Acta Didactica Napocensia

Volume 16, Number 1, 2023 - DOI: 10.24193/adn.16.1.7

Preservice Elementary Teachers' Mental Computation Strategy Use in Multiplication on One- and Two-Digit Natural Numbers

Anders MÅNSSON

Abstract: It is investigated if the mental computation strategies in the research literature are enough to satisfactorily categorize the mental computation strategy use by preservice elementary teachers in multiplication on one- and two-digit natural numbers. The preservice elementary teachers use of mental computation strategies is measured operationally with a written questionnaire. The paper indicates that there are few new strategies used by preservice elementary teachers which are not already contained in the research literature. However, the paper indicates that there is a theoretical need to clarify the strategy definitions, and that there several useful and valuable strategies on multiplication that do not occur in the research literature on mental computation.

Key words: mental computation strategy, multiplication, preservice elementary teacher, written questionnaire.

1. Introduction

Mental computation is part of the elementary school curricular content globally, and in recent years there have in many countries been an increased focus on mental computation in elementary school teacher education (Csíkos, 2016; Hartnett, 2007; Lemonidis et al, 2014). There are many advantages of becoming better at mental computation, for example that it improves number sense (Hajra & Kofman, 2017; Heirdsfield et al, 2002, 2011), it gives a better understanding of the place value system and elementary calculation rules (Gürbüz & Erdem, 2016; Maclellan, 2001; Reys, 1984, 1992; Sowder, 1990, 1992, 1994), and it is often involved in everyday use of mathematics (Baranyai et al, 2019a; Thompson, 2010). Mental computation strategies in connection to elementary school pupils have been well researched, but less so in connection to preservice elementary teachers (PETs). In this article the latter group's strategy use will be investigated. A brief background to mental computation is given in sections 2.1−2.4 before stating the research question in section 3.1.

2. Mental computation

2. 1. What is mental computation?

One can in the research literature find different definitions of mental computation, or synonyms such as mental calculation, mental math, and mental arithmetic (Lemonidis, 2016; Sowder, 1988; Thompson, 1999). An overarching trait among the definitions, and the one that will be used in this paper, is calculating without use of any equipment (Baranyai et al, 2019b; Lopez, 2014; McIntosh & Dole, 2000; Reys et al, 1995).

2. 2. What is a mental computation strategy?

Mental computation strategies are different ways that arithmetic problems are solved mentally (Hartnett, 2007; Threlfall, 2000, 2002). For example, one can calculate 6×23 as $6 \times 20 + 6 \times 3 = 120 + 18 = 138$, which is a mental computation strategy commonly referred to as "the distributive rule". Some strategies are more general like this, and others are more dependent on the numbers involved in the calculation such that multiplication by $2, 4, 8, \ldots$ can be done by repeated doubling. Mental computation strategies elementary school pupils use to calculate with natural numbers up to 100 have been well researched and documented for addition and subtraction (Beishuizen, 1993; Blöte et al, 2000; Heirdsfield, 1997, 2001; Klein et al, 1998). Some of these strategies have official names, such as SA

Received November 2022.

Cite as: Månsson, A. (2023). Preservice elementary teachers' mental computation strategy use in multiplication on one- and two-digit natural numbers. *Acta Didactica Napocensia,* 16(1), 99-108, https://doi.org/10.24193/adn.16.1.7

(standard algorithm done mentally), 1010 (separately adding tens and ones), N10 (stringing), N10C (stringing with compensation), A10 (bridging through multiples of ten), and B (balancing). There exist variants of these definitions and their names, but generally there is consensus on what they should mean (Heirdsfield, 2004; Varol & Farran, 2007). Elementary school pupils' use of mental computation strategies for multiplication and division has been researched to a lesser degree (Callingham, 2005; Heirdsfield et al, 1999; Murray et al, 1994; Oliver et al, 1991). There are few studies on PETs' mental computation strategy use, in addition and subtraction (Baranyai et al, 2019a; Hajra & Kofman, 2017; Månsson, 2022; Whitacre, 2007), multiplication (Baranyai et al, 2019a; Lemonidis et al, 2014; Whitacre, 2007), and division (Mutawah, 2016).

2. 3. Why learn mental computation strategies?

To do mental computation efficiently, one need to learn several different strategies and know when to use which strategy (Hajra & Kofman, 2017; McIntosh, 2003). Many mental computation strategies are possible for pupils to discover on their own, but one cannot presume that all pupils will be able to do so (Murphy, 2004). There is evidence to suggest that pupils are often not directly exposed to mental computation strategies in school but are rather left to themselves to devise strategies. Some pupils then get stuck in unwieldly mental computation strategies, such as doing the standard algorithm mentally, and therefore need to learn more efficient strategies in an organized and systematic way (Askew, 1997; Baranyai et al, 2019b; Hajra & Kofman, 2017; Joung, 2018; McIntosh et al, 1995). Hope and Sherrill (1987) highlight that pupils with less developed skills mostly use standard written methods for mental computation, while pupils with higher developed skills use a variety of mental strategies. Thompson (2009) stresses the importance of teaching and using mental calculation strategies, since the traditional methods are not effective enough to improve pupils' numeracy proficiency.

2. 4. Why should preservice elementary teachers study mental computation strategies?

Even though mental strategies are a desired focus for computational instruction in schools, Hartnett (2007) suggests that teachers have been slow to adopt such a focus in their classroom, and that a possible barrier to adopting a mental strategies approach is the teachers' own lack of knowledge about possible mental computation strategies. Since PETs are the next generation of teachers, it is important that they know and master mental computation strategies. They need a strong foundation of the mathematics of mental computation and the ability to apply this important calculation method as well as use efficient strategies of their own (Heirdsfield & Cooper, 2004; Lemonidis et al, 2014; Threlfall, 2002). Mental mathematics ability is considered a hallmark of number sense (Hajra & Kofman, 2017; Sowder, 1992), and good number sense is especially essential for elementary school teachers. Without it they are ill-equipped to make sense and take advantage of children's often unorthodox but very number sensible solution strategies (Whitacre, 2007).

3. Method

3. 1. Purpose of article and research question

In the light of section 2.1−2.4, it is proposed that it is important to know the current knowledge and proficiency base of PETs on mental computation. Knowing which strategies PETs are aware of and use, provides valuable information for continuing professional development and improving teacher content knowledge on mental computation, and for use in related research (Heirdsfield & Lamb, 2005; Valenta & Enge, 2013). In this article a written questionnaire is utilized to conduct research into PETs' mental computation strategy use. A written questionnaire has an advantage over interviews when it comes to gathering a large amount of data. One disadvantage is if some PETs calculate the exercises using pen and paper even though they are instructed not to do so. In Månsson (2022) it was demonstrated that using written questionnaires to survey and categorize PETs' strategy use can be a valuable and reliable method. This paper is based on the premise that to conduct research into PETs' strategy use it is important to first determine if the existing research literature in fact covers the strategies used by PETs. Further, when conducting research on PETs' strategy use it helps if different research papers use the same list of strategies. A search through the research literature reveals that there is no article containing an exhaustive and complete list of all strategies occurring in the research literature. The categories vary, and they do not always account for all possible strategies. Also, since most research on mental computation have been done with elementary school pupils it is not obvious that the mental strategies considered in research in connection with pupils are the same as those used by PETs. Since PETs have more schooling in mathematics than elementary school pupils it is possible that PETs use a different set of strategies than the pupils do. The availability and ability of PETs to participate in written questionnaires on mental computation also makes them suitable research participants. A relevant research question not emphasized in the research literature is therefore:

When preservice elementary teachers do mental computation in multiplication on one- and two-digit natural numbers, what strategies, beside those that can be found in the literature, do they draw on?

Depending on what the research reveals for the research question there could be a need to improve and extend the list of strategies in the research literature (presented in section 3.4). The answer to the research question is presented in section 3.6. Note that for practical reasons the investigation here is limited to one- and two-digit numbers. Two-digit numbers are more interesting than one-digit numbers when it comes to stimulating the development of number sense and insightful flexible number operations (Beishuizen et al, 1997; McIntosh et al, 1992), but a few one-digit numbers are included for completeness.

3. 2. Research participants

In 2022, a written mental computation strategy questionnaire was given to 271 first year PETs at a university in Norway. The PETs were chosen by availability, and they were not provided with any prior training on mental computation strategies prior to administering the questionnaire.

3. 3. Measures

The PETs' use of mental computation strategies was measured with a written questionnaire with the following 16 exercises (Figure 1):

1. 28×2	**2.** 3×19	**3.** 18×5	**4.** 9×35
5. 70×80	**6.** 22×8	**7.** 25×36	**8.** 15×15
9. 9×99	**10.** 21×40	**11.** 12×25	**12.** 15×16
13. 19×21	**14.** 14×35	**15.** 11×23	**16.** 19×19

Figure 1. *Questionnaire exercises*

The exercises were chosen so that many different strategies would be induced and used by the PETs. There is no general theory on how to find such exercises, and it is in the nature of things that is difficult to construct exercises inducing strategies that one is not aware of. However, the ambition was to construct exercises that at least could induce the strategies in Table 1. Figure 2 shows the questionnaire instructions and how each exercise was presented to the PETs.

Important instructions! For every exercise do these steps in order:

1. **Calculate the exercise in your head.** In this step you are not allowed to write anything.
2. **Write down the answer.**
3. **Explain mathematically how you were thinking when you calculated the exercise in your head.**

Exercise 1

$28 \times 2 =$ ______

Explanation:____

Figure 2. *Questionnaire instructions together with first exercise*

The PETs' written explanations of their strategy use were categorized by the author by referring to the list of mental computation strategies in section 3.4. This was demonstrated in Månsson (2022) to be an informative and reliable approach. It is an operational method, basing the categorization solely on the PETs' own written explanations, and not speculating on how they were "really thinking" when they calculated.

3. 4. List of mental computation strategies

Table 1 lists the mental computation strategies found from an exhaustive search in the research literature. This list was used in the categorization of the PETs' explanations. After each strategy a reference to where the definition can be found in the research literature is provided. If several articles agreed on a definition of a strategy, then only one of those articles were referenced. The strategies have been named after the most common and established name occurring in the literature. Strategies identified as being similar are placed together in the table, but other than that they are placed in no order.

Table 1. *Mental computation strategies for multiplication on one- and two-digit natural numbers*

Strategy	Definition	Source
Memory	Directly recall from memory of a known multiplication numerical fact or a production of a numerical fact. $6 \times 11 = 66$ or $4 \times 12 = 48$	(Lemonidis et al, 2014)
Basic fact or derived fact	Using a known or derived multiplication fact. $5 \times 9 = 45$ so $50 \times 9 = 450$ or $50 \times 90 = 4500$	(Hall, 2019)
Commutative rule	$3 \times 4 = 4 \times 3$	(Anghilieri, 1999)
Counting	4×15: $15, 30, 45, 60$	(Lemonidis et al, 2014)
Repeated addition	$3 \times 24 = 24 + 24 + 24 = 48 + 24 = 72$ $3 \times 24 = 2 \times 24 + 24 = 48 + 24 = 72$ $31 \times 3 = (30 + 30 + 30) + 3$	(Hall, 2019) (Lucangeli et al, 2003)
Mental analogue of standard algorithm	Doing the standard algorithm mentally: 7×23: $7 \times 3 = 21$ write 1 and carry 2, $7 \times 2 = 14$, $14 + 2 = 16$, thus the result is 161	(Whitacre, 2007)
Basic fact shortcuts	$\times 5 = 10 \times 1/2$ $\times 9 = \times 10 - \times 1$ $\times 11 = \times 10 + \times 1$ $\times 12 = \times 10 +$ double $\times 15 = \times 10 + \times 5$ $\times 25 = \times 100 \div 4$	(Wigley, 1996) (Hall, 2019) (Whitacre, 2007)
Distributive rule	$14 \times 3 = (10 + 4) \times 3 = 10 \times 3 + 4 \times 3 = 30 + 12 = 42$ $17 \times 23 = 10 \times 20 + 10 \times 3 + 7 \times 20 + 7 \times 3$	(Thompson, 2008) (Day & Hurrell, 2015)
Distributive rule units first	7×23: $7 \times 3 = 21$, $7 \times 20 = 140$, $21 + 140 = 161$	(Hall, 2019)
Distributive rule non-standard	$21 \times 23 = 11 \times 23 + 10 \times 23$ $46 \times 7 \rightarrow 40 \times 10 = 400 \rightarrow 40 \times 3 = 120 \rightarrow 400 - 120 = 280 \rightarrow 6 \times 7 = 42 \rightarrow 280 + 42 = 322$	(Cooper et al, 1996) (Erdem, 2017)
Distributive rule with compensation	$7 \times 19 = 7 \times 20 - 7 \times 1$	(Baranyai et al, 2019b)
Factorization	$15 \times 12 = (15 \times 4) \times 3 = 60 \times 3 = 180$	(Ding et al, 2017)
Doubling	$8 \times 5 = 4 \times 10 = 2 \times 20 = 40$ $25 \times 72 = 50 \times 36 = 100 \times 18 = 1800$	(Wigley, 1996) (Baranyai et al, 2019b)
Near doubles	$3 \times 5 = (2 + 1) \times 5 = 2 \times 5 + 5$ $5 \times 7 = 2 \times (2 \times 7) + 7$. So, double 7, double 14, then add 7	(McIntosh & Dole, 2005) (Heirdsfield et al, 1999)
Trial and error	25×19: Notices that $25 \times 4 = 100$. Since $4 \times 4 = 16$ (which is close to 19) one has that $25 \times 16 =$	(Heirdsfield et al, 1999)

	$25 \times 4 \times 4 = 100 \times 4 = 400.$ Remains 3, so add $25 \times 3 = 75$, giving $400 + 75 = 475$.	
Combination of strategies	Using two or more strategies. $416 \times 25 = 416 \times 2 \times 10 + (416 \times 10) \div 2$	(Baranyai et al, 2019b)

3. 5. Data collection

The questionnaire was administered as part of a mathematics lecture at the university. The PETs were not informed beforehand that they would take a questionnaire, so they had no way of preparing for it. The PETs' participation in the questionnaire were voluntary, anonymous, and by checking in a box on the test they gave their consent or dissent to that the test results were used anonymously in research purposes. There was no time limit to the test. The PETs were instructed to calculate each exercise mentally, write down the answer, and then write an explanation on how they were thinking when they solved the exercise (see Figure 2). The author categorized their explanations according to which mental computation strategy in Table 1 they used (if any).

3. 6. Results

To present the percent distribution of the PETs' strategy use in a clear way similar strategies are grouped together. *Memory* and *Basic fact or derived fact* are grouped as "Recall". *Counting* and *Repeated addition* are grouped as "Count". The four distributive strategies (*Distributive…*) are grouped as "Distributive". *Doubling* and *Near doubles* are grouped as "Double". (*Commutative rule* and *Combination of strategies* are not considered as strategies in this case since they are usually only part of other strategies.) Then, 59 % of the students used a Distributive strategy, 8 % used Recall, followed by 5 % *Mental analogue of standard algorithm*, 4 % *Factorization*, 3 % *Trial and error*, 3 % Count, 1 % *Basic fact shortcuts*, 0 % Double. The average success rate was 85 %, and 2 % gave an unclear explanation that was not possible to categorize unambiguously. Using a distributive strategy was also the most used strategy for all exercises except exercise 5 (70×80), where Recall by 93 % of the PETs. The dominant strategy was there to calculate 7×8 and add on two zeroes. The second most used strategy (with some ties) was *Mental analogue of standard algorithm* in exercises 4, 6, 9, 13, 15; Count in exercises 1, 2, 14; *Trial and error* in 8, 13, 14; *Factorization* in 7, 10, 11; Recall in exercise 12. Success rates varied from highest about 98 % in exercises 1 and 5, to lowest about 65 % in exercise 7 and 16.

Below are presented instances where the PETs used strategies that deviated from the list of strategies in section 2.4 are presented and commented on, thereby answering the research question in section 2.1.

3.6.1. One of the numbers ending with a zero. A common PET explanation to exercise 5 (70×80) is to do the multiplication 7×8 and then attach two zeroes to the answer, which is the strategy *Basic fact or derived fact* (or *Factorization*). However, in the following PET explanation to exercise 10 (21×40): "$21 \times 4 = 84,\ 84 \times 10 = 840$" it is unclear if $21 \times 4 = 84$ qualifies as a "basic fact or derived fact". The definition of the strategy *Basic fact or derived fact* is not clear on this, so there is a theoretical need to clarify or redefine this strategy.

3.6.2. Variant of Repeated addition. Exercise 2 (3×19) was calculated by a PET as "$20 + 20 + 20 =$ answer minus 3". This is a *Repeated addition* but with compensation. One could consider this as a new strategy, but it can also be considered as a variant of *Repeated addition*. In the latter case, it could be added as a defining example to *Repeated addition*. The explanation could also be considered as a *Combination of the strategies*, a combination of the strategies *Distributive rule with compensation* and *Repeated addition*.

3.6.3. Variants of Distributive rule. Exercise 15 (11×23) was calculated by a PET as "$11 \times 10 + 11 \times 10 + 11 \times 3$". It could be regarded as *Distributive rule non-standard*, or as a combination of strategies, for example using the *Distributive rule* twice. It could also be regarded as *Trial and error*, or as a variant of *Repeated addition*. In any case, the core idea in the explanation is using the distributive rule, so it is not a completely new strategy.

3.6.4. Overlap of strategies and imprecise strategy definitions. Several PETs noticed that exercise 12 (15×16) could be solved by looking back at exercise 8 (15×15) and add 15". As another example, one PET explained exercise 3 (18×5) as "Thought of $9 \times 5 = 45$, then I multiplied $45 \times 2 = 90$". Both explanations could be categorized as *Basic fact or derived fact*, or as *Memory*, and the second explanation could also be categorized as *Doubling* or *Factorization*. The strategy definitions in the literature are not precise enough to unambiguously differentiate and categorize explanations as strategies in these cases.

3.6.5. Combinations of strategies. The *Commutative rule* strategy seldom occurs alone, but usually and implicitly in combination with other strategies. For example, one PET calculated exercise 2 (18×5) as "$5 \times 10 + 5 \times 8$". Clearly the commutative rule has been used here in combination with the distributive rule. If combinations of strategies in Table 1 should be considered as unique strategies is questionable since that would give rise to many additional strategies.

3.6.6. Unreliable explanations. One PET explained most of his or her calculations as using the *Distributive rule* in the way defined by Day & Hurrell (2015): "$17 \times 23 = 10 \times 20 + 10 \times 3 + 7 \times 2 + 7 \times 3$". It is doubtful if this is how the PET thought since it is demanding on the memory to calculate in this way. One can suspect that the PET calculated in this way by writing down the calculation on paper even though being instructed not to do this. In any case, the explanation described above is a multiplication strategy that can be used, and it is already contained in the list, so it is not a new strategy.

3.6.7. Unclear strategy use. Some PETs gave the following explanation to exercise 13 (19×21): "$20 \times 20 - 1 = 399$". One can calculate in this way, but it is unclear which strategy the PETs have used. If they would have explained the calculation explicitly using the conjugate rule $(a - b) \times (a + b) = a^2 - b^2$ such as $20^2 - 1^2$ it would be a new strategy not included in Table 1.

4. Discussion and Conclusions

The results of this paper indicate that preservice elementary teachers (PETs) predominantly use distributive strategies in mental computational on multi-digit multiplication, where 59 % of the PETs used one of the four distributive strategies in Table 1. This is in close agreement with for example Baranyai (2019b) where 60 % of the PETs used distributive strategies. In Lemonidis (2014) most PETs used a mental analogue of a written algorithm, but also there a large part of the PETs used distributive strategies. One could however raise doubts to if PETs claiming to use mental analogue of standard algorithm really perform it purely mentally (i.e. without the use of any equipment) since that is demanding on the short-term memory.

Most of the core strategic ideas that preservice elementary teachers (PETs) use in mental computation are covered, or partly covered, by the research literature, but there are some exceptions, as presented in section 3.6. These are strategies or variants of strategies that could be added to Table 1, such as "Repeated addition with compensation", or using the conjugate rule as discussed in section 3.6.7. However, the main issue is that several of the strategy definitions in Table 1 are not clear and precise enough to enable unambiguous strategy categorizations. For example, as discussed in section 3.6.1, since the strategy *Basic fact or derived fact* is not limited to calculations involving zeroes, it could be a good idea to define this strategy of attaching zeroes as a new and separate strategy from *Basic fact or derived fact*. In some cases, the strategy definitions can be fixed by adding more defining examples to the strategies, but in other cases the strategies need to be reworked. The problem can in practical situations be avoided by grouping similar strategies, but theoretically seen it is not a completely satisfactory situation to have overlapping and unclearly defined strategies. There is thus a theoretical need to clarify the definitions in Table 1 to make them more precise and distinguishable.

Are there any strategies the PETs didn't use but could have arisen and that one could consider adding to Table 1? In fact, there are several. Some of them are more specialized, of more interest to so called "expert mental calculators" (that is persons trying to do mental calculations on multidigit numbers as quickly as possible), but there are those that could be useful for PETs and pupils. For example, as commented on in section 3.6.7, one can use the conjugate rule to do multiplications such as 19×21 or 38×42. This is a good way to introduce the conjugate rule, and it is a gateway to algebra. Rewriting the conjugate rule as $a^2 = (a-b)(a+b) + b^2$, here are two examples on how this rule can be used to effectively calculate squares mentally:

$$21^2 = (21-1) \times (21+1) + 1^2 = 20 \times 22 + 1 = 10 \times 44 + 1 = 440 + 1 = 441$$

$$17^2 = (17-3) \times (17+3) + 3^2 = 14 \times 20 + 9 = 28 \times 10 + 9 = 289$$

If one wants to calculate a square one can also use the quadratic rules $(a \pm b)^2 = a^2 \pm 2ab + b^2$. For example

$$21^2 = (20+1)^2 = 20^2 + 2 \times 20 \times 1 + 1^2 = 400 + 40 + 1 = 441$$

$$19^2 = (20-1)^2 = 20^2 - 2 \times 20 \times 1 + 1^2 = 400 - 40 + 1 = 361$$

It is straightforward to check that the following algebraic equivalence holds:

$$(x+a) \cdot (x+b) = x(x+a+b) + ab$$

This rule can be used when multiplying two numbers that are close together (such as in exercise 12 (15×16)). Here are two examples:

$$21 \cdot 23 = (20+1) \cdot (20+3) = 20 \cdot (20+1+3) + 1 \cdot 3 = 20 \cdot 24 + 3 = 480 + 3 = 483$$

$$103 \cdot 107 = 100 \cdot (100+3+7) + 3 \cdot 7 = 100 \cdot 110 + 21 = 11000 + 21 = 11021$$

Another multiplication strategy is when multiplying a two-digit number with 11. It can be done in following way (note the non-standard use of parenthesis here):

$$34 \cdot 11 = 3(3+4)4 = 374$$

That is, one inserts the sum of the two digits in between them. If the sum is ten or larger, one does it in the following way:

$$79 = 7(7+9)9 = 7(16)9 = (7+1)69 = 869$$

It is not surprising that these kinds of strategies did not occur among the PETs explanations, but since they are useful strategies for mental multiplication and give opportunities to use algebraic computational rules before going into algebra, one could consider adding them to the list of mental multiplication strategies so that they can been taught to PETs and pupils.

References

Anghileri, J. (1999). Issues in teaching multiplication and division. In I. Thompson (Ed.), *Issues in Teaching Numeracy in Primary Schools* (pp. 184-194). Buckingham, UK: Open Press University.

Askew, M. (1997). Mental methods of computation. *Association of teachers of mathematics*. MT160-09-97.

Baranyai, T., Egri E., Molnár, A. É. & Zsoldos-Marchis I. (2019a). Developing Preservice Primary School Teachers' Mental Computation Competency by Games. *Acta Didactica Napocensia*, 12 (1), 153-164.

Baranyai, T., Egri, E., Molnar, A. E. & Zsoldos-Marchis I. (2019b). Mental calculation strategies used by pre-service primary school teachers. 8717-8724. 10.21125/edulearn.2019.2167.

Beishuizen, M. (1993). Mental strategies and materials or models for addition and subtraction up to 100 in Dutch second grades. *Journal for Research in Mathematics Education*. *24*(4), 294-323.

Beishuizen, M., Van Putten, C.M., & Van Mulken, F. (1997). Mental arithmetic and strategy use with indirect number problems up to one hundred. *Learning and Instruction, 7*, 87-106.

Blöte, A. W., Klein, A. S, Beishuizen, M. (2000). Mental computation and conceptual understanding. *Learning and Instruction*, *10*, 221-247.

Callingham, R. (2005). A whole-school approach to developing mental computation strategies. In H. L. Chick et al. (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education, 2* (pp. 201-208). Melbourne: PME.

Cooper, T., Heirdsfield, A., & Irons, C. (1996). Children's mental strategies for addition and subtraction word problems. In J. Mulligan & M. Mitchelmore (Eds.), *Children's number learning* (pp.147-162). Adelaide, SA: Australian Association of Mathematics Teachers, Inc.

Csíkos, Csaba. (2016). Strategies and performance in elementary students' three-digit mental addition. *Educational Studies in Mathematics, 91*, 123-139.

Day, L., & Hurrell, D. (2015). An explanation for the use of arrays to promote the understanding of mental strategies for multiplication. *Australian Primary Mathematics Classroom, 20*(1), 20-23.

Ding, Y., Liu, R. D., Xu, L., Wang, J. & Zhang, D. (2017). Working memory load and automaticity in relation to mental multiplication, *The Journal of Educational Research*, 110:5, 554-564.

Erdem, E. (2017). Mental Computation: Evidence from Fifth Graders. *International Journal of Science and Mathematics Education*, 15, 1157-1174.

Gürbüz, R. & Erdem, E. (2016). Relationship between mental computation and mathematical reasoning, *Cogent Education*, *3*(1), 1-18.

Hajra, S. G., & Kofman, V. (2017). The effects of mental math strategies on pre-service teachers' self-awareness and computational skills. *Journal of Mathematics Education*, *10*(1), 92-108.

Hall, P. (2019). *There is more than one way to skin a cat: An exploration of flexible mental computation strategies with pre-service teachers*. Master in education research thesis. University of New England.

Hartnett, J. E. (2007). Categorization of mental computation strategies to support teaching and to encourage classroom dialogue. In J. Watson et al. (Eds.), *Proceedings 30th annual conference of the Mathematics Education Research Group of Australasia - Mathematics: Essential Research, Essential Practice* (pp. 345-352). Hobart, Tasmania.

Heirdsfield, A. M., & Cooper, T. J. (1997). The architecture of mental addition and subtraction. Paper presented at the annual conference of the *Australian Association of Research in Education*, Brisbane, Australia.

Heirdsfield, A. M., Cooper, T. J., Mulligan, J. & Irons, C. J. (1999). Children's mental multiplication and division strategies. In O. Zaslavsky (Ed.), *Proceedings of the 23rd Psychology of Mathematics Education Conference* (pp. 89-96). Haifa, Israel.

Heirdsfield, A.M. (2001). "Spontaneous" mental computation strategies. In N. A. Pateman et al. (Eds.), *Proceedings of the International Group for Psychology of Mathematics Education conference* (pp. 55-62). Honolulu, USA.

Heirdsfield, A. M. (2002). Inaccurate mental addition and subtraction: Causes and compensation. In B. Barton et al (Eds.), *Proceedings Mathematics Education in the South Pacific* (pp. 334-341), Auckland, NZ.

Heirdsfield, A. M. & Cooper, T. J. (2004). Factors affecting the process of proficient mental addition and subtraction: case studies of flexible and inflexible computers. *Journal of Mathematical Behavior, 23*, 443-463.

Heirdsfield, A. & Lamb, J. (2005). Mental computation: The benefits of informed teacher instruction. In P. Clarkson et al (Eds.), *Proceedings MERGA 28 - 2005 Building connections: Theory, research and practice, 2* (pp. 419-426). Melbourne, Australia.

Heirdsfield, A. (2011). Teaching mental computation strategies in early mathematics. *Young Children, 66*(2), 96-102.

Hope, J. A. & Sherill, J. M. (1987). Characteristics of unskilled and skilled mental calculators. *Journal for Research in Mathematics Education*, *18*(2), 98-111.

Joung, E. (2018). *A study of preservice teachers' mental computation attitudes, knowledge, and flexibility in thinking for teaching mathematics* (Doctoral dissertation, Southern Illinois university, Carbondale, USA, February 23, 2018). Retrieved from: https://opensiuc.lib.siu.edu/cgi/viewcontent.cgi?article=2518&context=dissertations

Klein, A. S., Beishuizen, M. & Treffers, A. (1998). The empty number line in Dutch second grades: Realistic versus gradual program design. *Journal for Research in Mathematics Education, 29*(4), 443-465.

Lemonidis, C., Tsakiridou, H., Panou, F. & Griva, E. (2014). Prospective teacher's efficiency and flexibility in prep and mental calculation of two-digit multiplications. *Journal of Educational Research, 12*, 110-125.

Lemonidis, C. (2016). *Mental computation and estimation: Implications for mathematics education research, teaching and learning*. Routledge, Oxon and New York.

Lopez, M. (2014). Development of working memory and performance in arithmetic: a longitudinal study with children. *Electronic Journal of Research in Educational Psychology*, *12*(1), 171-190.

Lucangeli, D., Tressoldi, P. E., Bendotti, M, Bonanomi, M., & Siegel, L. S. (2003). Effective strategies for mental and written arithmetic calculation from the third to the fifth grade, *Educational Psychology*, *23*(5), 507-520.

Maclellan, E. (2001). Mental Calculation. Its place in the development of numeracy. *Westminster Studies in Education*, *24*(2), 145-154.

McIntosh, A., Reys, B. J., & Reys, R. E. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics*, *12*, 2-44.

McIntosh, A. J., Bana, J. & Farrell, B. (1995). *Mental computation in school mathematics: Preference, attitude and performance of students in Years* 3, 5, 7 *and* 9. Perth: MASTEC, Edith Cowan University.

McIntosh, A. & Dole, S. (2000). Mental computation, number sense and general mathematics ability: Are they linked? In J. Bana et al. (Eds.), *Mathematics education beyond 2000* (pp. 401-408). Perth: MERGA.

McIntosh, A. (2003). Moving to mental: Seven steps along the computation path. In J. Burnett (Ed.), *Making sense of mental computation. Proceedings of a one-day mathematics conference for Australian teachers* (pp. 13-21). Brisbane: Origo Publications.

McIntosh, A., & Dole, S. (2005). *Mental computation: A strategies approach*. Hobart: Department of Education Tasmania.

Murphy, C. (2004). How do children come to use a taught mental calculation strategy? *Educational Studies in Mathematics, 56*(1), 3-18.

Murray, H., Olivier, A., & Human, P. (1994). Fifth graders' multi-digit multiplication and division strategies after five years' problem-centered learning. *International Group for the Psychology of Mathematics Education*, *18*(3), 399-406.

Mutawah, A. M. A. (2016). The Effect of Practicing Mental Calculation Strategies on Teacher Candidates' Numeracy Proficiency. *International Journal of Education and Literacy Studies 4*, 1-12.

Månsson, A. (2022). Categorization reliability of preservice elementary teachers' use of mental computation addition strategies on natural numbers using a written questionnaire. *Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)*, Feb 2022, Bolzano (en ligne), Italy.

Oliver, A., Murray, H., & Human, P. (1991). Children's solution strategies for division problems. In J. Ferrini-Mundy (Ed.), *Proceedings of the Sixteenth International Conference for the Psychology of Mathematics Education*, *2* (pp. 152-159). Durham, New Hampshire: PME.

Reys, R. E. (1984). Mental computation and estimation: Past, present, and future. *The elementary school journal*, *84*(5), 546-557.

Reys, R. E. (1992). Mental computation: Some ideas and directions for teaching. In C. Irons (Ed.), *Challenging children to think when they compute* (pp. 63–71). Brisbane: Centre for Mathematics and Science Education.

Reys, R. E., Reys, B. J., Nohda, N., & Emori, H. (1995). Mental computation performance and strategy use of Japanese students in grades 2, 4, 6, and 8. *Journal for Research in Mathematics Education*, *26*(4), 304-326.

Sowder, J. (1988). Mental computation and number comparison: their role in the development of number sense and computational estimation. In J. Hiebert et al. (Eds.), *Number concepts and operations in the middle grades* (pp. 182-197). Hillsdale: L.E.A.

Sowder, J. T. (1990). Mental computation and number sense. *Arithmetic Teacher*, *37*(7), 18-20.

Sowder, J. T. (1992). Making sense of numbers in school mathematics. In G. Leinhardt et al. (Eds.), *Analysis of arithmetic for mathematics teaching* (pp. 1-51). Hillsdale, NJ: L.E.A.

Sowder, J. (1994). Cognitive and metacognitive processes in mental computation and computational estimation. In R. Reys et al. (Eds.), *Computational alternatives for the twenty-first century*. Reston, VA: NCTM.

Thompson, I. (1999). Mental calculation strategies for addition and subtraction Part 1. *Mathematics in School, November*, 2–4.

Thompson, I. (2008). Deconstructing calculation methods, Part 3: multiplication. *Mathematics teaching, 206*, 34-36.

Thompson, I. (2009). Mental calculation, *Mathematics Teaching*, *213*, 40-42.

Thompson, I. (2010). *Issues in teaching numeracy in primary schools*. Open University Press McGraw-Hill.

Threlfall, J. (2000). Mental calculation strategies. *Research in Mathematics Education*, *2*(1), 77-90.

Threlfall, J. (2002). Flexibel mental calculation. *Educational Studies in Mathematics*, *50*, 29-47.

Valenta, A. & Enge, O. (2013). Student teachers' work on instructional explanations in multiplication – representations and conversions between them. *Nordic Studies in Mathematics Education*, *18*(1), 31-59.

Varol, F. & Farran, D. (2007). Elementary school students' mental computation proficiencies. *Early Childhood Education Journal*, *35*(1), 89-94.

Whitacre, I. (2007). Preservice teachers' number sensible mental computation strategies. *Conference paper: Proceedings of the Tenth Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education*. San Diego, California.

Wigley, A. (1996). Last chant for times tables. *Times Educational Supplement*. 24 May. Retrieved from https://www.tes.com/news/last-chant-times-tables

Authors

Anders Månsson, Oslo Metropolitan University, Oslo (Norway). E-mail: andersm@oslomet.no.