Received: 21 October 2020 | Revised: 26 May 2021 | Accepted: 21 June 2021

DOI: 10.1049/itr2.12098

IET Intelligent Transport Systems

The Institution of Engineering and Technology WILEY

ORIGINAL RESEARCH PAPER

# Reforming mixed operation schedule for electric buses and traditional fuel buses by an optimal framework

**Mengyuan Duan**[1] | **Geqi Qi**[1] | **Wei Guan**[1] | **Chaoru Lu**[2] | **Congcong Gong**[3]

[1] Key Laboratory of Transport Industry of Big Data Application Technologies for Comprehensive Transport, Ministry of Transport, Beijing Jiaotong University, Beijing, China

[2] Traffic Engineering Research Center, Department of Civil and Environmental Engineering, Norwegian University of Science and Technology, Trondheim 7491, Norway

[3] State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing, China

**Correspondence**
Wei Guan, Key Laboratory of Transport Industry of Big Data Application Technologies for Comprehensive Transport, Ministry of Transport, Beijing Jiaotong University, Beijing 100044, China
Email: weig@bjtu.edu.cn

**Abstract**

Bus scheduling plays a significant role in public transportation and supports the sustainable development of transportation systems. Challenges are beginning to appear with the newly emerging electric buses (EBs), as scheduling changes due to fleet composition make traditional fixed timetables no longer able to satisfy operational needs. Moreover, the fixed-trip time hypothesis has been inappropriate for large cities due to the variety of urban traffic statuses. This paper proposes an optimal framework for reforming the mixed operation schedule for electric buses and traditional fuel buses under stochastic trip times. Based on the primary grouping genetic algorithm (GGA), a straightforward framework with a Monte Carlo simulation is presented to optimize the scheduling scheme. Case studies based on the operating environment and service trips of real bus lines in Beijing are conducted to verify the effectiveness of the proposed model by considering both the composition of fleet types and time stochasticity. Additionally, the impacts of stochasticity, fleet composition, government subsidies and cost factors on operational costs are investigated. Considering stochastic trip times, the achieved scheduling strategies can provide the optimal proportion of electric and traditional fuel buses and make a crucial impact on operational costs.

## 1 | INTRODUCTION

Recently, considering climate change and health impacts, air quality has attracted more attention worldwide [1]. Therefore, reducing the use of fossil fuel is a commonly consentaneous measure [2, 3]. In public transportation, most conventional bus types (such as heavy fuel diesel buses) make significant contributions to air pollution and greenhouse gases due to high daily mileages [4, 5]. The introduction of electrified transportation solutions is part of a wide range of policy options worldwide [6, 7]. Compared with traditional fuel buses (CBs), electric buses (EBs) have considerable inherent advantages such as zero exhaust pipe emissions, lower energy costs, high comfortability, and low noise emissions [8, 9]. Therefore, the transformation of the public transport fleet from conventional fuel buses to electric or alternative energy buses can significantly reduce exhaust pipe emissions and improve air quality.

At present, EBs have been introduced into the market and operated in many countries (e.g. China, Norway, Sweden, and Germany). However, high ownership costs and limitations on driving range and charging speed make it hard for EBs to completely replace CBs [10]. Considering the trade-off between economic and environmental benefits, there will be a period of mixed operation modes composed of EBs and CBs. How to reasonably arrange the operation scheme of mixed fleets and reduce the operational costs during this period are crucial for the sustainable development of urban public transportation systems.

Compared with the CBs, EBs have their own propulsion technology and characteristics. Operating a full or partial electric bus fleet presents additional challenges to the bus planning process, especially bus scheduling [6]. First, CBs can usually operate for one day without refuelling, which means that on the premise of satisfying the timetable constraints, it can be assumed that all buses carry out their service trips at any time. In contrast, when considering electric buses, one night of charging is not enough to complete a one-day operation (even high-capacity batteries with 200+kWh), which means recharging in the daily operation becomes an important part of the planning process [11]. Therefore, the scheduling of the mixed bus fleet will have a great impact on the entire fleet operation. Second, environmental and economic benefits should be considered. To achieve the optimal scheduling strategy for a mixed bus fleet, the technological advantages of CBs and EBs must be comprehensively assessed. On the one hand, we should exploit the low energy cost per kilometre of electric buses and consider the conflicts between the execution of service trips, recharging and the limitation on driving distance. On the other hand, restricting the number of CBs is considered due to the serious emissions costs of fuel. From the environmental benefit and operational costs aspects, comprehensively formulating a dispatching strategy is necessary for evaluating the impact of the electrification process on the bus operating company.

Another problem that should be settled is the fixed-trip time hypothesis, especially for large cities. In bus scheduling, a predetermined timetable is usually given to cover all service trips within a fixed time for the starting and ending locations. However, due to the variability of road traffic and driving conditions, buses cannot generally be operated according to an established schedule and fixed-trip time (i.e. the time duration of a service trip), which may lead to starting delays for some service trips and poor punctuality rates [12]. When there is interference in the actual operation of buses, it will not only lead to the degradation of system performance but also bring additional operational scheduling costs [13]. However, to date, little research has been conducted on the environmental effects, operations costs and passenger service levels with mixed fleets.

To fill the research gap, this study comprehensively considers the environmental effects, cost benefits and passenger service levels, as well as setting the bus purchase costs, energy consumption costs, recharging costs, emission costs, and expected waiting time and delay time costs as objective values to establish a mixed fleet scheduling model under stochastic road conditions. For the problems related to bus operation optimization, many scholars have built a multi-objective optimization model. Considering costs related to passengers and operating agency, the transit network was optimized [14]. To produce a robust schedule, an MIP model was constructed incorporating the travel time and passenger demand uncertainty [15]. For meeting unbalanced demand patterns, some operating strategies were proposed which also reduced the required number of vehicles [16]. A two-objective optimization model is established considering the minimum waiting time of passengers and departure time of bus company [17]. In our study, all costs are monetized and then converted into a single-objective optimization model. This model is explicitly formulated to consider mixed bus operations under stochastic service trip times and can be solved efficiently using the grouping genetic algorithm (GGA) with a crossover and mutation strategy suitable for a mixed fleet. For the stochastic variable of the model, a Monte Carlo simulation method is used to solve the problem. Finally, based on a real bus line operating environment and service trips in Beijing, several case studies are conducted to verify the effectiveness of the model. Additionally, the influence of design variables such as randomness, fleet size and composition are evaluated.

The remainder of this paper is organized as follows. Section 2 provides relevant literature reviews. Section 3 presents the description of the studied scheduling problem. Section 4 proposes the mathematical formulation and Section 5 proposes the solved approach framework based on the GGA with Monte Carlo simulation. Section 6 conducts a case study to schedule the mixed bus fleet and conducts a sensitivity analysis of the stochasticity of trip time, fleet composition, Government subsidy and cost factors. Finally, some conclusions are provided in Section 7.

## 2 | LITERATURE REVIEW

Many studies have examined stochastic trip times, and good bus route and scheduling strategies can be designed. For example, Tang et al. [12] proposed robust scheduling strategies for electric buses, developed static and dynamic models, and showed that considering the stochastic trip times in a scheduling model effectively avoids en route breakdowns. Liang et al. [18] developed a stochastic linear programming model for bus line frequency and passenger path flow under demand and bus travel time uncertainty; its case study shows that this method could significantly benefit public transportation systems. In the joint optimization of school bus routing and scheduling, Babaei et al. [19] considered stochastic time-dependent travel times to guarantee that buses arrived on-time at a school with a required reliability level, and proved that the time-dependent character of travel times has a remarkable influence. He et al. [20] formulated the dynamic vehicle scheduling problem to tackle trip time stochasticity. Assuming that trip time follows a probability distribution, Shen et al. [21] designed a probabilistic model to minimize total costs and maximize on-time performance. By comparing and analysing the on-time performance of buses in the case of different trip times, they also found out that it was more intuitive and realistic for schedulers to design variable trip times than fixed-trip times [22].

While these studies provide valuable insights into bus route planning and operations scheduling, they usually research single-fleet operating environments and ignore the mixed operating conditions of CBs and EBs. To the best of the authors' knowledge, there has been no study on scheduling mixed fleets with stochastic trip times, although stochastic time in other fields has been studied by many.

Mixed fleets include the mix of different capacity vehicle types, as well as the mix of different energy consumption vehicle types. Several studies have been conducted that consider the mixed fleet in vehicle scheduling can significantly reduce operating costs [23–25]. In vehicle routing problems, many studies on mixed fleets have been studied. Murakami [8] focused on electric and diesel-powered vehicle routing. Hiermann et al. [26] studied the electric vehicle routing problem combining conventional, plug-in hybrid, and electric vehicles. Considering a mixed fleet with electric and conventional vehicles, Macrina et al. [27, 28] studied a specific version of the green vehicle routing problem. Goeke and Schneider [29] optimized the routing problem of a mixed fleet of electric and conventional vehicles with time windows.

In bus operations, the management and scheduling problems of mixed fleets have also been studied. Li et al. [24] proposed a new life additional benefit-cost approach to solve a mixed bus fleet management problem. The results showed that mixed fleet optimization is an important consideration in bus fleets, reducing their considerable operational costs. To investigate the extent to which electrification occurs, Rinaldi et al. [6] constructed a mixed-integer linear programming model (MILP). Zhou et al. [30] jointly optimized the vehicle scheduling and charging arrangements of mixed fleets and developed a multi-objective bi-level programming model. Yao et al. [31] established a model for the electric vehicle scheduling problem with multiple vehicle types. However, because electric buses have not fully penetrated the market in many cities, this model cannot be applied in practice at this stage. Rogge et al. [25] addressed problems associated with electric bus fleet size, mix and charging infrastructure optimization. Li et al. [23] constructed a mixed bus fleet scheduling optimization model under range and refuelling constraints and developed time-space networks for bus flow and passenger flow. However, the study described the energy consumption by discretizing the continuous variables, and the trip times of service trips were regarded as fixed constants. The above studies have fully studied the scheduling optimization problem of mixed fleets, but this problem has not been studied based on stochastic trip times.

In this work, our contributions can be summarized as follows.

a. Our problem covers the scheduling of bus fleets, fleet composition and stochastic trip times. We use the optimal framework to reform the mixed operation schedule for electric buses and traditional fuel buses on the basis of stochastic road conditions and explicitly consider the impact of stochastic trip time.
b. We used the GGA with a crossover and mutation strategy to solve the problem effectively. For the stochastic variables in the model, the expected values of all possible scenarios are optimized and simulated by the Monte Carlo method.
c. Taking the operating environment and service trips of real bus lines in Beijing as an example, the influences of design variables such as the stochasticity, fleet composition and cost factors are evaluated.

## 3 | PROBLEM DESCRIPTION

In this section, we first give a general overview of the bus scheduling problem. This study considers the mixed operation of EBs and CBs under stochastic trip time. We refer to this bus operation system as ECM-OS (i.e. EBs and CBs mixed operations system). For the convenience of discussion, a CBs only-operation system is referred to as C-OS (i.e. CBs operation system), and EBs-only operating system is referred to as E-OS (i.e. EBs operation system). Compared to previous studies on bus operations, this study considers the mixed fleets with EBs and CBs and the effects of the stochastic service time. In addition, more practical energy inputs are adopted for the energy consumption of EBs. Table 1 lists the parameters and variables used in this study. General notations describe the arcs and nodes contained in the directed graph of the scheduling

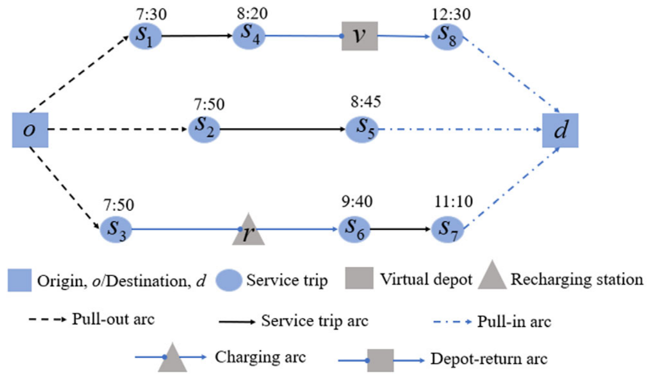


**FIGURE 1** Illustrative example of a scheduling problem

problem. Parameters are required to describe the operations system evolution (i.e. scheduled departure time, stochastic trip time, and energy consumption). Decision variables show the arrangement of bus types, current energy capacity and arrival time.

### 3.1 | The general overview of the problem

We defined a directed graph $G = (N, A)$ to model the problem as shown in Figure 1, where $N := o \cup d \cup S \cup R \cup V$ is the set of nodes and $A := A^o \cup A^d \cup A^S \cup A^R \cup A^V$ is the set of feasible arcs. The set of nodes $N$ contains the depot node represented by origin-depot node $o$ and destination-depot node $d$, the set of service trip nodes $S$, the set of recharging nodes $R$ and the set of virtual depot nodes $V$. The origin-depot node $o$ and the destination-depot node $d$ are where the bus begins and finishes its daily operations, respectively. The service trip node $i \in S$ corresponding to a departure time $a_i$, a stochastic trip time $t_i$, the start station $sp_i$ and the arrival station $ap_i$ is operated once by one bus. The recharging node $r \in R$ is where the electric bus recharges the battery between trips to avoid running out of energy. The virtual depot node $v \in V$ is distinguished from the depot node, and it is where the bus will rest at the depot if the idle time between service trips is too long.

The set of feasible arcs $A$ contains the pull-out arcs $A^o := \{(o, j) | j \in S\}$, pull-in arcs $A^d := \{(j, d) | j \in S\}$, service-trip arc $A^S := \{(i, j) | i, j \in S\}$, recharging arcs $A^R := \{(i, j) | (i, r) \rightarrow (r, j), i, j \in S, r \in R\}$ and depot-return arcs $A^V := \{(i, j) | (i, v) \rightarrow (v, j), i, j \in S, v \in V\}$. The pull-out arcs are the trips in which the bus departs from the origin depot to the start station of the service trip. The pull-in arcs are the trips that the bus travels from the arrival station of the service trip to the destination depot. The service-trip arcs are the trips in which the bus departs from the arrival station of the service trip to the start station of another service trip. The recharging arcs are the trips that the bus takes, first travelling from the arrival station of the service trip to the charging station for recharging and then continuing to the start station for the next service trip. Moreover, the depot-return arcs are the trips in which the bus travels from the arrival station of one service trip to the virtual depot for resting and then continues on to

**TABLE 1** Notations

| **General notations** | | | |
|---|---|---|---|
| $G = (N, A)$ | The directed graph | $V$ | Set of virtual depot nodes |
| $A = A^o \cup A^d \cup A^S \cup A^R \cup A^V$ | Set of arcs | $A^o$ | Set of pull-arcs |
| $N = o \cup d \cup S \cup R \cup V$ | Set of nodes | $A^d$ | Set of pull-out arcs |
| $o$ | Origin-depot node | $A^S$ | Set of service trip arcs |
| $d$ | Destination-depot node | $A^R$ | Set of recharging arcs |
| $S$ | Set of service trip nodes | $A^V$ | Set of depot-return arcs |
| $R$ | Set of recharging nodes | | |
| Parameters | | | |
| $E[\cdot]$ | Expected value of stochastic variable | $c_r$ | Fixed charging cost |
| $f_i(t)$ | Probability distribution function of stochastic trip time | $\tau_w^1$ | Waiting time cost of the bus out of the depot |
| $t_{ij}$ | Travel time between arcs $(i, j) \in A$ | $\tau_w^2$ | Waiting time cost of the bus in the depot |
| $a_i$ | Scheduled departure time | $\beta$ | Use ratio of different bus types |
| $\bar{t}_i$ | Expected trip time of service trip $i$ | $\tau_e^E$ | Energy cost per unit |
| $b_i^{ear}$ | Earliest arrival time of service trip $i$ | $\tau_e^{IC}$ | Fuel cost per unit |
| $b_i^{lat}$ | Latest arrival time of service trip $i$ | $c_e$ | Cost per unit emission |
| $\varphi$ | Maximum waiting time of bus out of the depot | $c_b^E$ | Fixed electric bus cost |
| $w_{ij}^1$ | Waiting time out of the depot | $c_b^{IC}$ | Fixed conventional bus cost |
| $w_{ij}^2$ | Waiting time in the depot | $g^{IC}$ | Fuel consumption per unit distance |
| $l_{ij}$ | Delay time | $z_i$ | Length of service trip $i$ |
| $\theta$ | Fixed charging time | $c_{ij}^{IC}$ | Cost of arc $(i, j) \in A$ performed by a conventional bus |
| $c_{ij}^E$ | Cost of arc $(i, j) \in A$ performed by an electric bus | $\lambda_1, \lambda_2$ | Percentage of minimum and maximum battery usage |
| $\tau_t$ | Emission per unit oil | | |
| Decision variables | | | |
| $t_i$ | Stochastic trip time | $e_{ij}^E$ | Battery consumption of arc $(i, j) \in A$ |
| $x_{ij}^{IC}$ | 1, if arc $(i, j)$ is traveled by a conventional bus; 0, otherwise | $x_{ij}^E$ | 1, if arc $(i, j)$ is traveled by an electric bus; 0, otherwise |
| $b_i^{arr}$ | Arrival time of service trip $i$ | $y_i^E$ | Current battery capacity after the trip$i$ |

the start station for the next service trip. Another set of arcs $(i, j) := \{(i, j) | (i, v) \rightarrow (v, r) \rightarrow (r, j), i, j \in S, v \in V, r \in R\}$, denoting the bus's return to the virtual depots for rest and recharging, is also feasible. However, virtual depots and recharging nodes are all in the depot, and the distance between them in our study is defined as zero. Therefore, the set of arcs is essentially the same as the recharging arcs.

Figure 1 shows an illustrative example of a scheduling problem. The blue squares with symbols $o$ and $d$ denote the origin-depot node and destination-depot node, respectively. The blue circles with $S_1 - S_8$ denote the service trip nodes. Additionally, the grey squares with the symbol $v$ denote the virtual depot nodes, while the grey triangles with the symbol $r$ denote the recharging nodes. Each service trip has a fixed departure time. Taking the path $o - S_1 - S_4 - v - S_8 - d$ as an example, the bus leaves the origin-depot $o$ for daily operations and goes through service trips $S_1$ and $S_4$. Due to the long wait between two adjacent trips, the bus returns to the virtual depot for rest and then leaves to complete the remaining service trips without recharging.

## 3.2 | Stochastic trip time for ECM-OS

Under stochastic traffic conditions, the trip times $t_i$ of service trips $i \in S$ are assumed to follow a normal distribution $f_i(t)$, with an expected trip time $\bar{t}_i$. The range of trip times is $[t_i^{min}, t_i^{max}]$. The arrival time of service trip $b_i^{arr}$ is constrained by Equation (1).

$$b_i^{arr} = a_i + t_i, b_i^{ear} = a_i + t_i^{min},$$
$$b_i^{lat} = a_i + t_i^{max}, b_i^{ear} \le b_i^{arr} \le b_i^{lat}, \forall i \in S \quad (1)$$

where $a_i$ is the scheduled departure time of service trip $i$, $b_i^{ear}$ is the earliest arrival time related to the minimized trip time $t_i^{min}$, and $b_i^{lat}$ is the latest arrival time related to the maximized trip time $t_i^{max}$. For each service trip $i \in S$, the trip time $t_i$ is a stochastic variable; therefore, $b_i^{arr}$, associated with the trip time, is also a stochastic variable. The travel time $t_{ij}$ between two trips is stochastic in actual operations. However, compared with the service trip time $t_i$, the stochastic influence of roads on the travel time $t_{ij}$ can be ignored, and $t_{ij}$ is regarded as constant [12, 21].

The feasibility of services arcs $A^S$ is constrained by Equation (2).

$$b_i^{arr} + t_{ij} \le a_j, (i, j) \in A^S \tag{2}$$

The constraint in Equation (2) indicates that the sum of the arrival time of service trip $i$ and the travel time $t_{ij}$ between two trips is less than or equal to the departure time of the next service trip $j$. Therefore, constraint in Equation (2) guarantees feasible service-trip arcs. The stochastic variable $b_i^{arr}$ makes it difficult to establish feasible arcs, and the expected trip time is used as substitute for the stochastic trip time [12]. Therefore, $b_i^{arr} = a_i + \bar{t}_i$.

The virtual depot and the origin/destination depot refer to the same place, but their meanings are different. Due to the long idle time between two service trips, if the bus stays outside the depot for a long time, it will occupy social resources and result in high operating costs. Therefore, the bus must return to the depot. Moreover, when the buses leave the depot, the state of charge (SOC) is the same as the SOC when the buses arrived. The recharging node is where the bus recharges the battery with a fixed charging time. In this study, we assume that the bus will leave the depot and that it is fully charged. The recharging arc ensures that the bus will not experience range anxiety during the operation period. The depot-return arcs and the recharging arcs are constrained by Equations (3) and (4), respectively.

$$a_j - b_i^{arr} - \left(t_{iv} + t_{vj}\right) \ge \varphi, (i, j) \in A^V, A^V := \{(i, j) \mid (i, v) \rightarrow (v, j), i, j \in S, v \in V \tag{3}$$

$$a_j - b_i^{arr} - \left(t_{ir} + t_{rj}\right) \ge \theta, (i, j) \in A^R, A^R := \{(i, j) \mid (i, r) \rightarrow (r, j), i, j \in S, r \in R\} \tag{4}$$

where $a_j$ is the scheduled departure time of service trip $j$, $b_i^{arr}$ is the arrival time of service trip $i$, $t_{iv}$ and $t_{vj}$ are the travel times between service trips and the virtual depot, $t_{ir}$ and $t_{rj}$ are the travel times between services trips and the charging station, $\varphi$ is the minimum waiting time at the depot, and $\theta$ is the fixed recharging time. For each depot-return arc$(i, j) \in A^V$, if the idle time between the two trips is equal to or greater than$\varphi$, the bus is forced to return to the depot for rest. For each recharging arc$(i, j) \in A^R$, if the idle time in between the two trips is equal to or greater than $\theta$, the bus can be recharged at the station.

## 3.3 | Energy consumption for ECM-OS

The energy consumption depends on the type of bus, road characteristics, and route length [32]. For EBs, uncertain factors such as traffic status and passenger load have a serious impact on battery consumption, affecting the subsequent trips [33]. Therefore, we use the energy consumption in the actual operations process as the direct input. The energy consumption of each trip is obtained through the historical data of electrical buses. By calculating the average energy consumption, we get the curve about the change tendency of SOC and the mileage travelled, shown in Figure 2. The energy consumption in the model is an input parameter. Giving the current SOC of the bus and the mileage to be travelled, the remaining SOC is outputted and then we can get the bus's energy consumption.

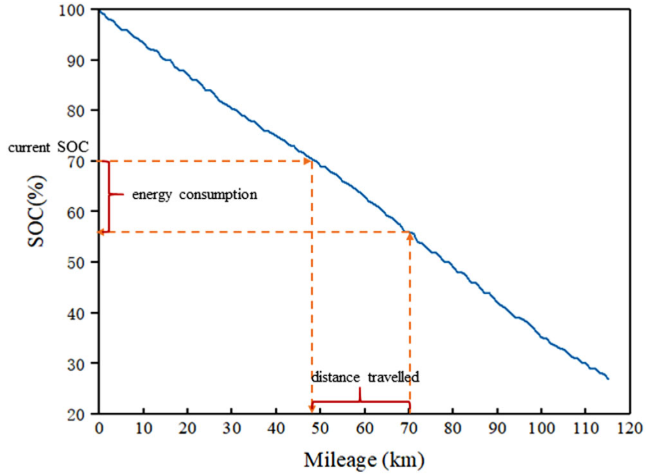


**FIGURE 2** The change tendency of SOC and the mileage travelled

CBs can complete a full day's journey with a full tank. Although the uncertainty will affect energy consumption, it will basically have no impact on the subsequent journey. Therefore, fuel consumption per distance is regarded as a fixed value.

To simplify the formulations, we have made the following assumptions.

1. In this study, a single route with one depot is considered in the mixed bus operation system. The basic information of the bus route is known, such as the length of the line, the type of bus, the deadhead distance and travel time. The service trip time of each schedule is unknown. However, the departure time and the distribution of trip time are provided.
2. All chargers are placed in the depot and they have the same charging rate. In addition, we always have available chargers.
3. The relationship between the fuel consumption of buses and the trip distance is assumed to be linear.

# 4 | FORMULATIONS

This section presents the scheduling model based on the stochastic trip time to optimize the total costs and generate an optimized fleet scheme. The objective function and constraints are presented in the following subsections. Figure 3 shows a diagrammatic sketch of the scheduling model, including the inputs, objective function, constraints and outputs.

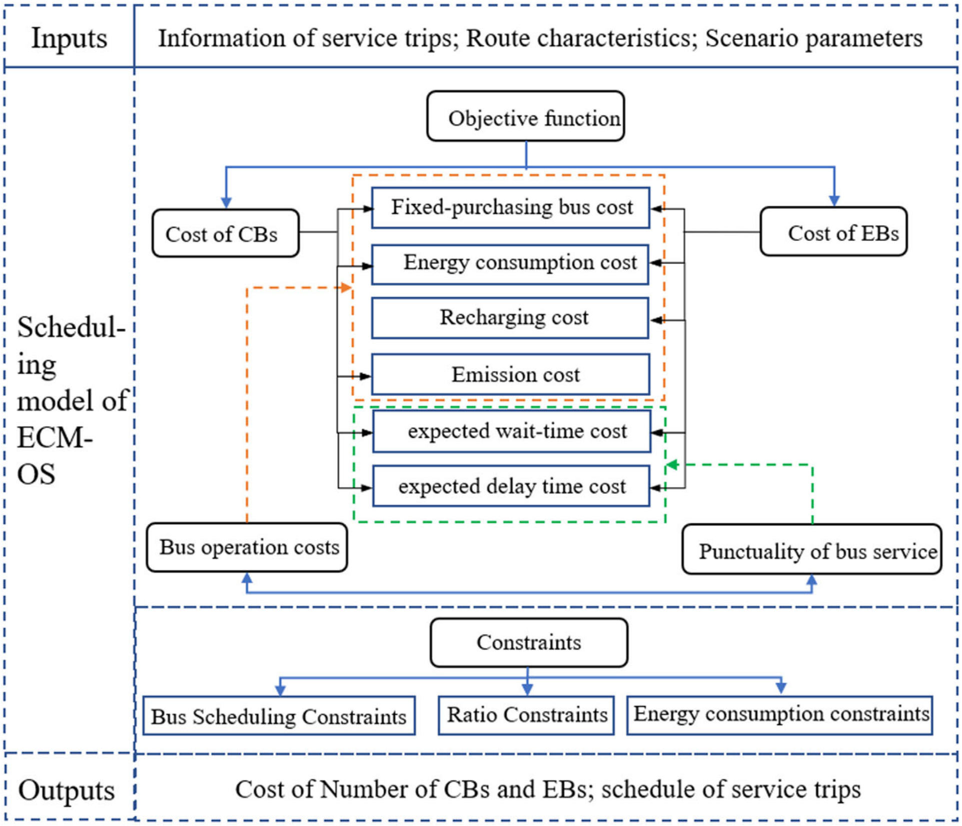


**FIGURE 3** A diagrammatic sketch of the scheduling model

## 4.1 | Objective function

In previous studies on conventional buses, the objective function was mainly to minimize the number of buses. Recently, with the emergence of alternative fuel vehicles, more realistic features such as energy consumption, infrastructure construction, operation time and emission costs have been increasingly emphasized. Li [34] minimized the operating costs. A model to optimize the distribution of charging stations for electric buses has been developed [35]. Tang et al. [12] considered the wait-time and delay-time costs in a dynamic scheduling model. This paper makes the scheduling arrangement from the perspective of the operating company. To better negotiate operations with a mixed fleet, the objective function considers the operational costs of EBs and CBs and the time costs, which reflecting the punctuality of bus services. The operational costs include the fixed-purchase bus cost, energy consumption costs, recharging costs of EBs and emission costs of CBs. The time costs include the expected waiting costs and delay costs. We describe this problem as a maximum flow and calculate the cost of all types of feasible arcs. The objective function is expressed as Equation (5).

$$\min Z = \sum_{(i,j)\in A} \left( c_{ij}^{E} x_{ij}^{E} + c_{ij}^{IC} x_{ij}^{IC} \right) \tag{5}$$

Pull-out arc: The cost of the pull-out arc $(i,j) \in A^{o}$ is formulated as Equations (6) and (7).

$$c_{ij}^{E} = c_{b}^{E} + e_{ij}^{E} \tau_{e}^{E} \tag{6}$$

$$\begin{aligned} c_{ij}^{IC} &= c_{b}^{IC} + e_{ij}^{IC} \tau_{e}^{IC} + e_{ij}^{IC} \tau_{t}\, c_{e} = c_{b}^{IC} + e_{ij}^{IC} \left( \tau_{e}^{IC} + \tau_{t} c_{e} \right) \\ &= c_{b}^{IC} + g^{IC} \cdot \left( d_{ij} + z_{j} \right) \cdot \left( \tau_{e}^{IC} + \tau_{t} c_{e} \right) \end{aligned} \tag{7}$$

where $c_{b}^{E}$ and $c_{b}^{IC}$ are the electric bus and conventional bus purchasing costs; $e_{ij}^{E}$ and $e_{ij}^{IC}$ denote the battery and fuel consumption for service trip $j \in S_{n+1}$ after trip $i \in S_{0}$, respectively, proportional to the distance travelled; $g^{IC}$ is the fuel consumption per kilometre; $d_{ij}$ is the deadhead distance of trips $i$ and $j$; $z_{j}$ is the distance of a service trip; $\tau_{e}^{E}$ and $\tau_{e}^{IC}$ are the unit prices for batteries and oil; $\tau_{t}$ is the emissions per unit of fuel; and $c_{e}$ is the price per unit emission. Pull-out arc costs consist of the bus purchasing cost and the energy consumption costs. If the arc is operated by a CB, it also includes the cost of emissions generated by the CB.

Pull-in arc: The cost of the pull-in arc $(i,j) \in A^{d}$ is formulated as Equations (8) and (9). Compared to the cost of pull-out arcs, the cost of pull-in arcs does not include the bus purchase. Since the bus departs from the depot and finally returns to the same place, the purchasing cost of the outgoing bus is only counted once. Consistent with most existing studies [22, 23, 36], the bus purchasing cost is included in the cost of a pull-out arc. For a pull-in arc, $j$ is the destination depot; thus, the energy consumption is just the energy consumed by the distance from trip $i$ to $j$.

$$c_{ij}^{E} = e_{ij}^{E}\ \tau_e^{E} \tag{8}$$

$$c_{ij}^{IC} = e_{ij}^{IC}\ \tau_e^{IC} + e_{ij}^{IC}\tau_t\ c_e = g^{IC}\cdot d_{ij}\cdot\left(\tau_e^{IC} + \tau_t c_e\right) \tag{9}$$

Service-trip arc: The cost of the service-trip arc $(i,j)\in A^S$ is expressed in Equations (10) and (11). The cost of the service-trip arc includes the cost of energy consumption, the expected cost of the out-of-the-depot waiting time, and the cost of delay time. If the arc is operated by a CB, it also includes fuel consumption and emissions costs.

$$c_{ij}^{E} = e_{ij}^{E}\ \tau_e^{E} + \tau_w^1 E\left(w_{ij}^1\right) + \tau_l E\left(\left(l_{ij}\right)^2\right) \tag{10}$$

$$\begin{aligned} c_{ij}^{IC} &= e_{ij}^{IC}\ \tau_e^{IC} + e_{ij}^{IC}\tau_t c_e + \tau_w^1 E\left(w_{ij}^1\right) + \tau_l E\left(\left(l_{ij}\right)^2\right) \\ &= g^{IC}\cdot\left(d_{ij} + z_j\right)\cdot\left(\tau_e^{IC} + \tau_t c_e\right) \\ &\quad + \tau_w^1 E\left(w_{ij}^1\right) + \tau_l E\left({l_{ij}}^2\right) \end{aligned} \tag{11}$$

$$w_{ij}^1 = \max\{a_j - a_i - t_i - t_{ij}, 0\}, \forall i,j\in S \tag{12}$$

$$l_{ij} = \max\{a_i + t_i + t_{ij} - a_j, 0\}, \forall i,j\in S \tag{13}$$

where $\tau_w^1$ and $\tau_l$ are the wait-time cost out of the depot per unit and the delay-time cost per unit, respectively; $E(\cdot)$ denotes the expected value of the stochastic variable; $w_{ij}^1$ is the wait-time out of the depot before service trip $i$ after trip $j$; and $l_{ij}$ is the delay time before service trip $j$. Similar to the study [37], a quadratic function is adopted to calculate the penalty cost for late arrival, reflecting the significant adverse effects caused by huge delays, where $w_{ij}^1$ and $l_{ij}$ are all stochastic variables because they are related to the stochastic service trip time $t_i$.

Depot-return arc: The cost of the depot-return arc $(i,j)\in A^V$ is formulated in Equations (14) and (15).

$$c_{ij}^{E} = e_{ij}^{E}\ \tau_e^{E} + \tau_w^2 E\left(w_{ij}^2\right) \tag{14}$$

$$\begin{aligned} c_{ij}^{IC} &= e_{ij}^{IC}\ \tau_e^{IC} + e_{ij}^{IC}\tau_t c_e + \tau_w^2 E\left(w_{ij}^2\right) \\ &= g^{IC}\cdot\left(d_{ik} + d_{kj} + z_j\right)\cdot\left(\tau_e^{IC} + \tau_t c_e\right) + \tau_w^2 E\left(w_{ij}^2\right) \end{aligned} \tag{15}$$

$$w_{ij}^2 = a_j - a_i - t_i - t_{iv} - t_{vj}, \forall i,j\in S, k\in V \tag{16}$$

where the cost of the depot-return arc includes the cost of energy consumption and the expected wait time. If the arc is operated by a CB, it also includes the cost of fuel consumption and emissions. The differences with the service-trip arcs are that there is no delay-time cost, and the wait-time cost is calculated by the bus waiting at the depot, where $w_{ij}^2$ is the wait time of the bus waiting at the depot, which generally needs to be greater than 3 h. The difference between $w_{ij}^1$ and $w_{ij}^2$ is the wait location.

Recharging arc: Finally, the cost of the recharging arc, $(i,j)\in A^R$ is formulated as Equation (17).

$$c_{ij}^{E} = c_r + e_{ij}^{E}\tau_e^{E} + \tau_w^2 E\left(w_{ij}^2\right) + \tau_l E\left(\left(l_{ij}\right)^2\right) \tag{17}$$

where $c_r$ is the fixed cost of recharging at the charge station and is equivalent to the depreciation cost of the charging station. The cost of the recharging arc includes the fixed recharging cost, the energy consumption cost, the wait-time cost in the depot and the delay-time cost. We assume that if the time has not yet reached the scheduled departure time of service trip $j$ after a full recharge, the bus is required to wait at the depot.

## 4.2 | Constraints

The constraints of bus scheduling, energy consumption, and investment ratio of different bus types are introduced in this subsection.

### 4.2.1 | Bus scheduling constraints

$$\sum_{(i,j)\in A}\left(x_{ij}^{E} + x_{ij}^{IC}\right) = 1, \quad \forall j\in S \tag{18}$$

$$\sum_{(o,j)\in A^o} x_{oj}^{E} - \sum_{(i,d)\in A^d} x_{id}^{E} = 0, \forall i,j\in S \tag{19}$$

$$\sum_{(o,j)\in A^o} x_{oj}^{IC} - \sum_{(i,d)\in A^d} x_{id}^{IC} = 0, \quad \forall i,j\in S \tag{20}$$

$$\sum_{(i,j)\in A} x_{ij}^{E} - \sum_{(j,k)\in A} x_{jk}^{E} = 0, \quad \forall j\in N \tag{21}$$

$$\sum_{(i,j)\in A\backslash A^R} x_{ij}^{IC} - \sum_{(j,k)\in A\backslash A^R} x_{jk}^{IC} = 0, \forall j\in N/R \tag{22}$$

$$x_{ij}^{E}, x_{ij}^{IC}\in\{0,1\}, \quad (i,j)\in A \tag{23}$$

Constraint in Equation (18) ensures that each service trip needs to be fulfilled exactly once by one bus, whether it is an EB or a CB. Constraints in Equations (19) and (20) guarantee that for each bus type, the bus starts from and eventually returns to the same depot. Constraints in Equations (21) and (22) are the flow conservation constraints for each bus type, indicating that the numbers of incoming arcs and outgoing arcs are equal at each node. Constraint in Equation (23) defines the binary decision variables for each bus type, representing a bus visit trip $i$ after trip $j$.

### 4.2.2 | Energy consumption constraints

Different from conventional buses, electric buses usually need to recharge the battery to finish daily service. The recharge schedule and duration depend on the time gap between adjacent trips. Since the CB refuel time can be ignored, only the energy consumption constraints are considered in this study. Energy consumption constraints are described by Equations (24)–(26).

$$x_{ij}^{E} = 1 \Rightarrow y_{j}^{E} = y_{i}^{E} - e_{ij}^{E}, \forall (i,j) \in A^{o} \cup A^{S} \cup A^{d} \tag{24}$$

$$x_{ij}^{E} = 1 \Rightarrow x_{ir}^{E} = x_{rj}^{E} = 1 \Rightarrow \begin{matrix} y_{r}^{E} = y_{i}^{E} - e_{ir}^{E} \\ y_{j}^{E} = E - e_{rj}^{E} \end{matrix}, \quad \forall\ (i,j) \in A^{R}, r \in R \tag{25}$$

$$x_{ij}^{E} = 1 \Rightarrow x_{iv}^{E} = x_{vj}^{E} = 1 \Rightarrow \begin{matrix} y_{v}^{E} = y_{i}^{E} - e_{iv}^{E} \\ y_{j}^{E} = y_{v}^{E} - e_{vj}^{E} \end{matrix}, \quad \forall\ (i,j) \in A^{V}, v \in V \tag{26}$$

where $y_i^E$ is the remaining energy of the EB after trip $i$. Constraint in Equation (24) restricts the energy consumption of the pull-out arcs, pull-in arcs and service-trip arcs. Constraint in Equation (25) ensures the energy consumption of the recharging arcs, and it consists of two parts. The first part is the remaining energy before the bus visits the charging station, and the second part is the remaining energy after the electric bus completes the service trip $j$. Once the bus has visited the charging station, it will be fully recharged. Constraint in Equation (26) ensures the energy consumption of depot-return arcs, and it also consists of two parts. The first is the remaining energy before arriving at the virtual depot, and the second is the remaining energy after the bus visits the service trip $j$.

$$y_{o}^{E} = E \tag{27}$$

$$\lambda_1 E \leq y_{i}^{E} \leq \lambda_2 E, \forall i \in S \tag{28}$$

$$y_{i}^{E} - e_{ir}^{E} \geq 0, \forall i \in S \cup V \cup \{d\}, \forall r \in R \tag{29}$$

where $E$ is the maximum battery capacity and $\lambda_1$ and $\lambda_2$ are the parameters for the minimum and maximum energy consumption ratios, respectively. Constraint in Equation (27) indicates that the electric buses have been fully charged when they depart from the origin-depot. Considering battery life and recharging speed, an EB usually maintains the SOC of the battery. Constraint in Equation (28) defines the battery available after the bus visits node ($\lambda_1 = 0.3$ and $\lambda_2 = 0.8$). However, even if the available battery of the bus cannot support the next service trip, the bus can be driven to the charging station after visiting the service trip or virtual depot by the remaining battery, which can be required in constraint in Equation (29).

### 4.2.3 | Ratio constraints

$$\sum_{j \in S} x_{0j}^{E} \Big/ \left( \sum_{j \in S} x_{0j}^{E} + \sum_{j \in S} x_{0j}^{IC} \right) \geq \beta \tag{30}$$

where $\beta$ is the use ratio of different bus types; constraint in Equation (30) guarantees the proportion of electric and conventional buses used in public transportation systems. When $\beta$ is 0, it means the primary operation plan with the conventional buses; when $\beta$ is 1, it means that the bus operation is purely electrified. As the investment ratio increases, it reflects the market penetration of electric buses.

## 5 | METHOD FRAMEWORK

To date, many heuristic algorithms have been proposed to solve bus scheduling problems, including genetic algorithms, iterative neighbourhood search algorithms and simulated annealing algorithms. Regarding solution algorithms for such problems, the group genetic algorithm (GGA) has been justified for obtaining a good operation scheme in the bus scheduling problem [25]. It mainly emphasizes the grouping aspects of the scheduling problem, and the schedule plan is uniquely defined by the service trips. Considering the specific characteristics of a mixed fleet, we design specific crossover and mutation strategies to apply to the GGA. To prevent falling into the local optimum, simulated annealing (SA) is used for cooling. For the expected term in the objective function, which makes the model difficult to solve, we use a Monte Carlo simulation to solve this term.

The input operating data consist of the information for service trips, route characteristics, and other scenario parameters. It is used to pre-process the deadhead distances and the energy consumption between trips. Each service trip $i \in S$ is described by scheduled starting time $a_i$, expected trip time $\bar{t}_i$, and the start and arrival stations. The route characteristics include the line length, the distance and travel time between the depot and start/end stations, and the distance and travel time between any feasible service-trip arcs. According to the input data, the deadhead distance and travel time of any two trips can be acquired. The energy consumption of each trip, assumed to be linear to the distance, can be calculated.

**Group Genetic Algorithm**

Step 1: According to the service trip information, route characteristics and scenario parameters, relevant data are extracted (such as the departure time, the departure and arrival station of trips, and unused buses). Arrange the service trips in ascending order in terms of departure time;

Step 2: Generate a number of initial solutions, which form the initial populations $S$;

Step 3: Sampling the probability distribution of stochastic trip time by the Monte Carlo method and calculate the total costs $C(s)$ of each initial solution according to Equation (5), then evaluate the fitness $f(s)$. Let $s^* = s$, $C^* = C(s)$, $f^* = f(s)$;

Step 4: Let the number of iterations $iter = 0$, with the solution after each optimization defined as $s_t$;

Step 5: The strategies of selection, crossover and mutation are applied to optimize the scheduling process and calculate the $C(s_t)$ and $f(s_t)$;

Step 6: If $f(s_t) < f^*$, $s^* = s_t$, go to Step 7; otherwise, go to Step 8;

Step 7: Let the number of iterations $iter = iter + 1$;

Step 8: A random number $p_0 = N(0,1)$ is generated. If $p_0 < p = \exp\{(f(s_t) - f^*)/T_n\}$, $s^* = s_t$, where $T_n = \alpha * T_{n-1}$ is the temperature at the $iter$ iteration, $T_0$ is the initial temperature and $\alpha$ is the cooling rate;

Step 9: If the termination condition $iter > N_0$ is satisfied, output the bus schedule and recharging plan; otherwise, proceed to Step 5. $N_0$ is the number of iterations.

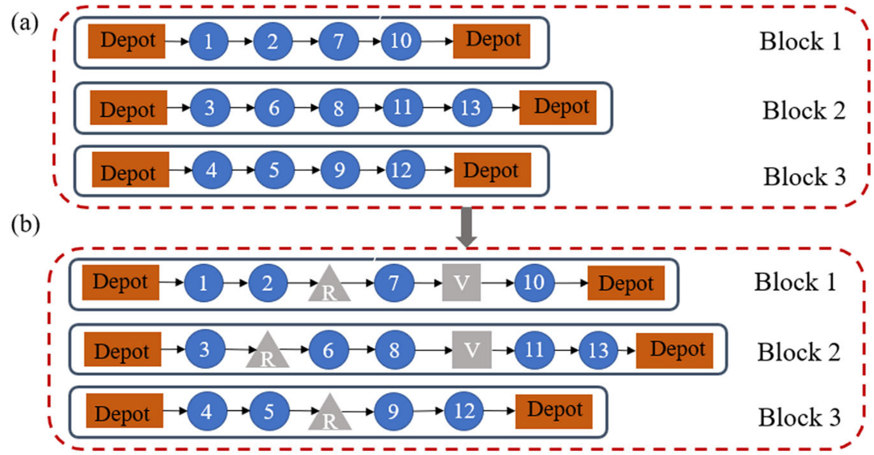


**FIGURE 4** Genetic problem representation: an individual consists of some blocks and the number of blocks is the number of buses required

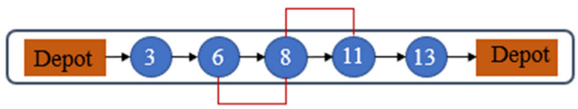


**FIGURE 5** A remediation strategy for non-feasible solution

## 5.1 | Generation of initial solutions

The initial solution consists of some blocks, and the number of blocks is the number of buses required. As shown in Figure 4, each row represents a block assigned to an EB or a CB, and the number of trips contained in the block is variable. A block represents a trip sequence in which a bus starts and ends at the depot. The starting time of the service trip is uniquely defined as the order of the block. The advantage of this representation method is that the number of various bus types used can be intuitively counted.

Random initialization is used to obtain an initial feasible solution satisfying time constraints. A series of processing steps is required to ensure that the trip sequences that make the recharging time and energy consumption constraints infeasible are removed from the initial solution.

**Generating Initial Solutions**

Step 1: Arrange the departure time of service trips in ascending order; select the first service trip as the initial trip of the first block, check the subsequent trips successively until the next trip that meets the time constraint, and take it as the second service trip of the first block until all the trips are checked; then construct the next block in the same way until all the trips are in the blocks; then the initial solution satisfying the time constraints is generated, as shown in Figure 4(a);

Step 2: According to the idle time between two adjacent service trips, check whether the virtual depot can be inserted;

Step 3: Check whether the block meets the energy consumption constraints; if not, whether the idle time is enough for the bus to replenish its electricity;

Step 4: For the block, if it satisfies the energy consumption constraints or it has enough time for the bus to replenish its electricity, the block can be performed by an EB; otherwise, it is performed by a CB;

Step 5: For all blocks of the initial solution, check whether the ratio between used EBs and CBs satisfy the ratio constraint; then, the initial solution is generated, as shown in Figure 4(b).

In step 3, check whether the bus has enough time to visit the charger between two service trips; if not, a strategy is conducted to remedy a non-feasible solution. The time to visit the charger can be advanced before performing the previous service trip. As shown in Figure 5, if the bus has not enough time to recharge between service trips 8 and 11, it can be checked whether the idle time between the trips 6 and 8 is enough to recharge.

### 5.1.1 | Strategy of selection, crossover and mutation

For the next generation, r to inherit optimized individuals, the selection criterion is applied to the population. The selection is a fitness-based process. Due to our minimization problem, we adopt the following selection criterion: each individual in the population is ranked in descending order according to their objective function. The square of the rank is taken as the fitness value, and then several individuals are randomly selected to form a new population according to the roulette selection method. The expected term of the objective function is simulated by Monte Carlo simulation. By sampling the probability distribution of the stochastic trip time, a discrete distribution with a sample size $K$ randomly generated by the distribution function can be approximately replaced. Therefore, the expected term in the objective function is expressed as the mean of the sum of $K$ sample objective values.

In the process of crossover and mutation, the solution is destroyed. If the destroyed solution is feasible, the fitness evaluation will be continued; if not, the destroyed solution needs to be repaired. The generation of a new individual is a combination of selected parents. The visual representation of the crossover is shown in Figure 6. The crossover strategy of is as follows: (1) randomly select a sequence from a block of parent 1 to

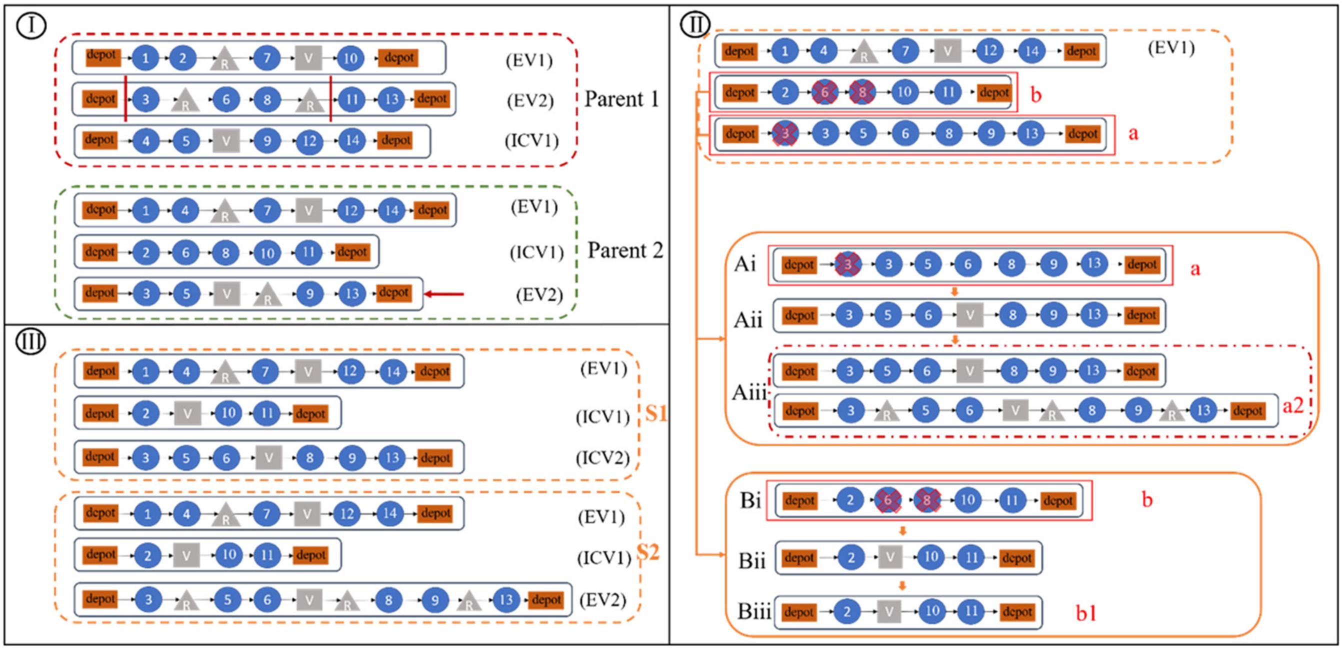


**FIGURE 6** Illustration of crossover process

combine with one block of parent 2. As shown in Figure 6-I, the two yellow lines in parent 1 are the defined crossover points. The sequence between the two points is copied into the block indicated by the yellow arrow in parent 2, and a new block is generated. Then, sort the new blocks in ascending order of their scheduled departure times. (2) Repair the new block and other affected blocks. As shown in Figure 6-I, the insertion from parent 1 caused the same service trip to appear twice in the new individual.

To repair the infeasible solution, the following steps are utilized. First, delete the same service trips in the new block as in the cross sequence. Then, check whether the arcs in the new block are all feasible and rearrange the virtual depot visits. The restoration of the affected blocks is similar to the new block. (3) Decide if the new block is operated by an EB or a CB. As shown in Figure 6-III, the final new individual is an S1 or S2. An S1 is a scheduling plan involving one electric bus and two conventional buses, while an S2 is a scheduling plan by involving two electric buses and one conventional bus.

For each block, the priority principle for EBs is given, and CBs performed when the energy consumption constraint is not satisfied. If there is no block whose energy consumption constraint is not satisfied, under the condition that the number of CBs is not exceeded and the ratio constraint is satisfied, the conversion principle of CBs shall be carried out following the cost-minimal conversation rule. The cost-minimal principle is as follows. Compare the changes in the total costs of operating a block from an EB to a CB. If the total costs decrease, put the cost differences in descending order and select the first $m$ for conversion; otherwise, put the cost differences in ascending order and select the same way. The variable $m$ is the number of blocks that can be converted.

## 6 | CASE STUDY

In this section, GGA and Tabu Search (TS) algorithms are applied to two scenarios that differ in their mode of operation. The following introduces the particularities of both scenarios and subsequently discusses the computed results. All experiments are implemented on the machine with an Intel(R) Core (TM) i5-8250U CPU @1.60G HZ, 8 GB of RAM, and the codes for all algorithms are written in Python.

### 6.1 | Scenario definition

The scenarios analysed below represent two different bus lines with different operational modes. As shown in Figure 7, Line A starts at terminal A and travels to terminal B, with a total of 22 stations. The depot is close to terminal A, and the route length is 17.83 km. Its operation time is between 5:00 to 23:00. The schedule of service trips is a total of 191 trips throughout the day, among which 96 trips start from terminal A and end at terminal B, and 95 trips start from terminal B and end at terminal A. We set the expected, minimum and maximum service times as 52, 49 and 55 min, respectively. Line B is a circle line with 25 stations. The route length is 22 km, and its operation time is between 5:30 and 24:00. The schedule of service trips is a total of 163 trips. The expected, minimum and maximum service times are set as 57, 52 and 62 min, respectively. The deadhead distance and time between the depot and the terminals are shown in Table 2.

The two bus lines are run entirely by traditional fuel buses. Thus, several mixed operation experiments are designed to validate our model. Assume that the charging station is located

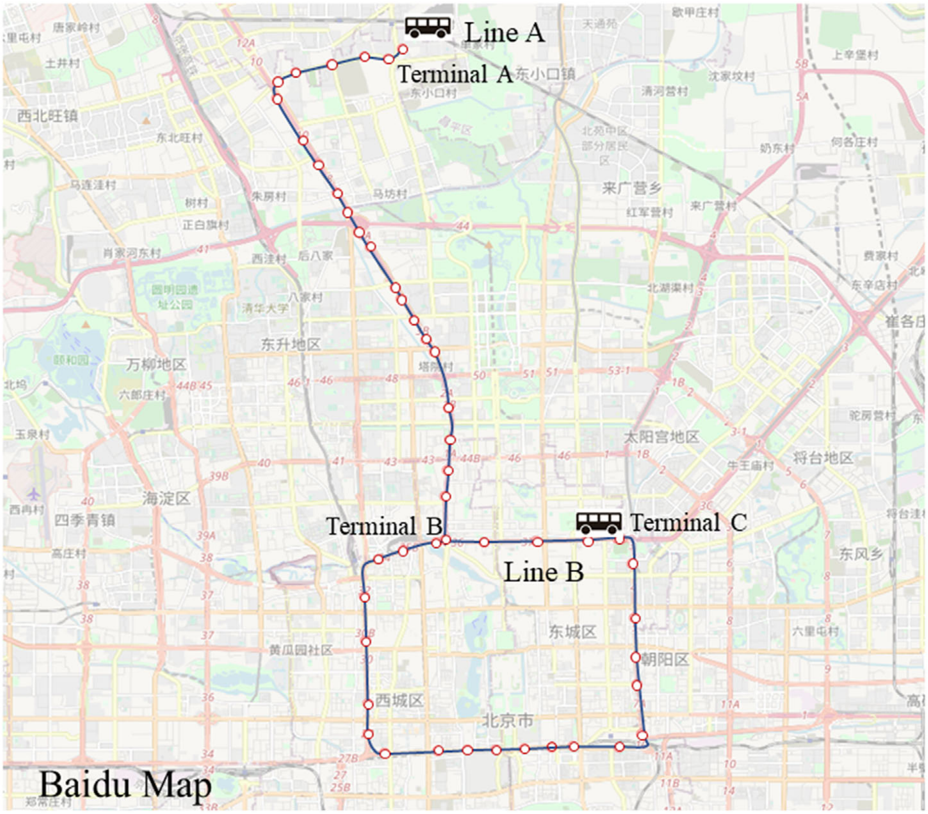


**FIGURE 7** Bus lines A and B in Beijing

**TABLE 2** Information of the deadhead distance and time

| | Terminal A | Terminal B | Terminal C |
|---|---|---|---|
| Depot A | 0.5 km/1 min | 15 km/30 min | \ |
| Depot B | \ | \ | 0.5 km/1 min |

inside the depot. The deadhead distance from the depot to the charging station is ignored. The maximum waiting time out of depot $\varphi$ is set as 3 h [21]. To truthfully reflect reality, we use the electricity consumed in the actual operation process as the input for EBs, and the recharging time is 20 min. For the CBs, we refer to the previous literature and consider a consumption rate of 0.63 L/km. To balance the purchase cost of electric buses, the emission cost of CBs is also considered, and a 40% subsidy is provided for the EBs. The impact of different subsidies on the overall operational plan will be investigated in the following sections.

To monetize these values, we considered the energy cost to be 0.16 \$/kWh [38], the fuel cost to be 1.27 \$/L and the exhaust cost to be 0.15 \$/L [39]. For the cost per recharge, we think like this. A fast charger costs \$120,000 and the charger life is 10 years [40]. The quick recharging time is 10 minutes, assuming that the charger works 8 h a day. So the times of a charger visited one day is $8 \times 60 / 10 = 48$ and the cost visiting a charger is $120{,}000 / (360 \times 48) = 7$ \$. There are also the land use costs and the maintenance costs, so the cost per recharge is set as \$10.

The time cost for waiting in the depot is 0.01 \$/min, out of the depot is 1 \$/min, and the delay time cost for a service trip is 1 \$/min. The battery capacity of the electric bus is 230 kWh, and the minimum remaining energy is set as 30% of the maximum battery capacity, i.e. 69 kWh. Because the route adjustments and service trips are not always the same, the objective function considers the daily operational costs, and the purchase cost of the bus is converted into a daily basis. The lifespan of an EB is 12-years, and that of a CB is 17 years; the purchase cost for an EB is \$790,000 [41], and that for a CB is \$321,143.

In addition, through the experimental analysis, the parameters of GGA are set as follows: the number of iterations is 1500, the mutation probability is 0.05, and the number of individuals per generation is 50. The TS parameters are set as follows: the tabu list length is 50, and the iterative step is 1000. To reduce the impact of the randomness of the heuristic algorithm on the results, we first conduct a robustness test of GGA by recording the objective values 20 times. As shown in Figure 8, the values at 20 times are not significantly different from each other. To test the stability of 20 values, we performed a two-sample $t$-test. The 20 values were divided into two groups of equal numbers of samples. The analysis results are $t = 0.11$, and $p = 0.9 > 0.05$, which shows that the mean of the two samples is the same and that the GGA proposed in the paper is robust. The trip time distribution is assumed to follow a normal distribution. To conveniently perform the Monte Carlo simulation, we take 10 s as a unit; that is, the expected value is 312. The 95% confidence level is always used, and the variance is set as 9. After analysing the

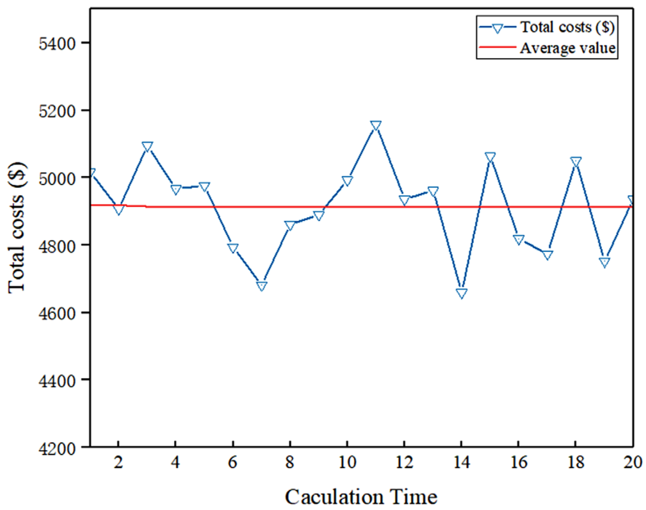


**FIGURE 8** Results of robustness tests

**TABLE 3** Results comparison of GGA and TS algorithms

| | Line A GGA | Line B TS | GGA | TS |
|---|---|---|---|---|
| Number of buses | 18 | 21 | 17 | 23 |
| Total costs ($) | 4442.32 | 6985.76 | 3713.586 | 4717.8 |
| Operation time (h) | 5.8 | 6.69 | 3.9 | 5.34 |

convergence of the Monte Carlo simulation, the sampling size $K$ is set as 1000.

## 6.2 | Solutions obtained by GGA and TS

In this section, we discuss the results obtained by GGA and TS in two bus lines. Table 3 shows that GGA's calculation results are superior to TS's. At the same time, GGA also shows obvious advantages in calculation time consumption. Therefore, GGA significantly improves the solution quality and efficiency. The optimal bus schedule is shown in Table 4, where numbers 1–191 are the service trips; $o$ and $d$ are the origin-depot and destination-depot, respectively; $r$ is the charging station; and $v$ is the virtual depot. The first column represents the bus types, and the second column denotes the bus schedule (blocks). The results show that 13 electric buses and 5 conventional buses are needed to complete all service trips.

## 6.3 | Model comparison

To verify the effectiveness of the proposed model by considering both the composition of fleet types and stochastic trip time, we conduct a series of experiments taking Line A as an example. The bus scheduling model with mixed fleets based on the stochastic trip time is taken as the standard model (denoted by M0), which is compared with the mixed fleets model under fixed-trip time (denoted by M1) and the single type fleets model under stochastic trip time (denoted by M2). Figure 9 shows that the number of buses required by the four models is equal. In terms of cost, the total costs of M0 are lower than those of M1, which indicates that stochastic time is considered to make the schedule plan more robust and reduce the expected time costs. Compared with M2-CBs, the total costs of M2-EBs are lower, which shows that the government's subsidies for electric buses can balance the high purchase cost. Even if the purchase cost does not give the advantage to electric buses, the lower energy consumption costs make the total costs relatively low. Compared with M2-EBS, there is no significant difference in total costs between partial electric buses and full-electric buses, but this is also due to government subsidies for electric buses. M2-CBs have the highest total costs among the four models, mainly due to high energy consumption costs and expected time costs. The introduction of electric buses can reduce not only fuel consumption but also protect the environment. Simultaneously, due to the limit of driving distance, the schedule implemented by electric buses can be arranged more compactly.

## 6.4 | Sensitivity analysis

In this section, we investigate the impacts of stochasticity, fleet composition, government subsidies and cost factors on operational costs. The following analysis of experiments is conducted on route line A.

### 6.4.1 | Impacts of stochasticity

In this section, we study the impact of varying road traffic stochasticity on operational costs. Take the operational data and results in Section 6.1 as the standard-variance scheme. To synthesize scenarios with different stochasticity, the method in the literature was adopted [12]. For scenarios with low-variance, the trip time generated by a Monte Carlo simulation was used and we reduced the difference between any time point and the expected time by 75%. Similarly, we increase the difference by 75% to synthesize a high variance. Based on the stochastic trip time of low-, standard- and high-variance, we further simulated the operations of the buses and compared their total costs and the other cost components. The larger the variance is, the greater the stochasticity will be. Figure 10 shows the cost components under different trip time variations. It can be observed that as the stochasticity of trip time increases, the total costs increase. Compared with the scenario of standard var, the effect of stochasticity on total costs is 2%. This indicates that the stochasticity of road traffic conditions will have an important impact on the operation of bus companies.

### 6.4.2 | Sensitivity analyses of fleet composition

This subsection assesses the benefits of mixed fleets. The use of ratio $\beta$ between electric and conventional buses is related to the initial investment. We set ten different scenarios to simulate

**TABLE 4** Optimal bus scheduling scheme

| Bus type | Scheduling scheme | |
|---|---|---|
| EBs | 1 | o → 2 → 12 → 26 → r → 44 → 55 → 73 → 88 → r → 102 → 119 → 133 → r → 150 → 166 → 176 → d |
| | 2 | o → 3 → 13 → 34 → r → 50 → 66 → r → 91 → 100 → 115 → r → 147 → 160 → 175 → 183 → d |
| | 3 | o → 4 → 15 → 31 → 43 → 61 → r → 89 → 99 → 114 → 125 → r → 154 → 167 → 177 → 186 → d |
| | 4 | o → 5 → 14 → r → 46 → 57 → 68 → v → 112 → r → 136 → 156 → 171 → 180 → d |
| | 5 | 0 → 6 → 24 → r → 58 → 71 → 81 → 96 → 107 → r → 127 → 145 → 161 → d |
| | 6 | o → 7 → 22 → 35 → 48 → 59 → r → 82 → 95 → 106 → 116 → 138 → r → 162 → 173 → 182 → 190 → d |
| | 7 | o → 8 → 21 → 33 → v → 80 → r → 98 → 110 → 129 → 143 → 155 → r → 172 → 187 → 191 → d |
| | 8 | o → 9 → 28 → r → 52 → 63 → 78 → 93 → r → 130 → 144 → 163 → 174 → 184 → d |
| | 9 | o → 10 → 23 → 45 → r → 67 → 83 → 92 → 105 → r → 128 → 140 → 157 → 168 → 178 → d |
| | 10 | o → 17 → 37 → r → 56 → 72 → r → 90 → 101 → 111 → 124 → 139 → 151 → r → 170 → 181 → 189 → d |
| | 11 | o → 20 → 40 → 51 → 64 → r → 87 → 97 → 108 → 120 → r → 141 → 153 → 164 → d |
| | 12 | o → 25 → 38 → 49 → 62 → 77 → r → 103 → 121 → 135 → 148 → 159 → d |
| | 13 | o → 29 → 41 → 54 → 65 → 76 → 86 → r → 109 → 122 → 142 → v → 185 → d |
| CBs | 1 | o → 1 → 11 → 32 → 53 → 69 → 79 → 94 → 104 → 117 → 132 → 152 → 165 → 179 → 188 → d |
| | 2 | o → 16 → 36 → 47 → 60 → 75 → v → 118 → 131 → 149 → d |
| | 3 | o → 18 → v → 74 → 84 → v → 134 → d |
| | 4 | o → 19 → 30 → 42 → r → 70 → 85 → r → 113 → 126 → 146 → 158 → 169 → d |
| | 5 | o → 27 → 39 → v → 123 → 137 → d |

Notes: *o* and *d* represent the depot; 1 − 191 represent the service trip, *v* is the virtual depot and *r* is the recharging station.

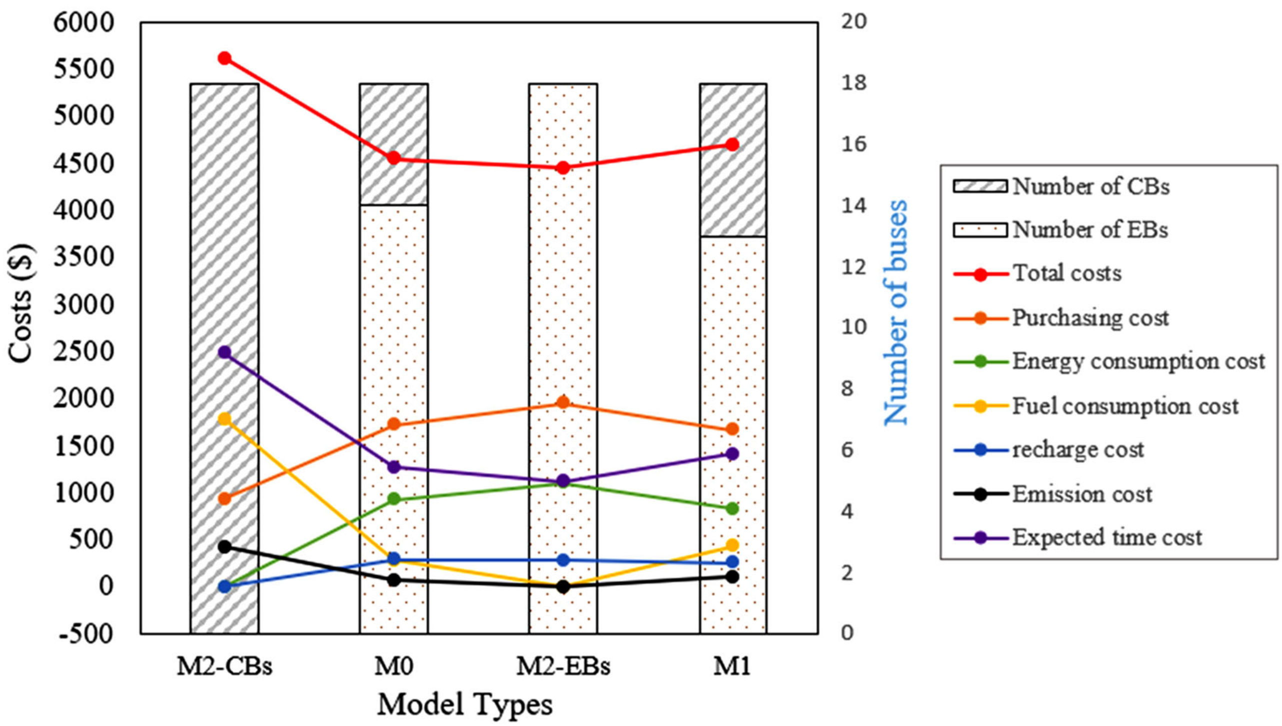


**FIGURE 9** Costs comparison with M2-CBs, M0, M2-EBs and M1: based on the stochastic trip time, M0 is the scheduling model with mixed fleets, M2-CBs is the scheduling model with the conventional buses, and M2-EBs is the scheduling model with electric buses; based on the mixed fleets, M1 is the scheduling model with fixed trip time

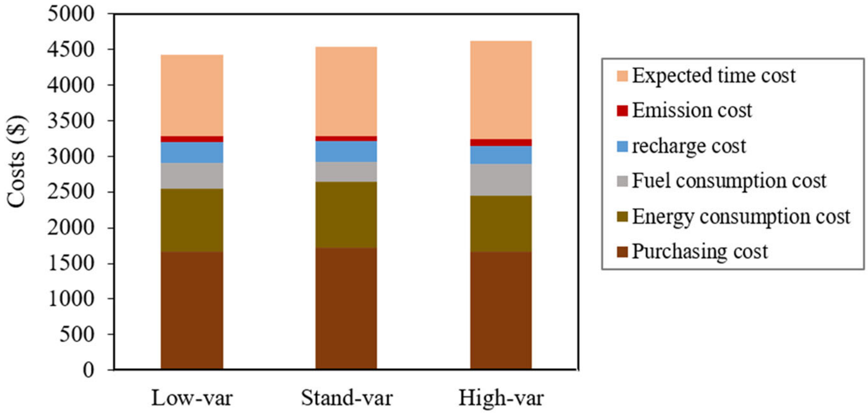


**FIGURE 10** Cost components under different trip time variation

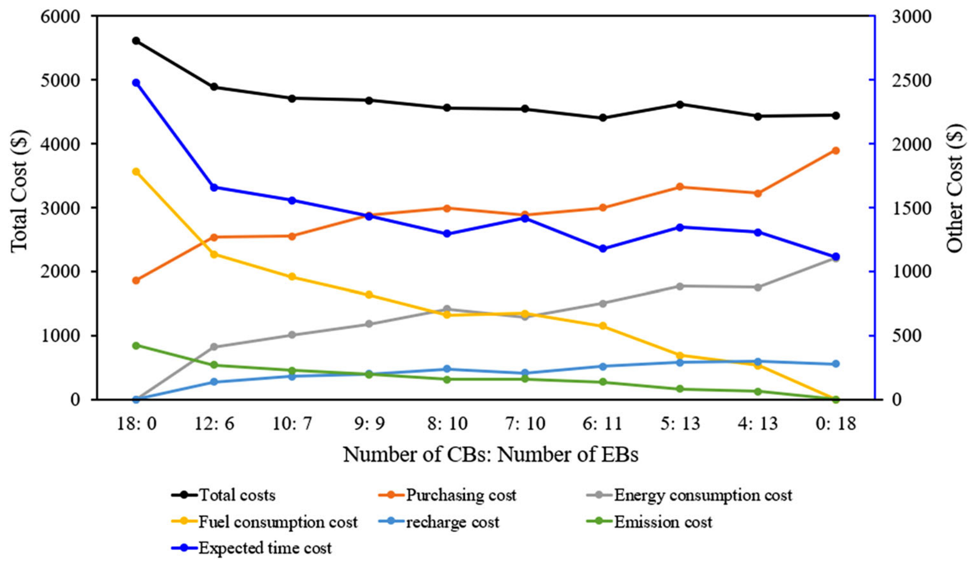


**FIGURE 11** Cost component with the change of the ratio between the number of EBs and CBs

the penetration process of the electric buses and $\beta$ is 0, 33%, 41%, 50%, 56%, 59%, 65%, 72%, 100%, respectively. Figure 11 shows the cost component with the change in the ratio between electric buses and conventional buses. By analysing the cases of ten different ratios between the number of CBs and EBs, we study their influence on the bus fleet structure and cost components. The $X$-axis represents the ratio between the number of CBs and EBs used. The $Y$-axis represents the total costs and other cost components, including the daily cost allocation for purchasing, energy consumption costs, fuel consumption costs, recharging costs, emission costs and the expected wait-time and delay-time costs. One special case is that where only CBs are used (the total costs are the highest). As the number of conventional buses decreases, the number of used electric buses increases and the total costs decrease. Compared with the scenario of full-CBs, the total costs of the basic scenario (Number of CBs:Number of EBs = 5:13, $\beta = 65\%$) and the full-EB scenario decrease by 18% and 21%, respectively. This shows that with the electrification of the public transport system in the market, the total costs are gradually reduced, which is on the premise of government subsidies.

### 6.4.3 | Subsidy for purchasing EBs

This subsection is devoted to modelling the different government subsidies and studying the influence on the bus fleet composition and total costs. As shown in Figure 12, the left

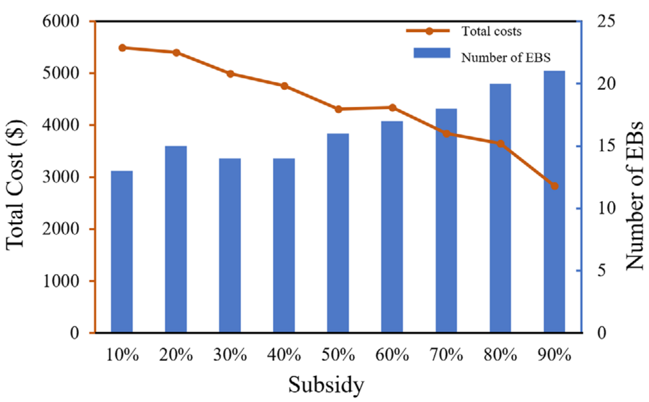


**FIGURE 12** Total costs and the number of EBs with the change of subsidy

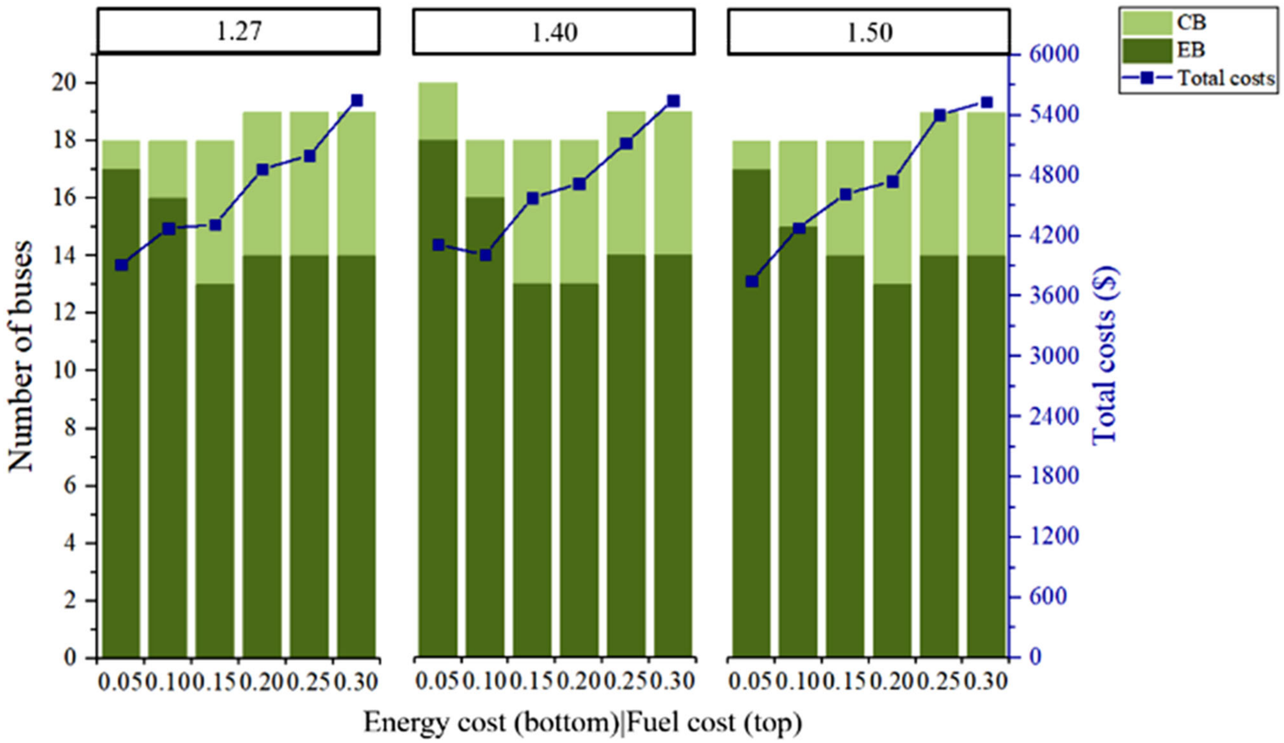


**FIGURE 13** Change in the fleet composition and total costs for different cost values

*Y*-axis is the total cost, and the right *Y*-axis is the number of electric buses. As government subsidies increase, the total cost decreases, and the number of EBs increases. This can be explained by the fact that when the subsidy for electric buses is increased, operators are more inclined to use electric buses, and the total costs are relatively low.

### 6.4.4 | Sensitivity analyses of cost factors

This subsection studies the impact of different scenarios combined with battery and fuel costs on the fleet composition and total operational costs. From Figure 13, we can see that when the fuel cost is fixed, as the energy cost increases, the total operating cost increases rapidly. An anomalous example is that in scenario ($\tau_e^{IC} = 1.40$), when the energy cost increases from 0.05 to 0.10, the total cost decreases instead. This may be due to the rearrangement of the operational schedule when the energy cost is increased. Service trips operated by electric buses are arranged more closely, reducing the number of electric buses used. Another finding is that as the energy cost increases, the number of electric buses generally decreases. Although the number of electric buses fluctuates up and down, it eventually shows a downward trend. Trend fluctuations can be explained by the fact that when the optimal schedule plan is generated, it is not only aimed at the minimum purchasing costs but also tends to choose the schedule plan with the minimum total operating costs.

## 7 | CONCLUSIONS

In this paper, we propose a methodology for solving the bus scheduling problem of mixed fleets with electric buses and conventional buses under stochastic trip time. The operational costs of EBs and CBs and the time costs that reflect the punctuality of bus services are included in the objective function. Based on the primary GGA, a straightforward framework is presented to optimize the scheduling scheme. The stochastic trip time is assumed to follow a certain distribution and is simulated by the Monte Carlo method. Case studies of two bus lines in Beijing are conducted to verify the effectiveness of the proposed model by considering both the composition of fleet types and stochastic trip time. Finally, we investigate the impacts of stochasticity, fleet composition, government subsidies and cost factors. The achieved scheduling strategies can provide the optimal proportion of electric and conventional buses while considering stochastic extent of the trip time, which has a crucial impact on the operational costs.

The results of these examples show that the stochasticity extent of trip time has a crucial impact on operational costs. Under the condition of an electrification market that has not been fully popularized at the present stage, reasonably arranging the proportion of electric and conventional buses can greatly reduce the total costs in the operation scheme. For certain service tasks, with the introduction of electric buses, traditional buses are gradually replaced. In particular, we considered a different ratio of electric and conventional buses that simulated the gradual penetration of electric buses into the market to observe their impact on total costs.

This mixed fleet formulation model only considers the single depot case. It is necessary to consider multiple depot mixed fleet scheduling problems, which will have more challenges to solve, especially for stochastic scheduling problems. Then, in this study, we failed to consider the time cost totally from the side of passengers. A thorough investigation on the value of time of varied passengers is needed. We will consider this factor in the future studies and try to build a multi-objective optimization model from both the company and passenger sides. Additionally, we only consider the stochastic trip time of the whole service trip and do not consider the trip time of each trip between two stations. In future studies, more road traffic stochasticity will be considered. Finally, most bus operations companies have bus types with different passenger capacities, increasing the model complexity and bringing computing challenges. Future research will focus on these problems.

## ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grants No. 71961137008; the Fundamental Research Funds for the Central Universities No.2020YJS096; the National Natural Science Foundation of China under Grants No. 71621001 and 91746201; JPI Urban Europe project 'SMUrTS' under Grant No. 299078.

## ORCID

*Mengyuan Duan* https://orcid.org/0000-0001-9506-9390
*Wei Guan* https://orcid.org/0000-0003-0038-4369
*Chaoru Lu* https://orcid.org/0000-0001-8418-7658

## REFERENCES

1. Wang, H., et al.: Metro timetable optimisation for minimising carbon emission and passenger time: A bi-objective integer programming approach. IET Intel. Transport Syst. 12(7), 673–681 (2018)
2. Sen, B., Ercan, T., Tatari, O.: Does a battery-electric truck make a difference? - Life cycle emissions, costs, and externality analysis of alternative fuel-powered Class 8 heavy-duty trucks in the United States. J. Cleaner Prod. 141, 110–121 (2017)
3. Wang, W.Y., et al.: Improving fuel efficiency of connected and automated transit buses on signallised corridors. IET Intel. Transport Syst. 13(5), 870–879 (2019)
4. Guo, R.G., Guan, W., Zhang, W.Y.: Route design problem of customized buses: Mixed integer programming model and case study. J. Transp. Eng. Part A. Syst. 144(11), 04018069 (2018)
5. Guo, R.G., et al.: Customized bus routing problem with time window restrictions: Model and case study. Transportmetrica A-Transport Sci. 15(2), 1804–1824 (2019)
6. Rinaldi, M., et al.: Mixed-fleet single-terminal bus scheduling problem: Modelling, solution scheme and potential applications. Omega-Int. J. Manage. Sci. 96, 22 (2020)
7. D'ovidio, G., Masciovecchio, C., Rotondale, A.: Hydrogen fuel cell and kinetic energy recover systems technologies for powering urban bus with zero emission energy cycle. IET Intel. Transport Syst. 10(9), 573–578 (2016)
8. Murakami, K.: A new model and approach to electric and diesel-powered vehicle routing. Transp. Res. Part E Logist. Transp. Rev. 107, 23–37 (2017)
9. Mouhrim, N., El Hilali Alaoui, A., Boukachour, J.: Vehicle routing problem with mixed fleet of electric and conventional vehicles under emissions allowances. In: Proceedings of 2018 4th IEEE International Conference on Logistics Operations Management. New York (2018)
10. Mahmoud, M., et al.: Electric buses: A review of alternative powertrains. Renewable Sustainable Energy Rev. 62, 673–684 (2016)
11. Boren, S.: Electric buses' sustainability effects, noise, energy use, and costs. Int. J. Sustainable Transp. 14(12), 956–971 (2020)
12. Tang, X.D., Lin, X., He, F.: Robust scheduling strategies of electric buses under stochastic traffic conditions. Transp. Res. Part C Emerg. Technol. 105, 163–182 (2019)
13. Furth, P.G., Muller, T.H.J.: Service reliability and optimal running time schedules. Transp. Res. Record 2034, 55–61 (2007)
14. Chen, J.X., et al.: Continuum approximation modeling of transit network design considering local route service and short-turn strategy. Transp. Res. Part E Logist. Transp. Rev. 119, 165–188 (2018)
15. Gkiotsalitis, K., Alesiani, F.: Robust timetable optimization for bus lines subject to resource and regulatory constraints. Transp. Res. Part E Logist. Transp. Rev. 128, 30–51 (2019)
16. Tang, C.Y., et al.: Vehicle scheduling of single-line bus service using operational strategies. IEEE Trans. Intell. Transp. Syst. 20(3), 1149–1159 (2019)
17. Tang, J.J., et al.: A data-driven timetable optimization of urban bus line based on multi-objective genetic algorithm. IEEE Trans. Intell. Transp. Syst. 22(4), 2417–2429 (2021)
18. Liang, J.P., et al.: Bus transit network design with uncertainties on the basis of a metro network: A two-step model framework. Transp. Res. Part B Methodol. 126, 115–138 (2019)
19. Babaei, M., Rajabi-Bahaabadi, M.: School bus routing and scheduling with stochastic time-dependent travel times considering on-time arrival reliability. Comput. Ind. Eng. 138, 15 (2019)
20. He, F., Yang, J., Li, M.: Vehicle scheduling under stochastic trip times: An approximate dynamic programming approach. Transp. Res. Part C Emerg. Technol. 96, 144–159 (2018)
21. Shen, Y.D., Xu, J., Li, J.P.: A probabilistic model for vehicle scheduling based on stochastic trip times. Transp. Res. Part B Methodol. 85, 19–31 (2016)
22. Shen, Y.D., Xu, J., Wu, X.Y.: Vehicle scheduling based on variable trip times with expected on-time performance. Int. Trans. Oper. Res. 24(1-2), 99–113 (2017)
23. Li, L., Lo, H.K., Xiao, F.: Mixed bus fleet scheduling under range and refueling constraints. Transp. Res. Part C Emerg. Technol. 104, 443–462 (2019)
24. Li, L., et al.: Mixed bus fleet management strategy for minimizing overall and emissions external costs. Transp. Res. Part D Transp. Environ. 60, 104–118 (2018)
25. Rogge, M., et al.: Electric bus fleet size and mix problem with optimization of charging infrastructure. Appl. Energy 211, 282–295 (2018)
26. Hiermann, G., et al.: Routing a mix of conventional, plug-in hybrid, and electric vehicles. Eur. J. Oper. Res. 272(1), 235–248 (2019)
27. Macrina, G., et al.: An energy-efficient green-vehicle routing problem with mixed vehicle fleet, partial battery recharging and time windows. Eur. J. Oper. Res. 276(3), 971–982 (2019)
28. Macrina, G., et al.: The green mixed fleet vehicle routing problem with partial battery recharging and time windows. Comput. Oper. Res. 101, 183–199 (2019)
29. Goeke, D., Schneider, M.: Routing a mixed fleet of electric and conventional vehicles. Eur. J. Oper. Res. 245(1), 81–99 (2015)
30. Zhou, G.J., et al.: Collaborative optimization of vehicle and charging scheduling for a bus fleet mixed with electric and traditional buses. IEEE Access 8, 8056–8072 (2020)
31. Yao, E.J., et al.: Optimization of electric vehicle scheduling with multiple vehicle types in public transport. Sustainable Cities Soc. 52, 101862 (2020)
32. Fotouhi, A., et al.: Electric vehicle energy consumption estimation for a fleet management system. Int. J. Sustainable Transp. 15, 40–54 (2019)
33. Kunith, A., Mendelevitch, R., Goehlich, D.: Electrification of a city bus networkAn optimization model for cost-effective placing of charging infrastructure and battery sizing of fast-charging electric bus systems. Int. J. Sustainable Transp. 11(10), 707–720 (2017)
34. Li, J.Q., Head, K.L.: Sustainability provisions in the bus-scheduling problem. Transp. Res. Part D Transport Environ. 14(1), 50–60 (2009)

35. Xylia, M., et al.: Locating charging infrastructure for electric buses in Stockholm. Transp. Res. Part C Emerg. Technol. 78, 183–200 (2017)
36. Li, J.Q.: Transit Bus Scheduling with Limited Energy. Transp. Sci. 48(4), 521–539 (2014)
37. Huisman, D., Freling, R., Wagelmans, A.P.M.: A robust solution approach to the dynamic vehicle scheduling problem. Transp. Sci. 38(4), 447–458 (2004)
38. Noel, L., Mccormack, R.: A cost benefit analysis of a V2G-capable electric school bus compared to a traditional diesel school bus. Appl. Energy 126, 246–255 (2014)
39. Clark, N.N., et al.: Transit Bus Life Cycle Cost and Year 2007 Emissions Estimation, carbon dioxide. (2007)
40. An, K.: Battery electric bus infrastructure planning under demand uncertainty. Transp. Res. Part C Emerg. Technol. 111, 572–587 (2020)
41. Dickens, M., Neff, J., Grisby, D.: APTA 2012 Public Transportation Fact Book, Environmental Protection. (2012)


**How to cite this article:** Duan, M., et al.: Reforming mixed operation schedule for electric buses and traditional fuel buses by an optimal framework. IET Intell. Transp. Syst. 15, 1287–1303 (2021). https://doi.org/10.1049/itr2.12098