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COST-OPTIMAL DESIGN OF FLEXURAL CONCRETE BEAM REINFORCED WITH FRP REINFORCEMENTS

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Abstract

Because of the non-ductile nature of FRP reinforcement, the convention of designing cross-sections with the flexure strength limited by reinforcement yield, as for steel, is not adequate. Although ACI 440.1R-15 gives easily fetched explanations on how to design FRP-reinforced concrete (FRPRC) sections for flexure, an understanding of the different parameters economic influence is required for FRP to be a competitive alternative to steel. In this paper, an analytical tool for cost-optimally analyzing and designing FRPRC-cross-sections according to ACI 440.1R-15 is presented. To this aim, by optimizing the functions for flexural strength and for the approximated price pr. meter of a cross-section – with respect to both fiber cross-section area and effective depth of concrete cross-section – a formula for the most economical ratio of these parameters in regard to flexural strength is presented. For a given FRP and concrete type, the optimal ratio proves to be the same for all desired flexural capacities, and can for rectangular cross-sections be presented as a function of the cross-section width.

1. Introduction

Along with higher initial costs - the lack of experience, standards and guidelines may scare many engineers from entering the unknown landscapes of fiber reinforced polymer (FRP) reinforcement [1]. Despite providing durable concrete structures, free of deterioration caused by corrosion [2], FRP bars as reinforcement has not yet managed to become a major competitor to steel in Europe [3]. Chloride initiated corrosion on steel reinforcement is the number one cause of reinforced concrete (RC) bridges not being serviceable, making up 2/3 of all recorded failures on German bridge constructions [4]. Regardless - the low weight, high tensile strength, superior resistance to deterioration from aggressive environments [5] and

significantly smaller environmental impact [6] has not been sufficient to penetrate the European mainstream market.
Depending on the matrix, as well as the fiber type and their alignment, some important properties of FRP products are generally associated with a drawback in comparison to those for steel [4]. Considering design for flexural strength of rectangular FRP-reinforced concrete beams, the relatively low modulus of elasticity of the reinforcement and its lack of ductility in many cases sets limitations for the possibility to exploit its high tensile strength.
Because of FRPs resistance to electrochemical corrosion, the total life cycle cost of a non-metallic reinforced structure is nevertheless likely to be lower than those for steel-reinforced structures when situated in corrosion-aggressive environments [7], taking realistic costs of repairs and traffic delay in to consideration [3]. This long-term economical viewpoint is part of the reason for an increase in the use of non-metallic reinforcement for new bridge structures in North America. As of 2015 more than 200 Canadian concrete bridges have been designed and built using FRP in slabs, barriers or girders, without showing any signs of deterioration after 10 years [8]. Meanwhile the corresponding marked in Europe is gradually growing importance, and the expanding and extensive construction of new concrete structures reinforced with FRP is motivating research on ways to design FRP structures more efficiently [10]. Because of this, the European Committee of Standardization (CEN) in 2016 published *Prospect for new guidance in the design of FRP: Support to the implementation, harmonization and further development of the Eurocodes* [9], as a step in the direction of a Eurocode governing guidelines for the design and construction of structural concrete reinforced with FRP.
With the industry's concerns of non-metallic reinforcements high first costs, the significance of developing ways to optimize costs is crucial to encourage further use for the future. According to authors' knowledge, there is no guiding to obtain cost-optimal FRP-reinforced concrete cross sections. In this paper, a method for determining the cost-optimal ratio of reinforcement to concrete area in a cross section is therefore presented.

2. Flexural design of FRP reinforced concrete (FRPRC) beams

Designing traditional steel RC beams, it is desirable to take advantage of the ductile nature of the steel reinforcement to obtain a non-brittle failure of the RC element [10]. This is done by designing RC beams so that the strain level in the reinforcement is beyond what is the yield strain of steel, when concrete crushing occurs. This limits an engineer's needs and possibilities to experiment with different reinforcement ratios. FRP reinforcements on the other hand does not inherent this ductile behavior [10]. Instead, the stress/strain of FRP reinforcement bars develops linear elastically, with sudden termination by brittle failure [10].

2.1 Reinforcement ratio and reduction factor for flexural design

Because of the non-ductile behavior of FRP, shown in Fig. 1, the conventional way of designing cross-sections with flexure strength limited by the reinforcement, as for steel, is not adequate [10]. ACI 440.1R-15 [10] refers to researches [11-12] proving failure by FRP-reinforcement rupture to be sudden and catastrophic, with compression-controlled cross-sections being marginally more desirable, exhibiting some inelastic behavior prior to failure.

To compensate for the lack of ductility using FRP, ACI [1] introduces a reduction factor to the nominal flexure strength, ϕ, which is reliant on which limiting state is controlling.

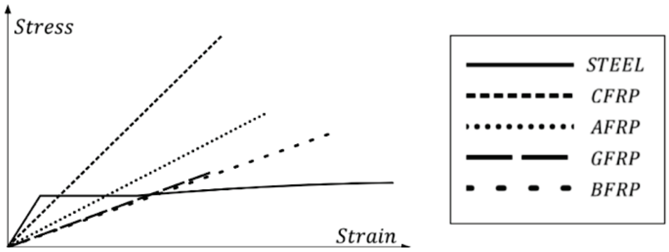

Figure 1: Stress-strain diagrams for steel, GFRP, CFRP, AFRP and BFRP, showing linear-elastic strain development with sudden failure for FRP reinforcement.

$$\phi M_n \geq M_u \quad (1)$$

where ϕ is strength reduction factor, M_n is nominal flexural capacity (kNm) and M_u is factored moment at section (kNm). Comparing a beams reinforcement ratio to a balanced reinforcement ratio, one can determine whether its failure will be controlled by FRP-rupture or concrete crushing, giving a corresponding reduction factor.

$$\rho_f = \frac{A_f}{b \cdot d} \quad (2)$$

where ρ_f is FRP reinforcement ratio, A_f is area of FRP reinforcement (mm^2), b is width of rectangular cross-section (mm) and d is effective depth of cross-section (mm). In ACI 440.1R-15 [10] is presented an equation for the balanced reinforcemenet ratio – the ratio where concrete crushing and FRP rupture will occour simultaniously. The equation does not include any geometric parameters and is along with Eq. (2) also valid for T sections as long as the depth of the compression zone is not larger than the thickness of the flage of the section. If the reinforcement ratio from Eq. (2) is greater than the value of the balanced reinforcement ratio, ρ_{fb}, the section will theoretically be controlled by the concrete crushing limit state. However, for a section with $\rho_{fb} < \rho_f < 1.4\rho_{fb}$ a linearly reduced value of ϕ is imposed in case the member as constructed does not fail accordingly, see Fig. 2.

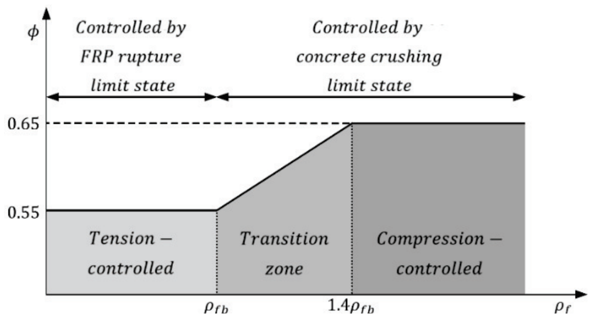

Figure 2: Strength reduction factor as function of reinforcement ratio, showing values for tension-controlled ($\rho_f < \rho_{fb}$), compression-controlled ($\rho_f > 1.4\rho_{fb}$) and a linear transition zone ($\rho_{fb} < \rho_f < 1.4\rho_{fb}$) [10].

3. Cost-optimization method

Having the option to design a cross-section to be controlled by different limit states creates a demand for determining the most economically optimal design with respect to reinforcement ratio. Based on methods from ACI 440.1R-15 the factored nominal flexural strength of a FRP reinforced concrete cross-section can be presented as a function of FRP reinforcement area and effective depth. This is obtained by locking the width of the cross-section, looking at a specific type of FRP.

3.1 Price of cross-section per meter

To consider the material costs of a rectangular FRP reinforced beam, a function of the same variables can be approximated based on the geometry of the cross-section. For the same specific width, the function representing price pr. meter beam can be presented as follows.

$$P(A_f, d) = P_{FRP}A_f + P_c(b(d + d_{c,m}) - A_f) \quad (3)$$

where P is price of cross-section per meter (price/m), P_{FRP} is average price of specific FRP type (price/mm^2/m), P_c is price of concrete (price/mm^2/m) and $d_{c,m}$ is thickness of concrete cover measured from extreme tension fiber to collective center of reinforcement. Since the price of FRP bars varies within the different available diameters, the expression is based on an average price of the most current diameters. Not knowing how many lateral layers of reinforcement will be necessary, an approximated distance from extreme tension fiber to collective center of reinforcement is assumed.

3.2 Minimizing by Lagrange multiplier

According to the method of Lagrange multipliers [13] the price function is minimized by analyzing the dot product of the two functions' associated gradients, when the factored nominal flexural capacity is constrained to a desired value. The pair of input values giving parallel gradients, is the most economically optimal.

$$\nabla\phi M_n(A_f, d) = \lambda \cdot \nabla P(A_f, d) \quad (4)$$

$$\phi M_n(A_f, d) = i \quad (5)$$

where ∇ is the del operator giving the functions gradients [14], λ is the Lagrange multiplier scalar [13], and i is the equality constraint which can be set to any current factored moment at section, M_u [1], or any desired flexural capacity.

Plotting the optimal pairs of input when varying the equality constraint, i, the nature of the two functions gives a straight line, proving a fixed ratio (A_f/d) minimizing the price function for all flexural capacities, see Fig. 3.

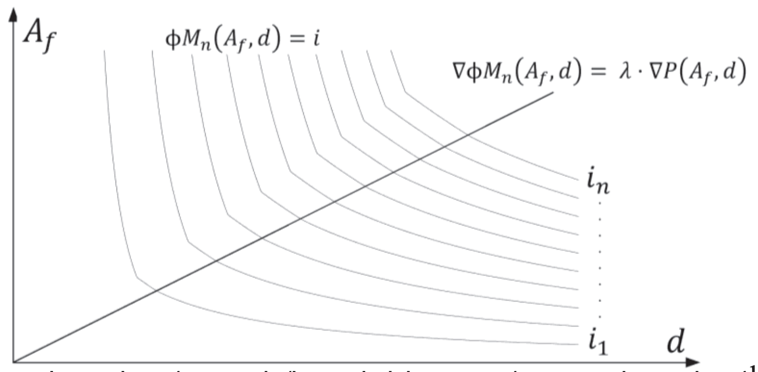

Figure 3: Price optimal combinations of A_f and d for a section reinforced with a specific FRP type, with fixed width. The straight line is obtained by plotting every optimal A_f and d pair which satisfies a flexural capacity $M(A_f, d) = i$, varying i continuously.

With a fixed cost-optimal reinforcement to effective depth ratio for a given width, we can easily generalize the concept to all cross-section widths. Repeating the operation of the optimization, varying the cross-section width discretely, the cost-optimal reinforcement to effective depth ratio as a function of the width, $\Lambda_{FRP}(b)$, is obtained by regression analysis.

$$\Lambda_{FRP}(b) = \frac{A_{f,opt}}{d_{,opt}}(b) \tag{6}$$

where $A_{f,opt}$ over $d_{,opt}$ represents the fixed relationship of values that together fulfills Eq. (4) and Eq. (5). Note that $A_{f,opt}$ and $d_{,opt}$ does not exist independently.
In the case of a GFRP-reinforced cross-section of C45 concrete with the width b, the cost-optimal ratio of A_f over d is presented in Eq. (7). The specific GFRP reinforcement has a modulus of elasticity of 46 000 MPa, guaranteed tensile strength of 724 MPa and price of 0,03 Euro/mm^2/m. The calculation is based on 142 Euro/m^3 as the price of C45 concrete.

$$\Lambda_{GFRP}(b) = 0{,}405b - 0{,}367 \tag{7}$$

4. Conclusion

This paper presents an analytical method to minimize material costs of rectangular FRP reinforced concrete beams designed for flexural strength. Establishing a function for price per meter beam, the cost-optimal relationship between FRP reinforcement area and the cross-sections effective depth is derived. This function, Λ_{FRP}, proves to be invariant of demanded flexural capacity, and can for a specific FRP and concrete type be presented as a function of the cross-section width, b. The high first costs associated with FRP makes this method very useful - specifically when material costs in a project are significant.
Further research will be concerned around generalizing the results and looking into the possibility of an appliance to deflection limit state.

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