Researching representation of diversity in mathematics pedagogical texts: Methodological considerations

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National exams in mathematics are important pedagogical texts which have been published annually for decades. They provide an interesting corpus for a project investigating the development of representation of diversity in mathematics tasks. Also, there are many reasons for including several diversity dimensions when researching diversity, e.g., gender, ethnicity, religion, functionality, sexuality, and class. Based on previous research on diversity in pedagogical texts, I discuss some of the methodological considerations that must be made when designing such a project.

Introduction

The aim of this article is to discuss some methodological considerations needed when investigating the development of representation of diversity in pedagogical texts over time. As these methodological questions are often strongly dependent on context, I will discuss one example: mathematics exams in Norway. While many details will be different in other contexts, many of the main ideas may still be useful.

Mathematics is traditionally a subject preoccupied with tasks (Walls, 2005). National exams have a special role: their tasks are published by the government, and are studied by teachers as signals of what should be prioritized (Burkhardt & Schoenfeld, 2018). Earlier exams are often used to prepare new groups of students for exams (Andresen et al., 2017). Rubel and McCloskey (2021) argues that exams most likely also influence teachers in how they include contextualization.

Exams are promising sources for studying change over time, as they are annual events with a long history. The national exam at the end of compulsory school in Norway has been arranged every year since 1962. In this period, Norwegian society (as many others) has been through profound changes in terms of diversity, with more gender equality, more ethnic diversity, and less discrimination of the LGBT population, to mention just three examples. The role of diversity in exams has probably also changed. *How* diversity has changed over time, however, is a matter for research.

I acknowledge my position as a CIS-gendered, male, white, urban professor of mathematics education to be a position of power, which leads to blind spots when

researching power issues that never affected me personally. My identity as gay and atheist have given some personal experiences of oppression. In my research, an interest in LGBT issues has gradually evolved into a more general interest in diversity. Analyses of data should preferably be performed by a group with diverse perspectives.

There are many possible approaches in mathematics education to combat oppression and injustice, including e.g., using mathematics to investigate injustice, or involving the local community in making mathematics relevant. This article is based on the premise that representation is important and that being made invisible is harmful. Representation is certainly not a panacea for social justice, but one of a range of factors that need to be researched.

Theoretical background

The importance of representation

Rubel and McCloskey (2021) argues that there are four rationales for contextualization of mathematics: 1) support the learning of mathematics, 2) motivate students, 3) support the teaching of functional literacy (both for private and for professional life), 4) support the teaching of critical literacy. Seeing oneself depicted may give a feeling of being included and accepted as opposed to excluded and rejected. Problems that feel relevant to one's own situation may lead to experiencing mathematics as relevant. Moreover, seeing oneself depicted in different contexts may serve to expand or limit one's foreground (one's image of one's own opportunities) (Rubel & McCloskey, 2021). In addition to this function as a 'mirror', diversity in texts also functions as 'curricular windows' (Luecke, 2011), letting children get to know minorities that are not physically present in the particular classroom or school.

Diversity dimensions

Many diversity dimensions could be discussed. I have chosen to include gender, ethnicity, culture, religion, geography, functionality, sexuality, and class. Age is excluded for reasons of space. I want to look at how researchers have studied the dimensions in exam tasks in mathematics when it comes to representation. Many other aspects of mathematics' relation to the different diversity dimensions could be examined, for instance different groups' relation to the subject and reasons for this. Such aspects are outside the scope of this article.

There are many reasons to investigate several such dimensions at the same time. Firstly, intersectionality research illuminates the danger of looking at diversity dimensions one at a time (Bowleg, 2008). The experiences of black women are not simply a product of the experiences of blacks and the experiences of women. Secondly, the different dimensions may well be treated differently in the exam tasks, and I would like to compare how they change over time. Thirdly, it is possible that some dimensions are rarely or not at all visible in the exam tasks, which makes it difficult to base a project on such dimensions alone. If diversity

dimensions are treated one by one, there is a clear danger that some dimensions will never be researched.

Bowleg (2008) notes that when doing intersectionality research, researchers must commit to a transdisciplinary approach, and they must examine "any contradictions or tensions relevant to these intersections" (p. 318).

I have argued that to provide an overview, we should include many dimensions and study long-term development. As some dimensions may rarely occur, we need to analyse a large sample of the exam tasks. Therefore, the analytical tool needs to be rather simple and with little nuance, for instance by just counting the numbers of occurrences of certain 'diversity markers'. This leads us to the topic of 'tokenism'.

Tokenism

The exclusion of a group from a text can be remedied by turning some people into members of that group. This can be done by changing the names or appearances of people in the text, or the information given about them. Such an approach will be criticized as *tokenism*, that is superficial inclusion without changing the underlying structures. While famously used in cases of women with non-traditional jobs (Kanter, 1977), tokenism is also used of people included in texts to improve the appearance of diversity. If a text is written from a white, middle class, straight perspective, it does not magically include more perspectives even if some names or skin colours are changed. Groups' everyday lives and their experiences need to influence the texts.

Stereotypes and normativities

Consistently placing people from a culture in contexts particular to that culture, will rightly be criticised for preserving stereotypes. This is the opposite of tokenism, the inclusion of people from a culture without changing the context. In some cases, tasks can even uphold pejorative stereotypes (Rubel & McCloskey, 2021).

Normativities, for instance heteronormativity, is relevant in two directions. On the one hand, a lack of representation contributes to upholding the normativities and the oppression they contribute to. On the other hand, such normativities mean that characters not described as belonging to a minority, are automatically seen as being part of the majority. In addition to analysing how many occurrences there are of the minorities and the majorities, the many characters that do not have a description are also relevant, as these may unconsciously be seen as part of the majority.

Luminescent or invisible

Ohnstad (1992) coined the concept pair *luminescent or invisible* to describe the dilemma a lesbian woman faces when deciding whether to come out: when coming out, the sexual orientation is treated as the most visible aspect of the person's identity, which feels unsafe, but when not coming out, that aspect of the identity stays invisible. I consider this concept pair useful for the discussion of diversity in exams (see below).

Different diversity dimensions

Despite my commitment to intersectionality, in this part, I will look at my choice of diversity dimensions one by one and provide examples of how each has been researched, as relevant articles where intersectionality is invoked, are rare. Even Macintyre and Hamilton (2010), analysing textbooks for representations of gender, ethnicity, disability and sexual orientation, are unable to analyse them together, as disability and sexual orientation were invisible in the data.

Norwegian research will be included when available. Where I found little research on exam tasks, I refer to research on textbooks as well, as both are pedagogical texts, albeit in different settings. As I look at many dimensions, each discussion must be concise.

Gender

Boaler (1997) claimed that earlier, "mathematics textbooks dealt only with male experiences" (p. 287). Among recent studies are Norén and Boistrup (2016), who counted boys and girls in Swedish mathematics textbooks for 8th and 9th grade, and analysed their roles. There are more boys, they are more active and more often have jobs, while girls to a larger degree are caregivers in the home. Using somewhat similar methods, other studies give a similar picture in different countries: Palestine (Karama, 2020), Brazil (Pereira, 2019), Greece (Chassapis, 2010), Britain (Macintyre & Hamilton, 2010) and the US (Piatek-Jimenez et al., 2014), finding that males are more mathematical and have more – and different – careers.

Ethnicity

Through the last 50 years, Norway became a more multicultural and multireligious society. International research on race/ethnicity is therefore increasingly relevant also for Norway. Analysing the ethnicity and activity of persons in tasks, Piatek-Jimenez et al. (2014) found that in US textbooks, "whites are portrayed as being more mathematical and more active and are shown in more careers than minorities" (p. 55), while Pereira (2019), in a study of Brazilian exams, found that nearly all characters could be socially perceived as white. In Norway, Flottorp and Poorgholam (2003) analysed mathematics textbooks from a multicultural perspective, and showed that Asian or African names and looks were underrepresented.

Culture

Flottorp and Poorgholam (2003) searched for ways in which Norwegian textbooks mirrored a multicultural reality. They found that tasks connected to buying food often fit with a traditional Norwegian cuisine, with little oil, rice, garlic, or olives. Holiday destinations were often the cabin or Mallorca. When places in other parts of the world were mentioned, it was often economically important places. Currencies mentioned were European currencies or US dollars. A Norwegian way of living, with a nuclear family in a house, was often portrayed. Mathematicians mentioned were often Western.

Fyhn and Nystad (2013) took another approach. They analysed how the 2009 Norwegian mathematics exams (of the same series of exams mentioned above) reflected four key Sami values. One of the values included were Sami traditional knowledge, in which knowledge is seen as a process, not a product. Therefore, tasks with process focus would be considered in harmony with Sami values regardless of the tasks' contexts. They concluded that the exam reflected Sami values to a certain degree, but that more emphasis on modelling would improve the fit with Sami values.

Religion

In Flottorp and Poorgholam (2003), little religion was found in textbooks, and when included, it was often Christianity. Few other studies have investigated the religious content in mathematics textbooks or exams. Yilmaz and Ozyigit (2017) studied one Turkish high school mathematics textbook from each of three eras, and found that religious contexts varied over time, with no instances found in the newest textbooks.

Geography – "urban bias"

Textbooks in many subjects have been criticized for an *urban bias*, reflecting authors' urban context, ignoring the rural way of life (Nawani, 2010). Fyhn and Nystad (2013) mentions that in the 1990s, Norwegian exams were criticised for the same. For instance, tasks on tram timetables may be of little interests to students with no access to trams. An urban bias would most likely lead to fewer tasks on rural activities such as farming and fishing, and more tasks on urban activities.

Functionality

In previous decades, attitudes towards functionality have changed, and visibility in society has increased. Through the Salamanca statement (UNESCO, 1994), disabled people's right to be included in school have been stressed. It does not seem, though, that this visibility has reached textbooks to the same degree. Macintyre and Hamilton (2010) looked for representation of disability in Scottish mathematics textbooks, but found nothing, neither in text nor images. Hardin and Hardin (2004) found that US Physical Education textbooks rarely included photographs of persons with disabilities. Hodkinson (2012) studied resources for primary-aged students in England. Very few included any mention of disabilities. Both Hardin and Hardin and Hodkinson combined a count of photographs or mentions with a more detailed study of the instances where disabled people were visible. In both studies, portrayals were stereotypical.

Sexuality

Family structures have also changed throughout the period. For instance, in Norway, gay sex was forbidden until 1972, while same-sex marriage was introduced in 2009. Dubbs (2016) notes two ways queer perspectives could be included in mathematics education: by including queers or by using the mathematics to investigate inequalities. Macintyre and Hamilton (2010) found no representation of sexual orientation in Scottish mathematics textbooks.

Smestad (2018) showed that while lack of inclusion was the normal finding in other countries, in Norway, LGB issues are more often included in textbooks in some subjects (not including mathematics). In some of the research finding a lack of inclusion, researchers go on to analyse in which parts of the textbooks it would have been reasonable to include queer perspectives.

Class

The last diversity dimension I will include, is class. The research I have found is not concerned with the representation of people from different classes as much as with how the tasks' contexts are relevant to the students. Macintyre and Hamilton (2010) point out that different social groups have different expectations and experiences to draw on. Harper (n.d.) gives a convincing example of an assessment task that "unfairly relies on assumptions that middle-class students would make", such as people normally working five days a week. Flottorp and Poorgholam (2003) found tasks asking students to calculate the area of their own room, thus assuming they all had one.

Chisholm (2018) calls for more historical – and comparative – studies of representation in textbooks, while Niehaus (2018) recommends a mixed-method approach, to "first lay out an overview of how certain diversity-related categories are represented before going into more depth through a text-based content analysis" (p. 334).

Discussion

It does seem that in most of the diversity dimensions, researchers have been counting representation as part of their analysis. These counts in themselves have given rise to interesting conclusions – not least in pointing out how certain groups are excluded. However, researchers tend to want to go beyond this, to analyse the roles and contexts in more detail. In this way, research has not only shown that girls are underrepresented in Swedish textbooks, but also that they are given a less active role. Research has also shown the portrayal of the disabled to be stereotypical. An analysis where inclusion is identified, can therefore both give an important count and a material to discuss in more detail. In Table 1, I provide a summary of ideas/examples of what to look for.

Höhne and Heerdegen (2018) point out that it is common to use a binary view of gender in such research. Even if we use a more inclusive gender concept, there are problems with assigning gender to persons based on names and appearances. However, the point is not to assign gender, but to analyse whether the attributes persons in such tasks are given, contributes to the perpetuation of gender norms. Thus, if persons with traditional male names are overrepresented in tasks, that is still an interesting finding.

Similarly, when counting 'ethnicities', we do not accept that there are clear boundaries between ethnicities, but rather analyse how different ethnicity markers are represented. Such markers could be whether the names are traditional Norwegian or not, or phenotype (perhaps skin colour), as in Pereira (2019).

Dimension	Count	Further analyses
Gender	Males/females/other (based on names, pronouns, and appearances)	What they do and how they are positioned
Ethnicity	Ethnicities (based on names and appearances)	What they do and how they are positioned
Culture	Places (European vs. non-European). Food. Currencies.	Looking for examples of "immigrant culture"
Religion	Religions	How they are portrayed
Geography	Place names and activities (rural/urban)	
Functionality	People with apparent disability vs. people with apparent non-disability.	How the two groups are portrayed.
Sexuality	Contexts having to do with families (traditional vs non-traditional). Mentions of minority sexualities.	
Class	(Occupations, large amounts of money)	Class-specific assumptions

Table 1: Suggested points to look for.

Relevant markers when it comes to class could include which jobs people have, the amounts of money they handle, or class-specific occupations or hobbies. Operationalisation would be difficult, however, as the markers not only varies across (and probably within) countries, but also over time. At which point in time did buying a bicycle turn from being a luxury to becoming normal in Norway, for instance? The research cited above suggests that in the case of class, analyses of middle-class assumptions are more fruitful than a counting of representations.

I have argued that counting representation provides both valuable quantifications and material for further analysis. Some aspects complicate this. As pointed out in research on queer representation, it is also of interest to identify contexts in which queer people could naturally be included, but where they (we) are not. In Fyhn and Nystad (2013)'s work on Sami values in exams, the issue is not to look at representation, as tasks that are in harmony with Sami values do not have to include Sami representation. If we just identify instances of representation and then analyse these, key perspectives are lost.

In addition, the dimensions are different. Some diversity markers are visible, others are not. Probably, more persons in tasks will have an explicit gender or ethnicity, either in texts or images, than a sexuality or religion. If there is a tradition of not being explicit about, say, people's sexuality, functionality or religion, the first occurrence would probably stand out and seem distracting – even luminescent (Ohnstad, 1992). By giving an overview over a long period of time, we will shed light on such traditions.

Another issue is fragmentation; We want to avoid ending up with a series of results only concerning one diversity dimension at a time. Every person's identity includes several

dimensions, and simple contexts, such as having your own room, touches upon several dimensions at once: it may be connected to class, the urban/rural divide, and culture. As pointed out by Bowleg (2008), an important phase of an intersectional analysis is to examine "any contradictions or tensions relevant to these intersections" (p. 318). In the context of exam tasks, this would entail studying the diversity contexts together. Regarding the counts, pivot tables should be made and studied. If representation of different ethnicities and functionalities is present, we can easily check whether disabled people tend to be portrayed as white.

More controversially, we may ask whether all dimensions are equally important, considering other societal developments. I will argue that a major equity issue in Norwegian schools is that working class immigrants are disadvantaged in mathematics, in terms of economic factors (e.g., no PC at home), lower quality education, and language demands of the tasks (Bjørnset et al., 2020). Including more tasks set in rural settings may increase the disadvantage of an already disadvantaged group. While diversity in general benefits all, each effort does not necessarily benefit all equitably.

Counting occurrences of different diversity markers helps study inclusion on one level. There is a risk that the diversity markers are just tokenism. There is also the thorny issue of tokenism and stereotypes. If including a person of Chinese appearance and name, should they go skiing (would not that be tokenism?) or doing something central to Chinese culture (would not that be perpetuating stereotypes?) Also, just including farm contexts do not assure that a rural child will feel included, as not all rural children have an interest in farms. For real diversity to occur, there must be diversity in the portrayal of Chinese, and of other identities. No such diversity is possible if the number of occurrences is low. Thus, counting still gives important information.

Concluding suggestion

I have considered some methodological considerations needed when investigating the development of the representation of diversity in mathematics exams over time. Although some operationalisations needed are difficult, it seems feasible and fruitful to analyse the whole corpus of mathematics exams (in Norway's case: 60 years of exams) based on counting occurrences of inclusion (Table 1). This could provide an overview of the development of the exams in terms of diversity.

Such an overview would not cover all important perspectives. However, it would be a foundation for further, more detailed analyses, of selected (combinations of) diversity dimensions and time periods. Cross-country quantitative analyses are feasible, to allow comparisons and then the further cooperation on analyses of interesting differences or similarities. This would take careful work on the operationalisations, however, as differences between classes, for instance, are not the same for all countries. Such an analysis would be in line with recommendations from Chisholm (2018) and Niehaus (2018) and provide new opportunities for insights.

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