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Operational Hedging in a Mean Reverting Environment

A Real Option Approach

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Abstract

This thesis investigates the value of operational hedging in the form of being able to temporarily close the production of silicone. With the use of real option theory, we developed a switching option model that estimates the value of being able to switch between open and closed production. The underlying risk factor in the model is the power prices, which has a great influence on the profitability in the production. Being a commodity, we argue that the power price follows a mean reverting process due to its circumstances. By making use of theories based on the Ornstein-Uhlenbeck process, the possible movements of the underlying asset have been predicted. The valuation of the real option is done through the use of binomial trees, risk-neutral probabilities and backward induction. By accounting for switching costs as well as fixed and other costs that depend on whether the plant is opened or closed, our model reveals the optimal strategy of production. The model reveals that introducing the flexibility that follows from the real option adds considerable value to the plant. Finally, we perform a sensitivity analysis which shows that small changes in some of the underlying factors can have a significant impact on the overall value, which is consistent with option theory.

Preface

This thesis marks the completion of our master's degree in business and administration at Oslo Metropolitan University. The major of our degree is finance which has given us some interesting topics to research.

The motivation for our thesis is based on courses on risk management and derivatives that we had during our degree that both touched upon hedging and real option analysis. These are topics we found particularly interesting and wanted to examine further.
Moreover, the Nordic power market has experienced extreme spot prices as of late 2021. This makes our study particularly relevant for business decisions in the current market anomalies.

During our work on this thesis, we have gained valuable knowledge that will aid us in the challenges we will face in our future careers. Our experience with real option valuation will be useful when faced with a decision on whether a project is valuable or not. We have also honed our skills in MS Excel, which undoubtedly will be valuable in our professional life.

We would like to thank Terje Omland and Ole-Andreas Sandnes from Elkem ASA. They have contributed with valuable insight to the smelter industry and the power market, as well as provided us with applicable data for our analysis.

Finally, we would like to thank our supervisor Johann Reindl for guiding us in the right direction as well as sharing valuable experience and knowledge on the topic. His guidance has been invaluable in overcoming challenges we met during the study. This has undoubtedly improved the quality of our work.

Oslo, May 2022

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Erik Langseter Solberg Stian Mørk Madsen

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1 Introduction

1.1. Introduction

As of late 2021, the Nordic power prices began to fluctuate heavily. The prices more than quadrupled in some areas, leaving the consumers with a similar increase in their power costs. The high prices have continued into 2022, leaving the businesses with large power consumption in tough financial conditions. Big industry production facilities, especially smelters, need a great amount of power to reach the high temperatures needed in their melting-production. When electricity is their main source of input, a heavy increase in the power-prices can easily obliterate the margins from production. Ferroglobe PLC is a Spanish world-leading supplier of silicone metals and alloys. In late 2021, they decided to shut down several furnaces at their facilities to cope with the increasing power-prices in Spain (Ferroglobe, 2021). Continuation of production was simply not profitable with the current circumstances at the time. Another example is Rec Solar, a Norwegian-based company producing polysilicon for solar panels. Due to a greater conversion in their input-factors in recent years, the power consumption was completely unhedged and reliant on the spot price in the market. This led to their power cost being 617% higher than their estimate for December 2021 (Hovland, 2022). The excessive power prices at the time makes an interesting aspect of consideration in similar cases. If Ferroglobe and Rec Solar were dependent on maintaining a maximum production capacity in order to meet their delivery obligations, could it leave them in bankruptcy? The ability to shut down the furnaces can be crucial for these firms when the power prices reach unsustainable levels. Furthermore, this leaves us with the question of the option value to seize production in volatile times. In this thesis we will examine how valuable such an option is for a large power consuming firm. We will take inspiration from one of the largest smelters in Norway that faces similar challenges and create a real option model. The goal of the model is to investigate how much value is added to a firm when given the option to shut down production in volatile times.

1.1.1. Background

The smelter industry is one of the largest contributors to Norway's annual power consumption (Statistisk sentralbyrå [SSB], 2022). The extreme price fluctuations in the power prices in recent months has had a big impact on this industry. This applies also to

Elkem, one of the biggest actors in the Norwegian smelter industry. Elkem is a producer of various advanced material solutions for natural raw materials, with history dating back to 1904 (Elkem ASA, 2022d). The raw materials used in their production are both excavated from their own mines and purchased from other actors. By 2022, Elkem has more than 6,800 employees and 29 different production sites across the world, including several smelters that are located across Norway. They are mainly producing silicones, silicone products and carbon solutions (Elkem ASA, 2022a). Their smelters have a high rate of power consumption which greatly influences their profit margins. Elkem stands for nearly 4 TWh of Norway's total annual consumption that amounts to approximately 140 TWH. This is equivalent to nearly 3% of the total power consumption in Norway. A generally higher consumption rate of power in Norway in recent years, accompanied with bad weather-seasons have given record-high power prices. Higher demand and lower supply is the general reason for the recent development with highly volatile prices. Further in this case we take inspiration from one of Elkem’s smelters for evaluating the choices these industrial companies are facing, in times of uncertainty.

By using a real option valuation method, one can get a better estimate of the value with having the option to stop or limit the production when one input factor is reaching extreme values. This option is often neglected in traditional valuation methods. Knowing its value can make it easier to find the optimal decision in volatile times that minimise the risk and maximise the profits. In addition, it can give more information about optimal hedging strategies by buying futures or other financial derivatives, and to what extent.

1.1.2. Elkem Bremanger

Elkem Bremanger is a plant started in 1928 and is located on the west coast of Norway in Vestland county. Elkem Bremanger has two production facilities: one for foundry products and one for silicone materials. The foundry department develops various ferrosilicon (FeSi) products and inoculants. FeSi is an alloy of iron and silicone and is widely used in manufacturing steel products (Elkem ASA, 2022b).

The silicone department is currently producing two products: Silgrain and Microsilica. Silgrain is a powder that contains a minimum of 99% silicone. It is used to develop polysilicon which is mostly used in various electronic devices and solar panels (Elkem ASA, 2022e).
Elkem Microsilica is a by-product from the silgrain production. It is a mineral additive for concrete that increases the performance and durability for the concrete and mortar. Microsilica is commonly used in the construction of bridges, tunnels and skyscrapers that need high performing concrete. Elkem's Microsilica can be delivered both as slurry and powder (Elkem ASA, 2022c).

Elkem Bremanger belongs to el-spot area NO 3 (Trondheim) which on average has lower power prices than the el-spot areas further south in Norway NO 1, NO 2 and NO 5. This gives Elkem Bremanger a competitive advantage compared to other smelters located in areas with higher power prices.

2 The Power Market

2.1. The Norwegian and Nordic Power Market

In 1991 the Norwegian power market changed from being regulated by the state, to being completely market driven. The power is today traded at a power-exchange called Nordpool, the successor of the original Norwegian power-exchange, Kraftbørsen Statnett Marked AS (Energifakta Norge, 2022). Nordpool has a licence from the Norwegian Water Resources and Energy Directorate, abbreviated NVE, to facilitate a marketplace for trading of power (Nordpool, n.d.a). The exchange started to expand to the Scandinavian countries in the late 1990s, and later expanded into the Baltic countries (Nordpool, n.d.b). Today Nordpool is the leading power market in Europe and offers their services in 16 European countries, including UK, Germany, and France (Nordpool, n.d.a). Nordpool offers efficient and liquid intraday and day-ahead markets. Unlike financial exchanges, Nordpool delivers actual assets (power) that are delivered upon purchase. Therefore, there is not much room to speculate on price-changes in the power-prices further than a day ahead in time. In the day-ahead market the hourly spot-prices for the next 24 hours are calculated daily at 12 p.m. (CET) (Nordpool, n.d.c). The prices are based on bid- and ask prices from suppliers and demanders of the

power. Since the market is freely regulated, Nordpool does not have a monopoly for trading power. Nevertheless, more than 90% of the power is purchased through Nordpool in the Nordic countries.
For the intraday market one can purchase power up until 1 hour before delivery (Brenna, 2018). Besides being heavily influenced by the demand for power, the supply is highly correlated to the weather conditions. In Norway, hydropower accounts for more than 90% of the total power production (Statkraft, n.d.). The hydropower plants use reservoirs to store water when there is an overload, and demand is low. This opportunity provides flexibility in adjusting the power production based on seasons, demand, and weather-conditions. In rainy periods, the water reservoirs will fill up and the power prices will naturally drop, whereas the opposite is true for dry periods. The prices increase during the winter period when more power is needed for heating (Fortum, 2015). The power consumption in the industry, however, tends to be stable across the seasons and unaffected by the weather conditions. Consequently, the firms will normally have higher production costs during the winter season, and opposite during the summer season. This is very much the case with Elkem, who is a large consumer of power.

The prices on Nordpool also vary between different areas, depending on where the power will be delivered. In Norway there are 5 el-spot areas which operate with different prices. (NVE, n.d.) These areas are illustrated in figure 2.1:

Figure 2.1 El-spot Areas in Norway

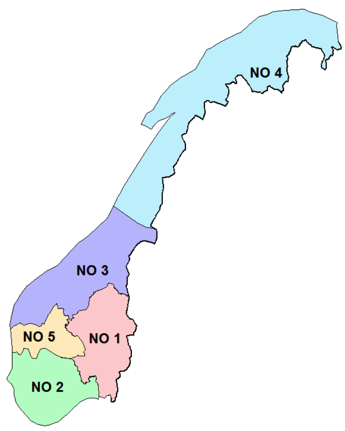


Figure 2.1. Elspot Areas in Norway as of March 2016, n.d., by NVE. (elspotområder-7-mars-2016_ny.bmp).

The difference in areas is because there is not a completely free flow of power without extra costs that occur from transportation limitations. Even though the power is mainly produced and consumed in one area, there is some exchange between the different el-spot areas. The amount of power that can be transported between the different areas is often limited by the capacity of the power lines between them. Areas with a power surplus and low prices will export some of their power to areas with a power deficit and high prices (Nordpool, n.d.c). Hence, the prices will slightly balance each other out between the areas.
Figure 2.2 illustrates the physical exchange of power between the Nordic areas and their neighbouring countries on January 1st, 2022 (Nordpool, 2022).

Figure 2.2 Physical Exchange of Power in Northern Europe

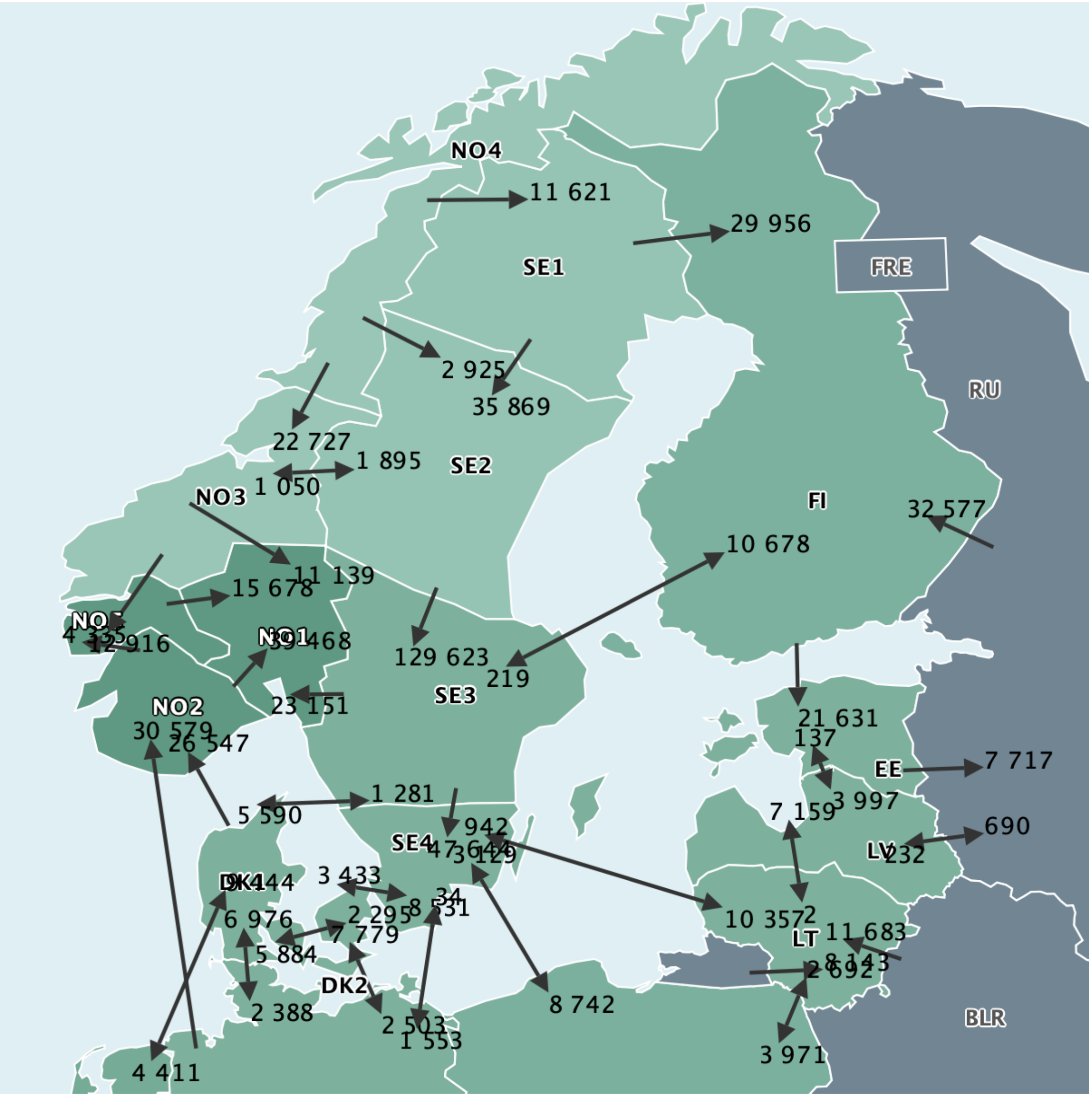


Figure 2.2 shows the physical deliverance and exchange of power in Northern Europe as of January 1st, 2022, 2022, by Nordpool. (https://www.nordpoolgroup.com/en/Market-data1/#/nordic/map)

2.1.1. Historical Development in the Norwegian Power Prices and the Nordic System Price

Historic monthly power prices in the 5 el-spot areas in Norway and the Nordic system price are visualised in figure 2.3.

Figure 2.3 Monthly Power Prices

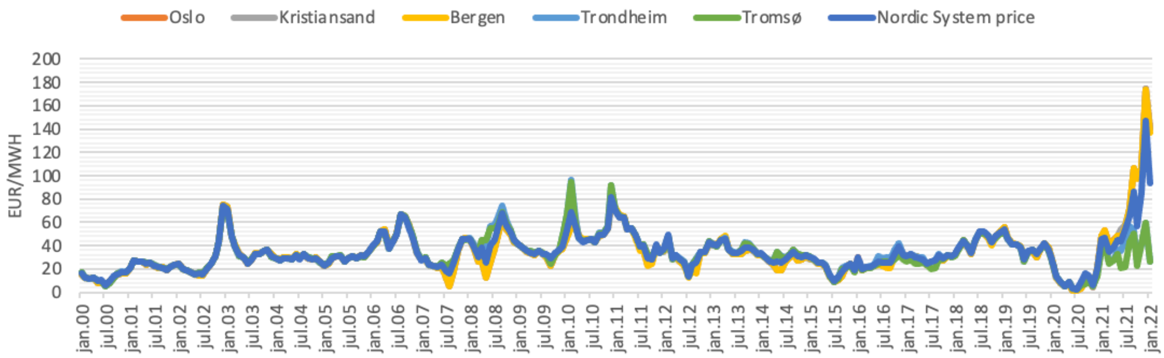


Figure 2.3 shows the monthly power prices in the different areas in Norway, as well as the Nordic System price in the period January 2000 – January 2022. Data is obtained from Nordpool, 2022. (https://www.nordpoolgroup.com/historical-market-data/).

Historically the system price has closely followed the Norwegian area-specific prices. From 2021 and onwards there has been a greater difference in the prices between the different el-spot areas. This has made the system price differ more from the area-specific prices as well. The power prices have generally been affected by high volatility in recent years. Going from record-low prices in 2020 to record-high prices in late 2021. Historically, the prices seem to stabilise themselves around a long-term mean, after price shocks like these. The average Nordic system price from 2000-2021 has been €34.16/MWh (Nordpool, 2022b).

2.2. Options in the Power Market

In order to speculate or hedge in future power prices, one can trade power securities either over-the-counter (OTC) or on a financial exchange. Nasdaq is providing a financial exchange for trading power-futures and other power-derivatives. There are several ways of trading and hedging power exposure in the open market using derivatives, which will be discussed in the next chapter.

3 Theory

3.1. Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) was introduced by William Sharpe (1964) and John Lintner (1965) and explains that all investments must follow the security market line (SML) in a competitive market.

Figure 3.1 Security Market Line

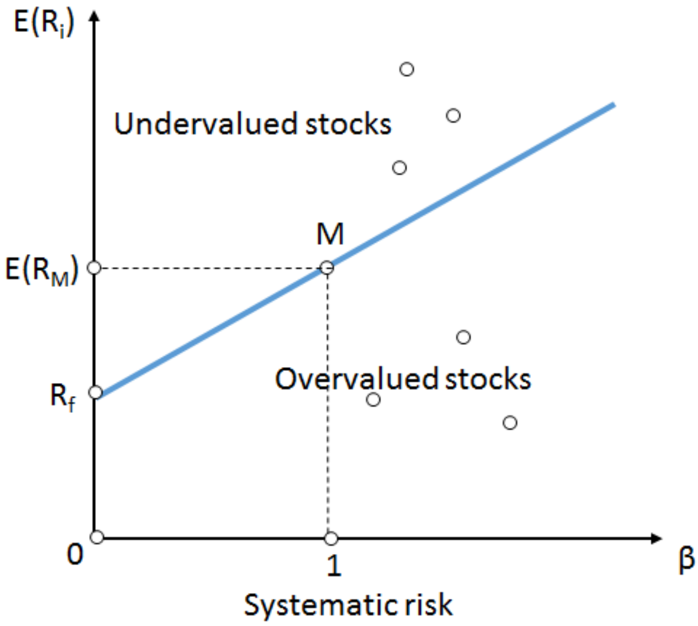


Figure 3.1 The Security Market Line shows the combination of risk and return that all investments must have in a competitive market. All stocks below the SML are overvalued and all stocks above the SML are undervalued, 2013, by Lamro. (https://en.wikipedia.org/wiki/Security_market_line#/media/File:SML-chart.png)

The model states that if all investors can lend or borrow money at the risk-free interest rate, one portfolio that grants the highest risk premium to volatility will be better than all the others. Since the investors can borrow or lend at the risk-free rate, they will adjust the weight in the optimal portfolio and risk-free rate, accordingly, making the portfolio optimal for all investors regardless of risk aversion. If the market is efficient and all investors have the same information, then everyone should hold the market portfolio.

Furthermore, we should not consider individual assets in isolation, but rather its contribution to the portfolio risk. The contribution depends on the assets sensitivity to changes in the value of the optimal portfolio, that is the market portfolio.

The CAPM is given by:

$$E(R_i) = R_f + \beta_i[E(R_M) - R_f] \qquad (3.1)$$

Where $E(R_i)$ is the expected rate of return, R_f is the risk-free rate and β_i is the asset's sensitivity to changes in the market portfolio.
The CAPM can be used to find the required rate of return on a stock for an investor, as well as the discount rate for new capital investment (Brealey et al., p. 207).

3.1.1 Beta

The risk of an asset can be split into two categories, systematic and unsystematic risk. The unsystematic risk is specific to the company or the industry. That is, it does not follow the movements in the broader market. The systematic risk on the other hand, explains exactly that. Events that affect the market, typically also affect individual assets. This effect is captured by the systematic risk of the asset and can be measured by its beta. The beta is found by running a regression on the assets return against the market return. The slope of a line through the data points in the regression represent the asset beta. Hence, it captures how much the asset follows the trend of the market.
A beta of 1 indicates that the asset is strongly correlated with the market, and that there is no unsystematic risk. That is, if the market moves, the asset will follow. If the beta is lower than 1, the asset will be less affected by movements in the market, whereas a beta of more than 1 indicates that movements in the market will have a larger impact on the individual asset (Brealey et al., p. 186).

3.1.2. Risk-free Rate

The risk-free rate represents the risk-free return investors can get in the market. For an asset to be risk-free, two conditions must be met:

1. There must be no default risk
2. The investment's actual return must equal its expected return, meaning there must be no reinvestment risk (Damodaran, 2012, p. 154-155).

3.1.3. Market Risk Premium

The market risk premium (MRP) is the excess return above the risk-free rate, required by investors for their investments in the market (Damodaran, 2012, p. 159). According to the CAPM, the risk premium will follow the SML visualised in figure 3.1 and vary depending on the beta of the investment or asset (Brealey et al., 2012, p. 205).

3.2. Options

An option is a financial instrument that gives the holder the right, but not an obligation to trade the underlying asset at a certain date for a specified price. There are two types of options; call option, which gives the holder the right to buy- and a put option which gives the holder the right to sell the underlying asset. The specified price that the underlying can be traded for is known as *exercise price* or *strike price*, and the date at which the option expires is known as *expiration date* or *maturity*. Options differ from forward- and futures contracts as the owner of the option has the right but not the obligation to buy or sell the underlying asset at expiration. Therefore, there is also a cost associated with acquiring an option, in contrast to a futures- or forward contract (Hull, 2018, p. 30-31).

We distinguish between two types of options: European and American. The European option can only be exercised at the date of expiration, whereas the American option can be exercised at any time up to the expiration date (Hull, 2018, p. 31). Given the nature of the options there are four different positions you can enter, as given in figure 3.2. However, these four positions can be combined to create numerous different strategies with unique payoffs (Hull, 2018, p. 288).

Figure 3.2 Payoff from Different Options

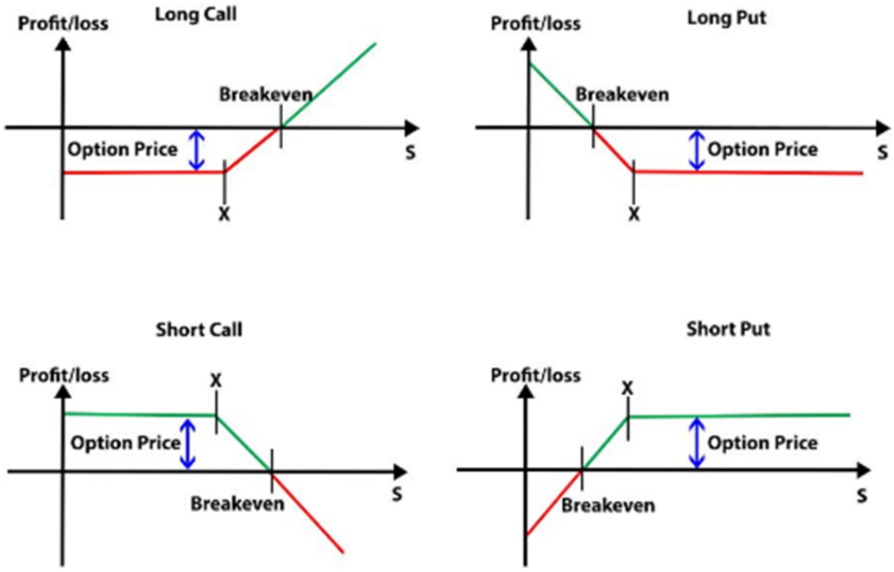


Figure 3.2 Payoff from long and short positions in call and put option, 2021, by Quantra. (https://quantra.quantinsti.com/glossary/Option-Payoff)

The value of the option is affected by six factors (Hull, 2018, p. 257):

Table 3.1 Six Factors Influencing Option Value

Increase in:		**Change in call value**	**Change in put value**
Asset price	S_T	+	-
Strike price	K	-	+
Time to maturity	T	+*	+*
Volatility of the underlying	σ	+	+
Risk free rate	r_f	+**	-**
Expected dividends		-	+

*Uncertain relationship, as this is always the case with American options, but not with European options

**Theoretical change, assuming that all other factors remain the same.

Considering a long position in a call option to see the relationship of the six factors (Hull, 2018, p. 257-259). The payoff at maturity is Max$(S_T - K, 0)$.

- **Asset price:** As the spot price of the asset increases, the payoff of exercising the option is higher, thus increasing the value of the option.
- **Strike price:** A higher strike price implies that the price of the underlying will have to increase more for the option to be valuable.
- **Time to maturity**: A longer time to maturity gives the holder more exercise opportunities than that of an option with a shorter time to maturity (for American options, that is) and therefore the value must be higher. For European options this assumption may not hold when adding dividends to the underlying.
- **Volatility**: measures the uncertainty of the future spot price. As the holder of the call will benefit from price increases, but has limited downside, the value of the option increases as the uncertainty of the future spot price increases.
- **Risk-free rate:** An increase in interest rates would increase the required rate of return for the investor and decrease the present value of cash flows. This increases the value of the call option. However, this is a theoretical change considering that all other factors are unchanged. In practice, increased interest rate would decrease the spot price and potentially decrease the value of the call option.
- **Dividends:** If the underlying stock pays dividend during the lifetime of the option, the call value will suffer as the stock price tends to decrease on the ex-dividend date.

3.3. Real Options

Myers (1977) introduced the term “real options” in his paper “Determinants of Corporate Borrowing”, where it referred to the options related to “real assets”. Besides discussing the so-called “debt-overhang problem”, Myers showed how to value non-financial assets with financial option-theory.

A real option, much like a financial option, gives the owner the right but not the obligation to decide based on a predetermined price. It differs from financial options in that they involve real physical assets, rather than securities that are traded on an exchange or over the counter (Hull, 2018, p. 857).

The traditional way of valuing a project is the net present value (NPV) approach. It values a project by discounting the future expected cash flows of a project by the required rate of return, often found by using the CAPM. If the NPV of the expected cash flows adds value to the firm, the project is undertaken. In this method, management is considered passive as there will be no changes in the project during its lifetime.
However, in practice there are often situations where the management can make decisions during the lifetime of the project that are crucial to the project's expected cash flows. In some cases, these embedded options are known before the project is undertaken and the value of real options can be used to value the project more accurately (Hull, 2018, p. 814-815).
The books “Investment under uncertainty” by Dixit and Pindyck (1994) and “Real options” by Trigeorgis (1996), both greatly contributed to the valuation of real options. They discarded the traditional NPV rule that dominated the investment-theories at the time. Instead, they focused on the various options that occur during an investment and how these will affect the valuation and decision-making process. In an economic environment with many uncertainties, being flexible to new information can have great influence in the investment decisions and valuation.

There are several approaches to value real options. Using Monte Carlo simulation for valuation of European options was introduced by Boyle (1977). Monte Carlo simulation is appropriate when there are one or more underlying market variables affecting the payoff. By making the option path-dependent, one can make Monte Carlo simulation suitable for American options as well. This approach was first developed by Tilley (1993). The usage of Monte Carlo simulation has later been developed to be more flexible and applicable to different situations. New and better computational programs have made it easier and quicker to solve more complex and numerous calculations required in a Monte Carlo simulation.

Longstaff and Schwartz (2001) introduced the usage of least square simulations for estimating expected payoff from continuation for American real options. Another simulation approach for American options is the Exercise Boundary Parameterization Approach, developed by Andersen (2000). Models involving random jumps can also be implemented into a Monte Carlo simulation, based on Merton’s Mixed Jump-Diffusion Model from 1976. Merton (1976) showed how to price European currency options which included discontinuous jumps and continuous changes. The theory from Merton’s model has later been adapted to be applicable in Monte Carlo simulation, where the jumps are sampled from random numbers generated via the simulation (Hull, 2018, p. 649-650).

In contrast to European options, valuation of American options cannot be solved analytically. Binomial trees are often used for valuing American options. The tree illustrates the life of the option, which is divided into several time intervals and paths based on the price of the underlying asset. Cox et al. (1979) greatly set the base for using binomial trees for option valuation where early exercise may be optimal. The trees can be constructed into being trinomial and numerous other paths, depending on the complexity of the option. The flexibility in path-dependent option trees is great and provides endless possible combinations and setups. “Real Options in theory and practice” from Gutherie (2009) further describes the possibilities available for using trees in option-valuations.

3.3.1. Real Options in the Smelter Industry

Real options can come in countless forms. One form can include an option to expand if the demand exceeds expectations. Another form is the possibility to defer or extend the life of a project depending on the current and expected payoffs. If a project is not as profitable as expected, it can be sold or closed and liquidated. This is known as an abandonment option (Hull, 2018, p. 818-820). A common type of real option is a switching option which can include the possibility to switch between various types of production, depending on the profitability from each product. A switching option can also be the option to alternate between two or more states.

In the case of a smelter like Elkem, the two states to alternate between can be producing and not producing. Temporarily shutting down the production for a limited time in case of extreme spot prices, while still having the option to restart when production turns profitable again will act as a switching option (Gutherie, 2009, p. 207-208).

3.4. Forward Contracts

A forward contract is an agreement to buy or sell an asset at a future time at a specified price. The contract is sold at the OTC market and requires one participant to go long in the agreement and one to go short (Hull, 2018, p. 28). The forward contract is typically used to hedge the risk of price movement in a certain asset. An example of such a hedging strategy consists of entering a long position in a forward contract on power to reduce the risk of extreme prices in the future. The payoff from the forward contract is the spot price at maturity minus the forward price (Hull, 2018, p. 29). Large power consumers will often agree upon area specific delivery directly with the power producers for future delivery of power. When setting the prices for these forward contracts, the public futures prices are often used as reference (T, Omland, personal communication, January 2022).

3.5. Futures Contracts

A futures contract is an agreement to buy or sell an asset at a future time at a specified price, much like a forward contract. It differs however, in that it is traded at an exchange (like Nasdaq) and involves standardised features. As the contract is traded on an exchange, the buyer of the contract has a guarantee that the contract will be honoured (Hull, 2018, p. 30). Nasdaq offers several different derivatives that can be used to hedge against fluctuations in the power market including base and peak load futures, Deferred Settlement futures, and Electricity Price Area Differentials (EPADs) (Nasdaq, 2022c).

The futures offered by Nasdaq are all financial instruments and there is no physical deliverance of power, but rather cash settlement on expiration date. The length of the futures range between daily, weekly, monthly, quarterly, and annually. The base load futures are covering all hours (00:00-24:00) of all the days in the delivery period (Nasdaq, 2022a, p. 76). Nasdaq does offer peak load (08:00-19:59) futures, but not for the Nordic power market (Nasdaq, 2022a, p. 2).

The EPADs are futures referencing the difference between area price and an Index. The contract base is the difference between the specific area and the daily el-spot system price for the Nordic region. Both prices are published by Nord Pool (Nordpool, 2022a). The length of the EPADs are ranging between weekly, monthly, quarterly and annually (Nasdaq, 2022a, p.

98). EPADs are used to hedge against area specific price risk. These contracts are not very liquid at public exchanges such as Nasdaq, and therefore often act as a poor reference. Hedging only against the system price can also be problematic, when the area specific spot price differs a lot from the hedged system price.

3.6. Binomial Trees

Constructing a binomial tree is a method to visualise different paths a price of an asset can take. The assumption is that the asset can take on only two different prices in each time-step (Hull, 2018, p. 472). The two possible prices in each time-step depend on what prices the asset has taken in previous time-steps. The price can either go up or down by a predetermined percentage change and based on an estimated probability of a movement. Given the probability *(p)*, of an up-movement *(u)*, the probability of a down-movement *(d)*, must be *(1-p)* since there are only two possible outcomes (Hull, 2018, p. 473). By following the different paths in the tree for the price of the asset, one can see the various possible prices at each state and the possibility for reaching each price.

Using binomial trees for option-valuation can be a great tool for valuing American options. Binomial trees make it possible to see what the option is worth at the different times or states. This can be useful if one wants to hedge risk in different unfavourable, or favourable, states during the life of the option. The limitation with a binomial model is that it is not a continuous model. It is made up of several time steps with only 2 possible paths at each state (up or down), but by adding enough small time-steps one can get very close to a continuous model.

The distribution of a binomial tree will be centred around the mean. There is only one path for the single most extreme outcome, while there are most paths for the centred outcomes. Given a binomial model with 2 time-steps, the following paths will be possible:

- *u, u*
- *u, d*
- *d, u*
- *d, d*

This gives 2 possible paths for reaching the middle state, and only one path for the most positive and most negative state. The probabilities for the different states will be 50% for the middle state, and 25% each for the upper and lower state. When expanding the model into many more time steps, the extreme outcomes will be less likely. The binomial model will have a normal distribution.

Figure 3.3 Two-step Binomial Tree

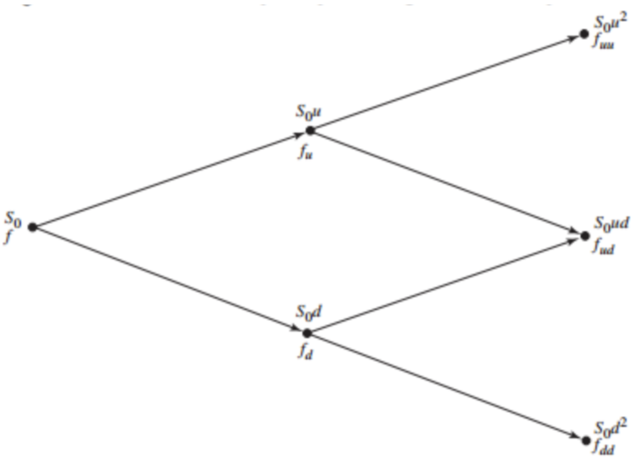


Figure 3.3 Two-step binomial tree, (Hull, 2018, p. 305)

Cox et al. (1979), contributed greatly to this method of option-pricing. They found that the probabilities for up or down movements would be determined by the rule of having no-arbitrage opportunities on the option. This is a key assumption for determining the risk-neutral probabilities in the tree. These risk neutral probabilities are estimated through lowering the expected growth rate by the risk-premium implied by the CAPM (Gutherie, 2009, p. 35-36). It follows that the expected return of the asset is equal to the risk-free rate. Hence, this risk-free rate can be used as the discount rate for the expected payoff from the option (Hull, 2018, p. 300). Setting up the risk-neutral probability distribution is covered in chapter 3.9.

For valuing American options through a tree, one uses backward induction. The future values are dependent on the actions done in the previous states. Hence, starting at the final values will provide the cash flows where the previous decisions already have been incorporated (Gutherie, 2009, p. 5-6). By working backwards from the end to the beginning- and making the optimal decisions at each state -one will end up with a single value for all the future cash flows. The expected future cash flows are usually discounted by a constant discount rate, to find their market value (Gutherie, 2009, p. 6).

3.7. Black & Scholes

When adding enough nodes so that the binomial tree becomes as good as continuous, we arrive at the Black & Scholes option pricing formula developed by Fisher Black and Myron Scholes (1973). The original B&S formula is based on European options and does not consider the possibility of early exercise. To price American options and estimate the optimal exercise strategy, one must use numerical techniques. As a reason, binomial trees are a more intuitive and flexible way of valuing American options with the possibility for early exercise.

3.8. Mean Reversion

Mean reversion is the assumption that the short-term variance and returns will revert towards the long run mean as time passes. Mean reversion applies to most commodity prices as their value is usually trending toward a central mean (Gutherie, 2009, p. 271-272).

3.8.1. Stochastic Processes

A variable whose future value is uncertain and varies uncertain over time follows a stochastic process. The changes for a variable following a stochastic process can both be discrete and continuous (Hull, 2018, p. 324). Stochastic processes are central in derivative-pricing and valuation. Stochastic processes can be derived in several ways depending on the variable's properties.

3.8.1.1. Markov Property

The memoryless property of a stochastic process is called a Markov property. A Markov process is using only the variable's current value as a base when predicting future movements. The probability distribution for X_{t+1} depends solely on the value of X_t (Dixit & Pindyck, 1994, p. 62-63). The idea is that the current value of the variable should reflect all the past information (Hull, 2018, p. 324). The history of how the variable came to its current value is therefore irrelevant in a Markov process. This property often applies to stocks, where all previous information is incorporated into the current value (Dixit & Pindyck, 1994, p. 63). This is consistent with Fama's "weak-form market efficiency" proposed by his Efficient Market Hypothesis (EMH) from 1970 (Fama, 1970, p. 414).

A Markov process can be applicable for describing the path for a single variable dependent on its own development. For variables depending on variations in underlying parameters, it will be necessary to survey more advanced processes.

3.8.1.2. Wiener Process

A Wiener process, also called a Brownian motion, is a kind of Markov process that has a mean of zero and a variance of 1 per year (Dixit & Pindyck, 1994, p. 64). The properties of a wiener process are the following:

1. $\Delta z = \epsilon\sqrt{\Delta t}$

   Δz represents the change

   Δt represents the time-period

   ϵ is the standard normal distribution with mean of 0 and variance of 1
2. Values of Δz at different intervals of time Δt are independent

The 2nd property makes z follow a Markov process, implying that forecasting future values are entirely depending on the current value.

A Wiener process can be generalised into having various distributions to be more applicable for different situations. Drift rate is the change of mean per unit of time, and the variance rate is the variance per unit of time (Hull, 2018, p. 327). When adding drift rate into the equation, the future expected value will be different from the current value. This process is called a *Brownian motion with drift*.

The Brownian motion with drift has the following equation:

$$dx = a\, dt + b\, dz \tag{3.2}$$

a and *b* are both constants representing the drift and the variance rate, respectively (Dixit & Pindyck, 1994, p. 65). *dz* represents the variability, also referred to as noise, from the Wiener process. This noise is multiplied by the variance rate, *b* (Hull, 2018, p. 329).

Figure 3.4 Generalized Wiener Process

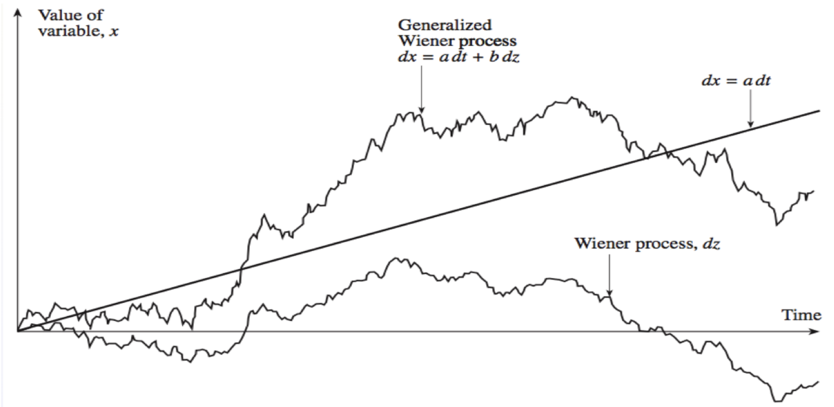


Figure 3.4 Generalized Wiener Process (Hull, 2018, p. 329)

The Brownian motion with drift can be further derived into an Itô Process, where *a* and *b* are not necessarily constants, but varies based on the current state *(x)* and time *(t)* (Dixit & Pindyck, 1994, p. 71). This gives the following extension to the previous equation:

$$dx = a(x, t)\, dt + b(x, t)\, dz \qquad (3.3)$$

3.8.1.3. Itô's Lemma

Itô's lemma is based on an Itô process where the stochastic process can be derived from the variable's stochastic process itself (Dixit & Pindyck, 1994, p. 79).
The value of a derivative is derived from the value of the underlying variable's price and time (Hull, 2018, p. 335). Itô's lemma is named after its discoverer K. Itô. It is based on the Itô process *dz* where *a* and *b* are functions of *x* and *t*. A function *F* of *x* and *t* will also include a Wiener process *dz*, and as a reason follow an Itô process (Itô, 1951, p. 2-3).
Itô's lemma uses differential calculus for the function of the stochastic processes. Furthermore, it provides the solution to these stochastic differential equations, which are widely used in derivative pricing models such as the Black & Scholes model.

3.8.1.4. Geometric Brownian Motion

Equation 3.3 is commonly known as a Geometric Brownian Motion. It is a continuous stochastic process which follows a random walk with drift (Dixit & Pindyck, 1994, p. 71). The process can be used in forecasting future values of an asset based on its historical data, given that the price is expected to follow a random walk (Gutherie, 2009, p. 265-266). When the price of an asset is not expected to follow a random walk, but rather mean reverting, the processes must be modified in order to capture the proper effect.

3.8.2. Ornstein-Uhlenbeck Process

The Ornstein-Uhlenbeck (OU) process, is an acknowledged mean reverting process used in forecasting the price-development of assets. The process can be written as the following (Dixit & Pindyck, 1994, p. 74):

$$dX_t = \kappa(\theta - X_t)dt + \sigma dz_t \tag{3.4}$$

In the equation, κ stands for the degree or rate of mean reversion. A larger κ means a stronger reversion effect towards the long-term mean. A low κ means a weaker reversion towards the long-term mean, represented by θ. If the X_t value is greater than θ, then (θ - X_t) will be negative and the drift will be negative. This will make the price revert down towards its long run mean of θ at a rate of κ. The actual changes in the level of X will depend on the speed of mean reversion and the amount of difference. If X_t is lower then θ, then (θ - X_t) will be positive and there will be a positive drift towards the long run mean at a rate of κ. The drift will therefore not be constant but vary depending on the level of X (Gutherie, 2009, p. 272). The last term in the process-equation is a Wiener process. It can be seen as white noise, generalised from a Gaussian process.
In general the first term of the equation represents the expected mean of the process, while the second term represents the expected variance of the process.

The OU-process equation can be rewritten at its ordinary differential form with respect to X, which is much used in derivative trading, which provides the following (Quantpie, 2018, 3:13):

$$dX_t + \kappa X_t dt = \kappa\theta dt \tag{3.5}$$

Divided by *dt* gives the following:

$$\frac{dx}{dt} + \kappa x = \kappa\theta \tag{3.6}$$

We can make the left side of the equation into an exact differential by taking the exponential of κ and *t*. Multiplying the previous equation with this factor, gives us an exact differential:

$$e^{\kappa t}\frac{dx}{dt} + e^{\kappa t}\kappa x = e^{\kappa t}\kappa\theta \tag{3.7}$$

$$\frac{d}{dt} + (e^{\kappa t}x) = e^{\kappa t}\kappa\theta \tag{3.8}$$

$$d(e^{\kappa t}x) = e^{\kappa t}\kappa\theta dt \tag{3.9}$$

The same technique can be added to the expanded general stochastic equation:

$$dX_t = \kappa(\theta - X_t)dt + \sigma dz_t \tag{3.10}$$

$$dX_t + \kappa X_t dt \ = \kappa\theta dt + \sigma dz_t \tag{3.11}$$

This is an exact stochastic differential equation, and by applying the exponential of κ and t we can make further use of an Itô process:

$$e^{\kappa t}dX_t + \kappa e^{\kappa t}X_t dt \ = \kappa\theta e^{\kappa t}dt + \sigma e^{\kappa t}dz_t \tag{3.12}$$

$$\rightarrow d(e^{\kappa t}X_t) = \kappa\theta e^{\kappa t}dt + \sigma e^{\kappa t}dz_t \tag{3.13}$$

By using Itô´s lemma and Itô processes, we arrive at an expected mean and variance formula for the integral solutions. Adding the integrals from zero to time *T* gives the following equation (Quantpie, 2018, 4:45):

$$\int_0^T \ d(e^{\kappa t}X_t) = \int_0^T \ \kappa\theta e^{\kappa t}dt + \sigma\int_0^T \ e^{\kappa t}dz_t \tag{3.14}$$

$$e^{\kappa T}X_T - e^0 X_0 = \kappa\theta \frac{e^{\kappa t} - e^0}{\kappa} + \sigma \int_0^T e^{\kappa t} dz_t \quad (3.15)$$

Making this equation as a function of X_T gives:

$$X_T = X_0 e^{-\kappa T} + \theta(1 - e^{-\kappa t}) + \sigma \int_0^T e^{-\kappa(T-t)} dz_t \quad (3.16)$$

Since the last term is a Brownian Wiener process with mean of 0 and variance of 1, we can remove it from the expected mean equation, giving the following equation (Dixit & Pindyck, 1994, p. 74):

$$E[X_T] = X_0 e^{-\kappa T} + \theta(1 - e^{-\kappa t}) \quad (3.17)$$

The variance of $[X_T]$ will then be the following:

$$V[X_T] = E[(X_T - E[X_T])^2] = E[(\sigma \int_0^T e^{-\kappa(T-t)} dz_t)^2] \quad (3.18)$$

We now have an Itô Integral that can be rewritten by making use of Itô's Isometry rule (Quantpie, 2018, 6:00). We then derive to the following equation for the variance:

$$V[X_T] = \sigma^2 \int_0^T e^{-2\kappa(T-t)} d_T = \sigma^2 \frac{e^0 - e^{-2\kappa T}}{2\kappa} \quad (3.19)$$

Since the exponential of zero equals 1, we can write it as the following (Dixit & Pindyck, 1994, p. 75):

$$V[X_T] = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa T}) \quad (3.20)$$

We can interpret what the long-term mean and variance will be for the OU-process as T approaches infinity.

When $\lim_{T\to\infty} E[X_T] = \lim_{T\to\infty}(X_0 e^{-\kappa T} + \theta(1 - e^{-\kappa T})) \approx X_0 * 0 + \theta(1 - 0) \approx \theta$ (3.21)

Since the exponential of a negative number approaches zero as the negative number increases, the expected long-term mean will approach θ as *T* increases (Dixit & Pindyck, 1994, p. 75).
The same will apply for the expected variance which for large *T* will be (Dixit & Pindyck, 1994, p. 75):

$$\lim_{T\to\infty} V[X_T] = \lim_{T\to\infty} \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa T}\right) \approx \frac{\sigma^2}{2\kappa}\left(1 - 0\right) \approx \frac{\sigma^2}{2\kappa} \qquad (3.22)$$

A high κ, will lead to a greater speed of mean reversion, making *X* vary less from the long-term mean of θ over time. The shocks to the price will not last for long, when the degree of mean reversion is high, thus making the price vary less around its long-term mean (Gutherie, 2009, p. 273).

To make these equations applicable for a real-life example where infinity is not an option, one can use the idea of half-life. For this we will need to produce an equation for the OU-process to be between its current value (X_0) and the long-term mean θ. The time *H* necessary for the process to reach half the way from X_0 to θ, will be:

$$E[X_H] = X_0 + \frac{\theta - X_0}{2} \qquad (3.23)$$

Written on its exponential form and abbreviating the equation, we get the expression for *H*, which is: $H = \frac{ln2}{k}$, meaning the necessary time to get to the half-way level between X_0 and θ is dependent on the rate of κ (Gutherie, 2009, p. 273).

The expected mean and variance of X_T can be written as as an arbitrary generalised function of $X_{t+\Delta t}$ giving the value of the OU-process as (Gutherie, 2009, p. 272):

$$X_{t+\Delta t} = X_t e^{-\kappa\Delta t} + \theta\left(1 - e^{-\kappa\Delta t}\right) + \sigma\sqrt{\frac{1 - e^{-2\kappa\Delta t}}{2\kappa}}\, N[0,1] \qquad (3.24)$$

The error term is normally distributed with a mean of zero and a variance of one.
This equation is similar to a first-order autoregressive process (AR(1)), which is generally written as:

$$y_{i+1} = by_i + a + \epsilon_{i+1} \tag{3.25}$$

We can see that it is similar to the following OU-process equations (Gutherie, 2009, p. 273):

$$b = e^{-\kappa \Delta t} \qquad a = \theta(1 - e^{-\kappa \Delta t}) \qquad \epsilon_{i+1} = \sigma\sqrt{\frac{1-e^{-2\kappa \Delta t}}{2\kappa}}\, N[0,1].$$

3.9. Risk Neutral Probabilities

To value the cash flows, Y_u or Y_d (depending on the movement) that will be received in the future periods, Cox et al. (1979, p. 231) found a simplified way of using the cash flow's replicating portfolio. This valuation method rests on the assumption that there are no arbitrage opportunities in the market and that the law of one price must hold. That is, two assets with identical cash flows must have the same price.
The replicating portfolio consists of a one-period risk-free bond with a price of 1 and a one-period risky spanning asset with a current price of Z. The risk-free bond will have a payoff of the risk-free rate (R_f) after one period. The spanning asset will have a payoff after one period of either equal X_u in case of an up-movement, or X_d with a down-movement. The replicating portfolio consists of A units of one-period risk-free bonds and B units of the spanning asset, and is given by (Gutherie, 2009, p. 40):

$$AR_f + BX_u = Y_u \tag{3.26}$$

and

$$AR_f + BX_d = Y_d \tag{3.27}$$

Rearranging and substituting the equations shows that the cost of the replicating portfolio must be given by (Gutherie, 2009, p. 41):

$$A + BZ = \frac{1}{R_f}\left[\left(\frac{ZR_f - X_d}{X_u - X_d}\right) Y_u + \left(\frac{X_u - ZR_f}{X_u - X_d}\right) Y_d \right] \tag{3.28}$$

If we substitute (Gutherie, 2009, p. 29):

$$\pi_u = \frac{ZR_f - X_d}{X_u - X_d} \tag{3.29}$$

and

$$\pi_d = \frac{X_u - ZR_f}{X_u - X_d} \tag{3.30}$$

We see that the replicating portfolio must cost (Gutherie, 2009, p. 32):

$$V = \frac{\pi_u Y_u + \pi_d Y_d}{R_f} \tag{3.31}$$

To arrive at the present value, the replicating portfolio is discounted by the risk-free rate. The same must be true for the real option if we adjust the probabilities of the up and down moves to equal that of the replicating portfolio. This provides the risk-neutral probabilities of up and down moves by π_u and π_d , respectively.

To calculate the risk-neutral probabilities we require the notion of the up- and down moves, which we covered in the previous section. The risk-free rate can easily be obtained by looking at the rate of return on the long-term government bonds. This leaves us estimating the parameter *Z*, which is the current price of the spanning asset (Guthrie, 2009, p. 28).

3.9.1 The Asset Pays Dividend

There are three different approaches to estimate the current price of the spanning asset. Firstly, if the traded asset generates a dividend or a convenience yield, we assume that the price *X* of a given time is measured right before the dividend *C* is paid. After one period, the price will either equal X_u or X_d. Since the dividend is paid immediately after acquiring the asset, the true cost of the asset is *X* - *C*. We can substitute *Z* with *X* - *C*, and rearrange the equation for risk-neutral probability to get (Gutherie, 2009, p. 31):

$$\pi_u = \frac{(1 - \frac{C}{X})R_f - D}{U - D} \tag{3.32}$$

and

$$\pi_d = \frac{U - (1 - \frac{C}{X})R_f}{U - D} \tag{3.33}$$

Since C is the return of the asset, we can measure the market's attitude towards the risk by how much the price changes when dividend is paid, namely $\frac{C}{X}$ (Guthrie, 2009, p. 30).

3.9.2 Using Forward or Futures Contracts

Another approach can be used when there are futures or forwards contracts that are tradeable on the asset. With a forward contract there is no immediate cash flow, but rather a cash flow at the end of the contract. If we enter a long position with forward price of F, we will receive a cash flow of either $X_u - F$ in the up state or $X_d - F$ in the down state. Since we don't have to pay anything now, we would invest $\frac{F}{R_f}$ in risk free bonds now and at the end of a contract receive (Gutherie, 2009, p. 33):

$$(X_u - F) + F = X_u \tag{3.34}$$

or

$$(X_d - F) + F = X_d \tag{3.35}$$

Since this payoff is the same as the spanning asset, and with the assumption that there are no arbitrage opportunities, the price of the forward-portfolio must equal the price of the spanning asset. That is: $Z = \frac{F}{R_f}$. Substituting in the equation for risk-neutral probabilities, we now get the following equations:

$$\pi_u = \frac{F - X_d}{X_u - X_d} \tag{3.36}$$

and

$$\pi_d = \frac{X_u - F}{X_u - X_d} \tag{3.37}$$

3.9.3 Using CAPM

Finally, Rendleman (1999, p. 109) proved that we can calculate the risk-neutral probabilities by using the CAPM to adjust for risk. This approach is convenient when we are not able to directly observe the traded asset. It consists of finding a portfolio that minimises the so-called mean squared tracking error. In other words, we want to find the portfolio that consists of risk-free bonds and the market portfolio that has the lowest expected difference between the payoff from the spanning asset (Gutherie, 2009, p. 34).

If the portfolio consists of A invested in risk-free rate and B invested in the market portfolio, the payoff must equal $AR_f + BR_M$, where R_M is the return on the market portfolio. If the asset we are tracking has a payoff of $\tilde{X}$, then the tracking error must equal (Gutherie, 2009, p. 42):

$$\tilde{X} - (AR_f + BR_M) \tag{3.38}$$

and the mean squared tracking error

$$\text{MSE} = \text{E}[(\tilde{X} - (AR_f + BR_M))^2] \tag{3.39}$$

Minimising the MSE we get

$$0 = \frac{dMSE}{dA} = -2E[(\tilde{X} - (AR_f + BR_M))R_f] \tag{3.40}$$

and

$$0 = \frac{dMSE}{dB} = -2E[(\tilde{X} - (AR_f + BR_M))R_M] \tag{3.41}$$

Simplifying and substituting the equations we reach

$$B = \frac{Cov[\tilde{X}, R_M]}{R_f} \tag{3.42}$$

and

$$A = \frac{E[\tilde{X}] - E[R_M]\frac{Cov[X, R_M]}{Var[R_M]}}{R_f} \tag{3.43}$$

Since the tracking portfolio has the same price as the spanning asset, $Z = A + B$ we get

$$Z = \frac{E[\tilde{X}] - (E[R_M] - R_f)\frac{Cov[\tilde{X}, R_M]}{Var[R_M]}}{R_f} \tag{3.44}$$

We now have an equation to retrieve *Z*, but this can be simplified further to make the calculations for risk neutral probabilities less complicated.

Recall that

$$\pi_u = \frac{ZR_f - X_d}{X_u - X_d} \quad (3.29)$$

and

$$\pi_d = \frac{X_u - ZR_f}{X_u - X_d} \quad (3.30)$$

If we adjust these formulas by dividing each node with *X*, we get (Gutherie, 2009, p. 36):

$$\pi_u = \frac{\frac{ZR_f}{X} - \frac{X_d}{X}}{\frac{X_u}{X} - \frac{X_d}{X}} \quad (3.45)$$

and

$$\pi_d = \frac{\frac{X_u}{X} - \frac{ZR_f}{X}}{\frac{X_u}{X} - \frac{X_d}{X}} \quad (3.46)$$

Since $\frac{X_u}{X}$ must be the size of an up move U, and $\frac{X_d}{X}$ must be the size of a down move *D*, we can rewrite the equation to

$$\pi_u = \frac{K - D}{U - D} \quad (3.47)$$

Where $K = E[\frac{X}{X}] - (E[R_M] - R_f)\beta_x$ (3.48)

Since *E[X]* is the expected value of *X*, we can rewrite $E[X] = p_u X_u$ where p_u is the actual probability of an up move.

Simplifying, we end up at the equation for risk neutral probability of an up move according to CAPM at:

$$\pi_u = p_u - \frac{(E[R_M] - R_f)\beta_x}{U - D} \quad (3.49)$$

If we have the actual probabilities, we can easily implement the risk neutral probability by only adding a risk-neutral factor of

$$\frac{(E[R_M]-R_f)\beta_x}{U-D} \tag{3.50}$$

that captures the exposure to risk in the market. By discounting the cash flows with the risk-free rate, we arrive at the present value (Guthrie, 2009, p. 37).

4 Estimating the Parameters

4.1. Determination of the Process for the Power Prices

The price development of an underlying asset in a binomial tree is generally assumed to follow a random walk process (Hull, 2018, p. 296). That is, the price movement of the asset changes completely at random at each time step. For commodities, however, it is normal for the price to follow a mean-reverting process as explained in chapter 3.8. When prices are high, production will increase, and the greater supply will pull the prices down and revert itself towards the mean. When prices are low, production will likely decrease, and prices will again rise towards its long-term mean. The power flow between the different el-spot areas also contribute to the prices being mean reverting. The theory of mean reversion is expected to apply for the power price development. This statement is backed up by the development of power prices during the last two decades as seen in figure 4.1.

Figure 4.1 Monthly Power Price Trend

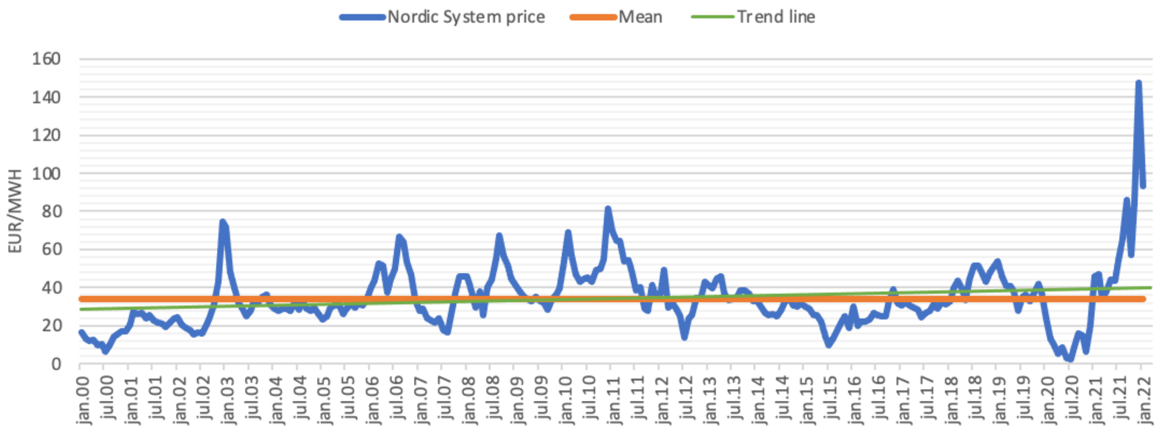


Figure 4.1 Historical Nordic System power prices with trend- and mean line. Trend line is slightly upward shifting due to recent price-levels. Data is obtained from Nordpool, 2022. (https://www.nordpoolgroup.com/historical-market-data/).

4.2. Determination of Power Price Reference

For a more general approach the Nordic system price will be used as a reference for the power prices in this thesis. It provides a general price that is not so vulnerable for local variations and events. The Nordic system price is calculated from the different area prices in the Nordic countries. It also considers the power-flow between the neighbouring countries like Germany, Netherlands, Poland, and the Baltic countries (Nordpool, n.d.c). The Nordic system price also has liquid futures-derivatives in the open market at Nasdaq, providing reasonable expectations for delivery of power many years ahead. Even though area specific prices greatly differ between the system price in certain periods, it still is the most used reference price for financial power contracts.

4.3. Calibration of the Parameters in the Ornstein-Uhlenbeck Process

We run an AR(1) regression with time-series data in order to find the estimated parameters for the OU-process. In our case we will use both historical and current systematic power futures prices for the Nordic market. The data for the power prices was provided by the Head of Power Department at Elkem ASA (T, Omland, personal communication, February 2022). Due to recent massive variation in the power prices, the futures prices are expected to better represent the power-price-development process without the extremely high variance of white noise. The futures prices are also expected to be the best current available predictor for the forecasted future power-prices. We use quarterly futures data for the Nordic system price from 2013 to 2022 for our estimation, as quarterly data is expected to capture the differences between the 4 seasons in a year. The AR(1) model can then be applied for modelling the time series. We obtain the following data from our AR(1) regression done in Microsoft Excel:

Figure 4.2 Regression Output from AR(1) Model on Power Prices

Regresjonsstatistikk	
Multippel R	0,85572877
R-kvadrat	0,732271728
Justert R-kvadrat	0,72415875
Standardfeil	0,079936874
Observasjoner	35

Variansanalyse

	fg	*SK*	*GK*	*F*	*Signifkans-F*
Regresjon	1	0,57674826	0,57674826	90,25930205	5,74886E-11
Residualer	33	0,210866827	0,006389904		
Totalt	34	0,787615087			

	Koeffisienter	*Standardfeil*	*t-Stat*	*P-verdi*	*Nederste 95%*	*Øverste 95%*	*Nedre 95,0%*	*Øverste 95,0%*
Skjæringspunkt	0,367052909	0,323265088	1,135454838	0,264364497	-0,290634859	1,024740676	-0,290634859	1,024740676
X-variabel 1	0,894188553	0,094120261	9,500489569	5,74886E-11	0,702699444	1,085677663	0,702699444	1,085677663

The slope-coefficient is equal to 0.8942 which represents parameter *b* in the equation. The intercept-coefficient is equal to 0.3671 which represents *a* in the equation. The standard error is equal to 0.0799. From these parameters we can calculate the normalised estimates for κ, θ and σ (Gutherie, 2009, p. 278):

Taking b = $e^{-\kappa \Delta t}$ and solving for κ provides:

$$\kappa = -\frac{ln(b)}{\Delta t} = -\frac{ln(0.8942)}{0.25} = 0.4474 \quad (4.1)$$

Solving a = $\theta(1 - e^{-\kappa \Delta t})$ for θ provides:

$$\theta = \frac{a}{1-b} = \frac{0.0367}{1-0.8942} = 3.4689 \quad (4.2)$$

Solving $\epsilon_{i+1} = \sigma\sqrt{\frac{1-e^{-2\kappa \Delta t}}{2\kappa}}$ for σ provides:

$$\sigma = SE\sqrt{\frac{-2ln(b)}{(1-b^2)\Delta t}} = 0.0799\sqrt{\frac{-2ln(0.8942)}{(1-0.8942^2)0.25}} = 0.1689 \quad (4.3)$$

κ is estimated to be 0.4474, which can be seen as a relatively strong rate of mean reversion. The logarithmic long-run mean is estimated to be 3.4689 which equals $e^{3.4689} = €32.10\ per\ MWh$. The error term σ, is estimated to be 0.1689 and reflects the white noise in the process.
These are the normalised estimates for the parameters.

We can then go from the normalised parameters to the ones that will be used in the binomial tree for the power price. The logarithmic price at node *(i, n)* is derived from the price at the starting point, and the number of increases and decreases by the time-equivalent σ in previous periods. The log price can be written as the following (Gutherie, 2009, p. 274):

$$x(i,n) = logP_0 + (n - 2i)\sigma\sqrt{\Delta t_m} \quad (4.4)$$

i represents the number of down-moves, and *n* represents the amount of time periods. Δt_m years is the time-period in each step of the binomial tree. This can be changed depending on what time intervals one finds appropriate to use in the given case. In the case of Elkem Bremanger, the task of finding the optimal time-steps must provide sufficient data as well as a suitable horizon to consider the option of temporarily closing the plant.
A time horizon of 10 years seems sufficient to capture the value of the real option. In an industry that faces several changes and conversions over time, a situation more than 10 years ahead will likely not face similar challenges as in the current situation. Therefore, we argue that a horizon of 10 years is suitable for our model. Furthermore, when we look at the fluctuations of power prices in bulks of 10 years, there seems to be a trend of a mean reverting effect as seen in figure 4.1.
The next step is to find the optimal time-steps. Daily or weekly data seems unreasonable as closing or opening the plant on a daily or weekly basis will be too costly. Furthermore, quarterly, or annual time steps seem to give us too little variation to be able to capture the true mean reverting effect, as well as give us too few observations. The monthly time-steps seem to both capture the mean-reverting effect and provide us with sufficient data. It also seems reasonable to evaluate whether one should close or open the plant on a monthly basis. Our binomial tree is therefore set to have monthly time steps, therefore Δt_m is set to equal $\frac{1}{12}$.

The size of the up moves in the log price is the exponential of σ times the root of $\Delta t_m \rightarrow$ $e^{\sigma\sqrt{\Delta t_m}}$. The size of down moves will be the negative exponential of the same term: $e^{-\sigma\sqrt{\Delta t_m}}$ (Gutherie, 2009, p. 274).

For monthly time steps in our model, we get the following sizes of up- and down moves:

- Up *(U)*: 1.05
- Down *(D)*: 0.95

The probabilities of having an up or down move will vary in a mean reverting process, depending on at which node we are in the binomial tree.

Assume that we have a probability (p) of an up move that at node *(i, n)* is equal to the following (Gutherie, 2009, p. 274-275):

$$p_u(i,n) = 0{,}5 + \frac{(1-e^{-\kappa \Delta t_m})(\theta - x(i,n)}{2\sigma\sqrt{\Delta t_m}} \tag{4.5}$$

The expected change in the log price will equal the sum of p_u * *U* and p_d * *D*. This can be expressed as:

$$E_{(i,n)}[x(\cdot, n+1) - x(i,n)] = p_u \quad (i,n)\sigma\sqrt{\Delta t_m} + (1 - p_u(i,n))(-\sigma\sqrt{\Delta t_m} =$$
$$\rightarrow (2p_u(i,n) - 1)\sigma\sqrt{\Delta t_m} = (1 - e^{-\kappa \Delta t_m})(\theta - x(i,n)) \tag{4.6}$$

This is the expected value for the OU-process (Gutherie, 2009, p. 295-296).

At some nodes where *x(i,n)* is sufficiently large, we get a negative probability for $p_u(i,n)$. Similarly, if *x(i,n)* gets sufficiently low, we get a probability greater than 1 for $p_u(i,n)$. This violates the condition that the probability for an up-move must be ≤ 1 and ≥ 0. Therefore, at these nodes the probability of an up-move must be set to equal 1 or 0 depending on if *x(i,n)* is sufficiently high or low (Gutherie, 2009, p. 276-277).

The binomial tree will be varying between its outer bounds.

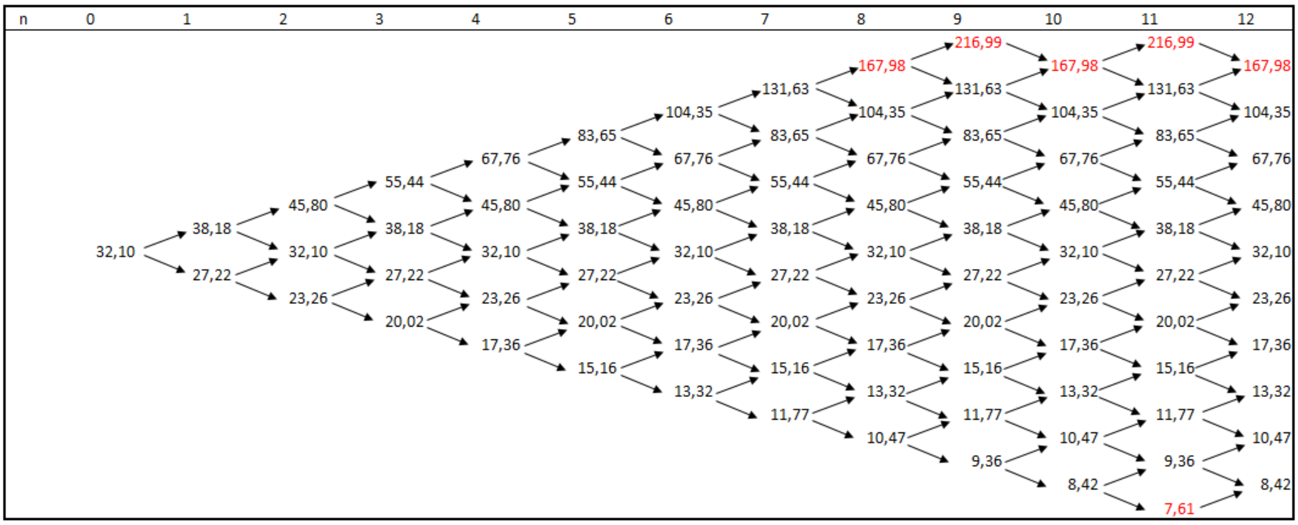
When $x(i,n) \geq \theta + \frac{\sigma}{\kappa\sqrt{\Delta t_m}}$, the next move is certain to be down, meaning $p_u(i, n+1) = 0$.

Similarly, when $x(i,n) \leq \theta - \frac{\sigma}{k\sqrt{\Delta t_m}}$, the next move is certain to be up, meaning $p_u(i, n + 1) = 1$.

This leads to some nodes in the binomial tree being impossible to reach, and the tree will be varying between an upper and lower bound, due to the mean reversion.

Figure 4.3 shows the possible values the power price can take on in € per MWh the first 12 months.

Figure 4.3 Binomial Tree with Possible Power Prices



The red numbers indicate that these power prices can never be reached in our model, as explained above. Thus, the highest possible value for the monthly power price is €131.63 and the lowest is €8.42.

For confirmation regarding the validity of the power price estimates, it can be appropriate to compare with the currently traded futures prices in the open market. Nasdaq Commodities provides yearly futures-data 10 years ahead in time. These prices will reflect the current general assumption about the expected prices in the future in the power-market. The forecasted prices derived from the regression at the different nodes, are varying around the currently traded quarterly futures prices in the Nordic system power market at Nasdaq as per April 2022 (Nasdaq, 2022b). The long-run mean estimate of €32.10 per MWh is roughly equivalent with the current yearly futures prices for delivery in 2024 and onwards to 2032. This indicates that the regression acts as a good estimate to use in the model.

4.4. Calibrating the Risk Neutral Probabilities Using CAPM

To value the real option, we will use the replicating portfolio method described in chapter 3.9. This method allows us to discount the future cash flows with the risk-free rate, given that we calculate risk neutral probabilities.

Since the asset used in the model does not provide any dividends or convenience yield, the dividend-method is not applicable to use in this case. The current futures and forwards available on the asset were not sufficiently traded to make the market prices reliable enough to use in our estimation of the risk-neutral probabilities. However, we can observe all the required market data explained in chapter 3.9.3 to calculate the risk neutral probabilities by using the CAPM-approach.

Recall that the risk neutral probability of an up move according to the CAPM is given by:

$$\pi_u = p_u - \frac{(E[R_M]-R_f)\beta_x}{U-D} \qquad (3.49)$$

Where p_u is the actual probability of an up move. p_u takes on a different value for every node in the binomial tree as is shown in equation 4.5. Therefore, we only need to calculate the risk neutral factor, which is given by the second part of equation 3.49 to arrive at the risk neutral probabilities. U is the size of an up move and D is the size of a down move, which we also found to be 1.05 and 0.95 respectively in chapter 4.3.

4.4.1.1. Beta

The next step is to calculate β for the asset. We retrieved daily data from the OSEBX and the power price data used in our model for the period 01.03.2013 to 01.02.2022.

Firstly, we estimated the daily log return of both OSEBX and the power price data.
Then we ran an Ordinary Least Squares (OLS) regression on the logged returns:

$$Power\ return(Log) = \beta_0 + \beta_1 OSEBX\ return(Log)$$

Regression in MS Excel gave us the results displayed in figure 4.4.

Figure 4.4 Regression Output for Beta Estimation

Regresjonsstatistikk	
Multippel R	0.174104823
R-kvadrat	0.03031249
Justert R-kvadrat	0.029875693
Standardfeil	0.011127079
Observasjoner	2222

Variansanalyse

	fg	*SK*	*GK*	*F*	*Signifkans-F*
Regresjon	1	0.008592215	0.008592215	69.39733267	1.39047E-16
Residualer	2220	0.274862386	0.000123812		
Totalt	2221	0.2834546			

	Koeffisienter	*Standardfeil*	*t-Stat*	*P-verdi*	*Nederste 95%*	*Øverste 95%*	*Nedre 95,0%*	*Øverste 95,0%*
Skjæringspunkt	-0.000083	0.000236	-0.349786	0.726532	-0.000546	0.000380	-0.000546	0.000380
X-variabel 1	0.140350	0.016848	8.330506	0.000000	0.107311	0.173389	0.107311	0.173389

Giving our OLS regression the following parameters:

$$Power\ return(Log) = 0.0000 + 0.1404 * OSEBX\ return(Log)$$

β_1 has a p-value of 0.0000 indicating that the parameter is statistically significant. The β for our CAPM-estimate is therefore given by 0.14.

4.4.1.2. Risk-free Rate

Government securities are likely to properly fulfil the conditions for a risk-free rate. Norway has a credit rating of AAA from S&P, which is the best possible rating. Accordingly, there is basically no risk of default by investing in Norwegian government securities (S&P Global, 2022; Trading Economics, 2022). Norwegian government bonds will therefore be a good proxy for our risk-free rate, for a business operating in the Norwegian market. A Norwegian 10-year government bond has been trading around 2% in the first quarter of 2022 (Trading Economics, 2022a). This is the same rate as the Norwegian target of annual inflation, and a risk-free rate of 2% seems sensible to use in the long run (Norges Bank, 2021).
In conclusion, the annual risk-free rate is set to equal 2% in our real-option model.

4.4.1.3. Market Risk Premium

The MRP can be estimated through several ways. One method is to see what the historical MRPs have been in recent years. Taking the average historical return from the stock-index, which in our case will be OSEBX. Damodaran suggests using geometric average, since arithmetic average tends to overestimate the return (Damodaran, 2012, p. 163). Using the annual historical average from 1996-2022, we get a geometric average of the return to be 10.01% (Euronext, 2022). Subtracting the average risk-free rate and the average tax-rate through the period, provides an estimated MRP of 4.59% (Norges Bank, 2021a; Trading Economics, 2022b).

Table 4.1 OSEBX Market Returns from 1996-2022

Date	OSEBX	Return OSEBX
02.01.2022	1211,32	25,4 %
02.01.2021	965,95	2,6 %
02.01.2020	941,3	17,2 %
02.01.2019	803,4	-1,4 %
02.01.2018	815,2	17,9 %
02.01.2017	691,5	15,1 %
04.01.2016	600,6	3,7 %
02.01.2015	579,4	5,9 %
02.01.2014	546,9	20,3 %
02.01.2013	454,5	16,7 %
02.01.2012	389,5	-12,1 %
03.01.2011	443,2	16,6 %
04.01.2010	380,2	58,9 %
02.01.2009	239,2	-51,3 %
02.01.2008	491,3	10,0 %
02.01.2007	446,5	34,1 %
02.01.2006	332,9	39,8 %
03.01.2005	238,1	37,3 %
02.01.2004	173,5	47,7 %
02.01.2003	117,5	-30,0 %
02.01.2002	167,9	-15,7 %
02.01.2001	199,2	3,4 %
03.01.2000	192,7	44,9 %
04.01.1999	133,0	-25,6 %
02.01.1998	178,8	35,8 %
02.01.1997	131,7	29,8 %
02.01.1996	101,5	

Table 4.2 Estimated MRP from Historical Market Return

Arithmetic average	0,1334
Geometric average	0,1001
Average risk-free rate	0,0373
Market risk premium pre tax	0,0627
- Average tax rate (26,77%)	0,0168
Market risk premium post tax	**0,0459**

A general agreement from users of return and risk models is that historical MRPs provide the best estimates for future MRPs, although there is no guarantee that the future will be like the past (Damodaran, 2012, p. 161). It can be beneficial to compare the MRP estimated from historical data to other estimates of the MRP as well.

PrimewaterhouseCoopers (PwC) is a global consulting company that each year provides an estimate for the MRP in the Norwegian market. Their estimate is based on a questionnaire with more than 100 Norwegian companies regarding their view of the current MRP. The result from the 2021 report is an MRP of 5%, which also has been the case for the last 10 years (PwC, 2021, p. 8).
Aswath Damodaran is a professor in finance that continuously provides estimates of MRP for each country. His estimate for Norway, updated 05.01.2022, is 4,24% (Damodaran, 2022).

The estimated MRP from the historical data shown in tables 4.1 and 4.2, lies between the two other estimated MRPs from PwC and Damodaran, and seems like an overall reasonable estimate for the future MRP. The annual MRP in our model will therefore be set to 4.6%. We adjust the MRP to monthly return to fit our model and get a monthly MRP of 0.38%.

4.5. Estimating the Risk Neutral Factor

Now that we have all the necessary parameters, we can calculate the risk neutral factor that is given by:

$$\frac{(E[R_M]-R_f)\beta_x}{U-D} = \frac{0.0038 * 0.1404}{1.05 - 0.95} = 0.0055 \qquad (3.50)$$

Finally, all we must do is subtract the probabilities calculated in the previous section with 0.0055 for each node to be able to discount all future cash flows with the risk-free rate.

5 Calibrating the Model

5.1. Silicone Price

As previously mentioned, silicone is a material that can be used in a variety of different areas as explained in chapter 1.1.2. An important factor for the usage, will often depend on the degree of silicone in the product. A common silicone-product is Ferrosilicon (FeSi) with 75% degree of silicone. This product has a range of possible applications and is much traded in the global market. As a reason, we will use FeSi 75% as the silicone product in our model. For the estimation of the FeSi 75% price, we will use historical price data obtained by CRU. CRUs prices for silicone metals and other ferroalloys are the most used benchmark in the European and North American market (CRU, 2022). The data on the silicone prices obtained from CRU, was provided by Elkem ASA (O, Sandnes, personal communication, 2022 February). We use CRUs monthly FeSi 75% EU spot prices in € per MT, 20 years back in time from 2002-2022.

Figure 5.1 FeSi 75% EU Spot Prices in €/MT

Figure 5.1 shows the historical FeSi 75% EU spot price in €/MT from 2002-2022.

The price has been quite stable in the period from 2007- 2021, ranging between 1,000-1,500 €/MT. There has been a significant change in the price in 2021. The price has more than doubled in certain periods, peaking in November at 4,100 €/MT. There are several factors leading to this massive increase in FeSi prices. Major silicone-manufacturers in China were suffering from production cuts as a result of power-shortage in Q4 2021 (SMM, 2021). China being one of the largest silicone-manufacturers in the world, this led to a significant decrease in the supply worldwide. Simultaneously, there was a great shortage in microchips after the outbreak of the pandemic. Silicone is a material used to produce microchips, leading to an increase in demand. After the peak in November 2021, the price has lowered again and is expected to stabilise itself again at a lower level (O, Sandnes, personal communication, 2022 May). The average FeSi 75% price in the time series is 1,095 €/MT and the median is 1,075 €/MT. We do not expect the recent price levels to last in the long term, and therefore we argue that the current spot price will fall towards the long-run mean. Hence, 1,100 €/MT seems like a reasonable estimate for the long-term average and will be used in our base case.

5.2. Profit Function and Costs

The costs for a smelter will consist of several elements; Raw material needed for production, cost of carbon used in the smelting process and cost of power needed for the production. Furthermore, there are other fixed and variable costs that include labour, maintenance, CO2 emission and other relevant costs.

Based on information provided by Elkem ASA, we have estimated the following costs included in the profit function for a silicone-producing smelter (O, Sandnes, personal communication, 2022 February). In the production of silicone products, quartz is the raw material needed. Quartz is estimated to stand for ⅙ of the costs in the production. In a normal situation, power would account for ⅓ of the total costs. However, the actual cost will vary a lot based on the power prices. What is certain, is the amount of MWh power required for production, which is 9.4 MWh per MT FeSi 75% produced. The total power costs would then equal the price of 1 MWh times 9.4 MWh/MT. Carbon-costs stand for ⅙ of the total costs. Fixed and variable costs would account for the remaining ⅓ with a normal rate of production. The annual production capacity is set to equal 40,000 MT of FeSi 75%. This equals 3,333 MT per month and will vary based on whether the production is operational at all times. The fixed costs are set to be a constant each month, while the variable costs are set to be based on the production rate.

These costs will vary between the different smelters based on their type of production and infrastructure. Even though there are differences between the smelters, our model will however give a rough estimate of the impact the different inputs have on the total costs. Power will generally be the input that varies most and can account for both more or less than ⅓ of the total costs. In our model we are evaluating the effect power prices have on the production. For simplicity, we will keep all other factors constant and let the power prices vary across time. This will reflect how the payoff differs between different states of power prices. The profit margin will also be a varying factor between different smelters, but as a base case, an overall profit margin of approximately 10% of the silicon price will be assumed. The margin will however vary considerably based on the power prices.

For a smelter producing silicone, the profit function can be generalised to the following:
Profit *(π)* = Silicon *(S)* - Quartz *(Q)* - Carbon *(C)* - Power *(P)* - Variable costs *(V)* - Fixed costs *(F)*

Given a silicone price of 1,100 €/MT we get the following values per produced MT of silicone:

Power costs: 9,4 MWh/MT * €/MWh
Quartz costs: 185 €/MT
Carbon costs: 185 €/MT
Variable costs: 300 €/MT
Annual Fixed costs: €2,400,000
Annual production capacity: 40,000 MT

6 Valuation & Results

6.1. Valuation Using Backward Induction

The value of the option can be found by comparing the firm-value without the option and the firm-value with the option. The option-value will be the difference between the two firm-values. To value the option this way, one will have to do two valuations: one including the option to close production, and another that will always keep production open.

6.1.1. Valuation of Firm Without Switching Option

Firstly, the value of the firm not having the option will be evaluated. Each node *(i, n)* represents a period that has a cash flow *Y(i, n)* related to it. The cash flow at each node *(i, n)* has the value *V(i, n)*, which will consist of the profit-function:

$$\pi(i,n) = (S - Q - C - V - 9{,}4P(i,n)) * MT - F \qquad (6.1)$$

The last cash flow in the model will be at date *N*. Since we use backward induction, our first calculated value will be *V(i, N)*. *V(i, N)* will simply be equal to the market value *Y(i, N)* (Gutherie, 2009, p. 65).

Next, we must consider what happens at all the earlier nodes *(i, n)* leading up to node *(i, N)*. The value *V(i, n)* at each node will be a combination of the cash flow received at node *V(i, n)* plus the value of all the possible future cash flows extending from that node. The market value of the future cash flows received at node *(i, n)* will equal:

$$\frac{p_u(i,n)V(i,n+1) + p_d(i,n)V(i+1,n+1)}{Rf} \qquad (6.2)$$

The formula gives the proportional value from the possible future up- and down-values based on the risk-neutral probabilities for reaching each node. The proportional value is then discounted with the risk-free rate to obtain the market value. Adding this equation with the cash flow received at node *(i, n)* gives the following value at each node *(i, n)*, where *Y(i, n)* represents the profit-function *π(i, n)* at the different nodes:

$$V(i,n) = Y(i,n) + \frac{p_u(i,n)V(i,n+1) + p_d(i,n)V(i+1,n+1)}{Rf} \qquad (6.3)$$

By adding this equation into each node in the binomial tree and using backward induction, we will arrive at the present value of all future cash flows at node (0, 0).

6.1.2. Valuation of Firm with Switching Option

For the valuation of the firm that has the option to close production in periods with negative profits, the valuation-function must be slightly adapted. Assuming that fixed costs will occur regardless of whether production is open or closed, fixed cost will be the bottom line for the payoff. As long as keeping the production open generates a less negative payoff than the fixed cost, the production will be kept open.

Going from having the production open to closed will induce some closing costs. Closing costs can be related to cleaning of the production line, storage of already purchased raw materials and other input factors. Furthermore, the smelter can terminate employee contracts if needed, which also will induce some costs. This value can be seen as the strike price *Kc*, for exercising the option to temporarily close production. The option will be like a put option with the following parameters:

- Underlying asset S_T= power price
- Strike price K = *Kc*
- Time to maturity T = time to next time-period (which for monthly time steps would equal $\frac{1}{12}$
- Risk-free rate Rf = 2%
- The volatility of the option would match the volatility of the underlying asset which is the power price.

The same will apply in the opposite situation. When having a closed production, there will be some opening costs *Ko* to restart the production. Opening costs will be heating costs related to reaching the operating temperature in the furnaces for production, re-hiring employees if needed and generally getting the site ready for production again. This would act as a call option, with the same parameters as in the put option, but with *Ko* as the strike price instead of *Kc*.

These options will occur at each time step in the binomial tree. Which option that is available, will depend on the current state, at each node. Note that the costs *Ko* and *Kc* will only occur when the production goes from being opened to closed and vice versa. In other words, the strike price occurs when there is a switch in the production-status.

The terminal value at *V(i, N)* will now be the maximum of the profit function at that node and the fixed costs accruing for that period represented by the node. This gives the following equation:

$$V(i,N) = max\,\{Y(i,N), -F\} \tag{6.4}$$

Recall that *Y(i, n)* represents the profit function *π(i, n)* at the different nodes, and will never be less than the negative fixed cost *(-F)*.
For the other values at nodes *(i, n)*, the two strike prices *Kc* and *Ko* must be implemented into the equation, depending on the current state. Equation *V(i, n)* represents the nodes where the same state of production is kept as the one in *V(i, n-1)*:

$$V(i,n) = Y(i,n) - F + \frac{p_u(i,n)V(i,n+1) + p_d(i,n)V(i+1,n+1)}{Rf} \tag{6.5}$$

Closing the production will make the closing cost *Kc* occur immediately, giving the following equation:

$$-K_c + \frac{p_u(i,n)Vc(i,n+1) + p_d(i,n)Vc(i+1,n+1)}{Rf} \tag{6.6}$$

$V^{o\to c}(i,n)$ represents the nodes where production goes from open to closed:

$$V^{o\to c}(i,n) = max\left\{Y(i,n) - F + \frac{p_u(i,n)Vo(i,n+1) + p_d(i,n)Vo(i+1,n+1)}{Rf}, -K_c + \frac{p_u(i,n)Vc(i,n+1) + p_d(i,n)Vc(i+1,n+1)}{Rf}\right\} \tag{6.7}$$

Reopening the production will make the opening cost *Ko* occur immediately, in addition to the cash flow from the operating production in the given period. This gives the following equation:

$$-K_o + Y(i,n) - F + \frac{p_u(i,n)Vo(i,n+1) + p_d(i,n)Vo(i+1,n+1)}{Rf} \tag{6.8}$$

$V^{c \to o}(i,n)$ represents the nodes where production goes from closed to open:

$$V^{c \to o}(i,n) = max\left\{\frac{p_u(i,n)Vc(i,n+1) \ + \ p_d(i,n)Vc(i+1,n+1)}{Rf}, -K_o + Y(i,n) - F + \frac{p_u(i,n)Vo(i,n+1) \ + \ p_d(i,n)Vo(i+1,n+1)}{Rf}\right\}$$

(6.9)

In the model, the different equations are added into each node-function using dynamic "IF-statements" in MS Excel. By starting on node *(i, N)* and going backwards in the binomial tree, the present value of the market value of the firm including the option will be presented in node (0, 0).

6.2. Results

Adding the parameters and the mean reverting price development for power into the profit-function, gave the following results over a 10-year horizon with monthly time steps. The total number of nodes in the tree is equal to 1,052, whereas ⅓ (351 nodes) of the payoffs are negative. ⅙ of the negative payoffs are more positive than the monthly fixed costs, meaning the production should be closed 28% of the time (293 nodes).

The value of the plant without the option to close the production over a 10-year period is €21.36 million. The total production would then be 400,000 MT in the time horizon.

The value of the plant with the option to close production over a 10-year period is €25.19 million. The total production is then expected to be 288,593 MT in the time horizon.

The difference in the value is €3.84 million, which is equal to a difference of 15.22 %. This represents the value of having the option to close production in periods with high power prices and is in our base case equal to 15.22 % of the total plant value.

7 Sensitivity Analysis

7.1. Sensitivity Analysis

Conducting a sensitivity analysis grants a good indication of which parameters have a considerable impact on the option value. It also considers whether our model is in fact consistent with dynamics from general option theory, which will strengthen its credibility.

7.1.1. The Greek Sensitivity Parameters

Measuring the risk in having an option position are commonly measured with the Greek letters delta *(Δ)*, gamma *(Γ)*, rho *(ρ)*, theta *(Θ)* and vega *(ν)*. Each letter measures the sensitivity of a parameter in the option. The sensitivity is measured by how much the option-value changes with a 1% change in the parameter. The Greeks are widely used for hedging purposes but are also used to explain where the option-value derives from.
Table 7.1 shows what each letter measures.

Table 7.1 The Greeks

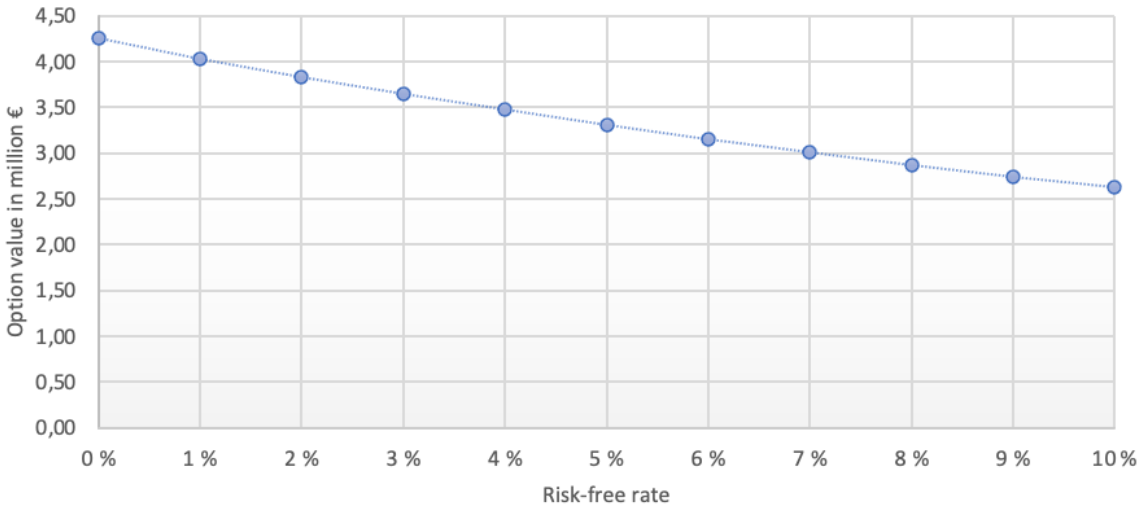
Letters:	**Changes in the option value with respect to the:**
Delta, Δ	Price of the underlying asset
Gamma, Γ	Sensitivity of the delta
Rho, ρ	Interest rate
Theta, Θ	Time to maturity
Vega, ν	Volatility

We have thoroughly investigated the effect of price changes on the option value in chapter 6, and so the sensitivity analysis provided by delta would not be of much value in our case. Furthermore, gamma and theta are not very applicable for this real option. However, rho and vega can be of interest regarding the sensitivity of the option value.

7.1.1.1 Rho

Rho measures the sensitivity of the option value with respect to changes in the risk-free interest rate (Hull, 2018, p. 439). The base case has a risk-free rate of 2%. A 1% decrease in the rate from 2% to 1% gives an increase in the option value of €0.20 million. This is equivalent to an increase of 5.21% in the option value. An increase in the risk-free rate from 2% to 3% decreases the option value with €0.19 million. This is equivalent to a decrease of 4.88% in the option value. The effect of the changes in the risk-free rate slightly decreases as the rate increases.

Figure 7.1 Option Value's Sensitivity to changes in the Risk-Free Rate



7.1.1.2. Vega

Vega measures the sensitivity regarding volatility of the underlying asset (Hull, 2018, p. 437). The underlying asset in this context is the power price. In the base case the sigma is set to be approximately 17% as derived from the regression-output in chapter 4.3. A 1% increase in the sigma from 17% to 18%, increases the option value with €1.05 million. This is equivalent to an increase of 26.91% of the option value. The option value is particularly sensitive to changes in the volatility. Given the fact that the value of an option mostly is derived from the volatility of the underlying asset, this result is consistent with that phenomenon. The effect on the option value derived from changes in the volatility, is decreasing for higher values of volatility. The changes are most present at volatility-values below 20%. When the volatility reaches 12%, the option value is close to zero. This is because the power prices will not fluctuate enough to make it profitable to suspend production. The more fluctuations around the mean, the higher prices will occur, thus making flexibility in the production more valuable.

Figure 7.2 Option Value's Sensitivity to Changes in the Volatility

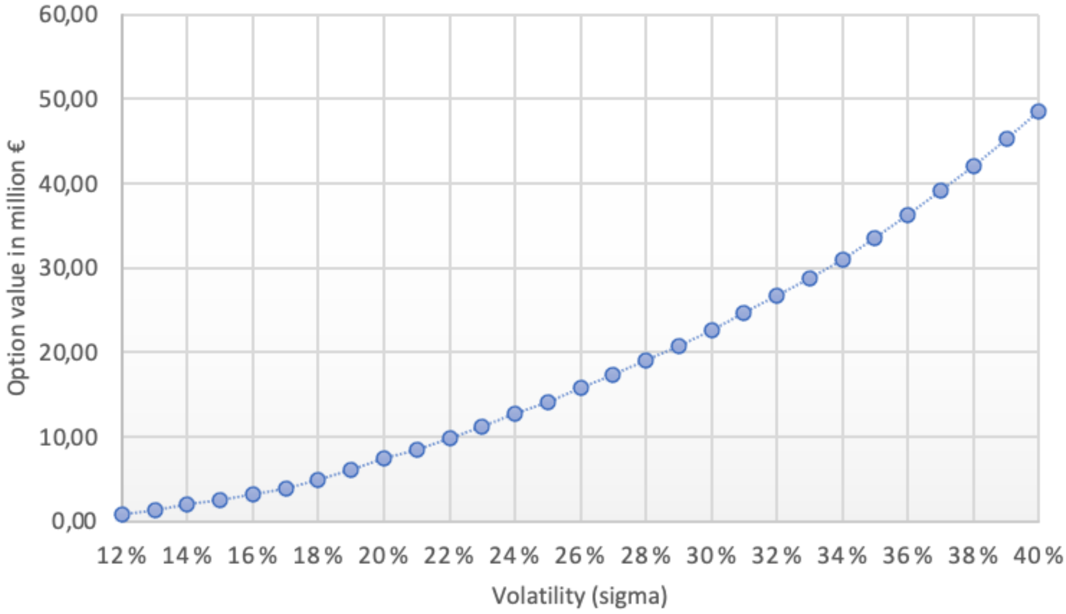
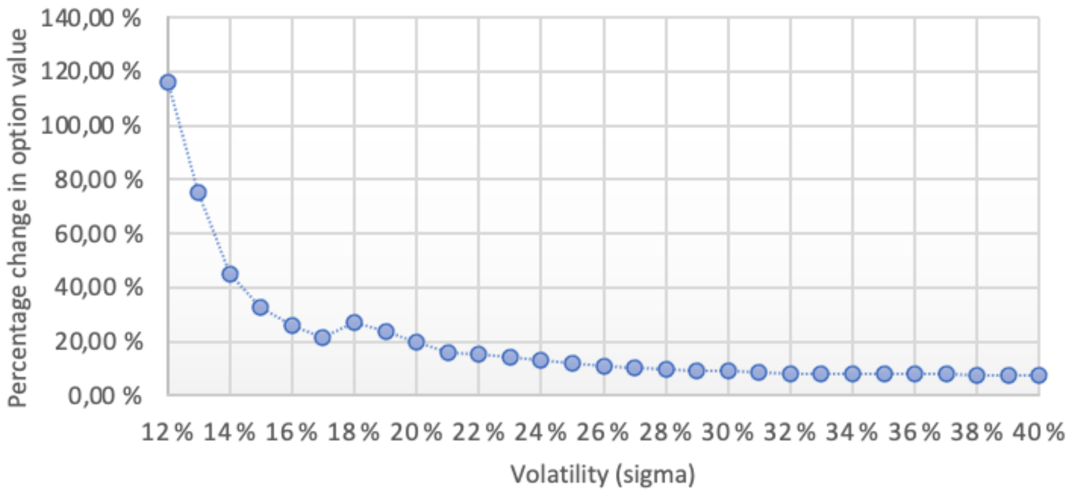


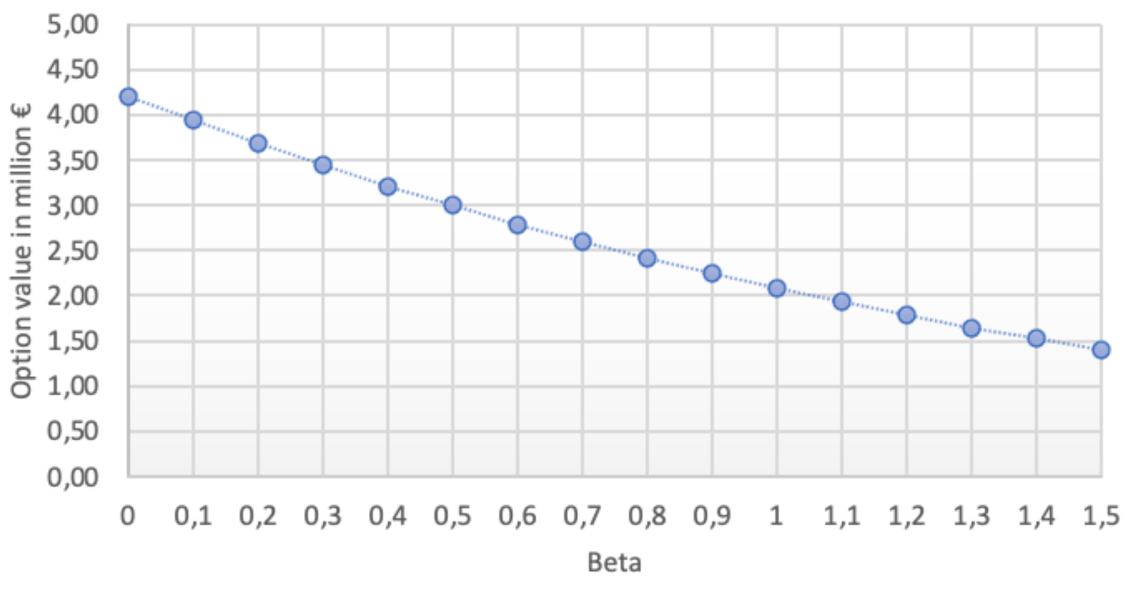
Figure 7.3 Vega



7.1.2. Sensitivity to Changes in the Beta

The risk-neutral factor reflects the risk associated with the cash flows. This factor is based on the overall risk in the market and other investment opportunities available for investors. The risk-neutral factor is derived from the β of the asset, MRP and the time step in the model. The base-case has a risk-neutral factor of 0.0055, whereas the β is 0.14. When β is zero, the risk-neutral factor is also zero. The MRP has been stable during the last decade as explained in chapter 4.4.1.3 and is not likely to suffer from large fluctuations. The β on the other hand is far more likely to vary, and so the effect of changes in the risk-neutral factor will be analysed based on changes in the β. A lower β indicates a lower systematic risk. This leads to the cash flows being more valuable, which again increases the market value of the option. The option value at the different levels of β between 0 - 1.5 are displayed in table 7.4. The results show a decrease in the market value of the option as the β increases. This is consistent with the expected effect of systematic risk in the cash flows.

Figure 7.4 Option Value's Sensitivity to Changes in Beta



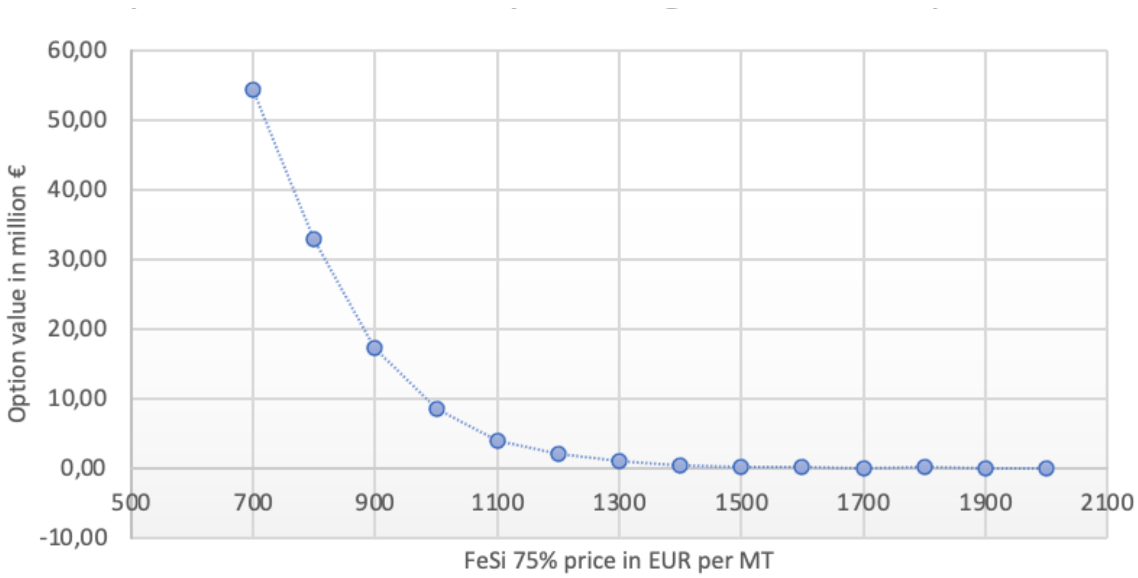
7.1.3. Sensitivity to Changes in the Margins

For the option of suspending production to have value, the profit-margins must be sufficiently negative in certain periods. The profit margins of the smelter will then have a great influence on how much fluctuations one can handle with various margins. Because of the complexity and various setups of costs in a production facility, the sensitivity in the margins will be based on changes in the market price of silicone (FeSi 75%).

The base case has a FeSi 75% price of €1,100. In the last decade, the silicone price has mainly stayed between €800 - €1,600, except for the recent periods with extraordinarily high prices, peaking at €4,000.

The option value is highly sensitive to changes at low levels of the FeSi 75% price. The value of the option more than triples between a price of €700/MT and €900/MT. Note that at these prices both the total value of the smelter with and without the option is negative. At FeSi 75% price-levels above €1,400/MT the option value is less than 0.5% of the total plant value. For high prices of FeSi 75% the option value is close to zero, as the high margins rarely make closing the production valuable.

Figure 7.5 Option Value's Sensitivity to Changes in FeSi 75% Price



7.2. Result of Sensitivity Analysis

Based on the performed sensitivity analysis, the main drivers influencing the option value consist of the volatility of the underlying and the profit margins in the production. Higher volatility mainly keeps the option in the money, hence making it more valuable. From option theory, there is a common knowledge that the value of the option increases with its volatility. Our sensitivity analysis is consistent with that theory.

The margins will vary from smelter to smelter, and due to the complexity of factors, it can be hard to predict precisely. The general sensitivity analysis proved the option to become extremely more valuable at low profit-margin levels. It also quickly lost its value when the profit margins increased sufficiently, making it non-valuable at adequately high profit-margins.

8 Summary & Conclusion

8.1. Implications of the Model

Maintenance costs, which is included as a variance cost in the profit function, will occur regularly during longer periods of production. During the maintenance, the production will occasionally be required to stop. The closed periods in times of high power-prices can be used for necessary maintenance on the furnaces. By making use of these periods without production to maintain the facility, the net maintenance costs can be lowered. This would require some timing issues, and it is not certain that the production will be closed due to high power prices when maintenance needs to be done.

Long-term contracts for delivery that need to be fulfilled will limit the flexibility in the production. This aspect is not considered in the model but will certainly influence the value of the switching option, depending on what degree of long-term contracts they have and their conditions.

Most smelters and other big consumers of power will often lock in the price for some of their future power consumption with derivatives. This makes them less vulnerable for extensive price jumps in the power prices, meaning they can stay operational even in periods when the spot prices exceed their break-even point. The companies will need to be proactive to benefit from the hedging. When the spot prices have already increased to a generally higher level, it will generally be too late to agree on long-term power contracts, as the case was for Rec Solar in December 2021 (Hovland, 2022). Entering long-term power contracts does provide some predictability for that certain input factor, but the future spot prices can both increase and decline in a similar manner. This means that while hedging against the worst-case scenario, they are exposed to missing out on potential gains in cases with low spot prices. Furthermore, there is no guarantee that the price for the other input factors, or output price, will remain constant. The break-even value for the power price might change depending on what the level of the other factors are. These hedging aspects will vary for each company, based on the degree of hedging done, and for which prices and time horizons. It is an interesting aspect that some companies have ended up with a bigger profit from their hedging positions than their actual production, due to the periodic high prices (Finstad & Kværnes, 2022). This phenomenon has proved difficult to model in general, due to the complexity.

8.2. Conclusion

We started by briefly introducing the smelter industry and Elkem Bremanger which is our inspiration to this thesis. Furthermore, we investigated the power market in Norway and the rest of Nordic Europe. How the prices have developed historically and the factors influencing the movements in the power prices in different areas. By using theories of stochastic processes and mean reversion, we developed a model for the possible future power price development. After estimating a mean reverting process of the power price, we could set up a binomial tree and make the price path dependent. From this tree, we could estimate various states for a smelter with the power price as the underlying asset.

Based on real option theories, and the use of backward induction, we derived at a present value of the possible future outcomes from our base case, both with and without the possibility for operational hedging. The operational hedge used in this thesis is the option to temporarily stop production in periods with high power prices. Our base case resulted in an option value of approximately 15% from the total value of the production. Furthermore, the sensitivity of relevant parameters in the model and valuation was examined. The analysis showed a high sensitivity to changes in the volatility in the power prices and changes in the profit margin of the production. Consistent with option theories, a higher volatility increases the option value.

Real option valuation can highlight aspects and their value that are not considered in other valuation methods such as the NPV. Using real option valuation can act as a value-enhancing supplement to other more regular valuation methods. It can also act as the main valuation method, depending on the situation and circumstances of a project or business. By not considering potential real options, net profitable projects and business-models can be foregone.

8.3. Suggestions for Further Research

The aspect of hedging the smelter's power exposure to reduce the risk of fluctuating prices using financial derivatives, would be an interesting topic for further research. By extending our model, one can obtain the necessary degree of the plant's total power exposure that needs to be hedged, to keep the production operational at a certain level.

Hedging using financial derivatives also opens another aspect of the real option. In contrast to buying power for physical delivery from the day-ahead market, the futures contracts are purely financial assets. The short position in the futures-trade, pays the difference between the spot price and the agreed upon futures price at the date of delivery. This gives the long position an instant profit if the spot price is higher than the futures price. The long position decides whether one wants to use the profit from the trade for buying the actual power in the spot market or keep the profit from the trade and cease production. This provides an opportunity of profiting from the hedging operations, when spot prices for the main input source makes the production margins unprofitable, if one can delay production to times with lower input costs.

Even though the situation with hedging would be complex, it could be quite beneficial for two aspects of the plant. Both the amount of hedging activities necessary regarding fulfilling contracts of delivery of the products, and the potential profits from the financial hedging.

Furthermore, it would be interesting to re-do the option valuation in a few years' time, when the market has potentially stabilised after the current volatile period. Whether the prices stabilise higher or lower than the historical average, can have a great impact on the big power-consuming industries. How the companies within these industries will adapt to the potentially new price standards, would be an interesting case to research.

The current Russian invasion of Ukraine can have a huge impact on the dynamics in the European heavy industry. The partly state-owned Russian energy company Gazprom provides approximately 40% of the gas in Europe (Solberg, 2022). Germany, Netherlands and Italy are some of the big consumers of Russian gas. On April 27th, 2022, Gazprom stopped their export of natural gas to Poland and Bulgaria (NTB, 2022). If Russia were to stop their export of gas to the other European countries as well, it can have dramatical consequences for the industry. The demand for power would likely increase as it is an alternative to natural gas. When power-capacity is already at its limit, we could see even more extreme prices than what we have seen in the winter of 2022. The effect on the European heavy industry that erupts from restrictions on Russian gas exportation to Europe would be a very interesting and currently hot topic to explore.

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