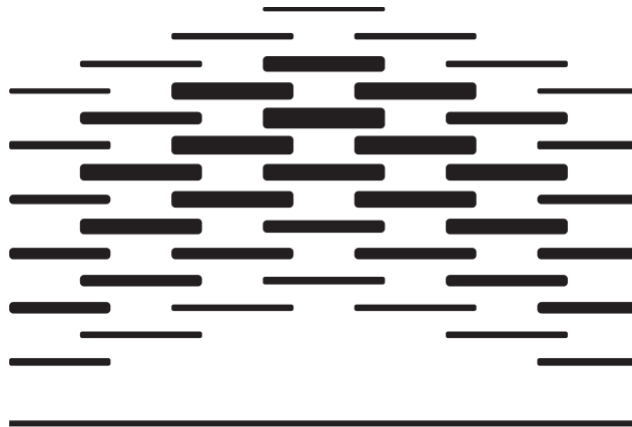


Teaching for Robust Understanding.

Students' accounts of learning mathematics in
problem-solving classrooms.



**OSLO AND AKERSHUS
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OF APPLIED SCIENCES**

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Abstract

The aim of this thesis is to explore connections between a problem-solving approach to teaching mathematics and student learning. The student learning has not been based on test results, but on measures of activity, knowledge and skills. Student accounts from problem-solving classrooms were used to measure this.

One teacher's approach to using problem-solving activities when teaching mathematics has been used as a base for the study. Using qualitative interviews, data was collected from students in the 7th and 10th grade classes at two different schools where he teaches. These interviews were analyzed through a framework designed to identify characteristics of *powerful mathematics classrooms*.

The theoretical perspective of this thesis connects research on knowledge types and knowledge quality to theory on what understanding is needed for- and obtained through mathematical problem solving. It is argued that there is a clear connection between the ability to solve problems, and the capability of connecting mathematical concepts, applying strategies and reasoning abilities.

A qualitative approach was taken in this study, conducting interviews to collect data. Although many lessons were observed, collecting data has not been done through observation. In order to acquire the students' perceptions on participating in problem-solving classes, using interviews were chosen as the most suitable method.

Concluding the thesis, there were several findings. First, it concluded that the students in these classes learned through a sociocultural approach to learning. Working in groups and engaging in productive struggles together with peers were connected to Vygotsky's ideas of *Zone of Proximal Development* and *scaffolding*. Collaborative work with problem solving helped the students engage with challenging tasks. Second, the students showed good abilities in applying strategies when solving problems. Although the students did not express that they had learned explicit strategies, they interacted with the tasks in ways that are connected with effective problem solving. Third, there seemed to be a correlation between the students who regularly worked with problem-solving tasks and the quality of knowledge they possessed. The interviewed students were all in possession of a variety of knowledge

and skills which they were able to connect and reason with through the problem-solving tasks, making the case that the students in these classes had achieved deep learning. Last, there are implications that problem solving should be a means towards mathematical competency, and not the end result of learning mathematics.

Acknowledgements

Finishing this thesis marks the end of what was supposed to be a one-year break from my job as a teacher in Bergen. It started out as one year, was extended for another year, before I ultimately handed in my letter of resignation this April. I am sorry to leave Nattland skole, having worked there for five years before moving to Oslo. Resigning was definitely not an easy decision to make. Thanks to my former colleagues for scaffolding me as my teaching developed and the all good times we spent together. My experiences with you will last a lifetime. A special thanks to Gunnhild Husebø Svendsen who taught me how to set high standards in the classroom, while still caring for every student. You will always be an inspiration to me.

Two teachers and five students were involved in the process of collecting data for my study. I appreciate that you were interested in helping me, and I am grateful for all the knowledge I gained through our conversations. Thank you for participating!

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Going into this study, I thought that I had understood a lot of what good mathematics teaching was. Now, after two years of studying mathematics teaching, I have realized that my knowledge is barely a tiny tip of an enormous iceberg.

Thanks to all of you!

Gunnar Voigt Nesbø

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List of abbreviations:

The following abbreviations are used in the thesis:

I	Interviewer
R1-5	Respondent number 1-5
RQ1-3	Research Questions 1-3
TRU	Teaching for Robust Understanding
TRU OG	Teaching for Robust Understanding Observation Guide
ZPD	Zone of Proximal Development

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1 Introduction

1.1 Background and motivation

This thesis is a result of enrolling in the study *Master of Educational Sciences for Basic Education: Specialization in Mathematics*. Having spent five years working as a teacher at the primary and upper secondary levels in Bergen, I was eager to expand my knowledge within mathematics teaching.

When teaching mathematics, I was constantly asking myself whether my teaching was the best I could offer my students. Did my teaching create critical mathematical thinkers; students with a solid and connected mathematical knowledge that would give them good opportunities for further studies and contributions in society?

I have always been curious to how I could develop and become better at the things I do. As a teacher in mathematics I am interested in the differences between teachers, teaching practices and student learning outcomes. Why do some teachers constantly deliver great results (and some teachers, unfortunately, regularly deliver poor results) compared to other teachers, even at the same school?

The first year of this study provided much insight on different types of knowledge for teachers and on the complexity of teaching mathematics. It was definitely both important and educational, making me want to dig even deeper for my master thesis. From the very start, I intended to find a topic for my thesis that would prove to be meaningful and relevant for the rest of my teaching career, and the aforementioned questions regarding quality of teaching once again emerged.

During my time teaching, I had heard of a teacher whose students on a consistent basis delivered exceptional results on 10th grade exams in mathematics (henceforth called upon by the pseudonym *Isaac Peterson*). Assuming that these results were not due to factors beyond mathematics (for example more mathematics lessons per week or sociocultural advantages), I figured that it would be meaningful to gain an understanding of how learning took place in his classrooms. I already knew that Peterson had a teaching-approach that evolved around

problem-solving tasks, and that one of the schools where he taught had a somewhat untraditional curriculum design (outlined in section 1.4). Aside from this, I had very little knowledge of Isaac Peterson, his teaching, or his students' learning before beginning my research.

1.2 Context of the study

The research reported in this study was conducted at two schools, one elementary (henceforth called Westwood) and one upper secondary (henceforth called Lakeside) in a large city in Norway. Isaac Peterson primarily works at Lakeside, and has one additional mathematics class at Westwood once per week. After meeting with and observing Peterson, the decision of conducting the study at two different schools was made for several reasons listed and argued for in Chapter 3.

At Westwood, the students in Peterson's class consisted of approximately 1/4 of the school's 7th graders. At the beginning of the 6th grade, the students at Westwood were given the choice of joining his class once per week, or to solely participate in regular mathematics classes. Westwood organized the classes so that Peterson's mathematics classes were run parallel to the other mathematics classes at the 7th grade. This way, Peterson's classes were considered an option to the regular mathematics classes that the students participated in. Since these classes only were once per week, the students joined their regular classes the remaining mathematics lessons over the course of a week.

Amongst the classes Peterson teach at Lakeside, he teaches two classes at the 10th grade once per week. These two classes consisted of approximately 50-60 students, where the students had abilities ranging from decent to great. These lessons replaced one of their weekly mathematics classes, and Peterson used these lessons primarily for problem-solving activities. The remaining mathematics lessons were taught by their regular teachers.

1.3 Research questions

As the title of my thesis reveals, problem solving is a key construct in this master thesis. Many researchers (Kilpatrick, Swafford, & Findell, 2001; Lester Jr, 2013; Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016; Ludvigsen, 2015; Putnam, 1987; Schoenfeld, 1985, 2013, 2014; Stacey, 2005, 2007; Wake, Swan, & Foster, 2016) argue for the important role of problem solving in mathematics classrooms. In their work *Adding it up: Helping Children Learn Mathematics*, Kilpatrick et al. (2001) describe the concerns that students in the American school system do not develop adequate knowledge and skills needed to apply the mathematics they have learned. Using the term *Mathematical Proficiency*, they look for a competency that enable students to become good mathematical thinkers. It is claimed that problem solving should be the site in which all of the strands of mathematics proficiency converge (Figure 1). It should provide opportunities for students to weave together the strands of proficiency and for teachers to assess students' performance on all of the strands (Kilpatrick et al., 2001. p. 421).

As Kilpatrick et al. (2001) claim, problem solving should be a main part of mathematics teaching in order to ensure students' mathematical competency. However, the teaching practices, and the learning that emerges from problem-solving classrooms is a field of research that has not received a lot of attention (Boaler, 2003; Lester Jr, 2013; Schoenfeld, 2014). Having the topic of problem solving as a key construct, my aim is to investigate students' interaction, participation and learning in classrooms that focus on problem solving, leading to a total of three research questions for this thesis.

Gaining insight on how students engage and participate in these classes is important in order to get an impression of students' activity and the effectiveness of the teaching, making me ask:

1. In mathematics classrooms where problem solving activities constitute the main part of teaching, how do the students engage? (RQ1)

With problem solving tasks at hand, investigating how students work with these tasks will provide understanding of the thoughts and strategies the students use, and directing the next research question to:

2. How do the students in these classes interact with problem solving tasks? (RQ2)

Learning outcomes can be represented in a number of ways. Though results from 10th grade exams were mentioned earlier, testing is not necessarily the only parameter of examining students' knowledge. Desired aims from mathematics teaching is that the students become critical thinkers who are independent, skillful and able to see connections in mathematics (Kilpatrick et al., 2001; Liljedahl et al., 2016; Ludvigsen, 2015; OECD, 2017; Stacey, 2005, 2007). These abilities are closely related to theory on deep learning (Pellegrino et al., 2012), ultimately leading me to the last research question:

3. To what extent does problem solving activities promote deep learning which supports development of powerful mathematical competency? (RQ3)

1.4 Lakeside's curriculum design

As mentioned, Peterson primarily works as a mathematics teacher at Lakeside. A few years ago, Lakeside implemented a new curriculum design intended to enhance students' learning in mathematics. In contrast to Bruner's spiral approach to curriculums, where topics are repeated from schoolyear to schoolyear and in this way increase in depth and difficulty for each year (Fried & Amit, 2005), they sought to create a curriculum where each topic was taught extensively and in-depth before moving on to the next topic(s). By strategically placing the mathematical topics in a specific order, each new topic would connect to previously attained knowledge. Ensuring that every topic was taught over a minimum of one semester, students would be given the opportunity to get much repetition and learn topics in-depth before moving on. In this way, Lakeside's idea was that five intertwining components; topics, concepts, methods, beliefs and procedures would constantly interconnect, ultimately leading to a connected and deep mathematical knowledge in their students. This information was given through an interview with another teacher in the mathematics section at Lakeside.

As this curriculum design is aimed for students at the upper-secondary level, it has not been developed for the 7th-grade students at Westwood. It is fair, however, to assume that the general ideas of this curriculum design is embedded in Peterson's teaching at Westwood as well. The observed lessons at Westwood focused on problem solving, but also required

Peterson to go through principles within the different mathematical topics. When the students were introduced to a mathematical concept, they were immediately given problem-solving tasks that highlighted these concepts.

The idea of intertwining these five components is founded in the theory on acquisition of *mathematical proficiency* (Kilpatrick et al., 2001). This term is used to capture the aspects necessary for anyone in order to become successful in learning mathematics (Kilpatrick et al., 2001; Ludvigsen, 2015). The important part of the five components is that they are both intertwined and interdependent when it comes to building mathematical proficiency (Figure 1). Cognitive research has shown that competence relies on mental representations and in what ways of which a topic is connected and structured (Kilpatrick et al., 2001; Mayer, 2011; Pellegrino et al., 2012). The idea that the five components are interwoven reflects this importance of connectedness. This ability of connecting knowledge is necessary in order to develop deep mathematical competency, which again can lead to effective problem solving (Kilpatrick et al., 2001).

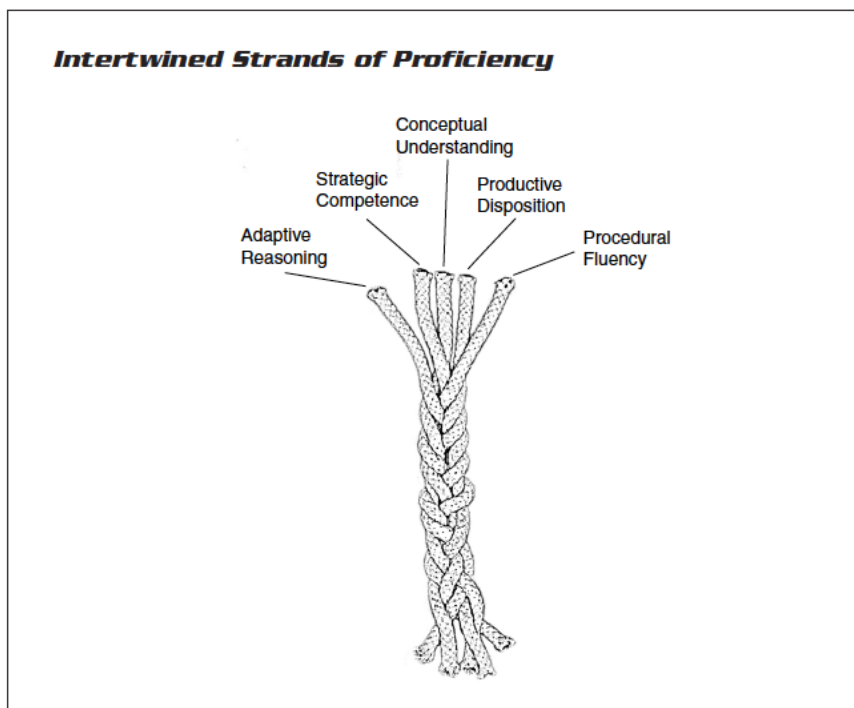


Figure 1. Five intertwined strands of mathematical proficiency.

Reprinted from "Adding it up: Helping children learn mathematics," by J. Kilpatrick, J. Swafford, and B. Findell (Eds.), p. 117.

The importance of spending sustained periods of time with each topic is also something Kilpatrick et al. (2001) mentions as an important factor in order to gain mathematical

proficiency. Students need time and practice to fully grasp new knowledge, and also to do this in a variety of ways; procedural and conceptual work, problem solving and developing essential skills (Kilpatrick et al., 2001; Ludvigsen, 2015; Putnam, 1987; Schoenfeld, 2014). This reflects the ideas implemented in Lakeside's curriculum design, as they spend a significant amount of time on every topic to ensure deep understanding. In that way, this curriculum design distances itself from the frequently applied spiral approach and concentrates on extensive teaching topic by topic, rather than learning more and more within each topic every year. Lakeside's practice of spending longer periods with each topic is consistent with what is called the *power law of practice*, that learning requires a lot of repetition over time (Newell, 1990; Rosenbloom, 1987). Developing knowledge over time can lead to skills becoming automated and fluent. As the skills are well-represented in a learner's long-term memory it will free capacity for the working memory, opening up possibilities to focus on other problems (Pellegrino et al., 2012; Putnam, 1987). To sum up, the idea is that the students at Lakeside will gain learning that gets represented in their long-term memory, which can easily be retrieved when this knowledge and these skills are needed in work with other mathematical topics.

1.4.1 Flipped classroom

An important aspect to include when discussing Lakeside's curriculum design is that the mathematics teachers should strive to avoid using large amounts of time in the classroom teaching. Instead, they have a large online database of instructional videos within all relevant mathematical topics that the students watch at home as homework, a method called *flipped classroom*. With flipped classrooms, students engage in preliminary learning before lessons to prepare for learning activities that will happen at school (Reidsema, 2017). Each video contains tasks that are linked to the content taught in the video, ensuring that the students have the opportunity to solve tasks immediately after they have watched a video.

Peterson used the flipped classroom method intensively both as homework and in class at both Westwood and Lakeside. This way, the students were active learners and problem solvers during class and he had more time to give feedback to each individual student. This does not, however, mean that he did not teach during classes: Problems arise and students

ask questions. But for the main part, the acquisition of new knowledge happened through the use of the flipped classroom method.

1.5 Outline of the thesis

This thesis is divided into five chapters. As already revealed, the current chapter shares background information, research questions and important aspects regarding the teaching practices of the teacher and schools selected. After this introduction, I will in Chapter 2 review relevant research on knowledge, learning and problem-solving. In Chapter 3, I explain the research process, where I argue for the choice of a qualitative method in this study. Chapter 4 contains an analysis of the collected data through a framework created to recognize characteristics of good teaching. Last, in Chapter 5, I will discuss the findings from Chapter 4, connecting it to the literature reviewed in Chapter 2. I will conclude Chapter 5 with implications of problem solving as a means approach in mathematics teaching, connecting it to aspects of deeper learning.

2 Literature review

In this literature review, I will examine research which is relevant for this thesis. The research questions state a position towards learning in classrooms that focus on problem solving. However, when researching learning, I also find it important to examine teaching characteristics. Describing what good teaching practices are gives a foundation to understand what leads to desired learning, and this thesis will look at these two factors in connection with each other. Considering this, parts of this literature review will cover aspects of good teaching practices in mathematics.

This chapter will be divided into three sections. First, in section 2.1, I will look at characteristics of what constitutes good mathematics teaching. Then I will give examples of what types of knowledge and understanding that students need to possess in order to become good mathematical thinkers in section 2.2. Last, in section 2.3, I will continue to look at literature within problem solving in mathematics teaching, and how this affects learning.

2.1 What is “good” mathematics teaching?

When it comes to teaching, it is essential to look for aspects of teaching that prove to be successful and which are less successful. Shulman (1986) called for what he referred to as *the missing paradigm*. This was explained as how a teacher’s knowledge transforms into something that a student will learn. With this, he asked questions to what types of teacher knowledge are required for teachers to be successful. “The professional holds knowledge, not only of how – the capacity for skilled performance – but of what and why. The teacher is not only a master of procedure but also of content and rationale, and capable of explaining why something is done” (Shulman, 1986, p. 13). What Shulman’s article does not emphasize, however, is what the characteristics of good mathematics teaching are. Obviously, there is a connection between teacher knowledge and how well he or she will teach, but how is this knowledge exercised in classrooms to create opportunities for learning? And are there specific traits that apply only to mathematics teachers?

Appropriately recognizing what is looked upon as “good mathematics teaching” is not necessarily easy, as adjectives in recent studies vary as to what has been researched. Is there

a difference between “good mathematics teaching”, as researched by Wilson, Cooney & Stinson (2005) and “effective” teaching suggested by the National Council of Teachers of Mathematics (Leinwand et al., 2014)? What about “teaching in powerful classrooms” (Schoenfeld, 2014), or “teaching for mathematical proficiency” (Kilpatrick et al., 2001)? One would assume that mathematics teaching resulting in high levels of student learning, where students are enabled to become good mathematical thinkers and given opportunities to use mathematics in a variety of ways, is what the goal of mathematics teaching is. The question remains however, to what type of teaching and learning that creates good mathematical thinkers. Using the term *Profound Understanding of Fundamental Mathematics*, Liping Ma (2010) gives examples of knowledge that successful Chinese teachers possess and how this affects student learning. This knowledge consists of four properties: Connectedness (a teacher’s intention of making connections between different mathematical concepts and procedures), multiple perspectives (the ability to use different approaches to the same topic to ensure flexibility among their students), basic ideas (possessing powerful basic knowledge of topics) and longitudinal coherence (understanding and connection of previous and future mathematical curriculum). These properties all point in the direction that teachers’ and students’ understanding should be comprehensive, both in depth and breadth, and should focus on the ability to interconnect topics, concepts and procedures (Kilpatrick et al., 2001; Ma, 2010; Pellegrino et al., 2012; Putnam, 1987). As a part of my work, I have looked at some teacher characteristics when it comes to learning mathematics and Ma’s (2010) notion of profound understanding captures many of the aspects which will be analyzed in section 4.2.

Within the discourse of good mathematics teaching, another frequently used term is *mathematical problem solving* (Halmos, 1980; Kilpatrick et al., 2001; Lester Jr, 2013; Liljedahl et al., 2016; Putnam, 1987; Schoenfeld, 1985, 2013; Skemp, 1976). There seems to be a consensus within mathematics research that developing good problem-solving abilities is an essential purpose of teaching, and this is closely related with good mathematical competence (de Corte, Greer, & Verschaffel, 2000; Halmos, 1980; Ludvigsen, 2015; Schoenfeld, 1985, 2013; Stacey, 2005, 2007). However, the question of whether mathematics should be taught in order for students to become good problem solvers (problem solving abilities being the end result) or problem solving should be taught to promote mathematics learning (a means for another goal) is an issue within mathematics education (Lester Jr, 2013). The relationship between

good mathematics teaching and a problem-solving approach to mathematics teaching seems to be important as many national curriculums highlight problem solving as an important part of students' learning (Stacey, 2005). However, research on *teaching* problem solving still lacks focus on teachers' behaviors, teacher-student and student-student relations as well as classroom atmospheres that evolve in classrooms where problem solving is taught (Lester Jr, 2013). As Lester claims in his article on problem solving: "(...)research is needed that focuses on the factors that influence student learning" (Lester Jr, 2013, p. 246). In this thesis, the perspective is teaching mathematics through the use of problem solving. Therefore, my research will be based in the thoughts of using problem solving as a means to develop mathematical competency.

2.2 Knowledge types and knowledge quality

A widely discussed topic within mathematics research, is the type of knowledge that students acquire, and should acquire, through mathematics teaching (Hiebert, 1986; Kilpatrick et al., 2001; Leinwand et al., 2014; Ma, 2010; Skemp, 1976; Stacey, 2007). In his 1976 article "Relational Understanding and Instrumental Understanding", Richard Skemp addresses the superficial understanding that many students have in mathematics, and what benefits come from teaching with a relational perspective on mathematics (Skemp, 1976). The questions that arise from Skemp's work are similar to what Hiebert (1986), Star (2000, 2005, 2007) and Star&Stylianides (2013) discussed later on in more recent research, drawing lines between procedural or conceptual knowledge types with deep or superficial knowledge qualities. Furthermore, an aspect within research on knowledge is the important distinction between knowledge type and knowledge quality. Within knowledge theory this distinction is difficult, because research has entangled these terms (Star & Stylianides, 2013; Star, 2000, 2005, 2007). The difference between type and quality is important to specify, as this has implications for the way teachers and researchers will understand educational studies (Star, 2005).

This section will be divided into subsections where I in subsection 2.2.1 and 2.2.2 will elaborate on research of knowledge quality and knowledge types respectively. In subsection 2.2.3 I will look at the distinction between the quality and type of knowledge and in what way they can

be understood in connection to one another, before I will discuss sociocultural perspectives to learning in subsection 2.2.4.

2.2.1 The quality of knowledge

In the 2012 report “Education for Life and Work” on what competencies will be important in the 21st century, *deeper learning* is considered as a key aspect of what is needed to be emphasized in schools in the future (Pellegrino et al., 2012). The concept of deeper learning is complex, and ranges from different perspectives of learning and thinking, domains of competency, and different types of knowledge a student is able to acquire: “Deeper learning occurs when the learner is able to transfer what was learned to new situations” (Pellegrino et al., 2012, p. 99). *Transfer* is the idea that knowing something within a domain, strengthens the learning of something else in that same domain if the two domains are somewhat relevant to each other (Pellegrino et al., 2012).

Transferable knowledge and skills are “(...)competencies that enable learners to transfer what they have learned to new situations and new problems” (Pellegrino et al., p. 70). In order to acquire transferable knowledge, the required knowledge needs to be based on a fundamental understanding of the principles of the knowledge, and it needs to be well-organized mentally (Pellegrino et al., 2012). These aspects of transferable knowledge can be seen in relation to the aforementioned properties Ma (2010) claimed were required in order to possess profound understanding of fundamental mathematics.

Given the fact that deeper learning relies on transferring current knowledge to new and unknown situations, deeper learning is strictly not needed if the end goal for the students is to solve and complete tasks that are identical to those encountered when learning. On the contrary, if the aim is for students to succeed in new and challenging situations that have a higher cognitive demand and the solutions are not obvious, a focus on deeper learning is important. Learning that focuses on deeper understanding and how to solve problems leads to this transferable knowledge (Pellegrino et al., 2012).

Referring to Mayer (2011), Pellegrino et al. (2012) give examples of five interconnected types of knowledge that are essential to develop deeper knowledge: Factual knowledge, concepts, procedures, strategies and beliefs about one’s learning. The important part of these types of

knowledge are that they are interconnected and dependent on each other to achieve deeper learning. Factual knowledge, consisting of isolated bits of information, is not enough in itself to develop a solid understanding within a subject. However, if these facts are integrated and seen in relation to each other, there is a higher chance that the knowledge can lead to transfer. These five types of knowledge mirror Kilpatrick et al.'s (2001) framework of mathematical proficiency used at Lakeside covered in section 1.4. This parallel connects the development of solid mathematical understanding with the idea of deeper knowledge, exemplifying the way their curriculum is designed to develop profound mathematical thinkers through teaching.

2.2.2 Teaching concepts or procedures? Building relational or instrumental understanding?

In “Fremtidens skole” (2015), Ludvigsen presented what they thought would be important to focus on in the Norwegian school system in the future. One of the main findings was that deeper learning within all school subjects would enhance students’ overall competency. Ludvigsen uses the term *taxonomy* in the discussion of deeper learning, and parallels are drawn between taxonomy and to what degree someone has learned something deep. The opposite of deeper learning; “surface learning”, requires a low level of taxonomy, as deeper learning requires a high level of taxonomy where for example connecting, application and analyzing are important elements. The development of deep knowledge is a necessary condition in order to re-structure and use current knowledge in new contexts (Ludvigsen, 2015).

Skemp (1976) explained what he means is a *faux amis*¹ in mathematics teaching. He argued that the word *understanding* is used with different meanings, and that the way teachers look at understanding plays an essential role in the mathematical knowledge acquired by students. He states that there are two different types of understanding; *instrumental* and *relational* understanding (Skemp, 1976). Instrumental understanding is the comprehension connected to rules, straight-forward calculations, and similar types of processes that do not require larger extents of reflection or explanations. Instrumental understanding was referred to as “rules without reasons”, expressing that there is some sort of emptiness to this form of

¹ Faux amis: French term used to describe words that are similar, but have different meanings.

understanding (Skemp, 1976, p. 20). Relational understanding, on the other hand, is connected to reasoning, explanations and the “why”-aspect of subject conception, characterizing this form of understanding as “knowing both what to do and why” (Skemp, 1976, p. 20).

These conceptions of understanding led to much research on the forms of knowledge acquired in schools, further developing the categories laid out by Skemp. Examples being studies by Hiebert (1986), Star & Stylianides (2013) and Star (2000, 2005, 2007). The initial terms *instrumental* and *relational* understanding were altered, taking the perspective of knowledge type as opposed to understanding, and the expressions procedural and conceptual knowledge emerged (Hiebert, 1986). This terminology is still used in educational research to refer to the type of knowledge that emerges from mathematics classrooms. Procedural knowledge is characterized by the knowledge of rules and the sequential actions necessary to complete tasks or reaching correct answers, closely linked to the features of instrumental understanding. Conceptual knowledge is considered to be knowledge of concepts; knowledge which is rich in relationships, leading to a deep understanding (Hiebert, 1986; Star, 2000, 2005, 2007; Star & Stylianides, 2013).

Although research supports teaching that leads to conceptual knowledge, the skills required to perform accurate and effective procedures should not be forgotten, and the acquisition of *procedural fluency* is considered an important part of developing mathematical proficiency (Kilpatrick et al., 2001). The abilities of performing accurate computations, for example, are needed to support conceptual understanding within numbers. The terms procedural and conceptual knowledge are interconnected and are related to the development of each type of knowledge (Kilpatrick et al., 2001). Proper understanding makes skill learning simpler, and within some concepts a procedural approach can promote understanding (Kilpatrick et al., 2001). The problem occurs, however, when procedures are not seen in relations with each other (Kilpatrick et al., 2001; Ludvigsen, 2015; Ma, 2010). Students who learn skills as isolated pieces of information, may experience difficulties when learning similar topics because they have no possibility of seeing connections to previously learned knowledge. Learning only procedural skills, and thereby being deprived of the opportunity to gain a connected understanding of mathematics, will limit students’ access to higher education and better jobs,

effectively condemning these students to a second-class status in society at best (Kilpatrick et al., 2001).

2.2.3 Connecting quality with knowledge types

The definitions given on procedural and conceptual knowledge has received criticism because there are no clear distinctions between the aspects of quality and types between them (Star & Stylianides, 2013; Star, 2000, 2005, 2007). Conceptual knowledge is not defined in regards to the *quality* of ones knowledge of concepts, but rather to what extent one has the ability to link and make connections within the particular knowledge: “(...)the richness of the connections inherent in such knowledge” (Star, 2005, p. 407). Star points to Medin (1989), claiming both that knowledge of a concept is not necessarily rich in relationships, and that connections within a concept can be superficial and limited. Procedural knowledge on the other hand is characterized as superficial and without the aspect of being able to make connections. There are many different kinds of procedures, and there are also different demands to quality of one’s knowledge to execute these procedures. On one side there are algorithms that guarantee a correct solution if executed properly, but on the other side there is heuristics which is an essential part of being an effective problem solver (Star, 2005). It is argued that the definitions of knowledge types are insufficient, and that there is for example need to discuss what characterizes deep procedural knowledge and superficial conceptual knowledge (Star, 2005). The perception of these terms have been discussed in literature on knowledge qualities and knowledge types (Star & Stylianides, 2013; Star, 2000, 2005, 2007). *Quality* would normally refer to the way something is known and how well someone understands something (Star & Stylianides, 2013). As mentioned, knowledge can exist in both the deep and superficial levels, and deeper knowledge is associated with the ability of solving tasks with higher cognitive demand in new and challenging situations, whereas surface-learning requires a lower level of taxonomy (Ludvigsen, 2015; Pellegrino et al., 2012). Knowledge *type*, on the other hand, implies *what* is known. Procedural knowledge should tell us something about how well a person knows procedures, and conceptual knowledge should give answers to what knowledge someone has about mathematical concepts (Star & Stylianides, 2013). A common view is that conceptual knowledge is something that is known

on a deep level, and procedural knowledge is superficial knowledge, leading to an entanglement of these two terms (Star, 2005).

Table 1 illustrates this conflict, clearly showing how the terms have become intertwined:

Knowledge type	Knowledge quality	
	Superficial	Deep
Procedural	Common usage of <i>procedural knowledge</i>	?
Conceptual	?	Common usage of <i>conceptual knowledge</i>

Table 1. Types and Qualities of Procedural and Conceptual Knowledge. Reprinted from “Reconceptualizing Procedural Knowledge,” by J.R. Star, 2005, *Journal for Research in Mathematics Education*, 36(5), p. 408.

When procedural and conceptual knowledge types and quality get tangled up, there will be difficulties for teachers to distinguish between the types and qualities associated with effective learning (Star & Stylianides, 2013). Within research, this confusion of terms will definitely make interpretations of the results difficult. In order to understand the results, clear definitions and assessments of these knowledge types are important to gain any form of knowledge from it (Star & Stylianides, 2013). With these considerations in mind, it can prove to be difficult to define what are traits of good mathematics teaching. The distinctions between what are important types and qualities of knowledge lead to important decisions made by teachers and teacher educators, causing impact in mathematics classrooms.

2.2.4 A sociocultural perspective to learning

Two elements of research within learning and thinking, a cognitive and sociocultural perspective, are worth emphasizing in the context of attaining deep knowledge. In the cognitive perspective, the focus is on the mechanisms of learning and how different types of knowledge are structured. The sociocultural perspective, on the other hand, focuses on how learning takes place when people take part in communities and learn through interaction with others (Pellegrino et al., 2012).

An important researcher within sociocultural perspectives to learning is Lev Vygotsky. Through his works (e.g. *Thinking and Speech* from 1934), Vygotsky emphasized that human activities happen in cultural surroundings, and cannot be understood as isolated events. Cognitive

structures and thought processes are created through social interactions, and development is considered the transformation from these social interactions into internal processes (Woolfolk, 2010). In short, improvement and growth are progressions from collaborative work to individual understanding. Two important terms in Vygotsky's work are *Scaffolding* and *The Zone of Proximal Development (ZPD)* (Baltzer, 2011; Hedegaard, 1993; Woolfolk, 2016). *Scaffolding* happens when adults or more capable peers assist and support learners in understanding something. This help is needed to gain a solid understanding, eventually leading to the learners being able to solve problems on their own (Baltzer, 2011). A learner's ZPD is an illustration of an area of learning where students are not able to solve a problem by themselves but might be successful with support from an adult or another more skilled peer (Baltzer, 2011; Hedegaard, 1993). Simply put, the ZPD is the difference between what a learner can do without help and what he/she can do with help (Hedegaard, 1993). These two terms are closely related as the cooperation and communication through scaffolding within a learner's ZPD is considered important aspects in order to promote understanding (Baltzer, 2011; Hedegaard, 1993; Woolfolk, 2010).

As my study focuses on problem solving, positions towards a sociocultural perspective to learning and thinking will be taken. A teacher's knowledge and decisions affect what students learn, but students play an important role in their own learning through expectations, knowledge and how they respond to the teaching. I will consider teaching and learning mathematics as a matter of the interactions between students and teacher and how they produce knowledge together. Through different contexts in mathematics classrooms these student-teacher and student-student interactions occur, and by recognizing these contexts analysis can be made on which classroom practices that promote and which do not promote mathematical proficiency (Kilpatrick et al., 2001). In this thesis I will investigate how, and to what extent, problem solving as a classroom practice promotes mathematical proficiency in students.

2.3 Problem solving

The aspects of knowledge types and quality can be related to features mathematical problem solving. Previously mentioned research emphasizes the importance of teaching approaches

that create opportunities for deeper learning (Ludvigsen, 2015; Pellegrino et al., 2012). In order for students to achieve deep knowledge, they need abilities to transfer current knowledge to new and unknown situations and the capability of transferring knowledge relies on making connections. Acquiring knowledge which is easily connected, labeled above as conceptual knowledge, is essential for learners to obtain in order to transfer knowledge (Kilpatrick et al., 2001; Ludvigsen, 2015; Pellegrino et al., 2012).

Problem solving tasks are often defined as tasks where the solution method is not known in advance by the solver (Schoenfeld, 1985; Wake, Swan & Foster, 2015), although this can be considered as a simple definition (Lester Jr, 2013). Some researchers have put critical thinking in relation to problem solving, as both critical thinking and problem solving require reasoning, analytical abilities, identifying relevant questions and using purposeful strategies to solve complex problems (Lester Jr, 2013; Ludvigsen, 2015; Schoenfeld, 1985; Stacey, 2007). Problem solving means that students need to analyze a problem and deciding how to attack it with use of expedient methods (Ludvigsen, 2015). A more thorough definition of problem solving posed by Lester (2003) gives a comprehensive explanation of the abilities needed in order to be effective at problem solving:

“Successful problem solving involves coordinating previous experiences, knowledge, familiar representations and patterns of inference, and intuition in an effort to generate new representations and related patterns of inference that resolve some tension or ambiguity (i.e., lack of meaningful representations and supporting inferential moves) that prompted the original problem-solving activity” (Lester Jr, 2013, p. 249).

Although there are many different definitions and thoughts on what mathematical problems are, there seems to be a consensus in the community of mathematicians that problem-solving skills are important to become good at mathematics (Leinwand et al., 2014; Lester Jr, 2013; Putnam, 1987; Schoenfeld, 1985; Stacey, 2005, 2007). In the Norwegian curriculum of mathematics, problem-solving abilities are emphasized as a key part of learning mathematics, because mathematical competency «(...)consists of using problem solving and modeling to

analyse and re-structuring a problem to mathematical form, solving it, and considering how valid the solution is" (The Norwegian Directorate for Education and Training, 2013). This addresses the fact that problem solving is a fundamental part of what is considered being a competent mathematics learner.

In this section, I will first, in subsection 2.3.1, look at the connection between transferable knowledge and problem solving in. The following subsection, 2.3.2, will deal with heuristics, the way one uses reasoning and methods in order to solve problems. Last, in subsection 2.3.3, classroom practices will be discussed through the lens of problem solving research, closing with Schoenfeld's framework for employing robust understanding through teaching.

2.3.1 Defining a knowledge base for problem solving

What are the important attributes of knowledge needed for mathematical problem solving? Section 2.2 looked at different types and qualities of knowledge, but is there a connection between a student's knowledge or understanding and the ability to solve problems?

The abovementioned explanations of problem solving are clearly linked with the concept of deeper learning. Problem solving gets associated with connection, critical thinking, analysis and inference, all of which being attributes of deep knowledge (Lester jr, 2013). The importance of connecting knowledge in new problems is especially important, as this is not considered an aspect of instrumental understanding (Skemp, 1976). This being said, one could go a long way in arguing that deep, conceptual understanding is a prerequisite in order to become good mathematical problem solvers: "Deeper learning requires students to use their analytical abilities to solve problems and reflect over their own learning to construct a lasting understanding" (Ludvigsen, 2015, p. 14). If the aim of (deeper) learning for the students is to acquire transferable knowledge, problem solving activities should be easier to work with as the goal for students is to use already-known mathematical content in non-routine problems (Wake, Swan & Foster, 2015).

Stacey (2005) looked for similarities and differences in curriculums when it comes to the view of problem solving. In this work, Stacey draws lines between Singapore's model of teaching mathematics and other European countries. In many curriculums (for example the UK's), the

goal is for students to achieve good mathematical knowledge, and the use of problem solving is a way to ensure this. In Singapore, however, this is turned the other way around. Stacey quotes Singapore's Ministry of Education which states that "The primary aim of the mathematics program is to enable pupils to develop their ability in mathematical problem solving (...) The attainment of this mathematical problem-solving ability is dependent on five inter-related components — Concepts, Skills, Processes, Attitudes and Metacognition." (Stacey, 2005, p. 345) These five components embedded in mathematics education in Singapore can easily be connected to Kilpatrick et al's (2001) view on how to acquire *mathematical proficiency*, through the intertwining of factors important to learning. These two ways of looking at problem-solving instruction in mathematics are consistent with Lester Jr's (2013) means- or end approaches which were mentioned in section 2.1.

There are several factors that come into play when working with mathematical problems. Building on Polya's (1945) work, Schoenfeld (1985) looked to identify traits of learners that were good at problem solving, taking a position where he accepts that solving problems depend on a learner's prior knowledge and understanding of the problem at hand. In doing so, Schoenfeld clearly defines limits to a learner's possibilities of solving problems (Liljedahl et al., 2016). Classifying this, Schoenfeld used four categories of knowledge and behavior necessary for students to become good problem solvers: Resources (possessed mathematical knowledge), heuristics (strategies and techniques), control (decision making) and belief systems (beliefs and attitude towards the subject, a problem etc). Resources is the foundation of what mathematical understanding and procedures a learner has access to, and the heuristics lay the ground to which extent the learner can use this mathematical knowledge in strategies. Control is the ability to make decisions that put the knowledge and strategies to good use. Last, a learner's view of the mathematics or his/her abilities within mathematics or a mathematical topic, affect the three first categories (Schoenfeld, 1985). Another more detailed look at required skills for problem solving is illustrated in Figure 2, and it gives an example of how problem solving is dependent on several contributing factors (Stacey, 2005). The elements in Figure 2 and Schoenfeld's four categories are similar, but there are a few points from Stacey's (2005) figure that separate from Schoenfeld's classification. Both emphasize having solid mathematical knowledge, and that they possess *strategic knowledge*, or heuristics, in order to solve problems. However, Stacey also looks to include communication

skills and capability of working with others as important parts of becoming good problem solvers. These points can be linked to the aspects of sociocultural learning mentioned in subsection 2.2.4, looking at problem solving as work involving social interactions.

In this thesis, I will focus on factors of deep mathematical knowledge. In addition, due to the connection shown through the literature reviewed, I will also discuss general reasoning abilities and heuristic strategies.

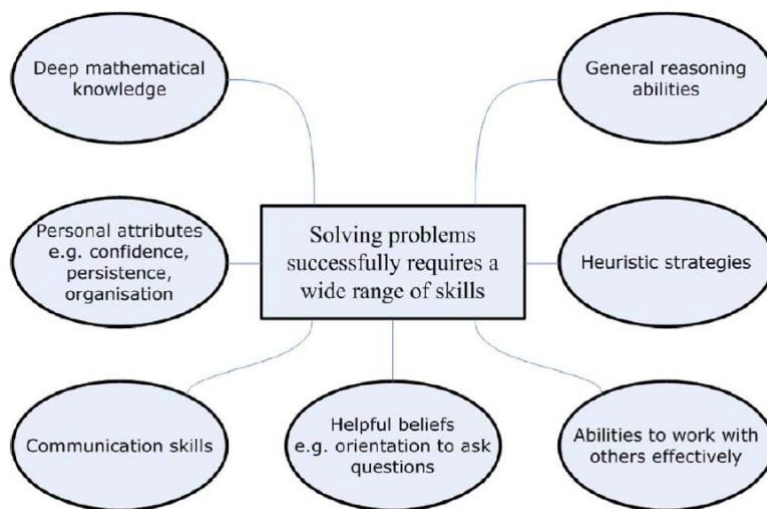


Figure 2. Factors contributing to successful problem solving. Reprinted from “The Place of Problem Solving in Contemporary Mathematics Curriculum Documents,” by K. Stacey, 2005, *Journal of Mathematical Behavior*, 24, p. 343.

An interesting question to ask is whether it is possible to achieve good problem-solving abilities through pure instrumental understanding? Would it be achievable to obtain a high degree of knowledge and behavior within Schoenfeld’s four categories if the knowledge base was founded merely in procedural knowledge? Are the factors shown in Figure 2 possible to obtain based on purely instrumental understanding? I would agree that knowing only the *how* and not the *why* could serve as a problem when developing several of the components in Figure 2, one example being general reasoning abilities. Reasoning is considered the ability to see logical relationships between concepts and situations, effectively connecting everything together. Whilst strategies for solving problems might require use of procedural knowledge in order to calculate or measure, reasoning is the component which is used to determine whether the selected procedure is applicable (Kilpatrick et al., 2001). This example supports

the claim I made above, that problem solving in mathematics relies on possession of deep knowledge.

2.3.2 Heuristics and its place in mathematical problem solving

Through his 1945-published work, *How to solve it*, George Polya tried to refresh heuristics to a new and comprehensible form. The book is considered a guide to problem-solving techniques, and has been used as a base for later work on the same topic (Schoenfeld, 1985). “Once nearly forgotten, heuristics have now become nearly synonymous with mathematical problem solving” (Schoenfeld, 1985, p. 23). But what are the essential steps one need to take when solving mathematical problems? Polya’s (2009) work from 1945 include four phases necessary for learners to master in order to become good at solving problems.

According to Polya (2009), it is first crucial that the learner understands the problem. The problem at hand should not be too difficult, nor too easy, and should be comprehensible for the learner. In order to understanding the problems, students could ask themselves questions like “What is the unknown?” or “What information is provided?”. Second, the learner should devise a plan for solving the problem. The process of coming up with a (meaningful) plan for solving a problem is considered the main achievement in finding a solution to the problem. The path to finding a plan can be a long struggle and might emerge gradually or through ideas that emerge. As teachers, Polya thinks the best approach is to procure bright ideas for the students, asking questions that might bait such thoughts. This can be seen in relation to Vygotsky’s ideas of scaffolding learners to help them develop (Baltzer, 2011; Hedegaard, 1993; Woolfolk, 2016). Third, learners must carry out the plan devised from phase two. Carrying out this plan is much simpler, as it relies mainly on patience and precision to be successful. Fourth, the learners should look back at the work they just completed in order to see if mistakes were made, if there are any improvements or further knowledge to be made. No problems are completely exhausted, and solutions could be improved and understood at a deeper level, or even solved in different ways (Polya, 2009).

Several other researchers have looked to build on Polya’s classic approach. Amongst them is Verschaffel, Dooren, Greer, & Mukhopadhyay (2010) who devised six steps that effective problem-solvers go through. These steps can be summed up as (1) understanding the

problem, (2) creating a model to derive results, (3) using the model to derive results, (4) critically evaluating the outcome of the procedures conducted, (5) critically evaluating if the mathematical outcome make sense and (6) communicating the solution of the problem. The first three steps are relatively similar in both Polya's (2009) and Verschaffel et al.'s works, as they both explain that students need to understand the problem, create a plan, and carry out this plan. Polya's fourth phase is covered in Verschaffel et al.'s fourth, fifth and sixth steps, as Verschaffel's last three steps explain a more thorough process after a plan has been carried out.

As one of my research questions directly relates to how students solve problems, I intend to use Polya's four phases when looking at students' interaction with problem-solving tasks. Although Verschaffel et al.'s six steps are more detailed, I will focus on Polya's (2009) four phases. As described, the three first steps are very similar in both approaches, and Polya's last phase captures the three last steps of Verschaffel et al. Therefore I think that Polya's approach also should be able to provide good indications of students' strategies in problem solving as his approach contains much of the same aspects whilst limiting itself to only four phases.

2.3.3 A theoretical framework for analyzing properties of good mathematics teaching

As mentioned earlier in this chapter, research identifies connected knowledge that transfers to new situations as an important part of learning (Kilpatrick et al., 2001; Ludvigsen, 2015; Pellegrino et al., 2012). As problem-solving activities are considered tasks where the learner does not know the solution immediately (Lester Jr, 2013), these claims for knowledge and problem solving as an important part of mathematics go hand in hand.

Using open-ended problem-solving tasks, as opposed to more "traditional" approaches where teacher instruction and textbook work is the main focus, has given indications that classrooms where problem solving takes place have a positive effect on mathematical competency (Boaler, 2003). Encouragement of students to represent problems, justification of answers and looking at problems from different angles support the development of profound mathematical knowledge, instead of over-scaffolding and lowering cognitive demands of tasks (Boaler, 2003).

Building on his work with problem solving and as part of the Algebra Teaching Study, Schoenfeld (together with Robert Floden) wanted to create a framework to analyze teacher performance and what results come from it (2014). “The question here is, can one identify the key aspects of powerful mathematics classrooms - classrooms that produce students who do well on tests of mathematical content and problem solving?” (Schoenfeld, 2013, p. 406). With this question, Schoenfeld defines what he believes is a “powerful mathematics classroom”, namely one which produces students that perform well when it comes to mathematical content and problem solving.

Trying to answer the questions of what classroom interactions and pedagogies which result in *robust understanding* of important mathematics, Schoenfeld and Floden created a scheme in order to analyze teacher performance and what results come from it. Robust understanding, defined by Schoenfeld (2013) as the ability of being effective at dealing with something, is in the context of mathematics a person’s capability of effectively solving complex and demanding tasks. Exploring the relationship between what is considered good classroom practices with the student outcomes that emerge from them was, according to Schoenfeld, not studied through research and needed further attention (Schoenfeld, 2013, 2014; Schoenfeld, Floden, R. E., & the Algebra Teaching Study and Mathematics Assessment Project, 2014; Schoenfeld & the Teaching for Robust Understanding Project, 2016a).

The framework, Teaching for Robust Understanding (TRU), would give implications to what extent students were given the possibility to enhance their mathematical abilities in the classroom. Five dimensions of classroom activity were identified, and to what extent these dimensions were applied affected what degree of proficient mathematical thinkers and problem solvers would emerge from those classrooms (Schoenfeld et al., 2016a). Each dimension has a rubric to scale a teacher’s ability within the dimensions and a more extensive observation guide was also developed, named the Teaching for Robust Understanding Observation Guide (TRU OG). This guide was intended to be used in planning, conducting and reflecting on observations in mathematics classes (Schoenfeld & the Teaching for Robust Understanding Project, 2016b).

Although it was created from a mathematical standpoint, the framework does not limit itself to mathematics only. The five dimensions that constitute the framework are general and could

easily be used to evaluate robust learning environments in other subjects as well. These dimensions are Content (in this case: The Mathematics); Cognitive Demand; Equitable Access to Content (in this case: Equitable Access to Mathematics); Agency, Authority and Identity; and Formative Assessment (Figure 3).

The Five Dimensions of Powerful Mathematics Classrooms				
The Mathematics	Cognitive Demand	Equitable Access to Mathematics	Agency, Ownership, and Identity	Formative Assessment
<i>The extent to which classroom activity structures provide opportunities for students to become knowledgeable, flexible, and resourceful mathematical thinkers. Discussions are focused and coherent, providing opportunities to learn mathematical ideas, techniques, and perspectives, make connections, and develop productive mathematical habits of mind.</i>	<i>The extent to which students have opportunities to grapple with and make sense of important mathematical ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with task difficulty ranging from moderate to demanding. The level of challenge should be conducive to what has been called “productive struggle.”</i>	<i>The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematical content being addressed by the class. Classrooms in which a small number of students get most of the “air time” are not equitable, no matter how rich the content: all students need to be involved in meaningful ways.</i>	<i>The extent to which students are provided opportunities to “walk the walk and talk the talk” – to contribute to conversations about mathematical ideas, to build on others’ ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and the development of positive identities as thinkers and learners.</i>	<i>The extent to which classroom activities elicit student thinking and subsequent interactions respond to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction “meets students where they are” and gives them opportunities to deepen their understandings.</i>

Figure 3. The five dimensions of powerful mathematics classrooms. Reprinted from “An Introduction to the TRU Dimensions,” by Schoenfeld, A. H., Floden, R. E., & The Algebra Teaching Study and Mathematics Assessment Project, 2014, p.2.

The mathematics covers the students’ experience of the content in mathematics classrooms. Is the mathematics experienced as isolated facts, procedures and concepts that are to be memorized and practiced, or is the mathematical content coherent, meaningful and connected? To what extent is the teacher able to give students an understanding of the subject that turns them into mathematical thinkers and gives the ability to link their existing knowledge to new learning? *Cognitive demand* asks the question: “Do students have opportunities to engage in *productive struggle*, through which they develop understandings of mathematics and the kind of perseverance in problem solving that is valued?” (Schoenfeld et al., 2014, pg 10). It is important to find the right balance between too easy and too difficult, and to make sure that students have the ability to work through problems. With the help of

scaffolding (as opposed to being told how to solve a problem), students will be given opportunities to learn, which gives them room to develop when facing difficulties (Schoenfeld et al., 2014; Schoenfeld et al., 2016a). This can be connected to Vygotsky's sociocultural perspectives to learning, where learning processes occur in interactions with adults of more skilled peers (Baltzer, 2011; Hedegaard, 1993; Woolfolk, 2016). By avoiding repetitive exercises, not giving step-by-step instructions and encouragement of productive struggle in problem solving activities, teachers help maintain appropriate levels of cognitive struggle (Schoenfeld et al., 2014; Schoenfeld et al., 2016a).

To ensure *Equitable Access to Mathematics*, it is important for teachers to encourage participation from all students in the classroom. By selecting tasks that all students can engage with, teachers can create equal opportunities for all students to take part in the learning process. It is also important to remember that all students should have the same opportunities to discuss the mathematical topics, regardless of their mathematical competency. *Agency, Ownership, and Identity* focuses on "the extent to which students have the opportunity to generate and share ideas, both in a whole class and in small group settings(...)" (Schoenfeld et al., 2016a, p. 9). In what way do students make contributions, and is there a safe environment for making such contributions? How does the teacher build on contributions from students to enrich the teaching? Last, *formative assessment* sheds light on a teacher's ability to assess a student's understanding during the learning process. Are there misunderstandings that need to be addressed, and does the teacher create classroom activities that build on this assessment? In order to keep developing students' knowledge, teachers need to meet the student's suitable abilities in order to give them opportunities to develop further (Schoenfeld et al., 2014; Schoenfeld et al., 2016).

These dimensions are fairly comprehensive and cover a lot of what goes on inside a (mathematics) classroom. The idea is that students in classrooms where these dimensions are constantly in play will become learners with a robust knowledge base, able to confront and overcome different problems and tasks in a series of ways (Schoenfeld et al., 2014; Schoenfeld et al., 2016a). Unlike many other frameworks, the TRU Framework does not say what to do or what not to do, but rather looks at learning environments where students excel, and specifies properties typical to those classrooms. Schoenfeld makes a point of seeing teaching as something complex where there are many right ways to create learning for students, and TRU

is a framework created in order to help teachers identify elements where they can develop (Schoenfeld et al., 2014, Schoenfeld et al., 2016a).

As mentioned in 2.1, teaching where students acquire broad and deep knowledge which they are able to connect, in addition to developing good reasoning abilities are indications of good mathematics teaching (Kilpatrick et al., 2001; Ludvigsen, 2015; Ma, 2010; Putnam, 1987; Stacey, 2007). The TRU Framework seems to capture these broad mathematical aspects. Within the *Mathematics* dimension, the framework emphasizes the importance of teaching the mathematical content in a way which is coherent, meaningful and connected, as opposed to memorizing isolated facts, procedures and concepts. This is consistent with Skemp's (1976) thoughts that students need to acquire relational understanding instead of instrumental understanding, and Star & Stylianides' (2013) and Star's (2000, 2005, 2007) ideas on obtaining deep levels of both procedural and conceptual knowledge. The framework also underlines the importance of giving students the ability to link existing knowledge to new learning and engaging them in the productive struggle. These aspects are connected to features of deep knowledge and sociocultural learning perspectives.

In light of the research questions of this thesis, the TRU framework appears to be a framework that could serve the purpose of identifying qualities of teaching and learning that goes on in problem-solving classrooms.

3 Methodology

In this chapter, I will discuss the process of using qualitative method in order to answer my research questions. In section 3.1, I will give an explanation of the chosen qualitative methods, and the aspects of reliability and validity that occur when conducting interviews and observation. Further, the collection and analysis of my data will be examined in sections 3.2 and 3.3. Last, the issue of ethics in qualitative methods is reviewed in section 3.4.

3.1 Qualitative methods

Gradually becoming more accepted within social sciences, qualitative approaches are recognized by a large amount of different data and a variety of analytical techniques. A primary goal of qualitative methods is to understand social occurrences, giving interpretation an important role in analysis and causing challenges in terms of ethics due to the close contact between researcher and the subjects being studied (Thagaard, 2013). Qualitative methods are flexible ways of gathering data, relying on open questions to get detailed descriptions from the subjects, as opposed to quantitative methods where answers are based on predetermined alternatives for responding (Christoffersen & Johannessen, 2012). Results from qualitative research look to emphasize things that are not measurable in quantity or frequencies and gather much information on a low number of units.

The research questions of this thesis require an interaction with the subjects being studied. Trying to answer questions linked to classroom practices and problem solving demands an approach that would give students opportunities to express themselves and elaborate on their experiences when learning. Therefore, conducting interviews were decided as the best way to gather valuable information. Although observation is another qualitative method of gathering data it has not been a chosen method in this thesis, which will be explained in subsection 3.1.2.

In this section, I will first give an introduction of what a qualitative interview is, and to what extent interviews have received recognition within research in subsection 3.1.1. Following this, a brief explanation of other qualitative methods is examined in subsection 3.1.2. In subsection 3.1.3, challenges with qualitative interviews are discussed, before I in subsection

3.1.4 state reasons for choosing qualitative interviews as my preferred method of collecting data for my thesis.

3.1.1 Qualitative interviews

The term *interview* has been used since the 17th century and means the exchange of meaning and thoughts between two people in a conversation about a subject that concerns them. Although interviews faced much resistance early on, in today's society interviews are considered useful and has influenced many social practices (Kvale, Brinkmann, Anderssen, & Rygge, 2009). Piaget and Freud, two of the most quoted psychologists in scientific literature, collected most of their empirical evidence from interviews, and in the last 50 years there has been a great expansion on methodological literature on how to conduct interviews (Kvale et al., 2009). With this expansion on literature and increased knowledge on how to carry out research through interviews, many disciplines now rely heavily on qualitative research, including psychology, sociology and education (Kvale et al., 2009).

Qualitative interviews are the most used approach when collecting qualitative data and gives the possibility of gathering detailed and insightful descriptions of a topic (Christoffersen & Johannessen, 2012; Jacobsen, 2005; Mason, 2002). Using qualitative interviews is considered suitable when researching a limited amount of people, and when the goal is to gain information of a person's ideas, thoughts and feelings (Jacobsen, 2005). As opposed to survey interviews, where the interviewer poses predetermined questions in a strict order, qualitative interviews are structured conversations with a purpose of gaining understanding. This contrasts the quantitative method, where expansion and numbers are more important than answering questions of what or why. Qualitative interviews are often considered more as a dialogue than just a questionnaire, and through interviews one intends to capture insights of a person's perceptions and produce knowledge through a social interaction between the interviewer and interviewee (Christoffersen & Johannessen, 2012; Jacobsen, 2005; Kvale et al., 2009; Thagaard, 2013). (Kvale et al., 2009) use the term *life world*, focusing on the subject's perceptions of their lived everyday world. Through interviews there are unique possibilities of obtaining access to what people encounter in their lives, and the researcher's goal is to get unprejudiced descriptions in relation to the world of science.

Interviews can be conducted in several different ways, ranging from an open conversation with little or no structure, to a carefully organized interview where questions are predetermined and the order is fixed as well (Christoffersen & Johannessen, 2012; Jacobsen, 2005; Mason, 2002; Thagaard, 2013). Open interviews give the researcher good opportunities to follow up stories and experiences revealed by the interviewee, but the relation between researcher and informant can affect the answers given by the informant. Structured interviews give more opportunities of comparing interviews and answers as the set questions might lead to information that can be easier to systemize, but they give little room for flexibility to adjust interviews to the individual informants. In between these two extremes, there is the semi-structured interview. The semi-structured interview is characterized by predetermined topics, but the questions posed, order and arrangement of questions and explorations underway will vary as the interview is conducted (Christoffersen & Johannessen, 2012; Jacobsen, 2005; Thagaard, 2013). The qualitative interview is based on this semi-structured form of interviewing, giving researchers flexibility, but still a somewhat narrative approach (Kvale et al., 2009; Mason, 2002).

Due to the nature of interviews, there are limitations to how many informants one can interview and how much data one can generate from field notes to a specific study. Quantitative research differs because there are almost no limits to the amount of data one can collect and process with the technology of this day and age. For a master thesis to be comprehensible, the number of subjects for interviews will most likely be below ten, and the information they can provide will give a detailed view of their life world, but it will not be as easy to generalize upon this data (Christoffersen & Johannessen, 2012; Thagaard, 2013).

In this thesis, a total of six semi-structured interviews were conducted. Five of the interviews were with students from Lakeside and Westwood, the last interview was conducted with a teacher from the mathematics section at Lakeside in order to gain insight on their curriculum design. The decision of choosing interviews as a method was made with a few considerations in mind. I wanted to research the teaching and learning in these classrooms, which is reflected in the research questions. Trying to gather information on the aspects of teaching and learning only from observation was quickly rejected as a possibility. Observation over time generates a massive amount of data (Christoffersen & Johannessen, 2012; Thagaard, 2013), and in order to get a good impression of the classroom practices there would have to be a substantial

amount of hours observed. Therefore, I determined that an approach with interviews would be taken.

The interview which was conducted with the teacher at Lakeside is definitely considered part of the study as she gave valuable information on the curriculum design. However, this has only been used as background information, and will not be analyzed.

3.1.2 Other qualitative methods

In this context, the other possibility of collecting data would be through the use of observation as a method. Observation is especially suitable in research when the aim is to record what people actually do (as opposed to what they might say that they do), gaining an insight in typical behavior in a given circumstance. Another setting where observation is appropriate would be to record human behavior in a particular context (Jacobsen, 2005). Both of these examples could have been relevant for use in my research. I am, indeed, interested in evaluating student behavior, both individually and in groups. However, I soon realized that there were limitations to only using observation. For one, I wanted to have the possibility of having the students voice their thoughts and opinions on learning. Secondly, I decided that the amounts of data would be very comprehensive if my goal was to get an idea of how problem-solving affected students' learning in mathematics. This requires a large number of lessons observed, leading to data could possibly be too comprehensive for this master thesis.

Within research, it is common to start off with some sort of reality or problem that one seeks information on (Christoffersen & Johannessen, 2012). In order to design my research, it was important to fully grasp what would be the main focus and what questions I wanted to answer. Because I had little prior knowledge of what was actually done in these classrooms, a large part of my preparation phase was done whilst observing in different classrooms. This was helpful in giving me an idea of how the teacher and the students worked and engaged throughout the different lessons. A total of 58 lessons over the course of 26 days were spent observing, giving me a solid impression of the practices of the different classrooms. Although this observation could have been a tool in analyzing the learning during those lessons, I decided that my observation would serve two purposes. First, it would give me a solid

understanding in order to explain the dynamics of the problem-solving lessons. Second, it would help me to choose interviewees for my data collection. This way, the observations served as a pre-study to the research conducted in this thesis.

3.1.3 Challenges in qualitative research

Although there are many advantages to qualitative data in research, there are also objections to the quality of the research which is based on information from interviews. Kvale et al. (2009) sum up this criticism in three categories; general conceptions of scientific research, stages of the interview- and analysis process, and validation/generalization of the interview-produced knowledge. The criticism through all three categories mostly evolves around the fact that interviews are too person dependent, and therefore difficult to label as scientific. The issue of objectivity is problematic within qualitative research; the researcher poses questions, and results or interpretations of interviews can be accused of being biased, subjective and explorative. Different researchers may find different meanings when analyzing interviews, and this calls for a claim that interview interpretation is not inter-subjective. Like quantitative data, *the Bartlett effect*, where researchers choose data that support their message, is also an issue worth mentioning (Brown, 1992). In qualitative interviews, the Bartlett effect can be seen in connection with posing leading questions to gain answers that give backing to the points a researcher is trying to make. Leading questions might affect a subject's answers, which is also a critique of the subjectivity of the interviewer. In addition to this, the low number of subjects makes it problematic to generalize findings from interviews, again raising the question of whether it can be regarded as scientific (Kvale et al., 2009). In my research, several of these aspects may be raised as challenging. There is a low number of interviews, making it difficult to generalize any possible results. Issues of objectivity is also relevant as I have both collected and analyzed the data myself. These are issues of validity and reliability that are discussed further in subsection 3.4.2.

Another aspect to remember in this setting is the inexperience of myself as an interviewer. The key research instrument of interviews is the interviewer, and when conducting interviews one should have qualities that include solid knowledge, sensitivity, ability to structure and steer the interview, as well as being both open and critical in the process of the interview. An

interviewer's qualifications will thereby have an effect on the quality of the interview (Kvale et al., 2009). I realize that I am not trained in conducting interviews, as this is the first time I have been through an interview process of this size. Some further thoughts on this is covered in subsection 3.2.3, where I elaborate on how the interviews were conducted.

3.1.4 Qualitative research as a method for this thesis

In this thesis, my aim is to look at the learning that emerges through classrooms where problem solving takes place. Though this could be achieved by testing and re-testing, I wanted to look at other aspects than just specifically right or wrong answers on a given test. What are the characteristics of these classrooms, what type of teaching promotes acquisition of connected knowledge, and how do the students engage in problem solving tasks? These questions would be impossible to answer through a quantitative approach. Therefore, qualitative interviews were chosen to gather the students' perceptions on the teaching and learning in these classrooms.

3.2 Data collection

This section starts with a brief discussion in subsection 3.2.1 of the terms *collecting* and *generating* data in qualitative interviews. Then an explanation of subject selection gets reviewed in subsection 3.2.2. Last, in subsection 3.2.3, the execution of the interviews will be described.

3.2.1 Collecting or generating data?

In qualitative methods, there is a distinction to be made from the terms *collecting* and *generating* data, and there are different perspectives to what information from interviews can provide science with. Collecting data suggests that the same information is accessible to everyone and that the researcher is neutral when it comes to collecting this information. Through this positivist standpoint, the subject will give descriptions of his or her knowledge of the outer world based on prior experiences (Thagaard, 2013), and the knowledge that

emerges from the interview is considered given facts that are to be quantified (Kvale et al., 2009). However, many qualitative perspectives reject this idea. From a constructivist position, data from interviews is considered a result from the social interaction between researcher and subject, and both are active contributors of the knowledge that will arise from the interview. Data is collected through active construction of knowledge in social interaction through one's epistemological position (Kvale et al., 2009; Mason, 2002; Thagaard, 2013). These positions are important to take into concern when analyzing data. For my research, I accepted that both views gave important insight. On the one hand it is important to keep in mind that the interview is in fact a social setting, and that the roles of both interviewer and interviewee affects the outcome, and on the other hand that the knowledge of the interviewee are reflections of his or her experiences. In this thesis I take both of these positions into consideration and realize that both views are relevant and important to acknowledge.

3.2.2 Subject selection

In qualitative interviews, subjects are chosen through *strategic selection*. The criteria for selection in qualitative research is not to what extent interview subjects are accessible, but rather how purposeful it would be to include a subject in the research (Christoffersen & Johannessen, 2012; Thagaard, 2013). Mason (2002) refers to the selection process as *sampling*, a strategic way for researchers to narrow down and focus the group of people to be interviewed. For practical and resource-based reasons, it is important to ensure that the amount of subjects is not overwhelming and comprehensible for the researcher. In addition to this, there are sampling reasons that deal with the focus of the research. It is likely that the researcher is not looking for shallow information on broad subjects, but rather a deeper focus on specific topics or occurrences. Through this sampling of subjects, the aim is to gain a range of contexts or phenomena which will improve the research.

Some interview subjects may have characteristics that can be considered better than other interview subjects. Good interviewees obviously give accurate answers and do not contradict themselves with inconsistent responses, and they tend to stay on topic whilst still giving examples of detailed stories, descriptions and experiences. Still, it is important to note that

informants who hold these characteristics do not necessarily provide the researcher with the most valuable information to answer the research questions at hand (Kvale et al., 2009). Therefore, the challenge is to recruit informants with desirable characteristics who also possess qualifications which may give material that enhances the study's quality.

There are several strategies to select subjects for interviews, and these methods depend on what the researcher considers valuable to the study. Identifying a strategy for selection will therefore ultimately decide the quality and relevance of the final sample for the research. One strategy is to sample informants that have extreme qualities in relation to each other and try to draw conclusions to what qualities within these informants that lead them to each extremity. The contrary would be recruiting a homogenous selection and try to label and locate specific traits that make them similar or different (Kvale et al., 2009; Thagaard, 2013). Another approach is convenience sampling, where researchers choose the subjects they find most convenient; an approach which is not very desirable for scientific research (Kvale et al., 2009; Thagaard, 2013).

For my research I decided to use *quota sampling* when recruiting interview subjects. This method is recognized by the creation of categories with certain properties to label potential interview subjects in (Thagaard, 2013). After this labeling, subjects from the different categories will be selected to the sample of interview subjects. By quota sampling, researchers are given the opportunity to compare special phenomena with subjects in different categories (Kvale et al., 2009; Thagaard, 2013). My categories were not comprehensive, but they included properties I found important for collecting valuable data. First of all, Peterson teaches a lot of classes in mathematics, and the idea was to get an impression of how students learn and perceive their learning in his problem-solving classes. Although one of the schools has progressed classes for particularly gifted mathematics students, I chose not to interview any of these. These particular students have chosen to take part in his classes, knowing they will take their 10th grade exams in the 9th or 8th grade, and one could probably argue they are more motivated than regular students, and some might even be considered more gifted than the average student. In addition to this, I thought that interviewing the students he only taught once per week could prove to be valuable. They were exposed to several teachers in mathematics over the course of a week, and my assumption was that these students were to a much larger extent than other students able to compare classes that focused on problem

solving to their regular classes. With this in mind, I chose to interview students from the 7th grade class at Westwood and 10th grade classes at Lakeside where Peterson taught once per week. Further, I wanted a span of mathematical abilities amongst the interview subjects to ensure information from students with complementing capabilities. Last, I decided that both boys and girls should be interviewed in both classes. Although my research questions do not include gender-specific concerns, including both sexes is something I valued as important to ensure that the study focused on more than only one gender.

Having spent a total of 26 days of observation at Westwood and Lakeside I started to get a decent impression of both the teacher, the teaching and the students in these classrooms. After categorizing the students, choosing potential subjects for interviews was not too difficult. I figured it would be an advantage to conduct interviews with students I had some pre-understanding of or if I had experienced some sort of interaction with them during these weeks. Connecting questions to relevant tasks I knew that these students had successfully solved, or having follow-up questions ready with my background knowledge in mind was something I believed would enhance the quality of the interviews. With these criteria, five students were carefully chosen and invited to take part in interviews. First, I talked to the students during class, asking them for permission to contact their parents regarding my study, and after that e-mails were sent to their parents. All parents gave consent to conduct interviews with their children.

My interview subjects were as follows:

R1: Boy from the 7th grade at Westwood

R2: Girl from the 7th grade at Westwood

R3: Boy from the 10th grade at Lakeside

R4: Girl from the 10th grade at Lakeside

R5: Girl from the 10th grade at Lakeside

In addition to the selection of students for interviews, a teacher at Lakeside was selected for interview on their curriculum design. She was asked to participate after teachers at the mathematics section at Lakeside suggested that she would be able to give solid information on the curriculum design and the ideas behind it.

3.2.3 Conducting the interviews

The interviews conducted had a semi-structured approach, directed by an interview guide which included general topics and suggestions to relevant topics. The two 7th grade students were interviewed in mid-December, and the three 10th grade students were interviewed in mid-January. The interviews took place in classrooms where there would be no interruptions. They ranged from thirteen to twenty-seven minutes in length, and my telephone was used to record the interviews. All the interviews started with a brief conversation where I gave information regarding the study and their rights as interviewees, in addition to talking about general school-related topics. It seemed that the interviewees were comfortable and relaxed in the situation, and as an interviewer I felt I was able to listen, and to ask relevant follow-up questions without leading them into certain paths.

My overall aim was to make the students share their thoughts, descriptions and experiences on taking part in mathematics classes that focused on problem solving. Additionally, I asked them, during the interviews, to engage in one or more problem solving tasks. This would serve two purposes. First, it would give me an idea on how these students confronted mathematical problems. Second, their approaches to solving these tasks could provide me with information on what knowledge types and qualities these students possessed. Key topics for the interviews included (but did not limit itself to):

- General thoughts on mathematics
- Motivation, engagement at school and at home
- Problem-solving activities
- Differences between problem-solving tasks and other tasks they would encounter over the course of a week
- Solving one or more problems

Although the interviews were well-prepared, the quality of the interviews seemed to get better as I also developed in my role as an interviewer. Asking relevant follow-up questions and effectively guiding the conversation was something that improved during the process. The respondents in the interviews were named R1, R2, R3, R4 and R5 which arranged the students alphabetically, making it easier for me to distinguish during analysis. However, the order of the interviews conducted were R1 (first), R2, R4, R5, R3 (last). In the analysis there is a higher

frequency of R3's statements than R1's, as R3's interview had a larger degree of relevance to the topics of this thesis than R1's. This is likely because the quality of the interviews gradually became better as I developed in my role as an interviewer.

3.3 Data analysis

Analyzing qualitative data consists of three elements; describing, categorizing and connecting (Jacobsen, 2005). Miles & Huberman (1994) looked to similar flows of activity. First, data gets reduced through a researcher's field notes or transcriptions. This reduction starts ahead of the data collection period, however, as a reduction starts when a researcher narrows down the topics for the study and research questions. The next step in analysis is displaying data, a process of displaying the data in ways that are easily comprehensible. Last, conclusions must be drawn and the study needs verification (Miles & Huberman, 1994). Describing the data in my research is the process of transcribing and interpreting the audio recordings from the interviews, and it will be covered in subsection 3.3.1. Categorization is the procedure used to create groups in order to place the data in different categories and general considerations of categorization is covered in subsection 3.3.2. Last, in subsection 3.3.3, the TRU Framework is presented and narrowed down for analysis. Necessary clarifications and explanations to the application of the framework will also be made.

3.3.1 Transcribing the interviews

As mentioned in subsection 3.1.3, there are challenges with determining the quality of qualitative interviews. The transcriptions' quality is also a topic worth mentioning. Transcribing from recordings requires interpretations, and the difference between speech and text can cause several issues. First of all, with the interview being a conversation between two people, all other interactions between the two is hard to recover from audio recordings. Second, tone of voice, intonation and enthusiasm disappear in the transformation from oral to written language, possibly leaving out important pieces of information in the process (Kvale et al., 2009).

Transcription of the data was done soon after the interviews were held to ensure that I would remember as many details as possible from the conversations. Furthermore, notes were taken during the interviews to help me remember things that occurred as the interviews progressed, limiting the fragments lost in the process of transcribing.

3.3.2 Interpretive readings and categorization

Analyzing qualitative data involves taking many decisions that ultimately decide the outcome of the study. The issues of generation or collection of data was covered in subsection 3.3.1, but following this process of retrieving data there are different ways for researchers to read the data at hand. With transcribed interviews, it is common to read the transcriptions using an *interpretive* technique (Mason, 2002). Interpretive readings are made when researchers construct a version of what they believe can be referred from the texts, using their own subjective interpretations of the transcriptions (Mason, 2002).

Furthermore, an approach of categorical indexing of the transcriptions was conducted. By using constant categories, data can systematically be positioned into specific sections of similar information. However, this type of cross-sectional indexing has three possible limitations. Creating categories for analysis of a text can be difficult, possibly making them too broad for application in more than one text. Interviews also tend to bring up more than one topic, possibly making serial indexing impossible. As mentioned in 3.1.1, qualitative interviews are characterized by a semi-structured form, which does indexing in sequences difficult due to its lack of order (Mason, 2002). Creating relevant categories and subcategories for indexing, which were both interrelated and unrelated, that could also be applied for all five semi-structured interviews, was something that was never considered an option for this thesis. If such categories were created, the outcome would likely be a poor analysis with little or no foundation in relevant theory.

Ryen (2002) explains what she calls analytical depth. Categories are often created based on the data description, leading to categories with minimal analytical basis. By linking the categories to relevant research, however, it will lead to depth in the analysis and better analytical quality (Ryen, 2002).

3.3.3 The TRU Framework as tool for analysis

As a part of creating the TRU Framework which is outlined in subsection 2.3.3, Schoenfeld et al. (2016b) developed an observation guide for collaborative partnerships between teachers and observers. Observations were meant to be done in series and should focus on one or more of the dimensions from the TRU Framework. As a tool for discussion after the series of observations, a conversation guide for reflection was also created (Schoenfeld et al., 2016a). The TRU OG includes aims for both students and teachers within each of the five dimensions of the TRU Framework, giving traits of what are desired actions, behaviors and engagement from teachers and students in classrooms that create proficient mathematical thinkers (Schoenfeld et al., 2016b). With the abovementioned challenges of creating categories for analysis, using this framework as a base for indexing appeared to be a suitable way of analyzing the interviews.

There is one critical issue with this thesis' use of the TRU Framework (and observation guide) as a tool in analyzing teacher- and student behaviors in the classroom. Schoenfeld et al. created the observation guide in order to give guidelines for observation which are pre-planned where focus points get created prior to observation. Observers are given points to assess and can actively search for traits as the teaching takes place. This thesis uses a very different approach, taking a student perspective to this analysis. The researcher in this thesis has observed classes and interviewed the five selected students, but the characterizations of the teaching and learning come solely from the students' point of view and the interviews should capture true and real stories from the students' experiences in the classrooms. Another point is that Peterson had no idea that his teaching would be analyzed through the TRU OG measures and no way to prepare specifically for this, which is contrary to the suggested usage of the framework. This use of the observation guide is definitely something which can provoke discussions. However, I believe that student perspectives to characterize measures of the TRU Framework could give assessments which are not necessarily discovered through peer-observation, giving a breadth to the conducted research. That Peterson did not know he would be assessed on certain measures ensures that the student accounts are consistent with what is the normal behavior in the classroom.

Considering that the analysis would be in light of the student accounts, there were elements of the TRU Framework that proved to be more informative for analysis through interviews than others, in addition to the fact that a complete analysis would be either too comprehensive or too shallow to complete in this thesis. Therefore, some dimensions were considered more valuable than others, and I will now explain why *The Mathematics, Cognitive Demand* and *Equitable Access to Mathematics* were chosen for analysis:

The aspect of *The Mathematics* was obviously important to capture the content taught and learned in these classes. To which extent these classes challenged the students' *Cognitive Demand* was also considered important, bearing in mind the previously mentioned importance of deeper learning. All students' *Equitable Access to Mathematics* also needs to be addressed, giving information on whether the teaching practices affect the whole classroom or if there are groups of students that do not benefit from the activities performed.

The dimension of *Agency, Ownership and Identity* proved to be a difficult item to measure using only student interviews as a source. This dimension looks to the students' opportunities to explore and reason on emerging ideas in the classroom, and being able to build identities through ownership over the content. These types of behaviors were definitely captured through observation, but as qualitative interviews have been the chosen method this has not been taken into consideration and will therefore not be covered through the data analysis. *Formative assessment* is also a dimension that would be easier to capture through observation, asking questions like "do students provide specific and accurate feedback to fellow students?", characteristics that seem more valuable if they are witnessed than through interviews.

After narrowing it down to the three dimensions *The Mathematics, Cognitive Demand* and *Equitable Access to Mathematics*, another reduction had to be made within the TRU Observation Guide. A total of fifteen measures of student characteristics and seventeen measures of teacher characteristics in powerful classrooms are listed in these three dimensions, giving a total of thirty-two points to analyze through these interviews. This would also prove to be very extensive, demanding me to prioritize certain measures ahead of others. The selected measures have been highlighted in the tables below which are gathered directly from the TRU OG. For reasons of transparency, the full list of measures from the three

dimensions have been included in the tables. This is in order to show that there are several measures that were not analyzed during this study.

THE MATHEMATICS	
<p><i>The extent to which central mathematical content and practices, as represented by State or the Common Core State Standards, are present and embodied in instruction. Every student should have opportunities to grapple meaningfully with key ideas and, in doing so, to become a knowledgeable, flexible, and resourceful mathematical thinker and problem solver. Teachers should have opportunities to consider and discuss how each lesson’s activities connect to the concepts, practices, and habits of mind they want students to develop over time.</i></p>	
<p>Each Student...</p> <ul style="list-style-type: none"> • Engages with grade level mathematics in ways that highlight important concepts, procedures, problem solving strategies, and applications • Has opportunities to develop productive mathematical habits of mind • Has opportunities for mathematical reasoning, orally and in writing, using appropriate mathematical language • Explains their reasoning processes as well as their answers. 	<p>Teachers...</p> <ul style="list-style-type: none"> • Highlight important ideas and provide opportunities for students to engage with them • Use materials or assignments that center on key ideas, connections, and applications • Explicitly connect the lesson’s big ideas to what has come before and will be done in the future • Support the purposeful use of academic language and of representations (e.g., graphs, tables, symbols) central to mathematics • Support students in seeing mathematics as being coherent, connected, and comprehensible

Table 2. Measures of The Mathematics in the TRU OG. Reprinted from “The Teaching for Robust Understanding (TRU) observation guide for mathematics: A tool for teachers, coaches, administrators, and professional learning communities,” by A.H. Schoenfeld and the Teaching for Robust Understanding Project, 2016. Appendix: p. 1.

COGNITIVE DEMAND	
<p><i>The extent to which students have opportunities to grapple with and make sense of important mathematical ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with task difficulty ranging from moderate to demanding. The level of challenge should be conducive to what has been called “productive struggle.”</i></p>	
<p>Each student...</p> <ul style="list-style-type: none"> • Engages individually and collaboratively with challenging ideas • Actively seeks to explore the limits of their current understandings • Is comfortable sharing partial or incorrect work as part of a larger conversation • Reasons and tests ideas in ways that connect to and build on what they know • Explains what they have done so far before asking for help • Continues to wrestle with an idea after the teacher leaves 	<p>Teachers...</p> <ul style="list-style-type: none"> • Position students as sense makers who can make sense of key conceptual ideas. • Use or adapt materials and activities to offer challenges that students can use, individually or collectively, to deepen understandings • Build and maintain classroom norms that support every student’s engagement with those materials and activities • Monitor student challenge, adjusting tasks, activities, and discussions so that all students are engaged in productive struggle • Supports students without removing the challenge from the work they are engaged in

Table 3. Measures of The Cognitive Demand in the TRU OG. Reprinted from “The Teaching for Robust Understanding (TRU) observation guide for mathematics: A tool for teachers, coaches, administrators, and

professional learning communities,” by A.H. Schoenfeld and the Teaching for Robust Understanding Project, 2016. Appendix: p. 2.

EQUITABLE ACCESS TO MATHEMATICS	
<i>The extent to which classroom activities invite and support the meaningful engagement with core mathematical content and practices by all students. Finding ways to support the diverse range of learners in engaging meaningfully is the key to an equitable classroom.</i>	
<p>Each student...</p> <ul style="list-style-type: none"> • Contributes to collective sense making in any of a number of different ways (e.g., proposing ideas, asking questions, creating diagrams...) • Actively listens to other students and builds on their ideas • Supports other students' developing understandings • Explains, interprets, applies and reflects on important mathematical ideas • Participates meaningfully in the mathematical work of the class 	<p>Teachers...</p> <ul style="list-style-type: none"> • Create safe environments • Use tasks and activities that provide multiple entry points and support multiple approaches to the mathematics • Provide opportunities for students to see themselves, and their personal and community interests, reflected in the curriculum • Validate different ways of making contributions • Build and maintain norms that support every student's participation in group work and whole class activities • Support particular needs, such as those of language learners, for full participation • Expect and support meaningful mathematical engagement from all students, helping them contribute and build on contributions from others

Table 4. Measures of Equitable Access to Mathematics in the TRU OG. Reprinted from “*The Teaching for Robust Understanding (TRU) observation guide for mathematics: A tool for teachers, coaches, administrators, and professional learning communities,*” by A.H. Schoenfeld and the Teaching for Robust Understanding Project, 2016. Appendix: p. 3.

The measures from the TRU OG include descriptions of student- and teacher behavior which are to be used for analysis of the interviews conducted. However, one of the student-measures seems difficult to analyze because it assumes specific knowledge not mentioned in the TRU Framework: “Each student engages with grade level mathematics in ways that highlight important concepts, procedures, problem-solving strategies, and applications” (Schoenfeld et al., 2016b. Appendix: p. 1). This requires me to define what problem-solving strategies are, in order to understand whether the students possess these strategies. Using Polya’s (2009) four phases of problem solving, I will analyze the students’ abilities in using problem-solving strategies (heuristics) when solving mathematical problems. Although Polya’s approach to problem solving is not mentioned in the TRU Framework or the TRU OG, Schoenfeld (1985) looks to Polya’s work as important in reviving heuristics as a key part of problem solving. Through the four steps to solving problems, Polya’s method will provide a tool for analysis that will be effective whilst not overshadowing the TRU Framework which the analysis primarily relies on.

One final clarification is needed before presenting the analysis. Having a total of five respondents and many measures to analyze, statements and data from all interviews have not been gathered for each measure, and not even for each dimension. In some subsections, the data has been retrieved from only two respondents. This has been done in order to create an analysis of the teacher- and student characteristics that is able to connect and describe with flow and precision. In addition to this, using statements from every respondent in each subsection would lead to the analysis becoming too comprehensive to complete in this thesis. However, all students are represented within each of the sections 4.2 (Teacher characteristics in light of TRU measures) and 4.3 (Student characteristics in light of TRU measures), ensuring that the analysis has a certain depth.

3.4 Ethical considerations in qualitative studies

Due to the nature of qualitative interviews, where interactions between subject and researcher lead to detailed descriptions of the subject's thoughts, ideas and experiences, there are many ethical issues and considerations important to address. One of the most important ethical aspects in qualitative research is that it should not cause any harm to the people being interviewed. The interviewer is responsible of maintaining trust between both parts through the process of the research, taking the subject's best interest into consideration whilst still conducting informative interviews (Thagaard, 2013). In order to secure anonymity, all projects that handle personal information of informants need approval from the Norwegian Center for Research Data before carrying out any part of such research. This process is explained in subsection 3.5.1. The aspects of validity, reliability and generalization are also important in terms of ethics in qualitative research. To what extent a study is trustworthy, has been carried out appropriately, and to what degree the research can make general claims are discussed in subsection 3.5.2.

3.4.1 Approval from the Norwegian Center for Research Data (NSD)

The National Committee for Research Ethics in the Social Sciences and Humanities (NESH) in Norway has developed ethical guidelines directed towards research, ranging from concern of

other individuals to funding and public debate participation. In master theses, the aspects that deal with personal data are the most relevant as many of the other guidelines are directed towards higher levels of research and science (NESH, 2016).

In order to perform research that deals with personal data, researchers are obliged to seek approval from the NSD. This process ensures researchers' knowledge of how to properly manage personal data. Researchers are required to gather informed consent from all participants, treat all gathered information confidential and store personal data responsibly and describe this process in the application (Christoffersen & Johannessen, 2012; Thagaard, 2013). Obtaining consent has to be done in a way that ensures both the free will of the participants, but also that they receive proper and sufficient information on what involvement in the study means. With children under the age of 15, informed consent from parents is needed. Through the process of gathering information it is important that personal data is processed confidentially, ensuring that the information the researcher possesses cannot be passed on to identify the informants. In respect to confidentiality, it is the researcher's responsibility to ensure individuals' anonymity when using data actively in research. In cases where storage of data is necessary, the data containing identifiable information must be kept safely and separately from other parts of the data (NESH, 2016).

Because subjects in this research were students, consents were retrieved from parents as well as students. Both students and parents received basic information on the study and the use of data and were notified that the withdrawal from the study could be done at any point in the research.

3.4.2 Validation and reliability

The evaluation of qualitative research relies on the factors of *validity* and *reliability*. These elements question the credibility and trustworthiness of the research and its processes (Thagaard, 2013). Reliability in research refers to whether another researcher would get the same results using the same methods. The issue of reliability concerns the accuracy of the interview process, transcription and analysis, affecting the research process as a whole. In order to make a study reliable, all aspects of the research process need to be as transparent

as possible, ensuring a fair predictability that a similar project will lead to similar results (Christoffersen & Johannessen, 2012; Kvale et al., 2009; Mason, 2002; Thagaard, 2013). Using detailed descriptions of how the research has been conducted, in addition to a solid base of theory, I hope that the research from this thesis will be considered reliable.

Questions of validity involve to what degree a statement (or in research: the results of the study) is true, correct and strong. Results derived from data need to be coherent with what has been studied, and as a researcher one should be able to demonstrate that outcomes can be measured as claimed (Christoffersen & Johannessen, 2012; Kvale et al., 2009, Thagaard, 2013). Using qualitative interviews is definitely something that makes for difficult reproduction of results, due to the specific nature of interview results. Researchers can never be completely objective (Christoffersen & Johannessen, 2012), and my own perceptions and thoughts could possibly lead to answers I was looking for. However, during the interviews I tried to remain as objective as possible, asking open questions without leading the students in any direction: The research has gathered results based on what was in fact studied.

4 Analysis

The findings in this chapter are based on the analysis of the qualitative interviews presented in Chapter 3. The connection between the TRU Framework, which was argued as traits of good mathematics teaching in 2.3.3, and the students' experiences and learning outcomes in Peterson's classrooms will be exemplified, giving a base for discussion in chapter 5.

In order to understand learning outcomes from these classrooms, it is essential to gain a general idea of how these lessons were structured and what goes on in these classrooms. Although my 58 lessons over the course of 26 days of observation are not considered a method used in this thesis, I did develop a good understanding of both teacher- and student activity, and how the lessons evolved. These subjective descriptions are used simply to give ideas of the behavior in the classroom. The analysis of the data is therefore mostly based on the students' account of the mathematics teaching, but a few aspects include examples of tasks from the classroom gained through observation.

The chapter will be divided into four sections. First, in section 4.1, I will give subjective descriptions and explanations of the teaching, student behavior and atmosphere in the classrooms observed. Then, teacher- and student characteristics in light of TRU OG measures will be analyzed in sections 4.2 and 4.3 respectively. The teacher characteristics will rely solely on statements which arose during the interviews. The student characteristics, however, include more of the dialogue between respondent and interviewer. Because many aspects of the student characteristics became visible through working with problem-solving tasks, this section will include several excerpts of the conversations in addition to statements. Last, section 4.4 will elaborate on the students' use of heuristics in interaction with problem-solving tasks, analyzing the students' problem-solving work through Polya's (2009) four phases of solving problems. As explained in 3.3.3, Polya's four steps will be used to cover one of the measures from the TRU OG, giving me a tool for analyzing the students' interaction with the problem-solving tasks.

4.1 Classroom behaviors

Although there were individual differences to every lesson, most of the observed lessons followed a similar pattern. The lessons would start off with students and teacher greeting each other before Peterson would present a problem on the board, giving little or no information on how to solve it. He would also rank the difficulty of the problem before the students would start, claiming for example “this problem is a grade 5 on the 10th grade exam” or “this is grade 7+” (relating it to the Norwegian scale for grading, which goes from 1-6, with 6 being the best), giving the students a general idea of how difficult the problems are. Next, the students were given some time to solve the task either individually or in groups as Peterson would walk around the classroom observing and giving feedback to the students on their progress in solving the problem. After a while, one or more solutions to the problem were shown on the board before another (similar) problem was given for the students to solve. These general observations of the lessons were shared by the interviewed students, as they had many of the same experiences as I had witnessed. Respondent number three (R3) said:

A typical lesson? It often starts with a problem on the board, where he gives us a limited amount of information and lets us work for 10-20 minutes on that task by ourselves or in groups. And if he notices that many of us get stuck, he will send us in the right direction. So even the students that didn't have a clue will be able to finish. (...) You're working with a task that you don't know exactly how to solve, but when you finally do you might be able to use that solution in other tasks as well (R3)

The students in the classrooms did not have assigned seats, instead they were given the choice to select who they wanted to work with, and the size of the groups they wanted to work in. This way, all of the students were given the option to work together with peers, individually or with other students they chose. In some lessons Peterson would start off with asking the students whether they had any questions regarding their homework, or other tasks they had encountered since the last time they met. These questions would be used as an entry for discussion in the classroom where the students were given the opportunity to work out solutions for themselves. Following this, Peterson usually followed up with a similar task to

the one being asked about. An example of this is these two tasks that were used during the same lesson:

“A large tent has a surface consisting of two isosceles triangles and three rectangles. The shortest side in the tent’s base makes up 5/6 of the tent’s height. Each triangle’s circumference is 55 meters. How high is the tent?”

There are many difficult parts to this task and how to go forward when solving it. But the Pythagorean theorem came back in all of the students’ solutions, ending with an equation similar to $h^2 + \frac{5}{12}h^2 = x^2$ with h representing the height and x representing the equal sides of the triangle. However, most of the students failed to apply proper use of parenthesis as a correct equation would have the whole fraction term squared: $h^2 + (\frac{5}{12}h)^2 = x^2$. Peterson took advantage of this mistake which most of the students made, and came up with a similar task which forced them to work further with the proper use of the theorem:

“A flat screen TV has the dimension of 50”. What is the size of the other sides of the TV if we know that the ratio between the longest and shortest sides are 16:9?”

This task would highlight the importance of proper parenthesis usage in each term, helping to reinforce the learning the students had just acquired through the previous task. Classroom discussions would arise with students saying things like “No, it is not supposed to be $16x^2$, it is $(16x)^2$!”

As R3 said, this way the students would learn something and immediately be able to use it in other tasks as well. R5 explained this process in the following way:

(...) there’s a problem on the board, a difficult problem. And then he says that we should try to solve it. And then you try, some are able to solve it, and some are not, and then after a while he will go through it and show different ways of solving it. And then he will give us a similar task, or a task that has similar aspects in it, so that you get to use the methods he just used, and then you almost always find ways to solving it

(R5)

Another example of this occurred in a lesson where the focus was work on similar triangles. Peterson drew a task on the board (Figure 4), explaining it like this:

“A group of boy scouts are out camping. Close to their camp site there is a river they want to cross, and they need to figure out if it would be possible to jump across it or not. They determine the dimensions surrounding the river and end up with the following measurements. Is it possible to jump across the river?”

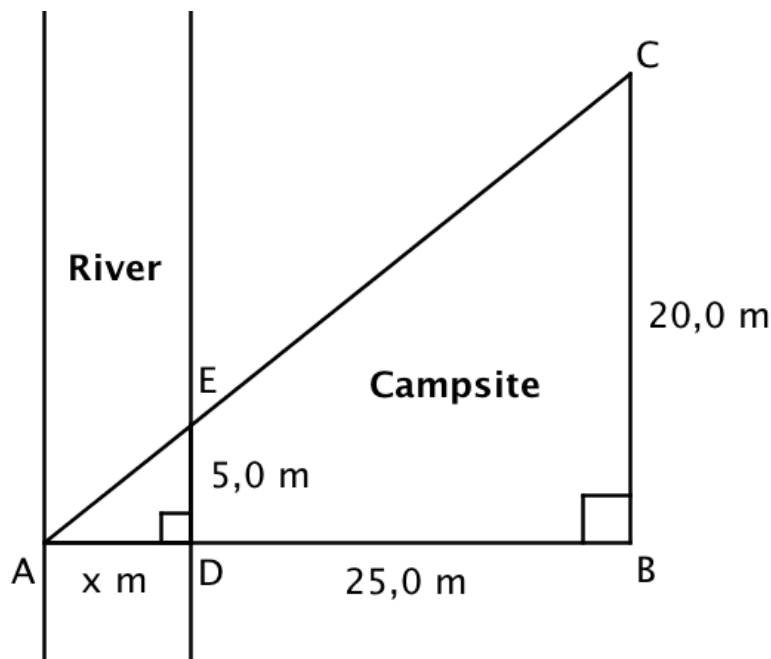


Figure 4. The River-task.

The task has several different solutions, resulting in the students solving it in a variety of different ways. Peterson took his time in letting the students solve the task, and as soon as a student had finished solving it he/she was encouraged to find another solution. Following the students' work with the task, different solutions were shown and explained on the board. Although the task has several solutions, one particular way of solving it was highlighted, which involved expressing side consisting of the end of the campsite to the edge of the river as “ $25 + x$ ”, using this in an equation of ratios.

Following this task, Peterson drew another task on the board; a trapezoid with diagonals connecting the opposite corners (Figure 5):

“ $AB = 9,0$ cm, $CD = 4,0$ cm and $AC = 6,5$ cm. AB is parallel to CD . AC and BD intersect at E . Find the value of CE .”

This task, just like the River-task, has several possible points of entry. In addition to the obvious part of proving that triangles ABE and CDE are similar, expressing sides CE and AE involves similar aspects to what the River-task had; one way being $EC = x$ and $AE = 6,5 - x$. Activities like this, and the build-up of these activities, constituted the major part of Peterson's lessons.

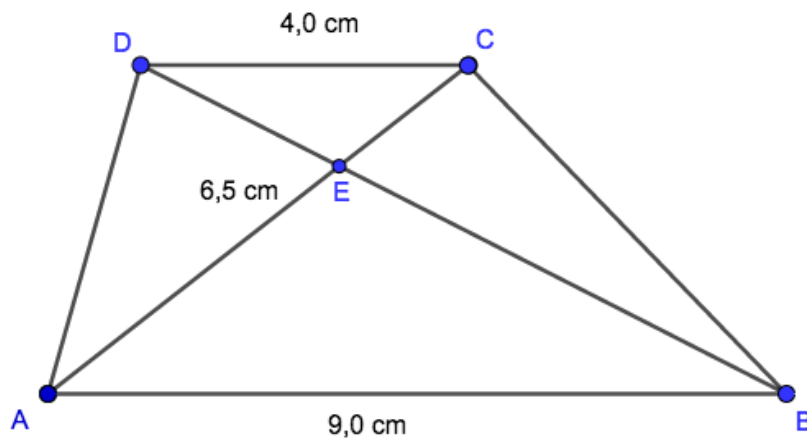


Figure 5. The Trapezoid-task.

4.2 Teacher characteristics in light of TRU measures

The research questions of this thesis, covered in section 1.3, are primarily directed towards student behavior and learning, as opposed to teacher characteristics and teaching. So what importance do teacher characteristics have in this analysis, when the aim is to answer questions regarding student behavior and learning? In order for the students to emerge with good mathematics competency, the teacher plays a crucial role in creating learning environments where the students can develop and flourish (Schoenfeld et al., 2016a). As Schoenfeld states, the answer is tied to the connections between Peterson's expectations and disciplinary ideas, and the learning outcomes that emerge from these factors. The teacher's approach, ultimately leading to good or poor learning outcomes, needs to be addressed in order to fully understand the students' learning process. Through an analysis of teacher characteristics of the TRU OG it will be possible to get an understanding of what classroom practices take place in these lessons, in addition to understanding the connections between teacher- and student behavior. This, together with the analysis of the student characteristics, will give me the opportunity of answering the three research questions of how the students

engage (RQ1), how they interact with tasks (RQ2) and whether problem solving develops powerful mathematical competency (RQ3).

In this section, teacher characteristics of the dimensions *Mathematics*, *Cognitive Demand* and *Equitable Access to Mathematics* will be analyzed in subsections 4.2.1, 4.2.2 and 4.2.3 respectively.

4.2.1 Teacher characteristics of *The Mathematics*

The goal of the dimension *The Mathematics* in the TRU OG is that “All students work on core mathematical issues in ways that enable them to develop conceptual understandings, develop reasoning and problem-solving skills, and use mathematical concepts, tools, methods and representations in relevant contexts” (Schoenfeld et al., 2016b. Appendix: p. 1). Through key characteristics from teachers this can be made possible.

Looking to the Observation Guide, the following two points lay the ground for a basis in mathematics teaching: “Teachers use materials or assignments that center on key ideas, connections and applications” and “teachers support students in seeing mathematics as being coherent, connected and comprehensible” (Schoenfeld et al., 2016b. Appendix: p. 1). These two measures go hand-in-hand, as they both focus on the teacher’s ability to connect mathematics. Through a focus on important ideas and applications, teachers should strive to help students experience mathematics as a coherent subject.

R1 had the following description of the approach to the assignments given in Peterson’s classes:

We kind of... We have to use different ways to solve the tasks. We have to look at them from different perspectives. ... We have to use several parts of mathematics in the same task too. So you have to look at the tasks in different ways (R1)

Through the use of problem solving tasks, the students are given opportunities to access materials that connect the different mathematical topics together. As R1 states, the tasks

given require them to look at tasks from different points of view and to use several mathematical topics in the same task. R5 underlines the same points:

You get to use a lot more of the mathematics, if that makes sense? ... Yeah, if you have to find the area you need to remember how to do that, and maybe put it into an equation, or maybe you have to use that knowledge in similar triangles, you know. You get to use all of it in one task. And it... It requires a lot more knowledge, if that makes any sense (R5)

In describing the challenging tasks she encounters in Peterson's lessons, R5's statements are related to important aspects of deeper learning. Solving the tasks is not necessarily just reproducing knowledge, but an active process of using current knowledge in tasks where this knowledge has not yet been applied. R1 and R5 do not only solve these tasks, they also reflect on what the essential parts of these tasks are. One aspect of working with problem solving is for students to be able to solve the tasks given, but in this context it is important to notice the fact that the students in Peterson's class also recognize mathematics as being *coherent, connected and comprehensible*, which according to Schoenfeld et al.'s framework are important features for teachers to accomplish in order for students to learn.

Furthermore, teachers should also "highlight important ideas and provide opportunities for students to engage with them" (Schoenfeld et al., 2016b. Appendix: p. 1). Using flipped classroom as a method for teaching, all students are given the opportunity to access and engage with important aspects of mathematics. It also ensures that students are able to participate in the solving of tasks, at the level they are, at any given time. R5's experiences from regular mathematics classes is an interesting contrast to Peterson's teaching:

I feel like I learn a lot more that one Monday per week than I have been over the duration of half a year. Namely because you get to try all these different possibilities. And Isaac kind of knows... He gives us challenging tasks that are useful (R5)

Peterson highlights important ideas, or as R5 names them; *possibilities*. R5 claims that these lessons are very valuable to her learning, as the tasks are considered *useful*. As she mentions

possibilities, she looks to important aspects of mathematics that are emphasized in these weekly classes. She further says:

Working with square roots, for example. We haven't worked with it a lot, but now we have to use it all the time in these tasks, and you get a lot better at it. As opposed to... For example working six months with simple algebra, which some people find very simple, but some think it's really difficult (R5)

Having observed these classes, I interpret R5's ideas on square roots not necessarily as how important this is by itself, but the principle of doing calculations with square roots (e.g. $\sqrt{2}$), pi or $\frac{1}{3}$ in order to get accurate answers. The key idea of this is not the calculation with square roots, but the ability to perform calculations (e.g. solve equations) without using numbers; which are important parts of being efficient in mathematics. Again, R5 looks to the application of these skills, as she compares them to what they have been doing in regular classes. Repetition and simple tasks within algebra create superficial procedural knowledge, as opposed to the usage of algebraic representations in problem solving activities. She looks to the constant use of these representations in relation to solving tasks as a way she has really developed a better understanding of the concepts.

4.2.2 Teacher characteristics of *Cognitive Demand*

Teachers should seek to challenge students with moderate to demanding tasks, giving room and support for growth. Promoting a productive struggle, a student's battle with solving a task with the perfect difficulty, is key in order to enhance learning and engaging student's in learning mathematics (Schoenfeld et al., 2016b. Appendix: p. 2).

Analyzing the interviews gave many examples of students who found themselves in the productive struggle during Peterson's classes. One example has already been mentioned in 4.1, where the students were given the task of finding the height of a tent. The TRU Observation Guide looks to teachers to "Monitor student challenge, adjusting tasks, activities and discussions so that all students are engaged in productive struggle" (Schoenfeld et al.,

2016b. Appendix: p. 2). With this example, Peterson showed a way of examining the students' work with the problem and adjusted his task to engage all the students towards a productive struggle within the same mathematical matter. R3 and R5's accounts mentioned in 4.1 of the teaching emphasize this as well. Both of them express the teaching as a process where tasks and activities are adjusted or altered in order to promote learning and deeper understanding. R2 relates the lessons of problem solving to regular mathematics classes, giving examples of the contrast of the activities done:

Here we get really difficult tasks and such, while in regular classes we get simple tasks where it is more like "It is okay if you can't solve them, it's fine!", and it's... We don't really get challenged, there's no "I really really want to solve this!", there's no challenge. And that's what is great with Isaac's lessons, you know? There is something grand about solving a task (R2)

R2's contrast of the problem-solving lessons and regular classes include interesting remarks towards motivation as well as insights on the cognitive demand of these different classes. First of all, her account on regular classes is that the tasks offered are simple with little demand of mathematical ability. In addition to this, she offers a perspective on what is expected of the students, expressing that it is widely accepted not being able to solve tasks. The idea that it is okay to fail and being unable to solve tasks are steps on the way to becoming good at mathematics is obviously important, but R2 expresses this in a way as it is not expected that the students struggle while working with mathematics. This is largely in contrast to her opinions of the atmosphere in the problem-solving classes, where she is constantly engaged and eager to solve demanding tasks. The attitude towards working with mathematics is fairly different, from "it is okay if you can't solve them" to "there is something grand about solving a task". In Peterson's lessons, the students get engaged in the productive struggle, and all the students are expected to actively participate and battle with the tasks.

Another measure within of the dimension of cognitive demand claims that the teachers should "support students without removing the challenge from the work they are engaged in" (Schoenfeld et al., 2016b. Appendix: p. 2). As mentioned in the previous paragraph, R2 finds the tasks challenging in the problem-solving lessons. However, R3's statement in 4.1 gives the

impression that students sometimes get stuck or have difficulties completing tasks. Unlike what is described in R2's regular mathematics classes where students aren't expected to solve all tasks, R3 portrays a classroom where Peterson supports students on their way to solving tasks. The support is not through providing the students a final answer, but guiding them along the way so that they get an opportunity of solving the tasks as well.

Finding the desired difficulty for tasks is definitely something that is demanding for mathematics teachers. Building on points from Chapter 2, the provided tasks should promote conceptual understanding whilst also leading to deeper learning. The work in the classroom should therefore have its nature in activities that connect existing knowledge in new situations. The TRU OG seeks out for teachers to "Use or adapt materials and activities to offer challenges that students can use, individually or collectively, to deepen understandings" (Schoenfeld et al., 2016b. Appendix: p. 2), also addressing the importance of stimulating deeper learning in mathematics. R3, again reflecting on the differences between problem-solving classes and his regular mathematics class, shared the following thoughts on the tasks given:

In regular lessons it's more like... Right now we have a lot of repetition, but usually we work on new things. And then we do a lot of small simple tasks, so we don't get to use it properly because it is the same pattern of solving them. And then you don't really learn how to use it in different situations. (...) It is different. ... Having problem solving instead of doing a bunch of tasks. Because you kind of work with it on a deeper level and learn different techniques (R3)

R3's descriptions of the contrasting work being done in the problem-solving classes and his regular classes are very closely related to the definitions of deeper learning. As mentioned in chapter 2, deeper learning has occurred when the learner is able to transfer what was learned to new situations. In R3's regular classes he works with many tasks that follow a similar pattern, which he thinks does not enable him to learn a topic properly. He links this to the problem-solving work where these skills are used in different situations, and even uses the phrase "work with it on a deeper level" realizing that this way of learning empowers him to

gain knowledge that has a larger relevance and higher quality than what he gains through repetitive learning of procedures.

4.2.3 Teacher characteristics of *Equitable Access to Mathematics*

Every classroom has a diverse range of learners, with different qualifications and abilities. Teachers should work to engage *all students* in activities that support learning within core mathematical topics. Using tasks with several solutions is a possibility of enabling all students to participate and access the mathematics (Schoenfeld et al., 2014; Schoenfeld et al., 2016a).

Isaac often starts with sharing a task on the board. Or he might say that the homework was this and that, and check if we paid attention. But mostly different tasks on the board. Sometimes equations, sometimes visual number patterns, or... Or something that we just learned. And then after a while he'll walk us through the task. And then he talks a bit, explaining if anyone has questions and such. It's not like you're afraid to ask questions, you know (R2)

R2 addresses several key features of what teachers can do in order to ensure equitable access to mathematics. For one, she points out how questions that arise in the classroom get explained and followed up, "Expect[ing] and support[ing] meaningful mathematical engagement from all students, helping them contribute (...)" (Schoenfeld et al., 2016b. Appendix: p. 3). The classroom practice opens up for the students to get involved and participate through task solving and asking questions. But there is also an expectation from Peterson that all students engage in their homework, using tasks that build on the topic(s) covered in the homework, both ensuring and expecting an active participation from his students. In addition to this, the safe environment in the classroom is brought up as R2 clarifies that students aren't afraid to ask questions. There is an atmosphere where students participate, and trial and error is considered a step on the way to learning.

R2's reflections described in subsection 4.2.2 also emphasizes this support and expectation towards the mathematical engagement. In her reflections, R2 draws a line between the

different mathematics classrooms she participates in. In her regular classes, she expresses that her teacher has low expectations towards the students solving tasks, basically removing the aspect of active engagement in difficult tasks. Lower-achieving students get an “easy way out” as they are taught that not being able to solve tasks is okay. Peterson’s classes differ in the sense that there is an expectation that students engage actively in the given activities whilst still being allowed to fail. R2 expresses the problem-solving classes as a place where there is a desire of solving tasks, but also the possibility of asking questions and getting support when stuck.

Other parts of the analysis show how Peterson has “buil[t] and maintain[ed] norms that support every student’s participation in group work and whole class activities” (Schoenfeld et al., 2016b. Appendix: p. 3). R5 states that:

There’s not only one student that takes up the teacher’s time to help. And it’s more like, you get one task to work with the whole lesson, perhaps two. Instead of what I mentioned earlier, working with booklets and such. And there’s more like... Cooperation in a way. You get to ask the students around you, and work together you know. (...) In lessons with other teachers we have to be really quiet. You can work together, but it has to be quiet. But with Isaac you work two-and-two, and you can talk to the ones in front of you if they solved it and such (R5)

Again, a contrast between the different mathematics lessons emerges. The norm of the problem-solving classes is that students are encouraged to participate in groups. Primarily two-and-two, but as R5 clearly expresses the group work between students is not limited to their closest classmate but they are also encouraged to work with other students close by. R5 relates this to the classroom norm she is most used to, where cooperation is stimulated, but limited to silent work between two students. In the problem-solving classes however, the accounts of R5 gives the impression of a practice where students actively engage with the content whilst working together with other students. The fact that the teacher is supportive in the students’ collaborative work is an essential part of building a classroom norm where cooperation between students takes place. This classroom practice can also be seen in

connection with the aforementioned measure, looking for teachers to “expect and support meaningful mathematical engagement from all students, helping them contribute and build on contributions from others” (Schoenfeld et al., 2016b. Appendix: p. 3). In these classrooms, where students are encouraged to work together, the students’ statements indicate that there are possibilities for learning how to contribute and how to build on other students’ contribution while solving problems.

A point that several of the respondents mentioned was how they were expected to solve the given tasks in several different ways.

I think it’s clever. That way you don’t limit yourself to one method all the time. Because I think it’s easiest to solve [geometrical] problems through the use of similar triangles, and I think it’s hard to solve them using equations. But then it’s nice that we get pushed into solving them that way so that you learn, you know? (...) You have to think about how to solve the tasks, and you have to think “should I use similar triangles”, “should I use ratios”, “should I use the Pythagorean theorem” or whatever. And there’s no... It’s not set for you. You kind of have to arrange it for yourself (R4)

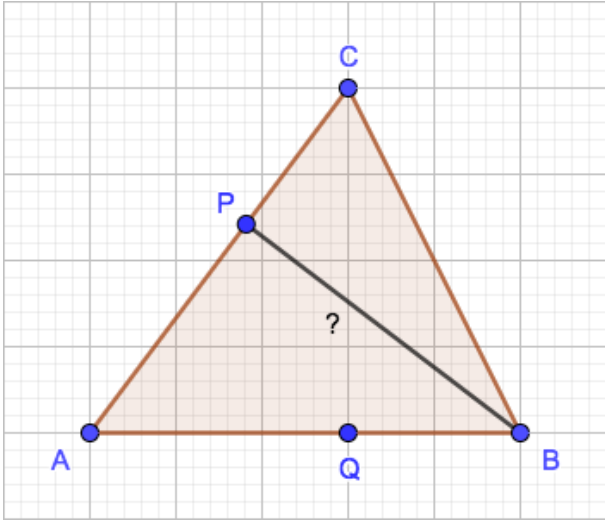
As R4 claims, she has preferred methods of solving tasks, which might often be enough. However, through tasks that have several entry points, the students are challenged and expected to solve them in ways that test more of their mathematical knowledge. Another feature of this is that using tasks that can be solved in a variety of ways opens up for a wider range of approaches, giving more students the possibility of solving tasks effectively. The importance of this is expressed through the measure “Teachers use tasks and activities that provide multiple entry points and support multiple approaches to the mathematics” (Schoenfeld et al., 2016b. Appendix: p. 3). The tasks described in 4.1 are good examples of this practice. The River-task can be solved in a large variety of ways, some are fairly easy, for example applying simple calculations with ratios. However, for the skilled students, this task could pose problems in terms of solving equations with three unknown variables, advanced use of similar triangles or solving with use of area between the triangles and the trapezoid

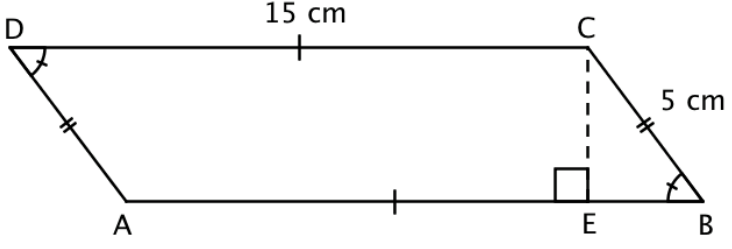
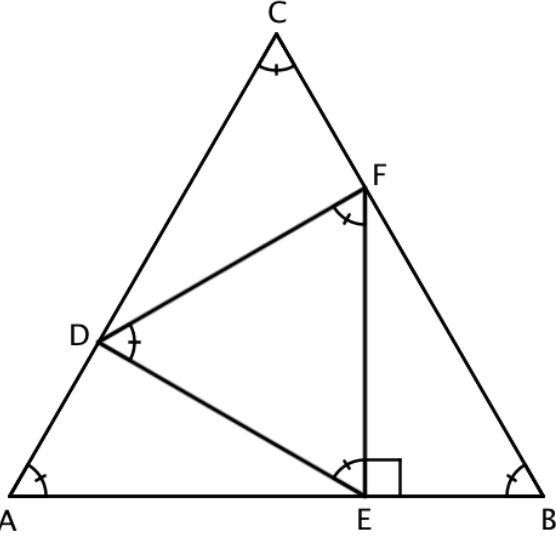
making up the campsite. The Trapezoid-task that followed also had many points of entry, enabling the students to use their newly acquired knowledge in the following task.

4.3 Student characteristics in light of TRU measures

Covering the characteristics of Peterson's teaching in these classrooms give an idea on how teaching can lay a foundation where learning can occur. However, the most important part is in fact what student outcomes that emerge from the teaching. How do the students engage and what are the characteristics of the students in these classrooms? Is there a correlation between the teacher characteristics and student engagement, or between the teacher characteristics and students' learning?

Some of the measures from the TRU OG require insight on what opportunities the students get or what abilities they acquire from these classrooms. In order to analyze this, parts of this analysis have been made on the foundation of the students' work with tasks during the interviews. The tasks presented were familiar to the students as they had worked with them before, enabling them to share their thoughts on the experiences from this work. The tasks used in the interviews were the following:

Task 1	
	$\angle AQC = \angle APB = 90^\circ.$ <p>Find the value of the unknown length.</p>
Figure 6. Task 1.	

<p style="text-align: center;">Task 2</p>  <p style="text-align: center;">Figure 7. Task 2.</p>	<p>The total area of the parallelogram is 60 cm^2.</p> <p>Find the value of the unknown length.</p>
<p style="text-align: center;">Task 3</p>  <p style="text-align: center;">Figure 8. Task 3.</p>	<p>Find the area ratio between triangles ABC and DEF.</p>

In this section, characteristics of the students in the problem-solving classrooms are analyzed through the dimensions of *Mathematics*, *Cognitive Demand* and *Equitable Access to Mathematics*. Subsections 4.3.1, 4.3.2 and 4.3.3 will cover these three dimensions respectively.

4.3.1 Student characteristics of *The Mathematics*

Activities in classrooms should provide opportunities of growth where students become good mathematical thinkers; being knowledgeable, flexible and resourceful in their approach to mathematics through working with key ideas (Schoenfeld et al., 2016b. Appendix: p. 1). In

what way did the students from the problem-solving classrooms give the impression that these activities and practices took place?

R5: I like algebra. And problem solving, like we do with Isaac. (...) We don't have time to do that (problem-solving activities) in our regular classes. But I think it's fun because it's challenging, and when you solve the tasks it is very fun. (...)

I: (...) What do you think separates problem-solving tasks from regular tasks?

R5: You get to use a lot more of the mathematics, if that makes sense? Kind of like... Uhm... Yeah, if you have to find the area you need to remember how to do that, and maybe put it into an equation, or maybe you have to use that knowledge in similar triangles, you know. You get to use all of it in one task. And it... It requires a lot more knowledge, if that makes any sense.

A measure within *Mathematics* in the TRU OG explains that "Each student engages with grade level mathematics in ways that highlight important concepts, procedures, problem solving strategies, and applications" (Schoenfeld et al., 2016b. Appendix: p. 1). R5's statements of problem-solving activities in Peterson's classes go a long way in suggesting that these features take place. First of all, it is obvious that the mathematics are relevant for the grade level, if not at a higher level (this is especially apparent with the 7th grade students from Westwood, frequently solving tasks from 10th grade exams). Further on, R5 shares her idea of the tasks and activities including a larger part of mathematics than regular tasks, claiming that they are fun due to the challenging nature of combining and drawing lines between different mathematical topics. This is a clear indication of R5's engagement towards the mathematics, being an active student and learner in these classes. With the nature of the problem-solving tasks being that students connect current knowledge in ways they have not done before, I also believe it is fair to say that they work with the mathematics in ways where important concepts, procedures and applications are highlighted. Supposing that the students apply problem-solving strategies during problem-solving classes is a fair assumption, and their use of strategies will be covered in 4.4.

Examples of work with important concepts, procedures, problem-solving strategies and applications was found during the interviews when the students were asked to solve different tasks. Working with Task 2 (Figure 7), R2 had the following reflections:

R2: Well, we're supposed to find that length. And we know the total area is 60cm^2 and that this length is 15cm and that one is 5cm , and those sides are the same and those angles are the same. And then... If you see... The whole length here is 15cm . So then you see that... This one is 5cm down, but we do not know the height. I know that it is 60 cm^2 . (...) 15 multiplied by 4 is 60 . That's how I would think. (...)

I: Okay?

R2: Yes, and then I know that the height is 4 . And then everything gets easier. Because then I know that the height is 4 , and I know the Pythagorean triples, so I know that that one is 3 . So... That's how I would solve it. Because now I know that the unknown side is 12 (R2)

Many key aspects of mathematics emerge through R2's work with Task 2. First of all, his work with the task shows that R2 is trained in reading tasks thoroughly and gathering important information. Her first action is identifying the information provided in the task, realizing that it is a parallelogram after detecting similar angles and sides on the figure. Further on she realizes that the height of the figure is not given, identifying that the area of a parallelogram is given by multiplying the base and height. With this knowledge, R2 is able to do a quick calculation ($15 \cdot 4 = 60$) in order to figure out that the parallelogram's height is 4 . The calculation she explains orally is a somewhat reduced solution of the following equation:

$$15 \cdot x = 60, \quad \frac{15 \cdot x}{15} = \frac{60}{15}, \quad x = \frac{60}{15}, \quad x = 4$$

After figuring out that the height of the triangle is 4 cm , R2 uses knowledge of Pythagorean triples ($3, 4, 5$) in order to find the length of the shortest side in the right triangle. However, using understanding of Pythagorean triples in that situation presupposes other pieces of

knowledge. First of all, R2 identifies that it in fact is a right triangle. Then she recognizes that the unknown side of the triangle was in fact the shortest side as well. Connecting these pieces of knowledge, R2 is successfully able to identify the sides in the triangle as Pythagorean triples, drawing the conclusion that the shortest side has to be 3 cm. The last part of the task is probably the easiest, subtracting 3 from 15 and finding the unknown side as 12 cm. Solving this task, R2 engages in several important aspects of mathematics, connecting *concepts, procedures, problem-solving strategies and applications* in order to being successful.

Another important measure of student activity in powerful classrooms, is to which extent each student “has opportunities for mathematical reasoning, orally and in writing, using appropriate mathematical language” and “explains their reasoning processes as well as their answers” (Schoenfeld et al., 2016b. Appendix: p. 1). Working with Task 1 (Figure 6), R1 reasoned in the following way with similar triangles:

R1: Maybe we could see if these triangles have similar shapes. See if they are similar triangles.

I: Okay, what can you tell me about that?

R1: We can check if two of the triangles have the same angles. And figure out if they are similar. (thinks) Both of these two triangles have a right angle. And they both share this one. So they are definitely similar (R1)

R1’s identification of the similar triangles is simple, but not necessarily easy. The task offers no information on the angles’ value aside from the right angles. R1 finds that two of the triangles have a right angle, and that one angle is common. His reasoning further includes that he classifies the triangles as similar: Because the last angle of each of the triangles is not shared, but because the value of the angles in a triangle always add up to 180 the last remaining angle must be the same as well. Working with Task 3 (Figure 8), R3 gives a similar reasoning process towards congruent triangles:

I: What about this task that you worked with yesterday, what can you tell me about that?

R3: Well, first of all, with the information given, the two large triangles are equilateral. And then we can say that because they are equilateral, the three smaller triangles are congruent.

I: Okay, how do you know that?

R3: Well, you can use calculations and figure out that the angles are similar. And then the longest side forming the right angle is equal in all three triangles because it is equal to the sides in the equilateral triangle DEF (R3)

The language used by R3 reveals several interesting points. First of all, through TRU measures, R3 shows confidence in mathematical terms, using them fairly precise. Secondly, R3 seems to be trained in sharing his reasoning, as he has little problems explaining the conclusions he made as he was working with the task. Last, when asked “what can you tell me about [Task 3]?”, R3’s response to the question does not include the answer, nor calculations or procedural matters. His focus is on the concepts and problems within the task, the important information and reasoning necessary to possess in order to successfully work with the task.

When it comes to explanation of reasoning processes, R2 explains that:

(...) we have to look carefully at the figure to understand the task. Not just guess or assume. In regular classes, if you had known the answer you would have just written it down and gotten it right. But here you kind of have to show what you’re doing, you know, and explain how you’re thinking. And show your calculations. (...) Sometimes you might find the correct answer, but your calculations might have been incorrect (R2)

R2 clearly addresses the expectation Peterson has for the students to reason when solving the tasks. Just solving tasks is not considered as sufficient, the students are required to properly understand and interpret the tasks, as well as being able to understand and explain their thought processes as they were solving the tasks. Engaging in the mathematics is not simply

solving tasks, but also an active use of reasoning orally and in writing, explaining thought processes as they go along.

4.3.2 Student characteristics of *Cognitive Demand*

Task 3, which was discussed in 4.4.1, was given during class the day before the interview was conducted with R3. During the interview it was revealed that he solved it another way in class than what he showed me during the conversation.

R3: Yesterday when I was working on it during class, I didn't realize that the smaller triangles had angles with values of 30, 60 and 90 degrees. So I ended up with something much more difficult.

I: So you realized that later?

R3: (Laughing) Yes! (R3)

How R3 continued his work with the task, even after class, is worth analyzing further. The task involves many difficult aspects which put R3 in the productive struggle, causing him to actively engage in the task even after the lesson had finished. The level of the task had a difficulty to where R3 was struggling with solving it, while still having the needed abilities and knowledge to solve it successfully. His work during class probably involved many other mathematical aspects while looking for a correct solution, which he found later after class had finished. First of all, this is an example of a student's continued "wrestle with an idea after the teacher leaves" (Schoenfeld et al., 2016b. Appendix: p. 2). Second, although the task was demanding and R3 probably could have solved it easily if support had been given (however, this probably would have removed the rich challenge R3 was experiencing), he continued to grapple with it on his own. According to the TRU Framework, this is a characteristic of a powerful classroom; students relentlessly working with difficult tasks in order to develop.

Another feature of the class and R3's work with this and similar tasks is how they "engage both individually and collaboratively with challenging ideas" (Schoenfeld et al., 2016b.

Appendix: p. 2). The collaborative nature of the work in Peterson's classes has been described in 4.2.3. How the students engage with challenging ideas has also been discussed, but can also be summed up by the accounts of R5's work with the same task (Task 3):

R5: I thought it was special that it had no numbers. No measurements or nothing. The angles in the triangles are the same, and these three triangles are congruent. And these two are equilateral.

I: You said that the task has no numbers, but are there other things that make it challenging?

R5: I didn't really know how to find the area of the large triangle. But then.. Yes, that was challenging, finding the area of both of them. Realizing how to do that.

I: But you did solve it?

R5: Yes, and I had tried solving it like three times and I had had some careless mistakes every time. But so much fun to finally nail it! (R5)

Through her explanation of the task, R5 shares many examples of her engagement with this particular task. It goes without saying that R5 is actively engaged with the challenging factors of Task 3. Trying and failing multiple times with the same task shows a committed participation in the mathematics. The *ideas* in this task, are the relation between the two equilateral triangles, and how to define the three smaller triangles as congruent. In addition to this, the sense of solving a task that has no numbers (the sides were not given any values) with only algebraic representations is also a challenging idea.

Another example is R4's encounter with similar triangles in Task 1:

R4: (...) I would think that... Maybe... That triangle (AQC) is similar to that triangle (ACP).

I: Yes, okay. Can you be certain?

R4: Yes. Both of the triangles have a right angle, and if you draw a line from C to Q...
They both share one additional angle. So yes, they are definitely similar.

This reasoning is very much alike R1's explanation of the same thing, which was covered in 4.3.1. The challenge in this reasoning is finding and identifying the similarities between the triangles, and ultimately proving whether or not they are to be considered similar or not.

Furthermore, in solving Task 3, R5 definitely explores "the limits of [her] current understanding(s)" (Schoenfeld et al., 2016b. Appendix: p. 3), as she encounters an unusual task without measurements, numbers or other valuable information, forcing her to step outside her common frame of task solving. The unusual nature of the task causes R5 to use her current mathematical knowledge in ways she has not done before, building what she already knows to this task with a challenging format. R4's insecurity of whether the two triangles in task 1 were similar or not can also be an indication of R4 moving towards the limit of her own understanding in order to explore the properties of the triangles. This process of reasoning of R4 and R5 in tasks 1 and 3 can be found as a measurement in the TRU OG, as "(e)ach student reasons and tests ideas in ways that connect to and build on what they know" (Schoenfeld et al., 2016b. Appendix: p. 2). These manners of reasoning have been exemplified and discussed several times in subsections 4.2.1 and 4.2.2, as the students actively work with tasks and try to connect them to current knowledge.

4.3.3 Student characteristics of *Equitable Access to Mathematics*

The dimension of *Equitable Access to Mathematics* tries to measure whether all students have the possibility of joining meaningful engagement within the mathematics. The goal is for *all* students to be engaged in the mathematical content, and it is interesting for teachers to figure out in what way more opportunities for participation can be made. Trying to look at the student characteristics is difficult, because of the dimension's focus on *all* students. The questions for the interviews were not designed to specifically answer these questions, and therefore the respondents have primarily answered with themselves in mind. However, some points did arise during the interviews, and were analyzed as aspects of *Equitable Access to Mathematics*.

The measures which consider student engagement have been described in several ways in subsection 4.2.3. R5's description on the contrast between the collaborative work in her regular mathematics classes and the problem-solving classes she has with Peterson provide valuable insight in the students' engagement as learners in these lessons. As the students interact (somewhat) freely in the investigative work of problem-solving tasks, the impression of the problem-solving lessons is one where each student "actively listens to other students and build on their ideas", and "participate meaningfully in the mathematical work of the class" (Schoenfeld et al., 2016b. Appendix: p. 3). This *meaningful participation* is further emphasized by R4's explanation of how she engages in the problem-solving classes: "A lot of his teaching is really a matter of getting and understanding everything he says and trying to learn as much as possible." (R4) If this was read out of context, R4 could be considered an inactive student who learns purely through listening to the teacher. In this setting, however, with the nature of Peterson's classes having students work in pairs or groups on challenging tasks, R4's explanation of "getting and understanding" involves an active engagement in the activities.

Through the use of problem-solving activities, the students were encouraged and expected to work in groups as mentioned earlier in this chapter. R5 further explained how she learned the best, giving examples of collaborative work with peers as her preferred approach to learning:

I feel like I learn a lot more than in these classes than I do in our regular classes. (...) I learn the most when I work in groups that have the same level as I do. And when you get to use several elements of the mathematics in the same task (R5)

This further underlines two of the traits with the classes observed; that the students were working with problem-solving tasks with several entry points, and that the students were collaboratively struggling to solve these tasks.

On being asked how he felt about working with problem-solving tasks, R1 provided the following thoughts: "It has been exciting. Because there are many different tasks that require you to use many different techniques to solve the tasks" (R1). R1's idea of work in class is explained through his excitement of the work with the problem-solving tasks. The tasks involve the use of several diverse techniques, forcing the students to connect knowledge from large parts of the curriculum in the same task. Stating that there are many different techniques

they can use, R1 describes that there are several ways of engaging and solving the different tasks, similar to what R5 mentioned in the previous paragraph. This can be considered an indication that there are many ways for the students to participate and make meaningful contributions in the classroom.

Furthermore, the four tasks explained in 4.1 were reviewed in order to give examples of the activity in the chosen classrooms. The descriptions of these tasks do, however, give valuable insight into what extent “Each student explains, interprets, applies and reflects on important mathematical ideas” (Schoenfeld et al., 2016b. Appendix: p. 3). The Tent-task, for example, is impossible to solve if they are not able to make use of important mathematical ideas. Solving the task is dependent on the understanding of key mathematical terms, the Pythagorean theorem, fractions, algebra and several other aspects. The River-task is a completely different task, but also requires the students to make interpretations and reflect on mathematical concepts. Examples from the analysis have covered that the students actively engage in the provided tasks. Evaluating the tasks as problems that require the students to interpret, reflect and explain, this can be considered implications that this measure is also fulfilled in these classrooms.

4.4 Using heuristics in problem-solving tasks

Sections 4.2 and 4.3 have covered teacher- and student characteristics in light of the TRU Framework. This section will look at one of the measures from the TRU OG, analyzing it with the help of Polya’s (2009) four phases of successfully solving problems. As mentioned in 3.3.3, this is merely a tool for helping me analyze how the students interacted with the problem-solving tasks, and is not a part of Schoenfeld et al.’s (2016a) framework.

Investigating the tasks given during the problem-solving classes could have made up a thesis in itself. Each of the mentioned seven tasks (four in 4.1 and three in 4.3) are complex problem-solving tasks, rich in concepts and designed to challenge the students in a number of ways. Going in-depth on the tasks is not my intention with this subsection. The idea is to look at how the students from the problem-solving classes worked with these tasks, and the knowledge and skills connected to their work.

Heuristics is considered an important part of becoming efficient problem solvers. Learning strategies, tools and principles to navigate through problems is of importance, as it decides how to use one's knowledge effectively when facing challenging tasks (Liljedahl, 2016; Polya, 2009; Schoenfeld, 1985). Definitions of problems were covered in Chapter 2, however solving problems is considered "(...)to search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable, aim" (Pólya, 1981, p. 117).

The first student measure within *The Mathematics* dimension of the TRU OG is that "Each student engages with grade level mathematics in ways that highlight important concepts, procedures, problem solving strategies, and applications" (Schoenfeld et al., 2016b. Appendix: p. 2). The analysis through the TRU OG gave many examples of the students' work with the mathematical content. The way in which the students faced the mathematics using problem-solving strategies, however, has not yet been discussed. Analyzing the interviews gave several implications of the students having strategic abilities when it came to solving problems. This was evident both from the students' work with problem-solving tasks, but also through their own reflections on how they usually worked in these lessons. Two of these general reflections were given by R2 and R5, generalizing the steps they made when facing problem-solving tasks. Although these reflections were cited in subsections 4.3.1 and 4.3.2, they are re-cited here for further reference:

(...) we have to look carefully at the figure to understand the task. Not just guess or assume. In regular classes, if you had known the answer you would have just written it down and gotten it right. But here you kind of have to show what you're doing, you know, and explain how you're thinking. And show your calculations. (...) Sometimes you might find the correct answer, but your calculations might have been incorrect

(R2)

R5: I thought it was special that it had no numbers. No measurements or nothing. The angles in the triangles are the same, and these three triangles are congruent. And these two are equilateral.

I: You said that the task has no numbers, but are there other things that make it challenging?

R5: I didn't really know how to find the area of the large triangle. But then.. Yes, that was challenging, finding the area of both of them. Realizing how to do that.

I: But you did solve it?

R5: Yes, and I had tried solving it like three times and I had had some careless mistakes every time. But so much fun to finally nail it! (R5)

In simple ways, R2 and R5 guide us through Polya's (2009) four phases of solving problems. First, it is important to fully understand the task at hand, properly analyzing what the goal of the task is and what information is given, which both R2 and R5 mentioned first. Then, there is a process of finding a way to the solution which might be difficult and strenuous, followed by the process of carrying out the plan for finding the solution. Explaining, applying and interpreting are important when devising a plan, as R2 emphasized the importance of explaining thought processes as she goes along and R5 stressed the challenging part of realizing how to find a way to solve it. Carrying out the plan is done as the problem has been solved and proper calculations and measurements have been made. R5's statement shows examples of how careless mistakes can affect the final answer, leading to wrong answers although the devised plan was correct. Finally, it is important to look back at the work done, as R2 explains that there might have been mistakes made along the way. Other aspects would be further understanding of the solution or being able to solve the problem in another manner (Polya, 2009), this practice has been explained in 4.2.1.

With these examples, R2 and R5 gave broad examples of how they strategically worked with problem solving tasks, navigating through Polya's (2009) four steps. In the analysis of the

mathematical content in 4.3.1, these steps were recurrent in all of the students' work with the tasks (although not all the steps in every citation).

Using Polya's (2009) first phase in his work with Task 3, R3 starts off identifying what the problem of the task is in addition to finding the important information to solving the problem: "Well, first of all, with the information given, the two large triangles are equilateral. And then we can say that because they are equilateral, the three smaller triangles are congruent" (R3). In doing this, he retrieved all the relevant and accessible information available from the task, having understood the problem and the important aspects of the task.

Furthermore, devising a plan and then carrying it out come as the next phases. These two phases are covered in R1's encounter with Task 1:

Maybe we could see if these triangles have similar shapes. See if they are similar triangles. (...) We can check if two of the triangles have the same angles. And figure out if they are similar. (thinks) Both of these two triangles have a right angle. And they both share this one. So they are definitely similar (R1)

Trying to devise a plan, R1 suggested that he could check if the triangles in Task 1 were similar. A part of Polya's second phase includes the possibility of using strategies from previous tasks, and it is possible that R1 uses that strategy in this case. From before he had encountered both the River-task and the Trapezoid-task from 4.1, both of which having solutions where using similar triangles is effective. R1 carries out his plan as he recognizes that two of the triangles have a right angle, and they have one angle in common as well. Although this is not the final solution to the task, when using this way to solve the task he had completed a major part of the solution when figuring out the two triangles were both similar. As mentioned in 4.3.2, R4 had a very similar encounter with the same task. In solving it, she also looked for similar triangles. Her strategies also showed that she looked for different ways of finding a solution, battling to devise a plan. Looking to the knowledge she had on similar triangles, she also found a plan she could use in order to solve the task.

R2's work with Task 2 sums up the whole process:

Well, we're supposed to find that length. And we know the total area is 60cm^2 and that this length is 15cm and that one is 5cm , and those sides are the same and those angles are the same. And then... If you see... The whole length here is 15cm . So then you see that... This one is 5cm down, but we do not know the height. I know that it is 60 cm^2 . (...) 15 multiplied by 4 is 60 . That's how I would think. (...) Yes, and then I know that the height is 4 . And then everything gets easier. Because then I know that the height is 4 , and I know the Pythagorean triples, so I know that that one is 3 . So... That's how I would solve it. Because now I know that the unknown side is 12 (R2)

Navigating through the task, R2 uses all of Polya's first three steps. First she investigates and ensures that she understands the task "we're supposed to find that length. And we know the total area is 60cm^2 and that this length is 15cm and that one is 5cm , and those sides are the same and those angles are the same". Moving on, she devises a plan for finding the unknown side in the parallelogram; by first finding the height of the figure. This devising of her plan is not transcribed, as she pauses and thinks. Her plan however, is to find the height of the parallelogram. In order to do this, she uses her knowledge of calculating the area of the parallelogram: "I know that it is 60 cm^2 . (Devises plan) 15 multiplied by 4 is 60 ". After this, carrying out the plan is easy mathematics for R2 as she uses her knowledge of Pythagorean triples and simple subtraction to solve the remainder of the task.

Last, it is important for the students to look back at their solution, figuring out if there is any further learning to be derived from the work done or if there are any other solutions to the problem. As I was conducting the interviews, this was not asked from the students. However, they did not express a request to do so neither. The accounts from R2 and R5 do however suggest that the students normally act in this manner, as they state that they have to show their solutions and try to solve the tasks in several different ways.

An important point to emphasize is that the selected tasks were known to some of the students, as they had already solved them prior to the interviews. With this not being their first encounters with the tasks, the validity of their answers can be discussed. However, if the tasks were to be unfamiliar and the students were to solve them on their own, it would first

of all be a lengthy process to cover in an interview. The fact that it would be different to the situation they usually would encounter the problems in is also important, as they usually had the opportunity of solving them in groups. Having the students solve problems they had encountered before provided a different valuable opportunity, however. Working on a task they had worked with before gave the students the possibility of giving detailed explanations of how they worked with it, analyzed it and went about in finding solutions.

5 Discussion and conclusion

In Chapter 4, my analysis of the data was presented. The analysis has given me the possibility of claiming something about teacher- and student characteristics in the two classrooms observed. In this chapter, I will use the findings to answer the three research questions given in Chapter 1. I will do this by connecting the findings from Chapter 4 with the literature reviewed in Chapter 2.

This chapter will be divided into four sections, where the three research questions from Chapter 1 will be answered in sections 5.1, 5.2 and 5.3 respectively. Last, in 5.4 I will conclude this thesis, giving implications of my study as well as suggestions for further research.

5.1 How the students engage

The first research question was: “In mathematics classrooms where problem-solving activities constitute the main part of teaching, how do the students engage?”. A topic that occurred frequently in the data analysis was the interactions that were made between the students as they were solving problems. This practice of collaborative work was mentioned by all the students during interviews, and the students shared details that underlined its importance. The best example of this is R5’s accounts discussed in subsection 4.3.3, as she states that she learns the most when given the opportunity of working in groups with other students that have similar mathematical abilities that she does. This practice goes hand-in-hand with a sociocultural perspective to learning, where learning takes place in social interactions (Pellegrino et al., 2012). The collective impression from the interviews was that there is a

larger degree of cooperation in the problem-solving classrooms compared to their regular classes, and as R5 stated in subsection 4.3.3 that the students were encouraged to work together, often in groups. Through moderate to demanding tasks, the aim for teachers is to position the students in a productive struggle (Kilpatrick et al., 2001; Putnam, 1987; Schoenfeld, 2013, 2014; Schoenfeld et al., 2014; Schoenfeld et al., 2016a; Stacey, 2007; Wilson et al., 2005). The term *productive struggle* seems to fit well with Vygotsky's idea of ZPD, a balance between easy and too difficult, where students understand the challenges they encounter whilst still giving them opportunities to succeed by themselves (Schoenfeld et al., 2014).

Approaching these productive struggles together with other learners would be a way for the students to enhance their learning through scaffolding each other. As mentioned in subsection 2.2.4, *scaffolding* is when adults or more capable peers support a learner in order to understand something. However, when peers who are considered equally competent in mathematics are working together in groups trying to solve problems, their diversity in abilities can also lead to scaffolding in itself. (Note: Students can be considered equally competent in mathematics, but still have different abilities within the mathematical topics.) Going from collaborative work to individual understanding is what Vygotsky considered development (Woolfolk, 2010), and devising plans for solving a problem is considered the most difficult part of problem solving (Polya, 2009). Coming up with plans together give learners the possibility of sharing thoughts, possibilities and ideas. When students actively work together with problem-solving activities, the exchange in ideas during the productive struggle may cause that each student's fundamental understanding leads to solution to problems. Using peers to scaffold is described by R5 as her preferred way of learning through her statement in subsection 4.3.3. Although there was a lot of emphasis on the student-student interactions in the analysis, there were also accounts of interactions between the teacher and the students. In section 4.1, R3 describes the teacher's scaffolding as an important part of giving all the students access to the mathematical content, as he would ensure that students that were struggling were assisted to come up with ideas that would guide them towards relevant strategies. This is related to what Polya (2009) explained as procuring bright ideas for the learners. The scaffolding from the teacher is not by giving the students a final

answer, but rather giving them ideas that help them devise proper plans in order to find solutions.

Schoenfeld et al. also use the term scaffold (Schoenfeld, 2014; Schoenfeld et al., 2014; Schoenfeld et al., 2016) in order to describe the right balance of difficulty related to the teacher's role, and how the students are to develop. For this scaffolding to occur, it implies meaningful participation from the students. This participation was analyzed as a part of an expectation from the teacher of an active involvement in the mathematics, based on accounts from R2 in 4.3.2 and 4.3.3. Boaler (2008), Cohen & Lotan (1997) and Schoenfeld (2002) (retrieved from: Schoenfeld et al., 2016a) emphasize the importance of the teachers encouraging participation, as this is a characteristic of effective teaching. These effective teachers use problems that engage all students, giving each learner access to the challenging content. Using problems with multiple entry points is one of the ways of accomplishing this (Schoenfeld et al., 2016a)

The students' involvement was described as active engagement with the content as they worked together with other students, in a classroom where this was considered the proper norm. The work was characterized by own contributions and building on others' contributions as they actively engaged in the mathematics, another important aspect of ensuring all students' equitable access to the mathematics (Schoenfeld et al., 2014; Schoenfeld et al., 2016a, 2016b). This activity of sharing contributions between peers can be related to the aforementioned thoughts of Vygotsky's ZPD and scaffolding.

The literature on knowledge types and quality mentioned in Chapter 2 described different ways and levels of understanding and learning. A desired effect of learning is that the learner receives knowledge which is diverse and can be transferred to new situations, also called deeper learning (Pellegrino et al., 2012). With a sociocultural perspective of learning in mind, a claim can be made that learning mathematics (at school) relies heavily on the relations between the students, and between students and teachers. Transferring current knowledge to other unknown domains is something that could be explained as taking place in the ZPD through scaffolding with other peers. As the students engage in groups with other students, they are able to transfer the knowledge they already possess from collaborative work to new understanding that they comprehend on their own.

This progression of learning was explained through the accounts of R3 in 4.4.2. While working on a difficult task (Task 3) with peers during class, he was unable to solve it completely. However, after the lesson was over, the learning that occurred in his interaction with the other students evolved and led to a new and better understanding of the problem. Working with his peers in class a series of cognitive structures were processed, and the internal processes that followed led to new knowledge. What started as collaborative work ended up with an individual understanding of the problem, which is what Vygotsky explained as development (Woolfolk, 2010).

R3's behavior in this case is not necessarily normal for all students, as his engagement went beyond the mathematics lesson at school. But the interviewed students did indicate an active engagement in the problem-solving classes. R5 gave examples of the active participation in Peterson's classes in subsection 4.2.3, describing the norm of these classrooms as environments where involved engagement takes place and is also expected. R2 addressed her enthusiasm of getting challenged in subsection 4.2.2, similar to the way R1 expressed excitement when solving problems in subsection 4.2.3. R3's reports of the problem-solving classes in section 4.1 underline these accounts as he describes students that are engaged in a productive struggle for 10-20 minutes at a time as they are working with tasks.

5.2 How students interact with problem-solving tasks

The second research question was: "How do the students in these classes interact with problem-solving tasks?". Analyzing how the students interacted with the three problems discussed in section 4.1 gave clear characteristics of how they dealt with difficult problems. Although no instruction on heuristics was observed nor documented through the interviews, the students provided answers that suggested that they had good abilities in strategies for solving problems, even being able to reflect on the necessary steps to take as they were battling with challenging tasks.

Because a student's strategic knowledge decides how well he/she will direct her knowledge and skills in appropriate ways when solving problems, it is essential that students have good heuristic abilities. This is an example of how Kilpatrick et al.'s (2001) different strands of

proficiency intertwine, as knowledge and skills serve little purpose if there are poor strategic abilities to accompany them. Building on Polya's (2009) four steps to solving problems, the analysis in section 4.4 pointed to the students' use of strategies when facing mathematical problems. The students seemed to be trained in carefully reading and interpreting tasks, in addition to gathering required and relevant information that the tasks posed. This helped them in gathering an idea of what needed to be done in order to solve the problem, giving them good possibilities of finding paths to solutions. When devising plans, the students looked for similarities between problems they had encountered before, in addition to using reasoning techniques to consider and control whether assumptions and suggestions were reliable and valid. Carrying out their plans, the students used their mathematical knowledge, relying on procedural fluency in order to calculate, measure and solving the problems without making mistakes. This is consistent with Kilpatrick et al.'s (2001) characteristics of being proficient in mathematics. Last, the ability of looking back at the problems was described by the students in the demand of explaining and showing calculations in order to fully understand what they had done, in addition to looking for other and better solutions to the same problems.

The students were able to use expedient strategies when facing problem-solving tasks. Although they did not clearly express that they were consciously applying strategies, the students seemed trained in solving tasks using certain steps. Their interaction was characterized by an initial investigation, followed by devising and carrying out plans for possible solutions. Through these steps the students were using their mathematical knowledge; doing calculations, connecting their knowledge in new situations and applying reasoning in order to solve the problems effectively.

5.3 How students develop powerful mathematical competency

The third research question was: "To what extent does problem solving activities promote deep learning which supports development of powerful mathematical competency?". To answer this question, it is important to first look at the analysis and the knowledge base possessed by the interviewed students. This will be addressed in section 5.3.1, before using these clarifications to answer the third research question in 5.3.2.

5.3.1 A knowledge base for solving problems

Skemp's (1976) thoughts on the word *understanding*, distinguishing the term between instrumental and relational types of understanding, in addition to Hiebert's (1986) Star's (2000, 2005, 2007) and Star & Stylianides's (2013) conceptions of knowledge are relatable to the findings from the analysis.

Understanding the important concepts and ideas of a subject is formed through students' experiences of the environments where the discipline is taught. In classrooms where the primary focus is memorizing techniques, following step-by-step procedures or acquiring automated knowledge, the students will probably not emerge with a solid understanding, nor an appreciation of the subject (Schoenfeld et al., 2016a). Learning mathematics should involve more than remembering isolated facts or formulas, as this rote type of knowledge is fragile (Kilpatrick et al., 2001; Ma, 2010; Pellegrino et al., 2012; Stacey, 2007). Forgetting how to encounter a mathematical issue can cause a student to getting stuck or simply giving up as there are no strategies for re-generating the required procedures (Schoenfeld et al., 2014). The essential understanding of a discipline should be the desired outcome of teaching, as this will lead to students having a deep comprehension and the ability to use their mathematical knowledge effectively (Schoenfeld et al., 2016a). In order for students to use the mathematics usefully, the teaching needs to be coherent and connected, with a focus on developing a comprehension of connections in mathematics and regenerative understanding (Schoenfeld et al., 2014).

Explaining the importance of making connections in the tasks in 4.3.1, R2 gave several examples of how the work in the lessons required conceptual knowledge. She clarified how it was necessary to carefully analyze the tasks to understand what was needed to be done; a clear indication that it wouldn't be possible to go through simple step-by-step procedures. Furthermore, R2 stressed the importance of explaining what she had done and what her ideas were in the same section. When students are able to explain their thought processes, the "why"-aspect of subject conception, students have acquired what Skemp (1976) explained as relational understanding or knowledge which can be defined as being conceptual (Hiebert, 1986).

Acquiring conceptual knowledge enables students to identify connections between concepts and methods. A student with conceptually based knowledge can be identified by the ability of representing mathematical situations in a number of ways and being able to use the different representations in the most beneficial way (Kilpatrick et al., 2001). As he explained the assignments he worked with during the problem-solving classes in 4.2.1, R1 reveals that the needed knowledge for working with the tasks as conceptual. Calling for the need of making connections between methods and different mathematical topics, he addresses the importance of possessing knowledge that is able to make these connections. In the same subsection, R5 made similar points whilst emphasizing the ability of using different representations when solving the tasks.

R5's answers from 4.3.1 provided information on how her procedural fluency was developed in these classes. Working with problem-solving tasks, she linked the solving with using proper procedures; in this case using square roots as a representation in different calculations. Kilpatrick et al. (2001) explained the importance of fluency within procedures being accurate and effective when performing them, as this frees capacity for the working memory (Pellegrino et al., 2012; Putnam, 1987). These procedural skills were emphasized as important parts of mathematics in the problem-solving classrooms. R5 compare her work in the problem-solving classes with the procedural work in her regular, describing her regular classes as working for lengthy periods of solving simple algebra. Solving simple tasks within algebra is, according to Skemp (1976), considered something which only requires instrumental understanding. Because it mainly consists of rules which only requires the learner to go through straight-forward calculations, being the very definition of instrumental understanding. Working with big ideas within mathematics, the students were able to work on their procedural fluency while still solving challenging tasks. In this process, they had the opportunity of attaining deep levels of knowledge within both procedural and conceptual knowledge types as described by Star (2005).

Using tasks and activities that had several points of entry was mentioned in subsection 4.2.3 as a way of ensuring that all students were participating in the mathematical work of the class. Using such tasks is also important in developing conceptual understanding on a deeper level. This was accounted for by R4 in subsection 4.2.3, explaining the practice as a way for her to certify her connections in knowledge. The students were expected to solve tasks in a number

of different ways, with several different representations and methods. In doing so, Peterson guaranteed that the students would look at the selected problems from different points of view and used a larger array of procedures when answering the different tasks. This is a way of ensuring that the students go through Polya's (2009) fourth phase of solving problems, as they review their current solution and find new and better ways of solving the problems. Through this manner of teaching, the students would develop their mathematical understanding within several domains and connect these pieces of understanding together in work with the same activity.

R3's descriptions in subsection 4.2.2 explain the cognitive struggle he encounters in the different mathematics classes he participates in. As mentioned in the analysis, his thought on the problem-solving lessons are closely related to aspects of deeper learning. The work he does in his usual classes can be looked upon as procedural in nature, describing amounts of repetitive tasks that follow the same pattern in solutions (Kilpatrick et al., 2001; Hiebert, 1986; Star, 2000, 2005, 2007). This is similar to R2's reflections in the same subsection; giving clear indications of the procedural work in her regular classes as opposed to the challenging conceptual work being done in Peterson's problem-solving classes.

The analysis suggests that the students in the problem-solving classes experienced mathematics that focused on key ideas and aspects of mathematics. The teacher- and student characteristics of the three TRU dimensions which were analyzed in Chapter 4 gave accounts from the students, where the mathematical content was examined as being connected and coherent. The content was described as activities where they needed to use several parts of the mathematics and connecting the topics together, emphasizing that the tasks and activities required "more knowledge" than regular tasks. This is consistent with claims that the teaching needs to be connected and coherent in order for the students to use their mathematical knowledge usefully (Schoenfeld et al., 2014; Schoenfeld et al., 2016a, 2016b), as well as Star's (2000, 2005, 2007) emphasis on attaining deep levels of both conceptual and procedural knowledge. The students faced tasks that had multiple points of entry and the activities focused on key ideas within the discipline, building on theory stating that mathematics needs to be centered around important ideas and provide access to all students in order for the teaching to be viewed as successful (Schoenfeld et al., 2014; Schoenfeld et al., 2016a, 2016b).

5.3.2 Deep learning as a result of problem solving

Schoenfeld (1985) narrowed down successful problem solving into four categories, with Stacey (2005) defining seven factors that all come in to play when working with mathematical problems. As mentioned in Chapter 2, this thesis has its focus on factors of (deep) mathematical knowledge, general reasoning abilities and heuristic strategies. The traits of the students' mathematical knowledge have already been argued in subsection 5.3.1, as well as the students' use of heuristics which were discussed in section 5.2. In this last subsection, I will take a look at the students' use, and need for, reasoning abilities in the problem-solving classes, connecting this to acquisition of deep knowledge.

After learning new concepts and procedures, students are required to practice them for a period of time in order to become fluent in using them, becoming able to validate and explain them in relation to other concepts and procedures they already possess (Kilpatrick et al., 2001; Newell, 1990; Rosenbloom and Newell, 1981). Reasoning is the process of steering through the different methods, procedures and concepts a learner has, trying to fit them together and making sense of the connections (Kilpatrick et al., 2001). It is far more comprehensive than just seeing connections, however. When solving mathematical problems, a student might use heuristics in order to find the correct strategy of solving a problem, but the reasoning process is what decides if a chosen strategy is considered legitimate for use in a given situation. This is also important when choosing procedures for calculating an answer, as reasoning plays a role in determining if an approach is believed to be suitable or not (Kilpatrick et al., 2001). In the TRU Framework, reasoning abilities play an important role in students' possibilities of becoming "knowledgeable, flexible, and resourceful disciplinary [mathematical] thinkers" (Schoenfeld et al., 2016b. Appendix: p. 1). Within the three selected dimensions of the TRU OG, reasoning abilities in students emerge through the measures "Each student has opportunities for mathematical reasoning, orally and in writing, using appropriate mathematical language", "Each student explains their reasoning processes as well as their answers" and "Each student reasons and tests ideas in ways that connect to and build on what they know" (Schoenfeld et al., 2016b. Appendix: p. 1-2).

A goal for future teaching is that students gain a deeper learning of the content being taught. Through transfer of what (knowledge) has already been learned to new situations, the effect of deeper learning occurs. If students develop this transferable knowledge, they will have a larger probability of applying using this knowledge to solve problems or to react well to new situations (Pellegrino et al., 2012). The idea of gaining transferable knowledge relies on the possibilities learners get to acquire this understanding. I will argue that there is a connection between mathematical reasoning abilities and deeper knowledge. Through reasoning, learners use and navigate their current understanding into new situations, much like the explanation of transferable knowledge's importance within deeper knowledge. Therefore, an essential part of being able to transfer mathematical knowledge is the acquisition of sound reasoning abilities, gaining the power to understand, explain and direct the different skills in ways that ultimately connect them in correct ways.

So, did the activity in the problem-solving classes promote the students' possibilities of developing good reasoning abilities? R1 and R4, working with Task 1 in 4.4.1, both reason in the same way when they try to determine whether or not two triangles are similar or not. Their reasoning involves investigation of the task at hand, examining and discovering properties of the triangles in order to draw conclusions. Their current knowledge involved that the angles in all triangles add up to be 180° . Furthermore, they discovered that both triangles had a right angle, and one angle was common between them. Through reasoning, they concluded that the last angle had to be the same as well. This reasoning process is similar to what can be used in the River-task covered in 4.1, but also elements of the Trapezoid-task covered in the same section.

As R2 was solving Task 2, she used reasoning consistently as she was working to find the answer. One example is her path to finding the height of the parallelogram. Recognizing that the total area was 60 cm^2 and the base was 15 cm, she quickly realized that the height had to be four. This is a clear indication that R2 has the ability to think logically between relationships in concepts and procedures, turning around the formula for finding the area of a parallelogram. If her understanding of the formula had been on a rote level, she probably would not have been able to determine the height through calculations.

Working with Task 3, reasoning is used by R3 in order to decide the properties of the triangles in the task. First of all, he identified that the two larger triangles are equilateral. On his further path to solving the task, he reasons his way to understand that the smaller triangles are congruent. This reasoning consists of two steps. First, like R1 and R4 did in task 1, he used the properties of triangles' sums to state that the three smaller triangles were similar. Following this, he correctly reasons that with the longest side in the right angle of all the three triangles being equal in length, the triangles must be congruent. In doing so, R3 was able to "navigate through the many facts, procedures, concepts, and solution methods and to see that they all fit together in some way" (Kilpatrick et al., 2001, p. 129).

Having a complex range of knowledge is essential in achieving deeper learning. "Deeper learning involves coordinating all five types of knowledge" (Pellegrino et al., 2012, p. 84). The third research question looked for the relationship between using problem solving activities and achieving deep learning. Through the analysis, I believe I have given examples that the students needed Kilpatrick et al.'s (2001) strands of mathematical proficiency when solving problems. One strand, *productive disposition*, has not found its place in the literature review or in the analysis; as it deals with students' sensemaking and beliefs within mathematics. This was not a topic during the interviews and has not received focus in this thesis. Aside from this, I believe the data has shown indications that solving problems puts students in situations where they apply and develop knowledge within procedures and concepts, strategic competence and reasoning abilities; and constantly work with these types of knowledge in relation with each other.

5.4 Conclusion

My research started as a curious investigation of Isaac Peterson and his teaching at Lakeside. Through a unique curriculum design, together with the use of problem-solving activities in his classrooms, his students seemed to consistently deliver excellent results on the 10th grade exams. Results in mathematics can be measured in a variety of ways, testing being one of them. The mathematics exams on the 10th grade should give an indication to how competent the students are at mathematics, but there are definitely abilities that cannot be measured through a single, written exam.

This thesis has tried to answer questions to what type of learning emerges from classrooms that focus on problem solving. Using literature that discusses a desired type of knowledge base to emerge from mathematics teaching, in addition to a quality of knowledge that promotes deeper learning, I have tried to make the case for important aspects of learning mathematics. Building on this, I have looked at research on problem solving and analyzing the collected data through a framework designed to specify attributes of classrooms that promote high levels of student learning.

The observed classrooms which were observed at Lakeside and Westwood received the same characteristics from the interviewed students on what type of student-involvement was expected and found place in the lessons. As mentioned in Chapters 2, 4 and 5, this thesis takes a sociocultural perspective to learning, a perspective that proved to be very fitting in the two problem-solving classrooms at Lakeside and Westwood. Through work with problem-solving activities, the students were given the opportunity and was encouraged to work in pairs or larger groups to enhance their chances of successfully solving problems. Accounts from the students described classrooms where the students would actively and openly listen and discuss ideas with other students, building on each other's ideas and approaches. Vygotsky's views on learning in social interactions, ZPD and the aspect of scaffolding (Baltzer, 2011; Hedegaard, 1993; Woolfolk, 2010) is important in this context, as the results showed that the learning that took place was relatable to these characteristics.

The degree of student involvement was part of classroom norms that had evolved from Peterson's expectations that all students should engage and contribute. But this also had a mathematical aspect, expecting all students to engage actively with the mathematical activities provided. The best example of this is the account of the student who brought his struggle with a mathematical problem outside the classroom and kept struggling with it after the lesson was over. All of the mentioned characteristics of classroom behavior are quality measures of powerful classrooms from the TRU OG.

Analyzing how the students interacted with the three tasks from section 4.1 gave clear characteristics of how they dealt with difficult problems. Although no instruction on heuristics was observed nor documented through the interviews, the students provided answers that suggested that they had good strategies for solving problems, they were even able to reflect

on the necessary steps to take as they were battling with challenging problems. As they were using strategies for solving problems, their mathematical competency was used to calculate, connect domains of knowledge and reason, all in order to solve the problems faced.

A focus for future teaching is for students to acquire deeper learning (Pellegrino et al., 2012). Deep levels of learning occur when students have knowledge in concepts, procedures, are able to reason, application (through strategies) and see sense in the discipline (Ludvigsen, 2015; Pellegrino et al., 2012). So what about the learning that emerged from the problem-solving classes at Lakeside and Westwood? I believe that the analyzed data showed that the students' knowledge in concepts, procedures, reasoning and strategies in mathematics were visible teaching outcomes Peterson's classrooms. The students seemed to acquire a profound understanding of mathematical connections, being able to apply their knowledge in challenging and comprehensive problem-solving tasks, consistent with Ma's (2010) ideas on mathematical knowledge. Having the ability to transfer current knowledge to new situations is considered a prerequisite for gaining deep knowledge, and this knowledge occurs when the transfer occurs together through reasoning and understanding in new situations. Through my analysis and discussion, I have tried show that these factors emerged from the classrooms at Lakeside and Westwood.

Schoenfeld (2013) defined a "powerful mathematics classroom" as a classroom where students emerge with good performances within mathematics tests and problem solving. Through the research questions, the question which would remain is whether there are sufficient indications that the observed problem-solving classrooms at Lakeside and Westwood could be considered powerful. The students did not go through tests as part of this thesis, so that part of Schoenfeld's definition is impossible to evaluate. I do believe, however, that there are indications of the students in those classes performed well when facing problems. Through social interactions with other students, they solved challenging mathematical tasks using appropriate knowledge and strategies.

Pellegrino et al.'s summary on research of American mathematics classrooms is a reasonable balance to the subjects discussed in this thesis:

These studies present a remarkably consistent characterization of mathematics teaching (...) in the US: Students generally work alone and in silence, with little

opportunity for discussion and collaboration (...) They focus on low level tasks that require memorizing and recalling facts and procedures rather than tasks requiring high-level cognitive processes, such as reasoning about and connecting ideas or solving complex problems” (Pellegrino et al., 2012, p. 128).

The students in the American classrooms, who consistently have delivered worse results than hoped (Kilpatrick et al., Ma, 2010; Pellegrino et al., 2012), appear to have opposite characteristics to the interviewed students from Lakeside and Westwood. They do not work in social interactions with other students, removing the aspect of learning in relation with others. This way, the tasks they encounter do not put them in their ZPD (Baltzer, 2011; Hedegaard, 1993; Woolfolk, 2010); as the classroom practices focus primarily on reproducing facts and methods, only giving the students opportunities of acquiring superficial procedural knowledge (Star, 2000, 2005), and a limited understanding of concepts, connections and reasoning within the discipline (Kilpatrick et al., 2001; Ludvigsen, 2015; Pellegrino et al., 2012).

5.4.1 Implications

Completing this study, what are the general results? The students gave accounts of challenging problem-solving lessons where they were expected to engage (in groups), and forced them to look at the problems from different angles and being required to use several parts of the mathematics in the same problem. This way they were constantly working with different topics and different domains of knowledge at the same time, as opposed to the mathematics being seen as disconnected fragments within the same discipline. Having this approach caused the students to see connections and being aware that they were in fact working with connecting the different parts together in the different tasks. Through problem solving, the students also needed to use proper strategies to solve the problems at hand. All of these traits are important factors in attaining deep mathematical knowledge.

Looking back to Stacey’s (2005) research on curriculums, it is interesting to discuss problem solving as a means or end result of learning mathematics. Lester jr (2013) argued for problem solving being a means for attaining mathematical competency, and this thesis can also imply

that problem solving as a means approach to mathematics teaching can result in students obtaining deep levels of knowledge and ultimately a powerful mathematical competency.

Another implication is the effect of student interaction when engaging in problem-solving activities. Through active collaboration with peers, the students gave accounts that they were able to discuss and learn from each other in a constant exchange of knowledge. This seemed to be an important aspect of the students' learning, as they benefitted from the joint knowledge in the group work.

5.4.2 Suggestions for further research

There are definitely limitations in this study, and other aspects within problem solving would have been interesting to research. Through this thesis, two classrooms have been observed and only five students were interviewed. In addition, the same teacher taught in both of these classes, possibly making the claim that the teacher characteristics could be the key to the students' learning and not the method used. Having no control group or comparisons with other teachers, students and classrooms that focused on other methods weakens the results of this study. In addition to this, the observed classrooms had no low-achieving students.

Lakeside's curriculum design mentioned in 1.4 is meant to build the students' mathematical knowledge in a way that constantly connects and intertwines different types of knowledge and skills, ultimately leading to the students becoming good at solving problems. Would the profit of problem solving be the same if the curriculum design had not been laid out in a way where good problem-solving abilities was a key objective?

Another factor worth researching is what type of motivation occurs in mathematics classrooms where problem solving is the main focus. Motivation has been mentioned briefly in this thesis, but in no means to the extent that could be done in a future study. Does the problem-solving approach lead to a higher degree of motivation in the students, ultimately leading to more learning?

5.4.3 Concluding remarks

I have tried to argue that a problem-solving approach to teaching has the potential to enable students in gaining deep, conceptual knowledge, and these learning outcomes could be interesting to pursue further.

As R5 stated when comparing her two different mathematics classes:

“[W]e don’t have time to do [problem solving] in our regular classes.”

Through my work with this thesis, I have learned that problem solving can lead to valuable effects on learning in mathematics. Ending a thesis where there are implications that problem solving should be more of a means- than an end result of mathematics teaching, I intend to find time for problem solving in my future mathematics teaching.

6 References

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Vår ref: 56313 / 3 / BGH

Deres dato:

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Vurdering fra NSD Personvernombudet for forskning § 31

Personvernombudet for forskning viser til meldeskjema mottatt 02.10.2017 for prosjektet:

56313

Behandlingsansvarlig

Daglig ansvarlig

Student

Høgskolen i Oslo og Akershus, ved institusjonens øverste leder

Annette Hessen Bjerke

Gunnar Voigt Nesbø

Vurdering

Etter gjennomgang av opplysningene i meldeskjemaet og øvrig dokumentasjon finner vi at prosjektet er meldepliktig og at personopplysningene som blir samlet inn i dette prosjektet er regulert av personopplysningsloven § 31. På den neste siden er vår vurdering av prosjektopplegget slik det er meldt til oss. Du kan nå gå i gang med å behandle personopplysninger.

Vilkår for vår anbefaling

Vår anbefaling forutsetter at du gjennomfører prosjektet i tråd med:

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Dersom prosjektet endrer seg, kan det være nødvendig å sende inn endringsmelding. På våre nettsider finner du svar på hvilke [endringer](#) du må melde, samt endringskjema.

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Vi tar kontakt om status for behandling av personopplysninger ved prosjektslutt

Dokumentet er elektronisk produsert og godkjent ved NSDs rutiner for elektronisk godkjenning.

Ved prosjektslutt 01.06.2018 vil vi ta kontakt for å avklare status for behandlingen av personopplysninger.

Se våre nettsider eller ta kontakt dersom du har spørsmål. Vi ønsker lykke til med prosjektet!

Marianne Høgetveit Myhren

Belinda Gloppen Helle

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Vedlegg: Prosjektvurdering

Kopi: Gunnar Voigt Nesbø,

Personvernombudet for forskning



Prosjektvurdering - Kommentar

Prosjektnr: 56313

INFORMASJON OG SAMTYKKE

Elevenes foresatte informeres skriftlig og muntlig om prosjektet og samtykker til deltakelse.

Informasjonsskrivet er godt utformet. Vi minner om at læreren må få tilsvarende informasjon, muntlig eller skriftlig.

BARN I FORSKNING

Deler av utvalget i prosjektet er barn og unge, og det er foreldrene deres som samtykker til deltakelse. Likevel bør barna få informasjon om prosjektet som er tilpasset deres ordforråd. Det er også viktig at barna og ungdommene får informasjon om at de kan velge å ikke delta i prosjektet hvis de ønsker det, selv om foreldrene har samtykket.

REKRUTTERING OG DATAINNSAMLING

Mens skole er en obligatorisk arena for elever og foresatte skal deltakelse i forskning være frivillig.

Forespørselen må derfor alltid rettes på en slik måte at de forespurte ikke opplever press om å delta, gjerne ved å understreke at det ikke vil påvirke forholdet til skolen hvorvidt de ønsker å være med i studien eller ikke.

Videre bør det planlegges et alternativt opplegg for de som ikke deltar.

I følge meldeskjemaet skal elever til intervjuene velges ut etter at studenten har gjennomført observasjon i klasserommet. Vi minner om at det under observasjonen ikke skal samles inn personopplysninger om elever, hvor foreldrene ikke har samtykket til deltakelse. Videre minner vi om at det bare er elever hvor foreldrene aktivt har samtykket som skal inviteres til intervjudelen.

INFORMASJONSSIKKERHET

Personvernombudet legger til grunn at forsker etterfølger Høgskolen i Oslo og Akershus sine interne rutiner for datasikkerhet. Dersom personopplysninger skal lagres på privat pc, bør opplysningene krypteres tilstrekkelig.

PROSJEKTLUTT OG ANONYMISERING

Forventet prosjektlutt er 01.06.2018. Ifølge prosjektmeldingen skal innsamlede opplysninger da anonymiseres. Anonymisering innebærer å bearbeide datamaterialet slik at ingen enkeltpersoner kan gjenkjennes. Det gjøres ved å:

- slette direkte personopplysninger (som navn/koblingsnøkkel)
- slette/omskrive indirekte personopplysninger (identifiserende sammenstilling av bakgrunnsopplysninger som f.eks. bosted/arbeidssted, alder og kjønn)
- slette digitale lydopptak

Forespørsel om deltakelse i forskningsprosjektet

”Bruk av problemløsning i matematikkfaget for å fremme læring.”

Bakgrunn og formål

Undersøkelsen er del av en masterutdanning ved Høgskolen i Oslo og Akershus. Formålet med studien er å se på hvordan problemløsning kan brukes som metode for å undervise i matematikk, samt hvordan dette påvirker motivasjon hos elevene. Problemløsning er noe som ofte gjøres i større grad på ungdomstrinnet, og det er derfor interessant å hente informasjon om hvordan elever på mellomtrinnet opplever slik undervisning også.

Det er ønskelig å ha med elever som mestrer faget på ulike nivåer og områder, og etter observasjon i undervisningen blir enkelte elever forespurt om å delta i intervju.



Hva innebærer deltakelse i studien?

Deltakelse i studien innebærer å la seg observere i undervisning, samt delta på et intervju. Intervjuet vil tas opp på lydopptak. Spørsmålene vil dreie seg om motivasjon i matematikkundervisningen og i hvilken grad de opplever utfordringer, mestring og utvikling. Det vil **ikke** innhentes informasjon fra andre kilder (registre, journaler, elevmapper etc), og svar fra intervju vil anonymiseres.

Som foreldre til barn som intervjues i en slik studie kan man på forespørsel få se spørreskjema/intervjuguide.

Hva skjer med informasjonen om deg?

Alle personopplysninger vil bli behandlet konfidensielt. Kun student vil ha tilgang til personopplysningene og svarene som kommer frem i intervju og observasjon. For å sikre anonymitet vil opptak anonymiseres ved hjelp av koblingsnøkkel som lagres adskilt fra øvrige data. Deltakerne vil ikke kunne gjenkjennes i publikasjon av masteroppgaven.

Prosjektet skal etter planen avsluttes 01.06.2018. Personopplysninger anonymiseres ved prosjektslutt.

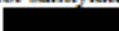
Frivillig deltakelse

Det er frivillig å delta i studien, og du kan når som helst trekke ditt samtykke uten å oppgi noen grunn. Dersom du trekker deg, vil alle opplysninger om deg bli anonymisert.

Dersom du ønsker å delta eller har spørsmål til studien, ta kontakt med Gunnar ~~Voigt~~ Nesbø på e-post s314761@stud.hioa.no. Om det er spørsmål til ansvarlige ved Høgskolen i Oslo og Akershus kan veileder i masteroppgaven, Annette Hessen Bjerke, nås på annette.hessen@hioa.no.

Studien er meldt til Personvernombudet for forskning, NSD - Norsk senter for forskningsdata AS.

Samtykke til deltakelse i studien

Det er ønskelig med tilbakemelding på e-post for samtykke til deltakelse i studien. Om man ønsker å gi samtykke på annet vis, ta kontakt på telefon  for å gjøre en avtale.

Appendix 4 – Interview guide with key questions

Intervjuguide elever

Bakgrunnsspørsmål:

Alder, tid eleven har gått på [REDACTED]. Enkle spørsmål rundt motivasjon, fag og skole generelt.

1. Hva liker du med matematikk? Er det noen emner i matematikk du er spesielt flink i?
2. Hvordan lærer du best i matematikk?
3. Du har nå hatt [REDACTED] i litt over et år. Hvordan er disse timene sammenlignet med undervisningen du har i andre timer?
4. Fortell om en typisk time med [REDACTED]. Hva gjør dere? Lærer du noe? Hvordan er aktiviteten din i timen?
5. Du har [REDACTED] som lærer en gang i uken. Hvordan er matematikkundervisningen lagt opp de dagene hvor han ikke er tilstede?
6. På hvilken måte arbeider du i matematikk på egenhånd?
7. Jeg la merke til at [REDACTED] gjorde _____ i timen. Fortell meg om det!
8. Denne oppgaven arbeidet dere med i en av timene jeg observerte. Hva kan du fortelle meg om oppgaven? Kan du forklare meg hvordan man kan løse den?