Ludvigsen, Tarjei
Mahmood, Osama

Valuing Convertible Bonds using Stochastic Dynamic Programming with Monte Carlo Based Regressions

An empirical study of the US convertible bond market

Abstract

Using Monte Carlo simulation combined with least squares regression to estimate continuation values and optimal exercise decisions in a stochastic dynamic programming framework, we estimate fair price for 40 convertible bonds in the US market. In contrast to most previous studies, we do not find evidence of systematic underpricing in the market. Our results show an average overpricing of 1.1 %, while deviations between observed and predicted prices seem related to coupon rate and credit rating. Furthermore, we find no evidence of a relation between price deviation and moneyness of the conversion option.

Master thesis, MSc in Business Administration
Oslo Business School at Oslo Metropolitan University
2018

This thesis is written as a part of the Master of Science in Business Administration at Oslo Metropolitan University. We would like to thank our supervisor Einar Belsom for excellent guidance and discussions.

1. Introduction

Convertible bonds are an important part of the securities markets, with approximately US$500 billion outstanding worldwide. The US market is the largest and accounts for half of the total convertible bond market. Attractive features of these financial instruments have made them popular among both institutional investors and issuing companies. The major investors in convertible bonds are institutional investors such as convertible arbitrage hedge funds (Xiao, 2014). Convertible bonds provide a different risk-return profile for the investor compared to straight bonds as the option component creates an unlimited upside potential. If the underlying stock performs poorly, the convertible bond offers a downside protection with the fixed cash flow protection from the bond component. However, the holder risks missing out on the interest and principal payment due to potential default of the issuing company. Thus, implementing credit spreads is important when pricing convertible bonds (Batten, Khaw and Young, 2014).

In this paper, we investigate whether convertible bonds are systematically mispriced in the US market. There is a wide range of pricing methods, but not one specific model stands out as superior. Several features need to be considered when calculating fair price, making these instruments more comparable to complex derivatives than standardized bonds (Kind and Wilde, 2005). This paper contains an empirical analysis contribution of the US convertible bond market. We analyze a data sample of current plain-vanilla convertible bonds. The purpose is to implement a model which considers the complexity of these hybrid instruments. We apply a least-squares regression model to price convertible bonds as proposed by Longstaff and Schwartz (2011), in combination with Monte Carlo simulation in a stochastic dynamic programming framework. We conduct stock price simulations to obtain 4.8 million data points and combine these with 1200 regressions to estimate the fair price for 40 convertible bonds.

Few studies have been conducted to price convertible bonds in the past ten years. This paper covers a gap in recent literature. It contributes by examining a larger data sample, with more homogenous and up-to-date convertible bond data. It is also worth noting that most previous studies examine historical convertible bonds, while we investigate current instruments. We attain a credible model by obtaining average credit spreads from Moody's Analytics on the explicit pricing date and up-to-date data from Bloomberg Terminal. Furthermore, this paper provides support to modern research such as Ammann, Kind and Wilde (2008) who find convertible bonds to be slightly overpriced. This contrasts to research prior to 2008, where convertible bonds are found to be systematically underpriced, e.g. King (1986), Carayannopoulos (1996), Buchan (1997) and Ammann, Kind and Wilde (2003).

The paper is structured as follows: Section 2 provides a brief overview of the complex structure of convertible bonds and convertible arbitrage. Section 3 discusses different theoretical approaches to pricing models. Section 4 presents the mathematical pricing procedure and a numerical example. Section 5 discusses our data and results, and Section 6 concludes and suggest future empirical research.

2. Convertible Bonds and Arbitrage Strategies

The holders of convertible bonds are entitled to receive regular fixed coupon payments and a repayment of the principal upon maturity, similar to regular bonds. The main difference is the added conversion feature, giving the investor the option to exchange the bond into a predetermined amount of stocks of the issuing company at certain times in the future. The number of shares an investor will obtain for one bond depends on the conversion ratio (Hull, 2012). If converted, the investor does not have any right to future coupons or redemption value, as the convertible bond ceases to exist after conversion. Some convertible bonds have additional embedded option features such as call and/or put options. However, we focus solely on plain-vanilla convertible bonds as we want to examine a homogeneous sample.

When valuing convertible bonds, accounting for credit risk is a vital factor. One obtain poor quality results if credit risk is ignored, as bond coupons and principal payments will be overvalued. Credit risk plays a major part as the debt holders only get the full refund if the asset value exceeds the debt value. If we consider a risk-free world, Figure 1 shows that the value of the straight bond is riskless and is therefore not affected by a change in the underlying stock price (Batten et al., 2014). The value of conversion, however, moves in line with the growing stock price as the value of the conversion equals the conversion ratio multiplied with the new stock price. As with standardized options, the option value is higher when the stock price is in-the-money (ITM). For convertible bonds, the option component is ITM when the value of conversion exceeds the value of continuation. There are several components that needs to be considered when valuing convertible bonds. Coupon rate, time to maturity, conversion ratio, face value, credit risk and the dividend yield of the underlying asset are examples of such.

The risk-free condition model is a simplification of how the straight bond value reacts to changes in the underlying stock price. The straight bond is affected by both interest risk and credit risk, while the option component is exposed to equity risk. Figure 1 must be adapted to these circumstances. If the company approaches default, the value of the convertible bond will be reduced to the value of the straight bond, adjusted for a recovery rate on the debt, as the value of the conversion option decreases to zero (Batten et al., 2014). The changes in value are illustrated in Figure 2.

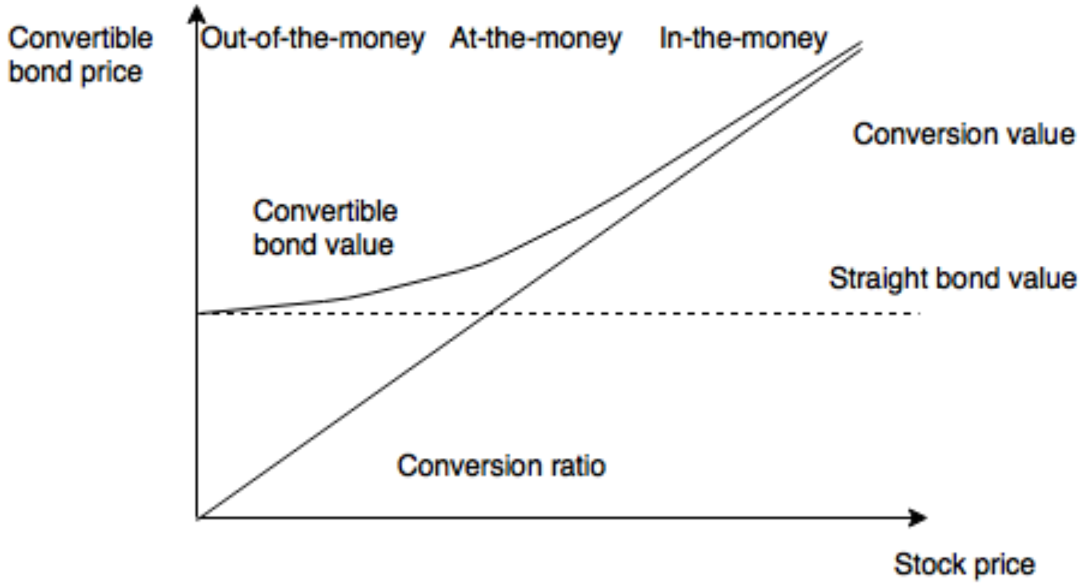

Figure 1 – Convertible bonds moneyness.
The figure illustrates the convertible bond price changes as the underlying stock price increases. It is assumed that the bond component is riskless and does not change as the underlying stock price moves. The conversion value is dependent on the conversion ratio and the underlying stock price, which means that it changes in line with the stock price. The investor is protected from downside risk through the straight bond component, and the option component gives the investor the advantage of a possible increase in the stock price of the company (Batten et al., 2014).

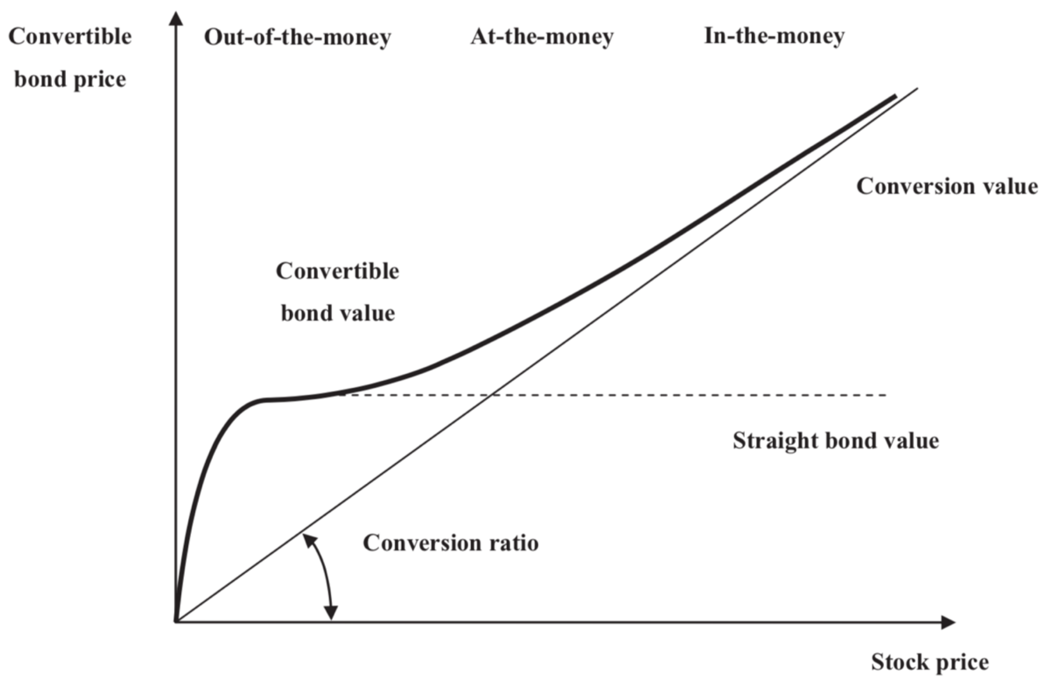

Figure 2 – The value of the option (Batten et al., 2014).
The figure illustrates that the option value makes the value of the convertible bond higher than the maximum of the straight bond value or the conversion value. The convertible bond is out-of-the-money (OTM) when the stock price drops relatively to the conversion value of the bond. This increases the probability of default (Batten et al., 2014).

The different components affect the pricing process, which can create uncertainty to what the actual price should be. This has led to arbitrage investors constantly searching for mispriced instruments. The complexity of convertible bonds creates a scenario where market participants can find differences between the market value and theoretical value. Arbitrage investors can profit by investing in mispriced instruments, e.g. by using buy-and-hedge strategies or buy-and-hold strategies (Agarwal, Fung, Loon and Naik, 2011). The buy-and-hedge strategy matches a long position in a convertible bond with a short position in the underlying common stock of the issuing firm. This is done at a current ratio delta (Δ), which is the measure of the sensitivity of the price of a convertible relative to the changes in the underlying stock. It is possible to estimate Δ by using a technique based on correlation, such as least squares regression. The strategy of delta hedging must be rebalanced constantly and is consequently an expensive trading method (Clewlow and Hodges, 1998).

To neutralize the convertible bond from equity risk, short selling an amount of the underlying stock is necessary. The amount to short sell is decided by multiplying the conversion ratio with Δ. To be able to lock in a profit, continuous rebalancing is necessary due to constantly changing stock prices. If the stock price decreases, Δ will also decrease. To rebalance this position the arbitrageur must buy the stock to cover some of the short position. If the stock price increases, the arbitrageur must short sell more of the stock to rebalance this position as Δ increases (Choi, Getmansky and Tookes, 2009). These rebalance actions contribute to an improvement of the market liquidity (Chordia, Roll and Subrahmanyam, 2002).

There are additional types of risks involved with hedging strategies, such as volatility risk, interest rate risk and credit risk. This would, however, involve buying and selling additional financial instruments. Hedging the interest rate risk would involve selling risk-free securities or futures. When hedging credit risk, short selling of non-convertible bonds or buying credit default swaps are required to hedge possible changes in credit ratings or credit spread. Volatility risk can be hedged by short selling stock options.

Hedging equity risk can be easier than hedging credit risk, as obtaining market prices for debt components are more difficult than for the equity markets (Asness, Berger and Palazzolo, 2009). Consequently, arbitrageurs prefer convertible bonds that are equity sensitive or convertibles that are in-the-money because these are more correlated with the underlying stock price. It is also found that convertible arbitrage activity is more significant for convertibles that are more equity-like than convertibles that are more debt-like, due to equity-like convertibles being more underpriced when issued (Loncarski, ter Horst and Veld, 2009).

Gamma trading is an additional way for investors to make money with a convertible arbitrage strategy by trading on the implied volatility. Gamma is the change in delta as the stock price moves. To make money on the stocks volatility within a convertible arbitrage strategy, the investor can implement a delta neutral hedge. This position will offset the money the investor make and lose on the bond and stock, and thus this position pursues to profit from the volatility of a stock.

A convertible bond with a delta of 0.5 means the convertible will rise and fall at half the rate of the underlying stock. If a company issue a convertible with a 2:1 conversion ratio and an investor purchase 100 convertibles, the initial delta of 0.5 tells the investor to short sell 100 shares of the underlying common stock. A move in the stock price means the investor is required to adjust the position when the purpose is to maintain a similar hedge. When the stock price falls, the gains from the short position should exceed the loss on the depreciation of the convertible bond price. The investor must then buy shares to still have a delta neutral hedge position. On the other hand, if the stock price rises the loss from the investor´s short position should be less than the investor´s gain from the appreciation of the convertible bond price. In summary, this means the investor is required to sell high and buy low continuously. High volatility provides many opportunities for the investor to sell and buy, as seen in Figure 3.

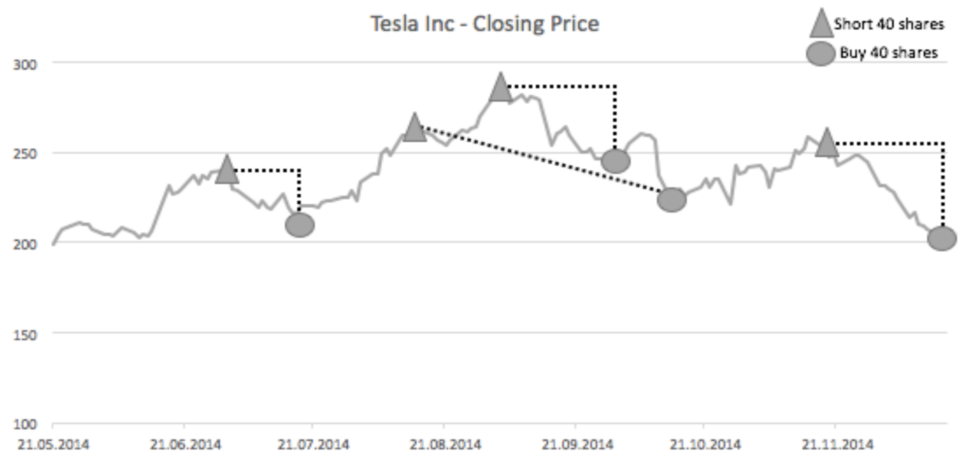

Figure 3 – Convertible Arbitrage
The figure illustrates a simplistic and theoretical approach to gamma trading on the stock price movement of Tesla Inc. The trading strategy can be used by convertible arbitrage investors to trade the underlying stock of an acquired convertible bond.

3. Theoretical Models for Pricing Convertible Bonds

Most research on pricing of convertible bonds is built upon a contingent claim approach, based on the work of Black and Scholes (1973) and Merton (1973). In previous literature, several different models and arguments have been developed to find the actual market specifications in the pricing of convertible bonds. Previous research has generated unreliable conclusions regarding applicable valuation processes due to the variation in inputs and different approaches of the models. Thus, these models may not reflect actual market behavior.

Theoretical research on convertible bond pricing is divided into three branches (Ammann et al., 2008). The first approach is the firm-value approach which finds a closed-form solution of the valuation equation. The approach is inspired by Black and Scholes (1973) and Merton (1973). It is used by Ingersoll (1977a) and Brennan and Schwartz (1977) who treats a convertible bond as a contingent claim on firm value. Both papers contribute to the initiation of the theoretical research on convertible bonds.

The second approach values convertible bonds numerically with lattice-based methods (Ammann et al., 2008). Commercially available models use this approach, e.g. Bloomberg Monis, OVCV, and SunGard. Brennan and Schwartz (1977) introduced the first theoretical model and later extended it with the use of stochastic interest rates in Brennan and Schwartz (1980). McConnell and Schwartz (1986) extend the method further by modelling the underlying stock as a stochastic variable. They apply a constant credit spread grossed up by an interest rate to account for credit risk. Bardhan et al. (1993) and Tsiveriotis and Fernandes (1998) split the convertible bond value into a stock component and a straight bond component. Ammann et al. (2003) extend the model by accounting for various trigger conditions in the call feature. Several other researchers use tree-based models (e.g. Hung and Wang (2002), Carayannopoulos and Kalimipalli (2003), Davis and Lishka (1999), Takahashi, Kobayashi and Nakagawa (2001), Andersen and Buffum (2004), Barone-Adesi, Bermudez and Hatgioannides (2003), and Ayache, Forsyth and Vetzal, (2003)). These models work well in theory, but have several drawbacks in practice. The computing time grows exponentially with a rising number of state variables, the flexibility is low modelling the underlying state variable, and it is not easy to incorporate path dependencies (Ammann et al., 2008).

The solution to these drawbacks is to use the third and final class of convertible bond pricing models, the Monte Carlo simulation model. Monte Carlo simulation is suitable to model dynamic features and call features that are path-dependent, as well as including realistic dynamics of a convertible bonds underlying state variables. The model considers that early redemption may be allowed in cases where stock price exceed an agreed level, and for only a pre-specified number of days in a pre-specified period (Ammann et al., 2008). Another advantage of Monte Carlo simulation is that there is an almost linear relationship between number of state variables and computing time, making the model flexible and suitable for future changes and extensions (Ammann et al., 2008).

Monte Carlo simulation in combination with dynamic programming has been discussed in several papers to address difficulties of pricing options with American features. Bossaerts (1989), Li and Zhang (1996), Grant, Vora and Weeks (1997), Andersen (2000), and García (2003) use a fixed number of parameters to account for the early exercise rule. They maximize the value of the option over the parameter space by finding the optimal exercise strategy and thus the price of American options. Carrière (1996), Tsitsiklis and Van Roy (1999), Longstaff and Schwartz (2001), and Clément, Lamberton and Protter (2002) use a backward induction technique to estimate the continuation value of the option. They identify the continuation value by using a linear regression to estimate future payoffs on a set of basic functions of the state variables. Tilley (1993), Barraquand and Martineau (1995), and Raymar and Zwecher (1997) also apply backward induction where the method satisfies the state space. For each subset of state variables, they find the optimal exercise decision. Broadie and Glasserman (1997) and Broadie, Glasserman and Jain (1997) use simulated trees to calculate prices of American-style options. Avramidis and Hyden (1999), Broadie and Glasserman (2000), and Broadie and Glasserman (2004) use a method based on stochastic-mesh for different mesh weights. Broadie and Cao (2003), Haugh and Kogan (2004), and Rogers (2002) priced Bermudan options using a simulation model with a duality approach. Buchan (1997, 1998) apply the simulation method by employing firm value as an underlying state variable as well as allowing senior debt to be accounted for. However, the conversion option is assumed to be European-style. Fu, Laprise, Madan, Su and Wu (2001) summarize different approaches with a comparison of various Monte Carlo approaches.

As most convertible bonds are American-style options, the bond value to be exchanged for the shares is not known in advanced. A straight call option has a known strike price in advance, while a convertible bond includes stochastic strike price in the option component. The strike price is a function of both changes in credit spreads and interest rates. Ammann et al. (2003) solve this issue by treating the option in the conversion as the option to exchange an asset for another. The solution is to treat the convertible bond as sum of a straight bond plus value of the option to exchange the bond into stocks. Margrabe (1978) introduced a closed-form solution to value exchange options, later named the Margrabe-model. Ammann et al. (2003) argue that the geometric Brownian motion used by Margrabe is not considered to be the best process specification for bond prices. Furthermore, they point out that the closed form pricing formula by Merton (1973), Black and Scholes (1973) and Margrabe (1978) refers to European-style options, and that the model fails to include potential embedded call and put features.

Due to these drawbacks, Ammann et al. (2003) implement an alternative approach, the binominal-tree model with exogenous credit risk. This model is more precise as it accounts for the embedded options and American-style options. They construct a univariate binominal-tree with 100 time steps following Cox et al. (1979) and base the nominal tree on the stock price described by McConnell and Schwartz (1986). They extend the approach further with numerous contract-specific boundary conditions to account for the many complex bonds characteristics such as embedded options and triggers in their data sample (Ammann et al., 2003). For each node, they calculate the convertible bond value regarding the optimal conversion behavior for the holder. Note that their study also includes a call option for the issuer, leading to four alterative outcomes: 1) The convertible does not get converted or called, and therefore continues to exist. 2) The convertible is called by the issuer. 3) The convertible is converted by the holder, or 4) the convertible is called by the issuing company and then converted by the holder, known as a forced conversion.

They compare the prices generated by the binominal-tree model with the prices available in the French convertible bond market to examine if any systematic underpricing exists. Furthermore, they include an exchange-option model and a simple component model as reference models and find underpricing of over 3 %. Like King (1986) and Carayannopoulos (1996), they find that the degree of underpricing depends on the moneyness of the convertible. ITM convertibles are less mispriced than ATM and OTM convertibles. They also find that longer time to maturity results in a higher underpricing.

In previous research, King (1986) find average underpricing of 3.75 % in a data sample of 103 American convertible bonds. Carayannopoulos (1996) investigate 30 American convertible bonds in a period of one year and finds an average underpricing of 12.9 %. Both authors find that deep OTM bonds are underpriced, while ATM and ITM bonds are overpriced. Using a firm-value model with a Cox-Ingersoll-Ross (CIR) term structure model, Buchan (1997) finds an average underpricing of 1.7 % on Japanese convertible bonds. Ammann et al. (2003) argue that drawbacks in these previous studies are too few data points. Finnerty (2015) finds a mean pricing error of 0.21 % in their closed-form contingent claim model. While Bunchan (1997) only use one calendar day, King (1986) two days, and Carayannopoulos 12 days, Ammann et al. (2003) expends the data set with 18 months of daily pricing data. This is the first empirical test with direct modelling of stock prices to value convertible bonds (Ammann et al., 2003). To account for complex bond characteristics, they implement a binominal-tree model with exogenous credit risk.

Ammann et al. (2008) provides a new theoretical and empirical contribution to the pricing process by using Monte Carlo simulation with parametric representation of the early exercise decisions. They reject the hypothesis of theoretical underpricing found in previous studies. They extend previous approaches by creating a model that is better to include and account for complex features reflecting real world convertible bonds. By using daily data for 69 months, they find theoretical underpricing of just 0.36 % in average for the US convertible bond market. In contrast to their previous studies they do not confirm any strong positive relationship between moneyness and price deviation.

4. Pricing Framework

The following section presents the mathematical pricing approach applied in this paper. We discuss the importance of including credit risk and how to apply the Monte Carlo simulation. Furthermore, we account for interest rates, coupons and dividend payments. Finally, we provide a numerical example to illustrate our pricing model approach.

4.1 Mathematical approach

If the convertible bond does not get converted it redeems at maturity *T* with pre-specified amount κN, where κ is the final redemption ratio of the face value and *N* represents the face value of the convertible bond with associated coupons. In most cases $\kappa = 1$. If the investors convert they receive $n_t S_t$, where n_t is the conversion ratio at time *t* and S_t is the spot price of the underlying stock at time *t*.

The optimal stopping time must be obtained to determine the investor's cash flows. The optimal stopping time is defined as τ^*. This may either be by a regular redemption when the bond reaches maturity, or an early conversion. The optimal stopping time is found by calculating $max(n_t S_t; N; V_t')$, where $n_t S_t$ is the payoff from a conversion in state X_t at time *t*, *N* is the appropriate discounted value of cash flows received at maturity, consisting of the face value and associated coupons, and V_t' is the value of continuation. Continuation is defined as the convertible bond being held for one additional time step. If $n_t S_t < V_t'$ or $n_t S_t < N$ there will be no conversion at the current time step.

The coupon payments occurring prior to τ^* must be accounted for. We apply the mathematical approach used by Ammann (2008), with adjustments to our plain-vanilla sample. We define $p(X_\tau^*, \tau^*)$ as the payoff at the optimal time of termination τ^*, discounted with risk-free rate r_f and $c(\tau^*)$ as the accumulated value of all coupons throughout the convertible bonds lifetime, i.e. before τ^*, discounted with the appropriate risk adjusted rate r_{adj}. The discount rate r_{adj} reflects the probability of default. The cash flows are discounted in a risk-neutral measure to obtain the convertible bond value:

$$V_0 = E^{\mathbb{Q}}\left[e^{-\sum_{t=0}^{\tau^*} r_f(X_t,t)} p(X_{\tau^*}, \tau^*) + e^{-\sum_{t=0}^{\tau^*} r_{adj}(X_t,t)} c(\tau^*)\right], \quad \textbf{(1)}$$

where V_0 is the convertible bond value in each path, τ^* is the optimal stopping time, expectation $E^{\mathbb{Q}}[\,\cdot\,]$ is the equivalent Martingale measure $\mathbb{Q}$ defined using the riskless security as the numeraire (Ammann et al., 2008). We define $r_f(X_t, t)$ as the interest rate between time *t* and *t* + 1. Note that if the investor does not convert but wait for the face value, the first part of the equation will also be discounted with the risk adjusted rate r_{adj} as the probability of default affects the face value.

The next step is to calculate the value of continuation. We apply the following equation as part of the regression approach:

$$\omega = E^{\mathbb{Q}}\left[e^{-\sum_t^{t+1} r_f(X_t,t)} CF_{t+1} + e^{-\sum_t^{t+1} r_{adj}(X_t,t)} cpn_{t+1}\right], \quad \textbf{(2)}$$

where CF_{t+1} is the expected cash flow received in the next time step, found by backward induction, and cpn_{t+1} is the coupon received for holding the convertible one more time step. As the investor does not have perfect knowledge about future stock prices, we use regression to estimate the expected value of continuation (V_t'), where the Y-value equals ω and the X-value is the simulated stock price at time *t*.

By applying the exercise strategies, we calculate the average discounted payoffs of all paths simulated by calculating:

$$V_0 = \frac{1}{N}\sum_{i=1}^{N}\left[e^{-\sum_{t=0}^{\tau^*} r_f(X_t,t)} p(X_{\tau^*}, \tau^*) + e^{-\sum_{t=0}^{\tau^*} r_{adj}(X_t,t)} c(\tau^*)\right], \quad \textbf{(3)}$$

where *N* is the number of paths simulated and X_t are realizations of simulated state variables. We provide a numerical example in section 4.5 to illustrate the progress method.

4.2 Accounting for Credit Risk

Credit risk is the risk that a borrower may default and hence not be able to repay the coupons or principal to the lender (Jarrow and Turnbull, 1995). There are two sources of credit risk associated with convertible bonds (Carol, 2008). First, convertible bonds are often issued by growth firms. Such firms are often expected to receive a high share price in the future but obtain low credit ratings today. Issuing convertible bonds is therefore a way to receive funding with relatively lower coupons compared to straight bonds, yet still be attractive to investors due to the conversion option. Second, convertible bonds are often classed as subordinated or junior debt (Carol, 2008).

Credit risk can be accounted for in different ways in a simulation such as Monte Carlo. Some studies apply the approach presented by Tsiveriotis and Fernandes (1998) by discounting cash flows separately. Cash flows that are not subject to credit risk are discounted with a different discount rate than cash flows subject to credit risk. Coupons and redemption payments are discounted with a risk adjusted rate (r_{adj}) when using a simulation-based approach (Tsiveriotis and Fernandes, 1998). It is also possible to implement credit spreads as a subsequent process correlated with other state variables, or as a constant. In our model, we implement credit risk similar to Tsiveriotis and Fernandes (1998). We apply constant credit spreads by discounting the coupons and final redemption with a risk adjusted rate, while we discount stock prices with the risk-free rate.

When implementing credit spreads, finding similar straight bonds with the same maturity as the convertible bonds can be a challenge. Studies such as Ammann et al. (2008) use constant credit spreads by obtaining monthly data from the Yield Book database. There are some drawbacks with this method as credit ratings change over time, and constant credit spreads only represent the rating at a certain point of time. Additionally, credit spreads data often represent only averages of bonds outstanding within a certain rating category. Errors when estimating credit spreads are therefore possibly significant when pricing convertibles bonds, given that most convertibles are rated relatively low and have high credit spreads. We obtain credit spreads from Moody's Analytics as of March 27, 2018, and apply a similar approach as Ammann et al. (2008).

4.3 Monte Carlo Simulation

An important aspect of pricing convertible bonds and derivatives in general is finding the optimal time of exercise. The investor cannot be certain what will happen in the future. Consequently, there must be some form of estimated expectation regarding the future value of a convertible bond to be able to make an optimal decision. The holder of a convertible bond will at any time convert the bond if the expected payoff today exceeds the expected payoff of continuation. Thus, the optimal point of exercise is decided by the conditional expectation on a convertible bonds value. To find the conditional expected value of holding the convertible bond for one more period we apply a least squares regression method to estimate a continuation value by using information from the Monte Carlo simulated paths, as done by Longstaff and Schwartz (2001). This provides an unbiased estimate of the expected conditional value of continuation from the cross-sectional information in the simulation, and thus helps to estimate the optimal stopping time for the convertible. Applying a Monte Carlo simulation and estimating expected continuation value on each possible exercise date will provide an optimal exercise strategy along all simulated paths.

We apply Monte Carlo simulation where we simulate 4000 paths with 30 time steps for each of the 40 convertible bonds in our data sample. Assuming a stochastic process and a discrete time model, we apply the following simulation formula:

$$\Delta S = \mu S \Delta t + \sigma S \in \sqrt{\Delta t} \quad \textbf{(4)}$$

In equation 4, stock price S follows a geometric Brownian motion as explained in Hull (2012), where ΔS is the change in stock price S in time interval Δt, μ is the expected rate of return, σ is the volatility of the stock price and $\in$ is the standard normal distribution.

The drift term μ and standard deviation are held constant in our model, where μ equals the risk-free rate r_f as we want the simulation of future stock prices to be independent of historical performance of the underlying stock. We simulate the stochastic process in a Monte Carlo framework. The Monte Carlo approach simulates random numbers between 0 and 1, resulting in numerous paths the underlying stock price can follow based on the applied risk-free rate and standard deviation. The randomized numbers are then used in an inverse cumulative normal distribution model.

We use ten years of adjusted historical stock prices or the maximum data available to calculate standard deviation. All historical stock quotes are obtained from Bloomberg Terminal. To obtain variance and standard deviation, we use daily logarithmic return. Yearly standard deviation is adjusted from calendar days to actual trading days. The risk-free rate is obtained from US Treasury Bills and is continuously compounded. As we use 30 time steps for each convertible bond, the number of days between each time step vary according to the different days to maturity. The model accounts days in each time step with a unique simulation for every convertible bond, where all parameters and inputs are adjusted. We achieve similar average values when conducting several simulations, proving that the number of paths and time steps in our model are sufficient for a simulation of high quality.

4.4 Coupon, interest rate and dividends

We adjust coupons to the time to maturity and number of time steps, providing a correct estimate of coupon payments in our simulation. When finding the optimal stopping time, we account for accrued coupons and risk adjusted value of future coupon payments.

Risk-free rates are obtained from the US Treasury and adjusted for every convertible bond's time to maturity. Obtained interest rates cover maturities from one month to thirty years. The convertible bonds have different number of days to maturity. We solve this by interpolating interest rates with a linear interpolation, providing accurate estimates down to the number of days to maturity. Some previous studies have discussed whether to use stochastic or constant interest rates. However, Ammann et al. (2008) find that the benefits of using stochastic interest rates are limited.

By using adjusted stock prices to estimate standard deviation we account for historical dividends in the simulation of future stock prices. Furthermore, the convertible bonds in our data sample are dividend protected meaning that the conversion ratio adjusts to dividend payments. Consequently, dividends do not affect the value of the convertible bond in our model.

4.5 Numerical example

This section provides a numerical example to illustrate our pricing procedure. To identify the conditional expected value of continuation, the key is to exercise the convertible bond at optimal time (Longstaff and Schwartz, 2001). At any point of time it is optimal to exercise the convertible bond if the conversion value exceeds both the expected value of continuation and the discounted face value with coupons, hereafter included in the term continuation value.

Consider the convertible bond issued by Tesla Inc., March 22, 2017. The convertible bond has a conversion ratio of 3.05, coupon 2.375, face value of US$1000 and maturity March 15, 2022. It is exercisable at any time step between $t = 1$ and $t = 30$. The paths are simulated as mentioned in section 4.3 and summarized in the following matrix:

Simulated stock paths

Timestep		t = 0	t = 1	t = 2	...	t = 29	t = 30
Path							
1		279,2	254,2	247,1		453,8	485,7
2		279,2	236,8	234,0		84,9	93,3
3		279,2	250,9	258,2		192,8	201,2
...							
3999		279,2	309,2	334,9		227,3	225,8
4000		279,2	306,8	304,3		200,2	203,2

We start our recursive algorithm in the final time step t = 30 and create 29 intermediate matrices. For each time step we condition no earlier exercise. Otherwise, there would be no expected cash flow in later time steps. The expected cash flows in t = 30 are:

Cash flow matrix at t = 30

Path	Simulated stock prices	Conversion value	Maturity value	Max
1	485,7	1483,0	1000	1483
2	93,3	285,0	1000	1000
3	201,2	614,2	1000	1000
...				
3999	225,8	689,4	1000	1000
4000	203,2	620,6	1000	1000

In the matrix above, conversion value is the stock price S_t at time t multiplied with the conversion ratio. The possible cash flows at t = 30 is the maximum of either the conversion value or the face value. For path 1, this would be the maximum of US$1483.04 (3.0534 x 485.7) or the face value of US$1000.

The next step is to calculate whether the investor should convert at one earlier time step, in this case t = 29, or continue holding the convertible for one more time step. We use least squares regression to estimate the expected value of continuation from t = 29 to t = 30. X denotes the stock prices at t = 29 for each path. Y is the corresponding expected discounted cash flow received at t = 30, conditioned on no conversion in t = 29. Note that this also includes accrued coupons for holding the convertible bond one more time step. The regression vectors X and Y at t = 29 are summarized in the matrix below:

Regression at t = 29

Path	Y	X	X^2
1	1481,7	453,8	205947
2	996,6	84,9	7211
3	996,6	192,8	37154
...			
3999	996,6	227,3	51661
4000	996,6	200,2	40089

We regress Y on a constant X and X^2 to estimate the expected continuation value conditioned on the simulated stock price at t = 29, resulting in the coefficients summarized below:

Regression at t = 29

	Coefficients
Intercept	961,41974
X-variable 1	-0,417675
X-variable 2	0,0027523

The coefficients provide the conditional expected function $E[Y \mid X] = 961.41974 - 0.417675X + 0.0027523X^2$. It is now possible to estimate the conditional expected value of continuation by substituting X with the corresponding stock value for each path. In path 1 the estimated value of continuation is US$1338.7. This provides a unique expected value of continuation for each of the 4000 paths.

In the following table the value of early exercise at t = 29 is compared with the value of continuation. If the exercise value exceeds the value of continuation the investor chooses to exercise at t = 29.

Exercise decision at t = 29

Path	Exercise	Continuation	Exercise or continue?
1	1385,7	1338,7	Exercise
2	259,3	945,8	Continue
3	588,6	983,2	Continue
...			
3999	694,0	1008,7	Continue
4000	611,4	988,1	Continue

For the paths illustrated in the previous table, the only optimal conversion is in path 1. In total, the convertible bond is exercised in 664 out of the 4000 paths in time step 29.

We repeat this process for all time steps to examine whether the convertible bond should have been exercised at an earlier time step. Remember that if a path is exercised in an earlier time step, the convertible ceases to exist and cannot be converted in a later time step. To keep the numerical example brief, we jump straight to time step 1.

The expected cash flows received in t = 2 is summarized in the following matrix. Again, the cash flows are conditioned on no exercising in t = 1.

Cash flow matrix at t = 2

Path	Simulated stock prices	Conversion value
1	247,1	754,4
2	234,0	714,5
3	258,2	788,3
...		
19	371,5	1134,3
...		
23	369,6	1128,6
...		
3999	334,9	1022,5
4000	304,3	929,1

We continue the recursive approach and investigate whether the convertible bond should be converted in t = 1. Y again denotes the discounted expected cash flows for the next time step.

Regression at t = 1

Path	Y	X	X^2
1	756,9	254,2	64603,6
2	717,1	236,8	56097,9
3	790,8	250,9	62935,6
...			
19	1136,5	357,4	127718,3
...			
23	1130,8	286,9	82291,7
...			
3999	1024,8	309,2	95599,0
4000	931,4	306,8	94155,3

We regress Y on a constant X and X^2 to estimate the expected continuation value conditioned on the simulated stock price at t = 1, resulting in the coefficients summarized below:

Regression at t = 1

	Coefficients
Intercept	60,38910578
X-variable 1	2,574124212
X-variable 2	0,000967214

The coefficients provide the conditional expected function $E[Y \mid X] = 60.38910578 + 2.574124212 + 0.000967214X^2$. The value of early exercise at t = 1 is compared with the value of continuation, and summarized in the following matrix:

Exercise decision at t = 1

Path	Exercise	Continuation	Exercise or continue?
1	776,1	777,1	Continue
2	723,2	724,3	Continue
3	766,0	767,0	Continue
...			
19	1091,2	1103,9	Continue
...			
23	875,9	878,4	Continue
...			
3999	944,1	948,7	Continue
4000	936,9	941,3	Continue

For Tesla, none of the 4000 paths gets converted in time step 1. By completing step 30 to step 1, we find the optimal stopping time for each path. In the following table, optimal stopping time is represented by the numeric value 1:

Optimal stopping time

Timestep	t = 1	t = 2	t = 3	...	t = 29	t = 30
Path						
1	0	0	0		1	0
2	0	0	0		0	1
3	0	0	0		0	1
...						
19	0	1	0		0	0
...						
23	0	1	0		0	0
...						
3999	0	0	0		0	1
4000	0	0	0		0	1

When optimal stopping time is found, it is straightforward to find the associated cash flow for each path. The following matrix illustrates the corresponding cash flows received at each optimal stopping time. Note that for t = 30, the cash flow is the maximum of the conversion value or the face value.

Cash flow at optimal stopping time

Timestep	t = 1	t = 2	t = 3	..	t = 29	t = 30
Path						
1	0,0	0,0	0,0		1385,7	0,0
2	0,0	0,0	0,0		0,0	1094,3
3	0,0	0,0	0,0		0,0	1094,3
...						
19	0,0	1134,3	0,0		0,0	0,0
...						
23	0,0	1128,6	0,0		0,0	0,0
...						
3999	0,0	0,0	0,0		0,0	1094,3
4000	0,0	0,0	0,0		0,0	1094,3

The final step is to determine the value in t = 0 by averaging the discounted cash flows based on the optimal stopping rule. Converted paths are discounted with the risk-free rate, while face values and coupons are discounted with the risk adjusted rates. In total 1702 of 4000 paths are converted for this convertible bond.

Convertible bond value

Path	Value at t = 0
1	1427,7
2	898,2
3	898,2
...	
19	1138,6
...	
23	1132,9
...	
3999	898,2
4000	898,2
Average	1052,3

By conducting this approach, we estimate a theoretical price of Tesla Inc.`s convertible bond of US$1052.3. The observed market price is US$1082.0, leading to an estimated overprice in the market of 2.82 %. This pricing method is applied to all 40 convertible bonds in our sample.

5. Data and Results

In this section, we present our data sample and discuss the results. Table 1 summarizes the convertible bond characteristics present in our sample and Table 2 provides the results from our analyzes.

5.1 Data

We examine all convertible bonds outstanding in the US domestic market per March 27, 2018. The US is the largest and most liquid market for convertible bonds. All convertible bond data and daily stock prices are obtained from Bloomberg Terminal and credit spreads are provided from Moody`s Analytics. The total number of convertible bonds outstanding in the US market is 504 per March 27, 2018, with at an average time to maturity of 1902 days, average coupons of 2.81 % and average market capitalization for issuing firms is US$39.7 billion.

Various requirements are made to ensure a quality data sample. We consider only plain-vanilla convertible bonds with market cap above US$100 million. We only examine convertible bonds rated by Standard & Poor´s Bond Guide as we need credit ratings to implement credit spreads. The latter two criteria reduce the sample to 83 convertible bonds. By excluding convertible bonds including call or put features, we are left with 49 convertibles. Two of these bonds are exchangeable, meaning that the holder can convert to stocks in a different company than the issuing firm. These are also excluded. Furthermore, we exclude four companies that have merged during the convertibles' lifetime, one company where the convertible bond is listed on London Stock Exchange and two convertible bonds with too brief history to compute a precise volatility estimate.

The final sample consist of 40 plain-vanilla convertible bonds, summarized in Table 1. This is a larger sample compared to previous research such as Ammann et al. (2008), who also include convertible bonds with call and put features. We find average coupons of 2.3 %, average time to maturity of 3.2 years and average market cap of US$12.23 billion in our sample. The oldest convertible bond is E*TRADE Financial Corp issued Match 19, 2009, and the most recent is Marriot Vacations Worldwide Corp issued September 25, 2017. Three convertible bonds are zero coupon bonds, while the remaining 37 are fixed coupon bonds. The credit ratings vary from BBB to CCC.

Issuer Name	Issue Date	Cpn	Maturity	Market Cap (m)	Coupon Type	Is Convertible	S&P Rating
Altaba Inc	26-Nov-13	0.00	1-Dec-18	62 607	ZERO COUPON	Y	BB
Ares Capital Corp	27-Jan-17	3.75	1-Feb-22	6 758	FIXED	Y	BBB
Cardtronics Inc	25-Nov-13	1.00	1-Dec-20	1 115	FIXED	Y	BB
Chart Industries Inc	3-Aug-11	2.00	1-Aug-18	1 833	FIXED	Y	B
Ciena Corp	28-Jul-17	3.75	15-Oct-18	3 746	FIXED	Y	B
Citrix Systems Inc	30-Apr-14	0.50	15-Apr-19	12 831	FIXED	Y	BBB
Cypress Semiconductor Corp	23-Jun-16	4.50	15-Jan-22	6 395	FIXED	Y	BB
DISH Network Corp	17-Mar-17	2.38	15-Mar-24	17 402	FIXED	Y	CCC
Dycom Industries Inc	15-Sep-15	0.75	15-Sep-21	3 238	FIXED	Y	BB
E*TRADE Financial Corp	19-Aug-09	0.00	31-Aug-19	14 710	ZERO COUPON	Y	BBB
Gogo Inc	9-Mar-15	3.75	1-Mar-20	776	FIXED	Y	CCC
Goldman Sachs BDC Inc	3-Oct-16	4.50	1-Apr-22	782	FIXED	Y	BBB
Hercules Capital Inc	25-Jan-17	4.38	1-Feb-22	1 026	FIXED	Y	BBB
Horizon Global Corp	1-Feb-17	2.75	1-Jul-22	209	FIXED	Y	B
Illumina Inc	11-Jun-14	0.00	15-Jun-19	35 671	ZERO COUPON	Y	BBB
Impax Laboratories Inc	30-Jun-15	2.00	15-Jun-22	1 449	FIXED	Y	B
Innoviva Inc	24-Jan-13	2.13	15-Jan-23	1 680	FIXED	Y	B
Integrated Device Technology Inc	4-Nov-15	0.88	15-Nov-22	4 222	FIXED	Y	B
iStar Inc	20-Sep-17	3.13	15-Sep-22	679	FIXED	Y	BB
Lam Research Corp	11-May-11	1.25	15-May-18	34 709	FIXED	Y	BBB
Macquarie Infrastructure Corp	15-Jul-14	2.88	15-Jul-19	3 128	FIXED	Y	BBB
Marriott Vacations Worldwide Corp	25-Sep-17	1.50	15-Sep-22	3 618	FIXED	Y	BB
Microchip Technology Inc	15-Feb-17	1.63	15-Feb-27	22 984	FIXED	Y	B
Nabors Industries Inc	13-Jan-17	0.75	15-Jan-24	2 275	FIXED	Y	BB
Newpark Resources Inc	5-Dec-16	4.00	1-Dec-21	749	FIXED	Y	B
Nuance Communications Inc	17-Mar-17	1.25	1-Apr-25	4 698	FIXED	Y	BB
NVIDIA Corp	2-Dec-13	1.00	1-Dec-18	148 606	FIXED	Y	BBB
ON Semiconductor Corp	8-Jun-15	1.00	1-Dec-20	11 074	FIXED	Y	BB
PDC Energy Inc	14-Sep-16	1.13	15-Sep-21	3 373	FIXED	Y	BB
Prospect Capital Corp	11-Apr-14	4.75	15-Apr-20	2 382	FIXED	Y	BBB
Red Hat Inc	7-Oct-14	0.25	1-Oct-19	28 393	FIXED	Y	BBB
Spirit Realty Capital Inc	20-May-14	2.88	15-May-19	3 463	FIXED	Y	BBB
TCP Capital Corp	6-Sep-16	4.63	1-Mar-22	827	FIXED	Y	BBB
Tesla Inc	22-Mar-17	2.38	15-Mar-22	50 203	FIXED	Y	B
TPG Specialty Lending Inc	1-Feb-17	4.50	1-Aug-22	1 123	FIXED	Y	BBB
TTM Technologies Inc	20-Dec-13	1.75	15-Dec-20	1 596	FIXED	Y	B
Twitter Inc	17-Sep-14	1.00	15-Sep-21	22 779	FIXED	Y	BB
Unisys Corp	15-Mar-16	5.50	1-Mar-21	544	FIXED	Y	B
Verint Systems Inc	18-Jun-14	1.50	1-Jun-21	2 443	FIXED	Y	B
Whiting Petroleum Corp	27-Mar-15	1.25	1-Apr-20	3 123	FIXED	Y	BB

Table 1 – Data Sample of Issued Convertible Bonds
The table provides a summary of the features present in our analyzed sample. The listed information is name of issuing firm, date of issue, yearly coupons, maturity date, market capitalization of the issuing firm, type of coupon for the convertible bond, whether the bond is convertible, and credit ratings from Standard & Poor's Bond Guide as of March 27, 2018.

5.2 Results

We find that market prices are on average 1.12 % higher than model prices, as summarized in Table 2. This is in contrast with previous studies using other pricing models and approaches, such as King (1986) and Ammann et al. (2003) who find convertible bonds to be underpriced. The results are however consistent with Ammann et al. (2008) who use a similar approach where they find an overpricing of 0.36 % in their sample of 32 convertible bonds. However, the samples in previous research consist of convertible bonds with additional call and put features, while our data sample contains only plain-vanilla convertible bonds and may therefore not be directly comparable. In total among all 40 convertible bonds, only two convertibles have mispricing exceeding 15 %. If we exclude these outliers, we find that market prices are on average 0.02 % lower than model prices.

The following table summarizes the mispricing contribution of each convertible bond:

Company	Ask price	Model price	Pricing deviation
Altaba Inc	1433	1367	0.0483
Ares Capital Corp	1024	1150	-0.1099
Cardtronics Inc	934	969	-0.0366
Chart Industries	1016	1038	-0.0218
Ciena Corp	1333	1287	0.0360
Citrix Systems Inc	1333	1285	0.0369
Cypress Semiconductor Corp	1412	1434	-0.0158
DISH Network Corp	877	781	0.1225
Dycom Industries Inc	1238	1085	0.1405
Etrade Financail Corp	5249	5235	0.0028
GOGO Inc	903	957	-0.0560
Goldman Sachs BDC Inc	1026	1111	-0.0764
Hercules Capital Inc	1011	1127	-0.1026
Horizon Global Corp	835	908	-0.0805
Illumina Inc	1108	1047	0.0581
Impax Laboratories Inc	998	890	0.1203
Innoviva Inc	1057	1043	0.0135
Integrated Device Technology Inc	1146	949	0.2075
iStar	953	1091	-0.1267
Lam Research Corp	3562	3393	0.0500
Macquarie Infrastructure Corp	993	1020	-0.0268
Marriott Vacations Worldwide Corp	1094	1037	0.0551
Microchip Technology Inc	1249	957	0.3051
Nabors Industries Inc	759	866	-0.1232
Newpark Resources Inc	1177	1162	0.0131
Nuance Communications Inc	995	908	0.0965
NVIDIA Corp	11093	11245	-0.0135
ON Semiconductor Corp	1488	1350	0.1029
PDC Energy Inc	977	962	0.0151
Prospect Capital Corp	1008	1056	-0.0453
Red Hat Inc	2112	2109	0.0017
Spirit Realty Capital Inc	997	1000	-0.0024
TCP Capital Corp	1014	1107	-0.0848
Tesla Inc	1082	1052	0.0282
TPG Specialty Lending Inc	1021	1136	-0.1015
TTM Technologies Inc	1683	1619	0.0395
Twitter Inc	940	934	0.0069
Unisys Corp	1293	1359	-0.0484
Verint Systems Inc	953	913	0.0437
Whiting Petroleum Corp	946	968	-0.0223
Mean pricing deviation			0.0112

Table 2 – Pricing Deviation by Issued Convertible Bond
The table provides a summary of price deviations between predicted model prices and observed market prices for the 40 convertibles analyzed in this paper. Pricing deviation represent to what extent observed market prices vary from the estimated model prices.

We investigate the relationships between pricing deviation and critical input factors by conducting several regressions. First, we examine the relationship between pricing deviations and coupons. In our sample, we find that convertible bonds with high coupons tend to be relatively more underpriced compared to convertible bonds with low coupons and zero coupons. The relationship proves to be statistically significant at 5 %-level. Next, we examine the relationship between credit ratings and convertible bond price deviations. The result implies that convertible bonds with low credit ratings tend to be more overpriced compared to convertible bonds with high credit ratings. This relationship also proves to be statically significant at 5 %-level. Note that the results are based on linear regression. It is not necessarily the case that investors interpret the relationship between the different credit ratings as linear.

The relationship between days to maturity and pricing deviation is examined, and proves to be statistically insignificant. The relationship between market cap size and pricing deviation also proves to be statistically insignificant. Furthermore, we find no statistical significant relationship between number of paths converted for each convertible bond and pricing deviations.

Test	Coefficient	p-value	R-squared
Coupon	-2.1748	0.0319	0.115
Credit Rating	-0.0656	0.0118	0.156
Moneyness	0.0004	0.9589	0.000
Days to Maturity	0.0000	0.3541	0.023
Paths Converted	0.0000	0.8839	0.001
Market Cap	0.0000	0.4276	0.017
Company in Finance Sector	-0.1007	0.0020	0.226
Company in Tech Sector	0.0711	0.0711	0.141

Table 3 – Regression table
The table provides a summary of coefficients, p-values and R-squared values for the conducted regressions, with the pricing deviation as the dependent variable. Company in Finance Sector and Company in Tech Sector are regressed as dummy variables.

Previous research, e.g. Ammann et al. (2003) find relationship between the degree of moneyness and mispricing, indicating that underpricing decreases when a convertible bond is further ITM. When examining for moneyness we find no significant relationship, consistent with Ammann et al. (2008). The relationship between coupon rates and price deviation implies that higher coupons leads to a higher estimated price in our model, as illustrated in Figure 5. However, it is important to note that some of these underpriced companies belong to the same sector and have similar credit ratings. If sector is a factor influencing the relationship between coupon and predicted prices, it is natural to think that the simulated stock prices in our simulation can be affected by sector specific volatility. Given that we use historical volatility when predicting stock prices, the historical data from companies within the same sector may lead to this relation. This is tested by conducting a dummy regression with the eleven companies belonging to financial sector against the rest of the data sample. We find a categorical effect of higher underpricing for the finance sector which is highly significant at 1 %-level. Similar regression is done for the thirteen tech companies, where we find the sector to be more overpriced with a statistically significance at 5 %-level, as shown in Table 3.

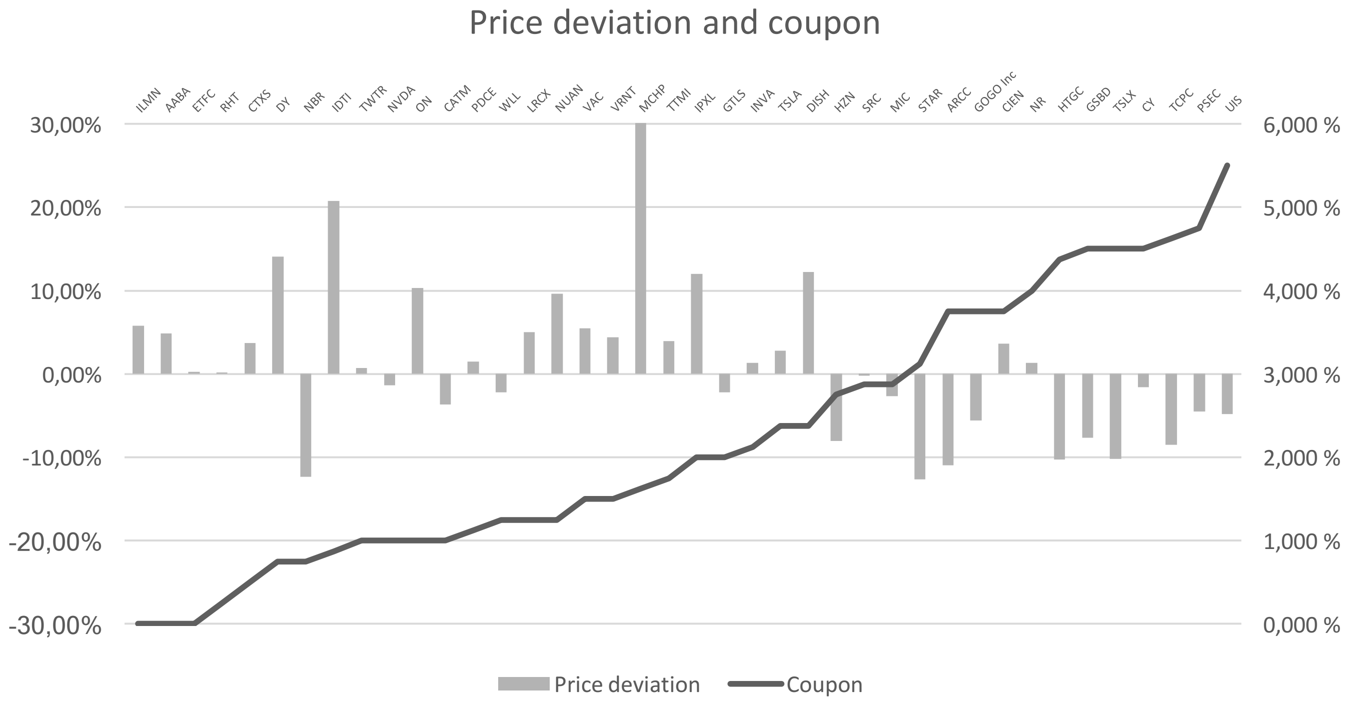

Figure 4 – Price Deviation and Coupon.
The figure provides an illustration of the relationship between coupon rates and price deviation between predicted prices and market prices for the convertible bonds in our data sample. The axis on the left side represent percentage of price deviation between estimated prices and actual market prices, while the axis on the right side represent the yearly coupon rate in percentage of face value.

Furthermore, the price deviation may also result from comparing estimated price with listed ask price for each convertible bond. It is not necessarily the case that all customers buy the convertible bonds at the listed ask price. Big financial institutions may purchase convertible bonds at a discount compared to smaller investors. Using a mid-price ranging between the bid and ask spread, we find an average misprice of 1.01 % compared to the initial price deviation of 1.12 %. Our findings may prove that it can be of interest for arbitrage investors to pursue mispricing in the US convertible bond market.

6. Conclusion

We present a simulation based approach to price convertible bonds where we find an average overpricing of 1.12 %. Price deviations seem related to coupon rate and credit ratings. The presented results are in line with recent literature such as Ammann et al. (2008), but in contrast with previous studies where other pricing approaches are applied. There is no clear consensus on one pricing model being far superior to others. However, the simulation based model is suitable to account for the dynamic state variables and features of a convertible bond (Ammann et al., 2008).

We investigate a relatively large data sample compared to previous studies. However, we exclude convertible bonds with additional call and put features from our data sample. For a more wide-ranging study, it would be natural to extend the pricing model by including these features. The investigated mispricing is conducted for one day only in this paper. It may be of interest to extend the investigated period, as done in some previous papers.

There are several model inputs that can be discussed. For instance, the implementation of credit risk is vital as minor adjustments impact the price. There are some disadvantages of implementing constant credit spreads, as we do in this paper. Studies such as Kind and Wilde (2005) model credit risk by finding a company-specific default probability and issue-specific recovery rate. These can vary over time, or be held constant as is done in Kind and Wilde (2005). This is a different approach to apply credit risk, and can be executed to investigate whether it influence prices significantly. Volatility of the underlying stock is another input which is implemented different in various studies. Our Monte Carlo simulation bases future volatility on historical volatility, and we implement a constant volatility over the entire simulation period. Studies such as Ammann et al. (2008) apply GARCH-type specifications to account for the stock volatility clustering. There is no consensus on how stock volatility should be applied when simulating stock prices, yet it may be of interest to examine how different approaches affect the result.

Finally, a potential extension to the literature is to examine additional liquid markets such as the French, UK and Japanese. It is of interest to test if the mispricing patterns we find in this paper are present in different markets, but we leave this to be investigated by future studies.

References

Agarwal, V., Fung, W. H., Loon, Y. C. & Naik, N. Y. (2011). Risk and return in convertible arbitrage: Evidence from the convertible bond market. *Journal of Empirical Finance,* 18(2), 175-194.

Alexander, C. (2008). Market risk analysis III: Pricing, hedging and trading financial instruments. West Sussex, England, John Wiley & Sons Ltd.

Ammann, M., Kind, & A., Wilde, C. (2003). Are convertible bonds underpriced? An analysis of the French market. *Journal of Banking & Finance,* 27(4), 635-653.

Ammann, M., Kind, & A., Wilde, C. (2008). Simulation-based pricing of convertible bonds. *Journal of Empirical Finance*, 15, 310-331.

Andersen, L. (2000). A simple approach to the pricing of Bermudan swaptions in the multifactor LIBOR market models. *Journal of Computational Finance,* 3(2), 5-32.

Andersen, L., & Buffum, D. (2004). Calibration and implementation of convertible bond models. *Journal of Computational Finance*, 7(2), 1-34.

Asness, C. S., Berger, A., & Palazzolo, C. (2009). The limits of convertible bond arbitrage: Evidence from the recent crash. In L. B. Siegel (ed.). *Insights into the Global Financial Crisis* (pp. 110-123). Charlottesville: The Research Foundation of CFA Institute.

Avramidis, A., & Hyden, P. (1999). Efficiency improvements for pricing American options with stochastic mesh. *Proceedings of the Winter Simulation Conference, New York: ACM Press*.

Ayache, E., Forsyth, P. A., & Vetzal, K. R. (2003). Valuation of convertible bonds with credit risk. *Journal of Derivatives*. 11(1), 9-29.

Bardhan, I., Bergier, A., Derman, E., Dosemblet, C., Kani, I., & Karasinki, P. (1993). Valuing convertible bonds as derivatives. *Goldman Sachs Quantitative Strategies Research Notes*, July 1993.

Barone-Adesi, G., Bermudez, A., & Hatgioannides, J. (2003). Two-factor convertible bond valuation using the method of characteristics/finite elements. *Journal of Economics Dynamics and Control*. 27, 1801-1831.

Barraquand, J., & Martineau, D. (1995). Numerical valuation of high dimensional multivariate American securities. *Journal of Financial and Quantitative Analysis,* 30, 383-405.

Batten, J. A., Khaw, K. L. & Young, M. R. (2014). Convertible bond pricing models. *Journal of Economic Surveys,* 28, 775–803.

Brennan, M. J. & Schwartz, E. S. (1977). Convertible Bonds: Valuation and optimal strategies for call and conversion. *The Journal of Finance*, 32, 1699-1715.

Brennan, M. J. & Schwartz, E. S. (1980). Analyzing Convertible Bonds. *Journal of Financial and Quantitative Analysis*, 15, 907-929.

Broadie, M., & Cao, M. (2003). Improved Monte Carlo methods for tighter price bounds of American options. *Working Paper, Colombia University*.

Broadie, M., & Glasserman, P. (1997). Pricing American-style securities using simulation. *Journal of Economic Dynamics and Control*, 21, 1323-1352.

Broadie, M., & Glasserman, P. (2004). A stochastic mesh method for pricing high-dimensional American options. *Journal of Computational Science,* 7, 35-72.

Broadie, M., Glasserman, P., & Jain, G. (1997). Enhanced Monte Carlo estimates for American option prices. *The Journal of Derivatives*, 5, 25-44.

Broadie, M., Glasserman, P., & Ha, Z. (2000). Pricing American options by simulation using a stochastic mesh with optimized weights. In: Uryasec, S.P. (Ed.). *Probabilistic Constrained Optimization, Methodology and Applications*, pp. 32-50.

Bossaerts, P. (1989). Simulation estimators of optimal early exercise. *Working paper, Carnegie Mellon University*.

Buchan, M.J. (1997). Convertible bond pricing: Theory and evidence. *Dissertation, Harvard University, unpublished*.

Buchan, M.J. (1998). The pricing of convertible bonds with stochastic term structures and corporate default risk. *Working paper, Amos Tuck School of Business, Dartmouth College*.

Carayannopoulos, P. (1996). Valuing convertible bonds under the assumption of stochastic interest rates: An empirical investigation. *Quarterly Journal of Business and Economics*, 35(3): 17-31.

Carayannopoulos, & P., Kalimipalli, M. (2003). Convertible bond prices and inherent biases. *Journal of Fixed Income*, 13, 64-73.

Carrière, J.F. (1996). Valuation of early-exercise price of options using simulations and nonparametric regression. *Insurance Mathematics and Economics*, 19, 19-30.

Choi, D., Getmansky, M. & Tookes, H. (2009). Convertible bond arbitrage, liquidity externalities, and stock prices. *Journal of Financial Economics,* 91(2), 227-251.

Chordia, T., Roll, R. & Subrahmany, A. (2002). Order imbalance, liquidity, and market returns. *Journal of Financial Economics,* 65(1), 111-130.

Clément, E., Lamberton, D., & Protter, P. (2002). An analysis of a least square regression method for American option pricing. *Finance and Stochastics*, 6, 449-471.

Clewlow, Les., & Hodges, Stewart. (1997). Optimal delta-hedging under transactions costs. *Journal of Economic Dynamics and Control,* 21(8-9), 1353-1376.

Davis, M., & Lishka, F. (1999). Convertible bonds with market risk and credit risk. *Working paper, Tokyo-Mitsubishi International*.

Finnerty, J.D. (2015). Valuing convertible bonds and the option to exchange bonds for stock. *Journal of Corporate Finance,* 31, 91-115

Fischer, B. & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy 81,* 3, 637-654.

Fu, M.C., Laprise, S.B,, Madan, D.B., Su, Y., & Wu, R. (2001). Pricing American options: A comparison of Monte Carlo simulation approaches. *Journal of Computational Finance,* 4, 39-88.

García, D. (2003). Convergence and biases of Monte Carlo estimates of American option pricing using a parametric exercise rule. *Journal of Economic Dynamics and Control,* 26, 1855-1879.

Grant, D., Vora, G., & Weeks, D. (1997). Path-dependent options: extending the Monte Carlo simulation approach. *Management Science,* 43, 1589-1602.

Haugh, M., & Kogan L. (2004). Pricing American options: a duality approach. *Operations Research*, 52(2), 258-271.

Hull, J.C. (2012). *Option, futures, and other derivatives (Eight Edition)*. Pearson Education Limited.

Hung, M. W., & Wang, J. Y. (2002). Pricing convertible bonds subject to default risk. *Journal of Derivatives*, 10(2), 75-87.

Ingersoll, J. E. Jr. (1977). A contingent-claims valuation of convertible securities. *Journal of Financial Economics*, 3, 289-321.

Jarrow, R.A. & Turnbull, S.M. (1995). Pricing derivatives on financial securities subject to credit risk. *The Journal of Finance,* 50, 53-85.

Kind, A., & Wilde, C. (2005). Pricing convertible Bonds with Monte Carlo simulation. Working Paper. Available at SSRN: https://ssrn.com/abstract=676507 or http://dx.doi.org/10.2139/ssrn.676507.

King, R. (1986). Convertible bond valuation: An empirical test. *Journal of Financial Research*, 9(1), 53-69.

Li, B., & Zhang, G. (1996). Hurdles removed. *Working paper, Merrill Lynch*.

Loncarski, I., ter Horst, J. & Veld, C. (2009). The rise and demise of the convertible arbitrage strategy. *Financial Analysts Journal*, 65(5), 35-50

Longstaff, F.A., & Schwartz, E.S. (2001). Valuing American options by simulation: simple-squares approach. *Review of Financial studies,* 14(1), 113-147.

Margrabe, W. (1978). The value of an option to exchange one asset for another. *The journal of Finance,* 33(1), 177-183.

Merton, R. (1973). Theory of rational option pricing. *The Bell Journal of Economics and Management Science,* 4(1), 141-183.

Merton, R. (1974). On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance,* 29, 449-470.

McConnel, J. J. & Schwartz, E. S. (1986). LYON Taming. *The Journal of Finance*, 41, 561-576.

Reymar, S., & Zwecher, M. (1997). Monte Carlo valuation of American call options on the maximum of several stocks. *Journal of Derivatives*, 5, 7-20.

Rogers, L.C.G. (2002). Monte Carlo valuation of American options. *Mathematical Finance*, 12(3), 271-289.

Takahashi, A., Kobayashi, T., & Nakagawa, N. (2001). Pricing convertible bonds with default risk. *Journal of Fixed Income*, 11, 20-29.

Tilley, J.A. (1993). Valuing American options in a path simulation model. *Transactions of the Society of Actuaries*, 45, 83-104.

Tsitsiklis, J., & Van Roy, B. (1999). Optimal stopping of Markov processes: Hibert space theory, approximation algorithms, and an application to pricing high dimensional financial derivatives. *IEEE Transactions on Automatic Control*, 44, 1840-1851.

Tsiveriotis, K., & Fernandes, C. (1998). Valuing convertible bonds with credit risk. *The Journal of Fixed Income,* 8(3), 95-102.

Xiao, Tim. (2014). A simple and precise method for pricing convertible bond with credit risk. *Journal of Derivatives and Hedge Funds*, Forthcoming. Available at SSRN: https://ssrn.com/abstract=2400101.