al., 2004) and increase mathematics self-efficacy (Harkness et al., 2007).

3.2 Efficacy beliefs about teaching mathematics

A study by Charalambous and Philippou (2010) revealed how teacher efficacy beliefs affect their task concerns (i.e., the daily duties of a teacher such as time and resource management) and their impact concerns (i.e., the consequences of the change to student learning). Mathematics teachers with high efficacy beliefs have been shown to benefit more from training on formative assessment than those with low efficacy beliefs (Schütze et al., 2017). Similar benefits of high efficacy beliefs were demonstrated in a study of prospective mathematics teachers, which showed that strong efficacy beliefs triggered motivation to persevere in problem solving (Voica et al., 2020).

Once established, however, teacher efficacy beliefs appear to be stable and resistant to change (Tschannen-Moran et al., 1998), even following targeted professional development experiences (Ross, 1994). However, initial teacher education has been identified as the context in which teacher efficacy beliefs increase and are more influential to change (Hoy & Spero, 2005; Hoy & Woolfolk, 1990; Wenner, 2001). Past experiences with mathematics influence prospective teachers' mathematics teaching efficacy (Swars, 2005). It is not surprising that a relationship between mathematics anxiety and mathematics teaching efficacy has also been found where prospective elementary teachers with high teaching efficacy demonstrate lower levels of mathematics anxiety (Bekdemir, 2010; Gresham, 2010; Swars et al., 2006). However, a welcome finding across the research literature is that increases in prospective teachers' mathematics teaching efficacy can occur as a result of interventions during initial teacher education for both early childhood (Gresham & Burleigh, 2019; Saçkes et al., 2012; Sancar-Tomak, 2015) and elementary teachers (Haverback & McNary, 2015; Hoy & Spero, 2005).

Some instruments have been developed to assess the mathematics teaching efficacy of prospective teachers; each serves as a valuable tool for use at different stages (entry versus exit level, for example) and phases (during fieldwork or school placement) of the initial teacher education programme. The Mathematics Teacher Efficacy Beliefs Instrument (MTEBI) developed by Huinker and Enochs (1995), based on modifications of their Science Teacher Efficacy Beliefs Instrument (STEBI) for in-service and prospective teachers (STEBI-B) (Enochs & Riggs, 1990), measures prospective teachers' mathematics teaching efficacy. Despite some issues with its reliability for those demonstrating below-average levels of efficacy (Kieftenbeld et al., 2011), the MTEBI is a popular measure in initial teacher education. Charalambous et al. (2008) developed a modified version of Tschannen-Moran and Woolfolk Hoy's (2001) Teachers Sense of Efficacy Scale (TSES) for use in conjunction with interviews to examine the development of prospective teachers' efficacy beliefs in mathematics during fieldwork or placement experiences. However, while these two instruments (Huinker & Enochs, 1995; Charalambous et al., 2008) provide valuable insights into prospective teachers' mathematics teaching efficacy beliefs, they both assume prior pedagogical experiences. Consequently, they are suitable for prospective teachers following completion of mathematics teacher education courses or school placement during or at the end of their programmes, and not for novice prospective mathematics teachers.

The notable absence of a measure to garner insights into mathematics teaching efficacy beliefs of entry-level (novice) prospective teachers led Bjerke and Eriksen (2016) to develop an instrument to measure self-efficacy in teaching children in primary mathematics (SETcPM). This instrument has been used to report on novice Norwegian prospective teachers' mathematics teaching efficacy (Bjerke & Eriksen, 2016) and to deepen the understanding of the nature and development of their mathematics teaching efficacy during teacher education (Bjerke, 2017). In this study, we use the SETcPM to identify entry-level prospective teachers' mathematics teaching self-efficacy and combine it with qualitative interviews to explore the factors that influence the development of mathematics teaching self-efficacy.

4 This study

School placement experiences, located within initial teacher education programmes, can enhance or threaten prospective teachers' efficacy beliefs (Charalambous et al., 2008; Tschannen-Moran & Woolfolk Hoy, 2001; Tillema, 2000). Awareness of these efficacy beliefs on entry to teacher education programmes, and prior to school placement experiences, can allow the redirection of supports to those who most need it and at a time identified as critical to the development of mathematics teaching efficacy beliefs. Furthermore, gaining insights into the formative experiences contributing to these beliefs can better equip teacher educators in providing needs-led interventions. This mixed-methods study uses the SETcPM (Bjerke & Eriksen, 2016) to identify entry-level prospective primary teachers' efficacy in teaching mathematics. Insights into the factors influencing these efficacy beliefs are further explored using qualitative interviews with participants selected based on their efficacy scores in the SETcPM. The study is motivated by two research questions:

1. What are entry-level prospective primary teachers' efficacy beliefs about teaching mathematics?
2. What is the nature of the relationship between prospective teachers' prior experiences as learners of mathematics and their mathematics teaching self-efficacy?

5 Methodology

5.1 Participants

Participants were undergraduate prospective primary teachers in the 1st year of a 4-year initial teacher education programme in Ireland. All 475 prospective teachers entering the programme were invited to participate in the study (371 females (78%) and 104 males (22%)), and 402 provided consent (329 female, 73 male).

All participants had studied mathematics throughout their 13 years of pre-tertiary education. For the vast majority of participants (age 17–18), selection was solely based on performance in the terminal pre-tertiary examinations called the Leaving Certificate examination. In Ireland, the college application procedure is centralised. Hence, instead of applying to individual institutions, the Central Application Office (CAO) is the organisation responsible for overseeing undergraduate applications. Within this process, points are allocated to each grade achieved for the student's best six subjects in the Leaving Certificate examination (CAO, 2022). Although the points required to receive a place vary annually depending on the number of places available and the demand, the high demand for primary teaching programmes in Ireland elevates the entry-level requirements for primary education. The cut-off for entry to this programme in the institution within this study was 474 points (out of a maximum of 600 points). Therefore, entrants to primary teacher education rank in the top 15% of all Ireland college entrants as measured by the CAO's point data (note: 78% of those who completed their Leaving Certificate were college entrants). A small number of mature students (age 23 +) achieve a place on the programme via an alternative route through a competitive interview process. As mathematics is mandatory in order to graduate from secondary school, and Ireland has one national curriculum for secondary mathematics education, all students had studied comparable mathematics content in secondary school. All entrants must also achieve the necessary stated minimum Leaving Certificate mathematics grade to gain entry to primary initial teacher education programmes; in reality, however, almost all entrants exceed this minimum requirement (Hourigan & O'Donoghue, 2015). The intake to the initial teacher education programme, which is the focus of this study, represents 50% of all students studying primary-level teacher education in Ireland.

5.2 Study design

In agreement with Stromquist's (2000) commentary that no one research instrument can guarantee a holistic grasp of the "truth", this study adopted a mixed-methods approach. While all participants completed the SETcPM instrument (quantitative), subsequently, a purposive sample of participants engaged in a semi-structured interview (qualitative).

5.2.1 Characteristics of the SETcPM instrument

The SETcPM is a 20-item instrument developed for prospective primary teachers (see Bjerke and Eriksen (2016) for details about its development and validation). Items focus on supporting a child with mathematics tasks and target mathematical number concepts aimed at grades 1–7 (ages 6–13) that are fundamental to primary mathematics curricula (Bjerke & Eriksen, 2016). Each item identifies a specific mathematics task that a primary student may be required to complete (see Appendix 1). Respondents indicate their confidence level, based on a 4-point Likert scale ("not confident", "somewhat confident", "confident", and "very confident"), to support a child with the task. While targeting one underlying construct (i.e., self-efficacy in teaching mathematics), the instrument consists of ten rules-items and ten reasoning-items. The ten rules-items frequently contain the verb "calculate" and focus on procedural or algorithmic knowledge. The following are three examples of rules-items (items 2, 10, 19 in Appendix 1):

Calculate 750/25.
Write $\frac{1}{5}$ as a decimal number.
Calculate $4.14 + 3.190$.

In contrast, the ten reasoning-items concentrate on understanding by predominantly using the verb "explain". Examples of three reasoning-items (items 5, 14, and 20 in Appendix 1) are as follows:

Explain why -1 is larger than -3.
Explain why, when subtracting, you can sometimes borrow from the place to the left.
Use a drawing to explain why $\frac{1}{8}$ equals 0.125.

Consequently, as argued and outlined in Bjerke and Eriksen (2016), the instrument addresses both the procedural and conceptual nature of teaching mathematics (i.e., the "hows" and "whys"), although a principal component analysis of residuals revealed no multidimensionality.

5.2.2 Quantitative data collection and analysis

The researchers met all prospective primary teachers in the cohort programme during an induction session in the first week of the 4-year teacher education programme. In compliance with ethical guidelines from the host institution, researchers described the purpose of the research and explained that participation was voluntary. A total of 402 students agreed to participate (329 female, 73 male) and were issued a unique project ID to ensure anonymity. They completed the 20-item SETcPM instrument (full version is given in Appendix

2 and in Bjerke and Eriksen (2016)) and provided three pieces of additional information: gender, Leaving Certificate mathematics level, and willingness to participate in subsequent interviews (alongside accompanying contact details where volunteered). Participant responses and information were input into a spreadsheet.

Traditionally, responses on the Likert scale were assigned numbers (e.g., 0 for not confident to 3 for very confident), and these numbers were added together to provide a raw score on the test. However, there are some problems when simply adding up a prospective teacher's responses on a Likert scale. For example, just as it is problematic to assume that all items in a test exhibit the same difficulty, all survey items should not be assumed to be equally agreeable (Boone, 2016). Also, Likert scale is ordinal data that cannot be assumed linear (Boone et al., 2014). The Rasch measurement, based on an equation developed by George Rasch, converts these ordinal data to linear measures, which is shown by a single vertical ruler that represents the construct being evaluated by a test. The items in the test are distributed along a vertical ruler and are used as measuring points for the persons undertaking the test. In this way, since both items and persons are distributed along the same vertical ruler, it allows comparisons between items, between persons, and between items and persons, in the form of establishing a person's probable answer on an item (given in logit units). For this reason, we chose the approach taken by Bjerke and Eriksen (2016) and applied the Rasch rating scale model (RSM) to analyse the quantitative data. WINSTEPS 3.81.0 software (Linacre, 2014) was used to test the compliance of the data with RSM.

5.2.3 Qualitative data collection and analysis

Qualitative interviews were incorporated into the study design, thus addressing Klassen et al.'s (2011) recommendations to incorporate qualitative approaches to reveal specific in-depth insights into the nature and sources of participants' efficacy in teaching mathematics. Hence, the focus of interviews was to add depth to the quantitative data already collected, triangulating the data and strengthening the findings. At the time of SETcPM administration, participants were invited to volunteer for interviews. The sampling for the interview was purposive in order to select "… information-rich cases" (McMillan & Schumacher, 2001, p. 400). Selection was based on SETcPM scores (e.g., low/medium/high scores). Out of 25 participants who were invited for interview, 22 agreed to be interviewed. Despite initial agreement (during the SETcPM administration), three prospective teachers did not respond to subsequent invitations to participate in the interview process. The remaining 22 interview participants represented a range of SETcPM scores across high ($n=3$), medium ($n=4$), and low ($n=5$) efficacy scores.

While the researchers developed a schedule of questions for the semi-structured interviews (see Appendix 2 for interview questions), five independent interviewers conducted interviews. The researchers did not administer the interviews, given that they also designed, taught, and evaluated the prospective teachers' mathematics education programme and feared that taking on the role of interviewer could result in prospective teachers responding conservatively. However, the researchers provided training for the five interviewers that explained the purpose of the study and examined the interview schedule. Each interviewer was allocated a subgroup of prospective teachers whose SETcPM scores suggested high, medium, or low efficacy. While the interviewers were encouraged to ask all of the questions on the schedule to each interviewee, a semi-structured approach was promoted, allowing interviewers to seek clarification or probe further as appropriate. Equally, this structure facilitated interviewees in raising issues of importance to them.

Interviews were conducted on-campus and at a time convenient for the participants. Interviews averaged 24.8 min and were digitally recorded and transcribed using transcription software. Interviewers carried out a quality check of all transcriptions, and corrections were made where necessary. Several efforts were made to enhance the reliability and validity of the process. For example, the same interview questions were used for all participants. Given that the use of more than one interviewer could lead to a different emphasis across interviews, the repetition of an interview question (Do you think you could teach mathematics effectively?) was deliberately used as a means of checking responses. "Member checking" was implemented for 20% of the interviewees, where interview participants examined their interview transcript to confirm it reflected their beliefs and to make changes or additions to their interview data if it did not match their intent. In addition, interviewers met with researchers to debrief and discuss the data arising from the interviews.

The constant comparisons approach (Silverman, 2005) alongside Miles and Huberman's (1994) tactics for generating meaning informed the qualitative data analysis. To circumvent individual researchers' personal biases (Suter, 2012), interviews were analysed separately by two of the researchers. Both these researchers analysed transcripts of all 22 interviews; this analysis involved selecting, applying, and describing a code for a piece of data. Codes were then compared with other data and codes developed to generate understandings of any potential relations between them. Subsequently, categories were established based on clusters of codes to represent common themes or patterns (e.g., teacher attitude, teaching approach, liking mathematics, difficulties in mathematics). When each researcher had completed their analysis, both researchers worked together to examine the tentative themes and associated evidence. Where necessary, the transcripts were revisited and analysed until dominant themes were agreed upon. A theme had to meet two criteria to be classified as a dominant theme. It had to occur within each of the three (high, medium, low) efficacy groups and appear in over 60% of the interviews. The three dominant themes identified were: universal experience of traditional teacher-centred approaches to mathematics teaching and learning (Theme 1), mastery matters (Theme 2), and formative experiences as a learner of mathematics (Theme 3).

6 Findings

In the next section, we describe the quantitative Rasch analysis of the data from the SETcPM implementation, interpret the findings of item-level and person-level differences, and outline how these contributed to selecting participants for interview. These findings respond to Klassen et al.'s (2011) recommendation for increased research in domain-specific teacher efficacy. Qualitative interview data are then analysed to reveal the factors that influenced participants' mathematics efficacy beliefs on entry to their initial teacher education programme.

6.1 Insights provided from the quantitative Rasch analysis of SETcPM data

We explored the degree of fit between the data from our sample and the Rasch rating scale model (RSM), i.e., examining whether the items in our SETcPM instrument were distributed evenly along a vertical ruler—revealing unidimensionality. The unidimensionality condition of the Rasch model held sufficiently well for the data, suggesting that the items in the instrument contribute to measure an underlying construct. However, initial analysis revealed some disordering of the average person abilities by category for items 8 and 11, meaning that there was some disordering in the thresholds, that is, the observed mean measure for respondents endorsing "not confident" was higher than that for "somewhat confident". This should be the reverse. Such findings suggest cleaning of the data is required, which is acceptable (and even recommended (see Boone and Noltemeyer (2017)). The rationale for choosing RSM was its emphasis on identifying and studying anomalies in the data disclosed by the Rasch model instead of selecting a model that best characterises the given data. By identifying the most unexpected answers on items 8 and 11 (WINSTEPS allows such analysis), these were removed (< 0.2% of the data were removed), and the disordering problem was resolved. The fit statistics for our SETcPM administration are given in Table 1, with mean square fit statistics (MNSQ) showing fit values within acceptable limits for all 20 items (between 0.6 and 1.4), with person means equalling 0.27 logits (SD = 1.07). The Rasch reliability estimates were 0.89 for persons and 1.00 for items, indicating a reproducible measure.

Once the unidimensionality of our data was established (i.e., through fit statistics and additional differential item functioning analysis which investigates the items, one at a time, to ensure that all groups of prospective teachers interpret the items in the same way), we took a closer look at the column in Table 1 that communicates the item

Table 1 Fit statistics for SETcPM administration ($N = 402$)

Item	SETcPM measure	Score	Std. error	Infit MNSQ	Infit ZSTD	Outfit MNSQ	Outfit ZSTD
1	0.52	532	0.07	0.94	−0.88	1.02	0.30
2	−0.51	776	0.07	0.97	−0.44	0.99	−0.14
3	−0.19	701	0.06	1.18	2.61	1.19	2.53
4	−1.04	881	0.07	0.93	−1.04	0.90	−1.15
5	−0.91	856	0.07	0.96	−0.60	1.13	1.48
6	−1.31	907	0.08	0.89	−1.56	0.99	−0.09
7	1.26	366	0.07	0.94	−0.79	0.94	−0.76
8	−1.60	975	0.08	0.72	−3.96	0.67	−3.40
9	0.33	571	0.06	1.18	2.65	1.14	2.03
10	−0.73	819	0.07	1.03	0.40	0.98	−0.17
11	−1.83	992	0.09	0.80	−2.51	0.72	−2.48
12	0.03	649	0.06	1.01	0.14	0.97	−0.35
13	0.74	482	0.07	1.01	0.19	1.01	0.20
14	1.09	402	0.07	1.06	0.82	1.05	0.73
15	0.23	591	0.07	0.92	−1.20	0.92	−1.20
16	0.41	551	0.07	0.87	−2.08	0.86	−2.15
17	1.61	302	0.07	1.02	0.27	0.98	−0.28
18	1.72	284	0.08	1.20	2.51	1.16	1.81
19	−0.74	825	0.07	0.94	−0.95	0.94	−0.74
20	0.94	440	0.07	1.36	4.88	1.33	4.24

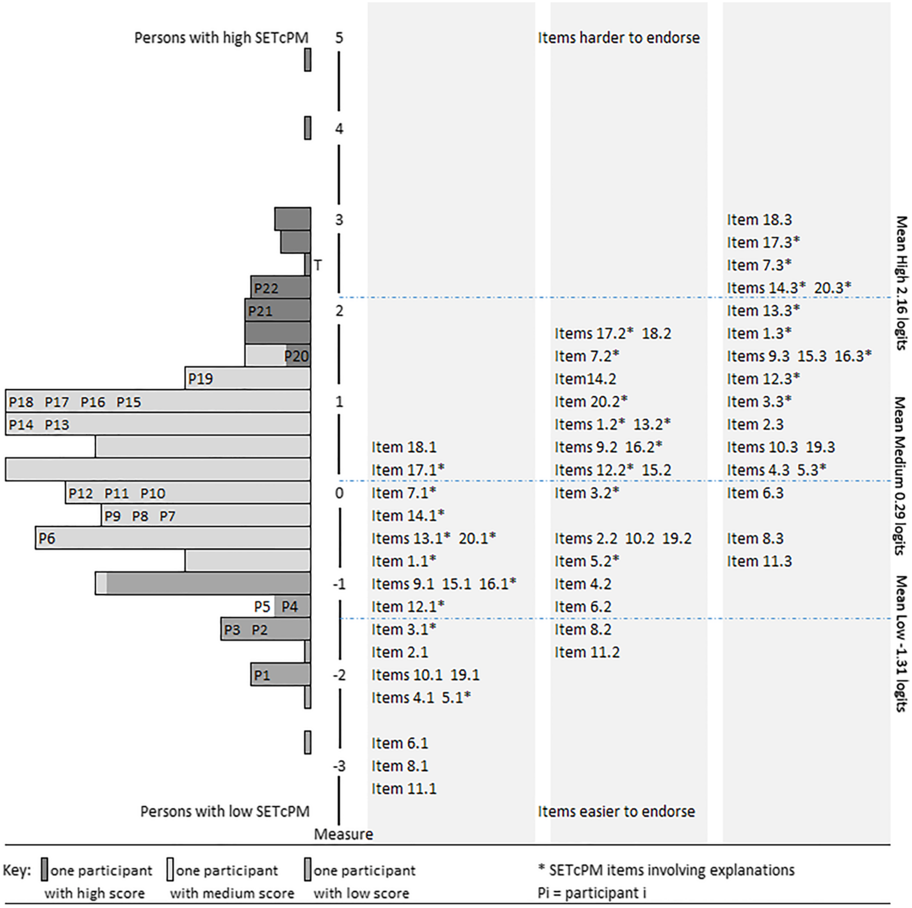

Fig. 1 Wright map for $N=402$ participants, showing the distribution of participants and item SETcPM scores in logits

measures (titled "SETcPM measure"). The higher values indicate the presence of greater SETcPM efficacy beliefs (and are placed on the top of the vertical ruler in Fig. 1), while the lower values indicate lower levels of SETcPM efficacy beliefs (placed on the bottom of the vertical ruler in Fig. 1).

6.2 Interpreting the Wright map

By monitoring the items and their nature, we searched for patterns and tendencies in our data. In the Wright map in Fig. 1, person measures are presented on the left, and item measures are on the right of the vertical ruler. Participants at the top represent those with the highest SETcPM (efficacy) scores and those at the bottom with the lowest SETcPM scores. The length of the horizontal bars (on the left of the map) indicates the number of participants located at that position (see key at the base of Fig. 1). The participants labelled in the distribution (P1–P22) are those who were selected for interview, where P1 was the interview participant reporting the lowest SETcPM score and P22 the one reporting the highest score.

Similarly, when examining the right-hand side of the Wright map, items positioned at the top are teaching tasks perceived as more challenging, whereas ones at the bottom are perceived as less challenging. Items are positioned at their Andrich threshold, the points of equal probability of adjacent categories (Bond & Fox, 2007). This means, for example, that the label "Item 20.2" (Fig. 1) represents item 20 at its second answer category threshold (between "somewhat confident" and "confident"). A participant with an estimate at this position on the scale (for example, P18 located at approximately 1.00) will have a 50% chance to choose either of the two categories. Since the thresholds are ordered, a respondent with an estimate above the label "Item 20.2" but below the label "Item 20.3" (for example, P21 located approximately at 2.00) will most probably endorse the category "confident" for item 20.

The distribution of scores was used to categorise participants as having low, medium, or high efficacy in teaching mathematics. Participants were determined to have low efficacy if their total score was more than one SD below the mean (SETcPM score < -0.8) (low group: P1–P5). Medium efficacy participants were located within one SD below and above the mean ($-0.8 <$ SETcPM score < 1.34) (medium group: P6–P19). Those with high efficacy had scores situated more than one SD above the mean (SETcPM score > 1.34) (high group: P20–P22). The mean measure for the three groups is given in Fig. 1. The presence of substantial differences in prospective primary teachers' efficacy scores before engaging in an initial teacher education programme merits further qualitative investigation regarding the profiles of these students and the factors that may have contributed to their efficacy ratings.

Our analysis reveals a clear tendency in the data. Across the sample, reasoning-items (marked with a * in Fig. 1) are reported harder to endorse than the rules-items. Moreover, most participants in the high group are above "confident" on most reasoning- and rules-items. In contrast, those in the low group are "not confident" or "somewhat confident" on all reasoning-items. From Fig. 1, we read that a "typical" prospective teacher (with SETcPM score equal to the mean) in the low group is "confident" on only two rules-items (items 8 and 11) and no reasoning-items. This finding, that prospective teachers with low mathematics teaching efficacy beliefs already perceive challenges with teaching mathematics reasoning-items and many rule-items, is a critical signpost for mathematics teacher educators in terms of where to focus efforts in the design of mathematics education experiences in initial teacher education.

The proportion of responses in each category, from "not confident" to "very confident" for each item was then analysed. Since the items together make up a unidimensional scale, all contributing to measuring SETcPM in individuals, we can investigate in more detail how the prospective teachers respond to different items, and hence, different subgroups of items. Hence, items were grouped according to the two types of tasks included in the instrument, with the top row in Fig. 2 presenting reasoning-items (explain-items demanding justifications) and rules-items (tell-items focusing on calculations) for the high group of prospective teachers and the bottom row representing the same data for the low group of prospective teachers. Examination of the top row reveals that, except for item 18, the most prevalent response for the high group was "confident" or "very confident" on all items. Nonetheless, we can see that while prospective teachers in the high group tend to use the category "very confident" most often, there is still a tendency that the other three categories are more often used in the reasoning-items (suggesting that reasoning-items are perceived as more difficult, even for the high group). Examination of the bottom row of Fig. 2, displaying responses of the low group,

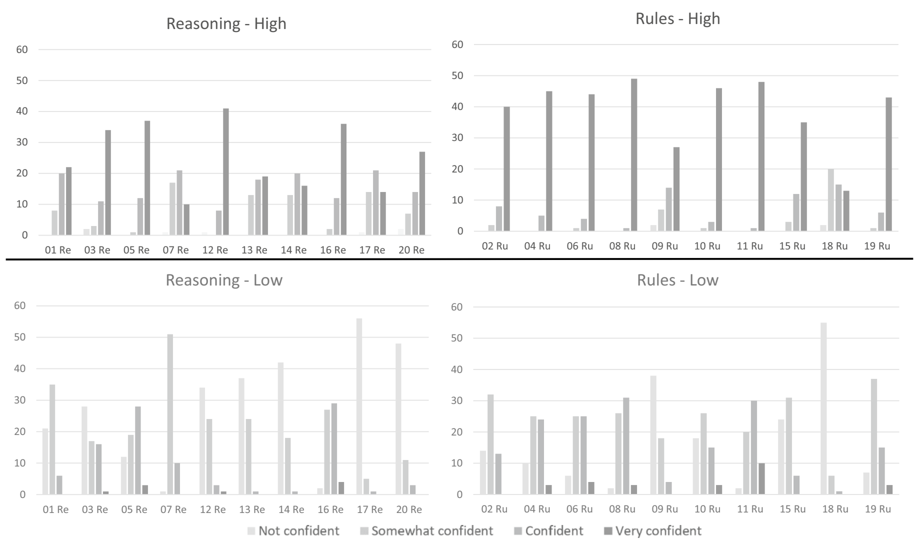

Fig. 2 Distribution of answering categories in high and low group of participants

reveals a higher proportion of "not confident" and "somewhat confident" responses on the reasoning- and rules-items than their peers in the high group.

6.3 Qualitative insights arising from interviews of a selection of participants

The use of qualitative interview within this study responded to all of Klassen et al.'s (2011) recommendations, providing qualitative insights into the sources of teacher efficacy within the domain of mathematics. The interview data gave voice to the school mathematics experiences of the 22 selected participants and how these connect with their levels of confidence around teaching mathematics. While the purpose of the interviews was not to generalise findings across the cohort, participants are located in relation to their responses on the instrument, thus allowing their data to be framed within the context of the larger surveyed group of 402 students (see Fig. 1). Figure 3 shows the precise SETcPM scores of the 22 interviewees, with the male respondents above the ruler and the female respondents below.

Analysis of the interview data revealed three dominant themes: universal experience of traditional teacher-centred approaches to mathematics teaching and learning (Theme 1), mastery matters (Theme 2), and formative experiences as a learner of mathematics (Theme 3). Each theme occurred within the low, medium, and high efficacy groups (see Table 2)

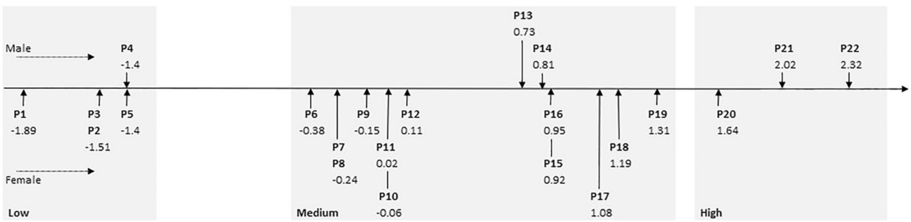

Fig. 3 The 22 interviewees' positioning in low, medium, or high group

Table 2 Distribution of themes across participants

High efficacy participants			Medium efficacy participants														Low efficacy participants				
22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
a	a		a	a	a	a		a	a	a		a			a	a	a			a	
b	b	b	b			b		b	b		b			b		b	b	b	b	b	
c	c		c			c	c		c			c	c			c	c	c	c	c	c

a, Theme 1; b, Theme 2; c, Theme 3.

and across at least 60% of the interview participants. These themes provide rich qualitative insights in the nature of novice prospective teachers' previous mathematics experiences and reveal sources of mathematics teacher efficacy.

6.3.1 Theme 1: universal experience of traditional teacher-centred approaches to mathematics teaching and learning

A somewhat unexpected finding was that all participants, regardless of mathematics teaching efficacy score, described traditional experiences of teaching and learning mathematics, with the teacher "writing something on the board and then we would do it out of the book, like a big list" (P10) and students "doing pages of sums in a copybook" (P17). Descriptions of a typical mathematics lesson depicted a teacher-led, book-based approach with "a lot of looking at the book" (P19), "workbook stuff" (P22), and being "shown how to do it" (P12) by the teacher who would "put a few sums up" (P16). Participants reported a "lack of games and activities" (P7) with many narrating experiences of rote learning of basic facts with P6 and P18 stating "we just learned them off" and "recited tables fast, it was all about speed". The terms "maths books", "workbooks", and "sums" occurred with greatest frequency across the interviews. There was some differentiation between experiences in the early years of primary school. For example, P21 described approaches that featured use of "unifix cubes… counters…. place-value mats" which contrasted with her senior class later experiences of "working from the maths book every single day". These experiences served as powerful sources of mathematics teacher efficacy for these novice prospective teachers. For example, their "apprenticeship of observation" served to implicitly communicate the nature of the mathematics classroom practice and role of the mathematics teacher and students within this context, thus providing vicarious experiences. The other themes reveal further sources of mathematics teacher efficacy.

6.3.2 Theme 2: mastery matters

While all participants experienced traditional mathematics teaching as students themselves, the impact of these experiences differed across participants, thus impacting their emotional states and performance accomplishment as mathematics learners. Those with high mathematics teaching efficacy recounted primary school experiences with a general fondness stating that they "really enjoyed maths" (P22) recalling their experience as "always very good" (P19). References were also made to experiences at secondary school. However, while some high mathematics teaching efficacy scorers found the Leaving Certificate examination a "challenge" (P22) and "hard" (P16), there was a clear sense that those with higher mathematics teaching efficacy had liked the subject as they wrote comments such as "It was grand like I could do most of it, I enjoyed maths" (P22) and "I always liked Maths in secondary school" (P19). Those with higher levels of teaching efficacy described themselves as mastering the mathematics content and achieving academically within this somewhat traditional, teacher-centred context. They reported experiencing success in primary mathematics tasks and described themselves as "very, very good at primary school Maths" (P22), "the brightest at maths" (P21), and finding it "really easy" (P16).

However, it was apparent for those with lower mathematics teaching efficacy scores that they did not experience the same feelings of mastery. This is evident in comments such as "I just always seemed behind" (P6) and "I never really 'got' long division and other maths that the others seemed to find easy" (P4). The following comment from P2 refers to how his difficulties understanding mathematics were not addressed:

> "…if you had any problems … you might ask a question it would be like 'oh we'll go through it in a minute' kind of way. All the way up along [primary school] but the minute might never really come, if you're struggling badly, you go to resource".

Hence, there were consistent reports of a strong relationship between mastery as mathematics learners and mathematics teacher efficacy among these novice prospective teachers. There was also evidence of the impact of social persuasion, where students compared themselves with peers. It is important to note how we conceptualised the notion of mastery within the context of these qualitative interview data. We mean it to refer to participants' own perception that they mastered the mathematics content. In that sense, we make no evaluative judgment regarding whether these participants achieved correct answers, or indeed whether their mastery extended beyond the successful completion of routine procedures and lead to conceptual understanding.

There is also evidence that these mastery experiences at the primary level supported participants' transitions into secondary school mathematics. A remark from P21, who recounted her experience of being the "brightest at Math" in primary school, illustrates the subsequent supporting influence on her transition to secondary school. She stated, "I remember going from sixth class into secondary school and thinking like 'Oh yeah, I'll be well able for higher level maths'".

6.3.3 Theme 3: formative experiences as a learner of mathematics

Participants consistently referred to their traditional experiences as mathematics learners throughout the interviews. Notably, the affective influence of these experiences on their emotional state alongside the degree to which they mastered mathematical content (performance accomplishment) appears to be related to their mathematics teaching efficacy score. Participants referred to these experiences as mathematics learners when asked about their opinions of mathematics itself and when thinking about themselves as future teachers of mathematics. A comment from P2 provides some support for this conjecture:

> "Shape and space with the teacher we had in 6^{th} class was good. We went out to the yard and we used to do a few bits there. There was something concrete to kind of help understand it, so it was more interesting as well. I'll do lots of this when I teach".

Those with higher SETcPM scores reported positive experiences as learners of mathematics and communicated positive dispositions towards mathematics. These dispositions were reflected in the emotive language used and references made to particular experiences of success. Phrases like "I could do the maths" (P16), "I was grand at maths" (P19), and "I would pick it up pretty fast" (P22) frequently occurred throughout individual transcripts. In contrast, those with lower SETcPM scores referred to negative experiences at primary school "I was always very bad at maths like I could never 'get it' in primary school" (P4), and experiences that were repeated, especially at Leaving Certificate level, stating "I found it quite difficult" (P9) and "I remember how so very difficult it was" (P6). Except for one low scorer, all referred to their disaffection towards mathematics and the pressure they felt preparing for the Leaving Certificate examination with P1 stating "I was never the best at maths. I worked hard at it, but it just was not really for me" and P3 saying "Maths was so hard…I was under like a lot of pressure with maths". Hence, performance accomplishment as mathematics learners, which in turn affected their emotional state, is reported as the main source of mathematics teacher efficacy among novice prospective teachers.

It became increasingly evident throughout interviews that participants' beliefs about mathematics and its teaching were rooted in their experiences as mathematics students. When asked what immediately came to mind at the mention of the word "mathematics", all referred to experiences as a learner of mathematics in primary or secondary school: "Definitely I think of Leaving cert maths experience in secondary school" (P5), "Leaving Cert maths is the first thing that came into my head" (P2), "learning addition, subtraction, like the numbers basically, in primary school" (P10), and "I remember in 3rd or 4th class doing short and long division and doing it on the board" (P9). Furthermore, for those with high mathematics teaching efficacy, their experiences as successful mathematics learners seemed to instil confidence in their ability to help a child with mathematics.

When asked about their responses on the SETcPM and their sense of confidence in teaching number, students stated "I was fairly confident [about my ratings] because I was always able to do the maths in school myself" (P19) and "I just was always very strong at number, it never posed any difficulty" (P22). P16 mirrored this in her description of the survey items as referring to "really easy maths stuff that we'd really be able to do—there would be no problems". Similarly, for those with lower SETcPM scores, their experiences of math as learners coloured their responses. P6 said that when answering the survey, she thought back to her experience of Leaving Certificate and consequently predicted that the teaching of mathematics would be difficult, "I wasn't great in secondary school and thinking about going back to the level [primary] and explaining it properly…I don't know will I be able to do it". Similarly, P13 stated "I am kind of nervous thinking about it because I did Ordinary [not advanced] level". P10 associated her experiences of secondary mathematics with her low expectations for her mathematics teaching ability. She mentioned getting a lot of private tuition throughout secondary school and when reflecting on her confidence in mathematics she said, "it all went down the swanny for me in secondary school". This frustrating past relationship with mathematics, characterised by negative mastery experiences, emotional state, and social persuasion, is what she attributed to her anxiety when thinking about teaching mathematics on school placement.

Overall, the findings evidence that novice prospective teachers' pre-tertiary mathematics experiences were powerful sources of mathematics teacher efficacy.

7 Discussion

Our study used the self-efficacy in tutoring children in primary mathematics instrument (SETcPM) (Bjerke & Eriksen, 2016) with 402 entry-level prospective elementary teachers. The instrument, designed specifically for use with entry-level (or novice) prospective primary teachers, asks them to identify their level of confidence when supporting a child with a series of 20 specific number tasks. The first research question sought to identify entry-level prospective primary teacher's efficacy beliefs about teaching mathematics. Analysis of the quantitative data revealed substantial differences in efficacy beliefs, thus supporting the finding of Bjerke and Eriksen (2016) that on entry to this competitive teacher education programme, prospective teachers demonstrate a broad range of efficacy beliefs in relation to teaching number. The second research question examined the nature of the relationship, if any, between prospective teachers' prior experiences as learners of mathematics and their mathematics teaching self-efficacy. Analysis of qualitative interview data supported the findings of Bandura (1977) in revealing the central role of performance accomplishment based on mastery experiences; those with mastery experiences as learners of mathematics possessed high efficacy beliefs and those with low mastery experiences demonstrated low efficacy beliefs. There was also some evidence of the role of social persuasion and emotional state, factors also identified by Bandura (1977), in influencing efficacy beliefs albeit to a lesser extent than performance accomplishment.

Previous research has identified the important role of experiences that occur while on school placement (Bandura, 1977, 1995; Charalambous et al., 2008; Hoy & Spero, 2005; Mulholland & Wallace, 2001) and has shown changes in teaching efficacy during initial teacher education and during school placement (Hoy & Spero, 2005; Hoy & Woolfolk, 1993). This has led to collective agreement that efficacy beliefs are most malleable early in learning (Bandura, 1977), with mathematics teaching efficacy subject to change during initial teacher education (Hoy & Spero, 2005; Mulholland & Wallace, 2001). A unique contribution of this study is that it provides much needed qualitative insights into factors that influence the development of efficacy beliefs about teaching mathematics (Klassen et al., 2011) prior to both mathematics teacher education and placement experiences. In other words, when prospective teachers first enter a teacher education programme. Examining these beliefs early in the teacher education continuum is important as these efficacy beliefs about teaching mathematics may mediate prospective teachers' performance (Bandura, 1977; Phelps, 2010). This is especially concerning for those prospective teachers who, at such an early stage, hold low efficacy beliefs that may result in "insidious self-doubts" (Bandura, 1977, p. 35) to negatively determine outcomes even for those with the best of capabilities and skills.

Analysis of the item-level responses revealed that prospective teachers differed in their confidence levels relating to the two types of tasks. They reported higher levels of confidence in relation to "rules" or tell-items that require making calculations and lower levels of confidence with "reasoning" or explain-items that require explanations. It is interesting to note that this pattern supports the findings of international comparisons, such as the Trends in International Mathematics and Science Study (TIMSS), and national assessments, which have historically reported significantly higher scores for the cognitive domain and subscales of "knowing" in mathematics among Irish students when compared to more higher order domains such as "Reasoning" (Eivers et al., 2007; Surgenor et al., 2006; Clerkin et al., 2016). Moreover, our Rasch analysis shows that those with lower efficacy beliefs demonstrated less efficacy with "reasoning" items than their higher-efficacy peers. Nonetheless, we caution about underestimating the challenge perceived with rules-items. As evident in Fig. 1, a small number of rules-items were harder to endorse than reasoning-items and those with low efficacy perceived challenges with both types of items.

The qualitative interviews with prospective primary teachers, selected purposefully arising from their scores on the SETcPM, provided several valuable insights into participants' efficacy in teaching mathematics and addressed the identified need for more qualitative approaches to explore teacher efficacy (Klassen et al., 2011). Those with high efficacy demonstrated confidence in their ability to do mathematics and expressed similar confidence in teaching mathematics. This suggests a relationship between high efficacy in doing mathematics and high efficacy in teaching mathematics (Bekdemir, 2010; Gresham, 2010; Swars, 2005; Swars et al., 2006). Their interview data suggested that positive experiences of school mathematics as a learner influenced the development of high efficacy beliefs in teaching mathematics. Conversely, all except for one participant selected for interview based on their low efficacy scores in the SETcPM reported negative experiences as school mathematics learners. Similar findings of lower efficacy towards mathematics have been found for prospective elementary teachers who had negative experiences of mathematics as learners and consequently higher levels of mathematics anxiety (Gresham, 2009).

A somewhat unexpected finding was the consistency of descriptions of experiences of traditional and teacher-centred mathematics teaching. These accounts do not reflect the intended primary mathematics curriculum in Ireland, which recommends constructivism and guided discovery methods (NCCA, 1999). These reports were prevalent for low SETcPM scorers and those with high efficacy scores who reported liking and enjoying mathematics. Of particular interest is the finding that many prospective teachers reported positive experiences of mathematics within quite traditional contexts that were devoid of opportunities to engage in constructivist-style experiences that appeal to the interests of the child (McMahon Giles et al., 2016; Hourigan & Leavy, 2019). Even though all participants reported similar traditional approaches, the importance of mastery experiences emerged as a possible contributing factor that supported the development of a high sense of efficacy among some participants. This importance of mastery experiences in mathematics supporting the development of high mathematics teaching efficacy for prospective teachers is supported in literature from several studies (Charalambous et al., 2008; Newton et al., 2012; Phelps, 2010; Usher, 2009). One participant referred to how his mastery of primary school maths supported his transition to secondary school mathematics. This observation was particularly insightful and is supported by literature suggesting that efficacy beliefs influence transitionary periods (Eccles et al., 1984) and supports the idea that mastery experiences are particularly influential in the development of efficacy beliefs in general (Tschannen-Moran et al., 1998) and in mathematics in particular (Phelps, 2010; Usher, 2009). For the low scorers, the Leaving Certificate process had a significant negative impact on most participants and influenced their perceptions of mathematics and its teaching. Similarly, Bjerke and Eriksen (2016) found that primary prospective teachers were strongly influenced by their secondary schooling experiences, regardless of whether or not they had positive primary school experiences.

One concerning finding arising from this study relates to the possible influence of school experiences of mathematics, for those with high mathematics teaching efficacy, on subsequent practices in teaching mathematics. Considering that these participants reported liking mathematics, communicated their confidence as learners of mathematics, and stated that experiences of school influenced their beliefs, this may lead these teachers to feel that traditional approaches, within which they thrived as learners, are appropriate for all children and emulate these beliefs in their pedagogical practices (Stipek et al., 2001). Consequently, not only does this study provide valuable domain-specific insights (Klassen et al., 2011) into the mathematics efficacy beliefs of prospective teachers and the factors that influence them; the interviews with high efficacy participants suggest that these same factors (i.e., success in traditional teaching environments) may not be supportive of the implementation of desirable teaching practices in these future teachers' classrooms.

The study has some limitations. As participation in the study was voluntary, the sample may not be representative of the student cohort. However, 402 of the 475 students within the cohort volunteered to complete the SETcPM scale. Equally, despite using a purposive sample for interviews, efforts were made to select participants representing a range of SETcPM scores. In addition, given the recency of the development of the SETcPM scale, there were few studies that utilise this scale, leading us to draw comparisons with studies that have used different mathematics teacher efficacy scales; there are limitations to comparing the findings of this study with the results of these studies. However, this study makes a significant contribution to the literature, providing insights in the nature and sources of novice student teachers' mathematics teacher efficacy.

In conclusion, this study provides insights into entry-level prospective teachers' efficacy beliefs about teaching mathematics. This is valuable information about a group of prospective teachers who are at a stage in their teaching journey where efficacy beliefs are malleable. Through identification of their mathematics teacher efficacy beliefs at this early stage of teacher development, we can identify those with low efficacy beliefs and provide support prior to their school placement experiences, thus avoiding the trend reported in Charalambous et al., 2008, p.141) study where prospective teachers "entered and exited the course with alarmingly low beliefs". The qualitative insights arising from interviews with participants holding low and high efficacy beliefs provide signposts alerting us not only to the powerful factors that influence the development of efficacy beliefs but also to the implications that these beliefs may have on the mathematical teaching practices of these future teachers.

Appendix 1. SETcPM scale

A	Imagine that you are going to help a child with their homework. How confident are you that you can help the child with the tasks listed below? When the verb «explain» is used, you are asked to help the child to be able to explain. Otherwise, you are just asked to help the child answer correctly.	Not confident	Somewhat confident	Confident	Very confident
A1	"Explain why you must find the common denominator when you add two fractions."				
A2	"Calculate 750/25"				
A3	"Explain why you can expect the result of 31× 0.5 to be less than 31."				
A4	"Calculate 100 000/100"				
A5	"Explain why -1 is larger than -3."				
A6	"Circle the largest number: 1.34 or 1.234?"				
A7	"Explain why you always get an even number when multiplying an even and an odd number."				
A8	"Calculate -17 + 5"				
A9	"Calculate 23 × 0,7"				
A10	"Write $\frac{1}{5}$ as a decimal number."				
A11	"Calculate 342 – 238"				
A12	"Explain why you can disregard the zero in 4.320 when calculating 5.23+4.320"				
A13	"How many 10s are there in 1 million? Explain your answer. "				
A14	"Explain why, when subtracting, you can sometimes borrow from the place to the left."				
A15	"Calculate $\frac{2}{3}+\frac{1}{5}$"				
A16	"Explain why 0.3 is ten times larger than 0.03."				
A17	"Explain why division doesn't always make a number smaller."				
A18	"Circle the integer divisions without remainder: 92/2 105/2 $(108 \cdot 3)/2$"				
A19	"Calculate 4.14 + 3.190"				
A20	"Use a drawing to explain why $\frac{1}{8}$ equals 0.125."				

Appendix 2. Interview protocol

A: MATHEMATICS BELIEFS AND EXPERIENCES

When you hear the word 'mathematics' what do you think of?

Tell me about your experience of studying mathematics to date

- What was your experience of mathematics prior to college?
- Was this experience different at different levels of your education?

Describe a 'normal' mathematics lesson when you were in school

- Was this experience different at different levels of education? How?

What do you remember about the topics you covered? The materials you used?

Opportunities to be involved in group work? Opportunities to share your strategies? Your experiences of testing?

- Was this experience different at different levels of education? How?

What is your level of understanding of the primary mathematics concepts (e.g. number, algebra)? Why do you think this? What evidence do you draw from?

B: BELIEFS REGARDING MATHEMATICS TEACHING

What do you think teaching mathematics will involve?

What do you consider the main factors that affect a child's ability to learn mathematics?

What influences your beliefs? Give an example of an experience to support your beliefs

What role do you believe the teacher plays in a child's level of success in learning mathematics?

Do you think you could teach mathematics effectively?

What rating would you give yourself out of 10? Why did you give yourself this rating? Why do you feel like that?

Do you feel you can be an effective teacher in the future?

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