On the relativity of rotation

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Abstract

The question whether rotational motion is relative according to the general theory of relativity is discussed. Einstein's ambivalence concerning this question is pointed out. In the present article I defend Einstein's way of thinking on this when he presented the theory in 1916. The significance of the phenomenon of perfect inertial dragging in connection with the relativity of rotational motion is discussed. The necessity of introducing an extended model of the Minkowski spacetime, in which a globally empty space is supplied with a cosmic mass shell with radius equal to its own Schwarzschild radius, in order to extend the principle of relativity to accelerated and rotational motion, is made clear.

1. Introduction

The extension of the principle of relativity from rectilinear motion with constant velocity to accelerated and rotational motion was an important motivating factor for Einstein when he constructed the general theory of relativity.

He was inspired by the point of view of E. Mach who wrote in 1872 [1]: “It does not matter whether we think of the Earth rotating around its axis, or if we imagine a static Earth with the celestial bodies rotating around it.” Mach further wrote [2]: “Newton’s experiment with the rotating vessel of water simply informs us that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative rotation with respect to the mass of the Earth and the other celestial bodies.”

Today physicists are somewhat ambivalent as to whether Einstein succeeded in constructing a theory according to which accelerated and rotational motion is relative. The question was discussed already in 1916, when Einstein presented the general theory of relativity. After having read Einstein’s great article [3] in 1916 the astrophysicist W. De Sitter was not in doubt. He wrote [4]: “Rotation is thus relative in Einstein’s theory. For Einstein, who makes no difference between inertial and gravitation, and knows no absolute space, the accelerations which the classical mechanics ascribed to centrifugal forces are of exactly the same nature and require no more and no less explanation, than those which in classical mechanics are due to gravitational attraction.” But, as mentioned by J. Illy [5], he warned us that the relativity of rotation does not imply that the fundamental difference between rotation and translation will disappear. Translation may be eliminated globally by a Lorentz transformation, but rotation may not (see below). Newton interpreted this difference by introducing the concept of absolute space; Einstein, however, by not distinguishing between centrifugal and gravitational forces.

In the present article I will discuss conditions that must be fulfilled in order that the principle of relativity as applied to rotating (and accelerating) motion shall be contained in the general theory of relativity.

2. Einstein, De Sitter and relativity of rotation

When Einstein presented this theory in 1916 [3], he wrote in the Introduction that the special theory of relativity contains a special principle of relativity, where the word “special” is meant to say that the principle is restricted to motion of uniform translation.

The second paragraph of Einstein’s great 1916-article is titled: “The need for an extension of the postulate of relativity”. He starts by writing that the restriction of the postulate of relativity to uniform translational motion is an inherent epistemological defect. Then he writes: “*The laws of physics must be of such a nature that they apply to systems of reference in any kind of motion*. Along this road we arrive at an extension of the postulate of relativity”.

Furthermore Einstein makes use of the principle of equivalence, according to which the physical effects of inertial forces in an accelerated reference frame K' are equivalent to the effects of the gravitational force in a frame K at rest on the surface of a massive body. Einstein asks: Can an observer at rest in K' perform any experiment that proves to him that he is "really" in an accelerated system of reference? He says that if the principle of equivalence is valid, then this is not possible. Then he states: "Therefore, from the physical standpoint, the assumption readily suggests itself that the systems K and K' may both with equal right be looked upon as "stationary"".

P Kerzberg [6] has given a very interesting discussion of "The relativity of rotation in the early foundations of general relativity". In particular he reviews and comments an article published in 1917 titled "On the relativity of rotation in Einstein's theory" by W. De Sitter [4]. Kerzberg writes "De Sitter thus maintains that rotation is relative in Einstein's theory, and even as relative as linear translation. Both rotation and translation are susceptible to being transformed away. Nonetheless a difference persists". He then cites De Sitter: "If a linear translation is transformed away (by a Lorentz transformation), it is utterly gone; no trace of it remains. Not so in the case of rotation. The transformation which does away with rotation, at the same time alters the equation of relative motion in a definite manner. This shows that rotation is not a purely kinematical fact, but an essential physical reality."

In this way one is lead to the point of view that according to Einstein's theory, rotation is always relative to observable masses, and that this is a sort of constraint on the very *content* of the universe. De Sitter says: "The condition that the gravitational field shall be zero at infinity forms part of the conception of an absolute space, and in a theory of relativity it has no foundation". Kerzberg comments: "In the theory of general relativity, such ideal boundary conditions should be replaced by physical conditions on space-time", and he further notes that in this theory distant matter is the physical support of the Minkowski spacetime. He then writes: "The search for a physical interpretation of boundary conditions is now seen as vital to the survival of Mach's principle". A necessary boundary condition for the validity of the principle of relativity for rotational motion will be given in section 8 below.

In a lecture in Leiden in 1920 Einstein seemed to give up the relativity of rotation [7]. He discussed the question whether the surface of the water in a bucket would change its shape as a result of a change of rotational velocity. Assume that the principle of relativity is valid for rotation. Then an observer in the water can consider the water as at rest and the mass of the universe as rotating. Inertial dragging due to the rotating matter would then be the cause of the changing surface of the water. But this requires, said Einstein, instantaneous action at a distance. Hence he argued for the existence of some sort of "ether" that conveys the inertial dragging effect. Also he argued that the water would change its shape even in the absence of the remote cosmic matter. Hence he reintroduced absolute rotation.

I will argue for the point of view that Einstein had in 1916 and which was supported by De Sitter, that the principle of relativity is valid for rotation. It is a consequence of Einstein's field equations that there is inertial dragging inside a rotating shell of matter, and that there is perfect dragging inside a shell with radius equal to the Schwarzschild radius of the cosmic mass inside the shell. But the dragging is smaller the smaller mass the shell has for a given radius, and vanishes if the shell is removed. In principle this can be tested experimentally by performing a "Gravity probe B experiment" inside a rotating shell. This inertial dragging is a prediction of the general theory of relativity which is what we are concerned about here – whether the principle of relativity is valid for rotation according to the general theory of relativity.

Let us first consider Einstein's 1920-argument about action at a distance. Assume that the angular velocity of the water relative to the cosmic masses increases, and consider the situation from the point of view of an observer following the water. He would then see the angular velocity of the cosmic mass increasing. However the actual increase of the angular velocity happened at the point of time when the received light was emitted, at the retarded time. Hence it is sufficient that the gravitational action is moving with the speed of light. Instantaneous action at a distance is not needed.

Let us then think about what happens, according to the theory of relativity if the cosmic mass is made less and then removed. The solution of Brill and Cohen [8] gives the dragging angular velocity inside a massive shell with radius r_0, Schwarzschild radius R_S, and which is observed to rotate with an angular velocity ω,

$$\Omega_Z = \frac{4R_S\left(2r_0 - R_S\right)}{\left(r_0 + R_S\right)\left(3r_0 - R_S\right)}\omega \ . \tag{1}$$

Assume that initially the Schwarzschild radius of the shell is equal to its radius, so that there is perfect dragging inside it. The surface of the water would be flat if the water is at rest in an inertial Zero Angular Momentum (ZAMO) frame. The shape depends upon the angular velocity of the water relative to the ZAMO inertial frame. An observer at rest in the water would observe that the cosmic shell has an angular velocity ω, and the ZAMO inertial frame has an angular velocity Ω_Z. Initially $\Omega_Z = \omega$ and the surface of the mass is maximally curved. The Brill-Cohen formula shows that if the cosmic mass is made less, the angular velocity of the ZAMO inertial frames would decrease relative to the water, and when there was no cosmic mass they would be at rest relative to the water. Hence the shape of the water would flatten out as the mass of the cosmic mass decreased.

The conclusion is that the general theory of relativity predicts that there would not be any change of the surface of the water if one tried to put it into rotation in an empty universe. Such an effort would not succeed. The water would not begin to rotate because there is nothing it can rotate relative to.

3. A cosmic time effect

In the Hafele-Keating experiment [9,10] the travelling time around the Earth as measured on a clock travelling in the same direction as the Earth rotates, a clock moving in the opposite direction, and one at rest on the surface of the Earth was compared. The result was that the clock travelling in the same direction as the Earth rotates showed shortest travelling time, and the one moving in the opposite direction with the same velocity relative to the surface of the Earth, showed the longest travelling time.

Particular focus was given to the 'East-West effect', i.e. that the travel time measured by a clock during circumnavigation of the Earth depends both on the direction of the circumnavigation and on the Earth's rotational speed.

Hafele deduced the proper time shown by the clocks by employing a non-rotating reference frame in which the Earth rotates [11]. In the Galilean approximation the velocity of the clocks in this frame is

$$u = (R + h)\omega \pm v \quad , \tag{2}$$

where R is the radius of the Earth, h is the height of the orbit, ω is the angular velocity of the diurnal rotation of the Earth, and v is the velocity of the airplane with plus for travelling westwards and minus eastwards. The East-West effect then comes from the usual kinematical time dilation factor $\sqrt{1 - u^2 / c^2}$.

The Hafele-Keating experiment may be thought of as a temporal version of the Foucault pendulum, making it possible to measure the rotation of the reference frame in which the experiment is performed, i.e. the rotation of the Earth. Hence, it invites to an interpretation where rotation is absolute.

Let us consider the situation with one clock A at rest and one B in circular motion in the Schwarzschild spacetime from the point of view of a rotating reference frame in which the clock B is at rest. A set of comoving coordinates $(t',\ r',\ \theta',\ \phi')$ in the rotating reference frame, is given by the transformation

$$t' = t \ , \quad r' = r \ , \quad \theta' = \theta \ , \quad \phi' = \phi - \omega\, t \ . \tag{3}$$

Here $\omega > 0$ represents the angular velocity of the reference frame. Note that the coordinate clocks showing t' goes at the same rate independent of their distance from the origin. For simplicity we assume that the clock B performs orbital motion at a constant radius r_0 in the equatorial plane for which $\theta' = \pi / 2$. Then the line element in the rotating reference frame along the path of the twins takes the form

$$ds^2 = -\left(1 - \frac{R_S}{r_0} - \frac{r_0^2 \omega^2}{c^2}\right) c^2 dt'^2 + r_0^2 d\phi^2 + 2 r_0^2 \omega\, d\phi\, dt \ . \tag{4}$$

For timelike intervals the general physical interpretation of the line element is that it represents the proper time $d\tau$ between the events connected by the interval,

$$ds^2 = -c^2 d\tau^2 \ . \tag{5}$$

It follows that the proper travelling time measured by the clock A is

$$\tau_A = \left(1 - \frac{R_S}{r_0} - \frac{r_0^2 \omega^2}{c^2} - \frac{r_0^2 \Omega^2}{c^2} - \frac{2 r_0^2 \omega \Omega}{c^2}\right)^{1/2} \Delta t' \ , \tag{6}$$

where $\Omega = d\phi' / dt'$ is the angular velocity of A in the rotating reference frame. The travelling time of B, having $\Omega_B = 0$ is

$$\tau_B = \left(1 - \frac{R_S}{r_0} - \frac{r_0^2\omega^2}{c^2}\right)^{1/2} \Delta t' \ . \tag{7}$$

The terms in eq.(6) have the following physical interpretations:

R_S / r_0 represents the gravitational time dilation due to the central mass.

$r_0^2\omega^2 / c^2$ represents the gravitational time dilation due to the centrifugal gravitational field.

$r_0^2\Omega^2 / c^2$ represents the kinematical, velocity dependent time dilation for clocks moving in the rotating frame.

$2r_0^2\omega\Omega / c^2$ is neither a gravitational nor a kinematical time dilation. It has not earlier been given any reasonable interpretation. Bræck and Grøn [12] have called it a *cosmic time effe*ct for reasons that will be explained below.

The expression for A's travelling time may be written

$$\tau_A = \left(1 - \frac{R_S}{r_0} - \frac{r_0^2\omega^2}{c^2} + \frac{r_0^2}{c^2} f(\Omega)\right)^{1/2} \Delta t' \ , \tag{8}$$

where $f(\Omega) = -\Omega^2 - 2\omega\Omega$. The graph of the function $f(\Omega)$ is shown in Figure 1,

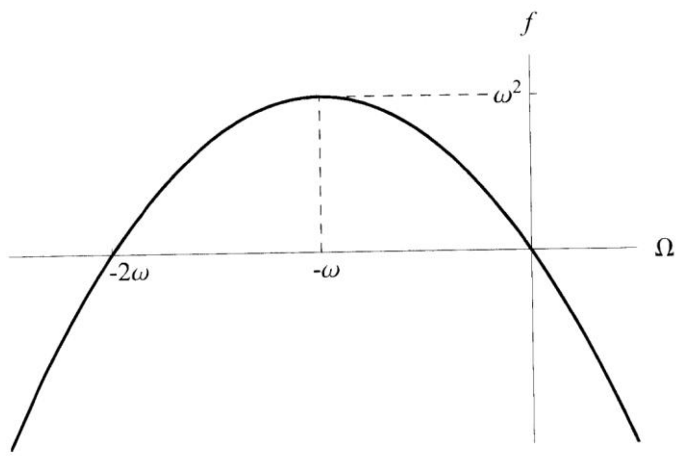

Figure 1. Sketch of the function $f(\Omega)$ introduced in Eq.(20) for different coordinate velocities Ω.

The graph shows that a clock with $\Omega = -\omega$ ages fastest. This clock is at rest in the non-rotating inertial frame. Naturally the graph is symmetrical about this angular velocity. Hence for clocks at the same height, the cosmic time effect acts so that the clock at rest in the non-moving, inertial frame ages fastest. The mathematical expression of the 'East-West effect', interpreted as a cosmic time effect, follows directly from eq. (6), giving the difference in travel time to lowest order, of a clock moving eastwards and one travelling westwards

$$\Delta\tau = \left(2R^2\omega\Omega / c^2\right)\tau \ , \tag{9}$$

where τ is the travelling time, $\tau = 2\pi / \Omega$. Hence the East-West time difference may be written

$$\Delta\tau = 4\omega A / c^2 \quad , \tag{10}$$

where $A = \pi R^2$ is the area enclosed by the paths of the clocks. The East-West time difference is independent of the velocity of the clocks, depending only upon the angular velocity of the reference frame and the area enclosed by the paths of the clocks.

This effect acts so as to make the clock that has the smallest angular velocity relative to the 'starry sky' age fastest. This means that the clock that travels eastwards, i.e. in the opposite direction to the rotation of the Earth, ages faster and will show a greater travelling time than the one that travels westwards. If the clock travels with the same velocity as a particle fixed on the surface of the Earth due to the Earth's diurnal rotation, i.e. about 1600 km/h, the clock would age faster than every other clock at the same height. In particular this clock travelling eastwards ages faster than a clock at rest on the Earth.

4. On the concept "gravitational field"

One may wonder whether the "cosmic time effect" is rather trivial and just the result of a coordinate transformation. However, that point of view does not take into consideration that the principle of equivalence is involved in a non-trivial way here. The statement would be like saying, when Einstein found that the frequency of light is increased if light moves in an accelerated reference frame in the opposite direction to that of the acceleration, that this is not particularly interesting, because it is only the result of a coordinate transformation. However, the important point was that Einstein further said: Hence the frequency of light is increased when moving downward in a gravitational field caused by masses. In the same way we say: Hence there is a cosmic time effect described by the expression $2r_0^2\omega\Omega / c^2$ in a gravitational field inside a rotating shell of mass.

This gravitational field is non-tidal [13] meaning that it exists in flat spacetime. The distinction between a tidal- and a non-tidal gravitational field is based on the geodesic equation and the equation of geodesic deviation. Consider two nearby points P_0 and P in spacetime, and two geodesics, one passing through P_0 and one through P. Let $\mathbf{n}$ be the distance vector between P_0 and P. The geodesics are assumed to be parallel at P_0 and P, so that $\left(dn / d\tau\right)_{P_0} = 0$. Using Eq.(53) of Ref.14 we find that a Taylor expansion about the point P_0 gives the following formula for the acceleration of a free particle at P,

$$\left(\frac{d^2x^i}{d\tau^2}\right)_P = -\left(\Gamma^i_{\alpha\beta}u^\alpha u^\beta\right)_{P_0} - \left(\Gamma^i_{\alpha\beta,\gamma}u^\alpha u^\beta\right)_{P_0} n^\gamma \quad . \tag{11}$$

The first term represents the acceleration of a free particle at P_0, and contains, for example, the centrifugal acceleration and the Coriolis acceleration in a rotating reference frame.

We define the gravitational field strength at the point P, $\mathbf{g}$, as the acceleration of a free particle instantaneously at rest. Then the spatial components of the four velocity vanish. Using the proper time of the particle as time coordinate gives $u^0 = 1$, and Eq.(8) simplifies to

$$g^i = -\left(\Gamma^i_{00}\right)_{P_0} - \left(\Gamma^i_{00,\gamma}\right)_{P_0} n^\gamma \quad . \tag{12}$$

Grøn and Vøyenli [13] has shown that in a stationary metric this equation can be written

$$g^i = -\Gamma^i_{00} + \left(\Gamma^k_{00}\Gamma^i_{jk} - \Gamma^\sigma_{0j}\Gamma^i_{\sigma 0}\right)n^j - R^i_{0j0}n^j \quad , \tag{13}$$

where the Christoffel symbols and the components of the Riemann curvature tensor are evaluated at the point P_0. The first term of this equation represents the acceleration of gravity at the point P_0, i.e. it represents the uniform part of the gravitational field. The second term represents the nonuniform part of the gravitational field which is also present in a non-inertial reference frame in flat spacetime, for example, the non-uniformity of the centrifugal field in a rotating reference frame. The last term represents the tidal effects, which in the general theory are proportional to the spacetime curvature.

This suggests the following separation of a gravitational field into a nontidal part and a tidal part

$$g^i = g^i_{NT} + g^i_T \quad , \tag{14}$$

where the non-tidal part is given by

$$g^i_{NT} = -\Gamma^i_{00} + \left(\Gamma^k_{00}\Gamma^i_{jk} - \Gamma^\sigma_{0j}\Gamma^i_{\sigma 0}\right)n^j \quad , \tag{15}$$

and the tidal part by

$$g^i_T = -R^i_{0j0}n^j \quad . \tag{16}$$

The nontidal gravitational field can be transformed away by going into a local inertial frame. The tidal gravitational field cannot be transformed away. The mathematical expression of these properties is that the nontidal gravitational field is given by Christoffel symbols, and they are not tensor components. All of them can be transformed away. But the tidal gravitational field is given in terms of the Riemann curvature tensor of spacetime, which cannot be transformed away.

It will be shown in section 6.3 that Eq.(15) gives the correct expression for the centrifugal non-tidal gravitational field inside a cosmic mass shell due to perfect inertial dragging.

5. Ageing in the Kerr spacetime

The rotation of a mass distribution changes the properties of space outside it. Inertial frames are dragged along in the same direction as the mass rotates. We shall consider circular motion in an axially symmetric space. Along the circular path the line element can be written

$$ds^2 = g_{tt}dt^2 + 2g_{t\phi}dt\,d\phi + g_{\phi\phi}d\phi^2 \quad . \tag{17}$$

The coordinate clocks showing t goe equally fast everywhere. Hence the proper time interval of a clock with angular velocity $\Omega = d\phi / dt$ is given by

$$d\tau = \left(-g_{tt} - 2g_{t\phi}\Omega - g_{\phi\phi}\Omega^2\right)^{1/2} dt \quad . \tag{18}$$

It can be shown [15] that an observer with zero angular momentum (ZAMO) has angular velocity

$$\Omega_Z = -g_{t\phi} / g_{\phi\phi} \quad . \tag{19}$$

A non-vanishing value of Ω_Z is an expression of inertial dragging. Let us find the angular velocity of the clock which ages fastest. One might think that it is the clock at rest, due to the kinematical time dilation, which tends to slow down. Putting the derivative of the function

$$F(\Omega) = -g_{tt} - 2g_{t\phi}\Omega - g_{\phi\phi}\Omega^2 \tag{20}$$

equal to zero, one finds, however, that the ZAMO ages fastest.

In the Kerr spacetime the angular velocity of a ZAMO is [15]

$$\Omega_Z = \frac{2mac}{r^3 + ra^2 + 2ma^2} , \tag{21}$$

where $m = GM / c^2$ is the gravitational length of the central rotating body, and $a = J / Mc$ where J is the angular momentum of the central mass (note that a has dimension length). The ZAMO angular momentum vanishes in the asymptotic Minkowski spacetime in the limit $r \to \infty$. If the central body is non-rotating there is Schwarzschild spacetime and the angular velocity of the ZAMO vanishes.

Our treatment of the clocks in the Schwarzschild and Kerr spacetimes seems to imply that rotating motion is absolute. For example one can decide which clock rotates by measuring how fast it ages. In the special theory of relativity rotational motion is absolute. However if the general principle of relativity is generally valid according to the general theory of relativity, rotational motion has to be relative. Below we shall see how the phenomenon of perfect inertial dragging plays a decisive role in this connection.

6. Inertial dragging inside a rotating shell of matter

6.1. The weak field result

As pointed out by Pfister [16] inertial dragging inside a rotating shell of matter was described already in 1913 by Einstein and Besso in a manuscript that was not published. This work was based on Einstein's so-called Entwurf theory of gravity which Einstein soon discovered had some serious weaknesses. The first published paper on inertial dragging inside a rotating shell based on the general theory of relativity was published by H. Thirring [17] in 1918. He calculated the angular velocity of Ω_Z a ZAMO inside a shell with Schwarzschild radius R_S and radius r_0 rotating slowly with angular velocity ω, in the weak field approximation, and found the inertial dragging angular velocity,

$$\Omega_Z = \frac{8R_S}{3r_0}\omega . \tag{22}$$

This calculation does not, however, remove the difficulty with absolute rotation in an asymptotically empty Minkowski space. Both the angular velocity of the shell and that of the ZAMO are defined with respect to a system that is non-rotating in the far away region. There is nothing that determines this system. The absolute character of rotational motion associated with the asymptotically empty Minkowski spacetime, has appeared.

6.2. Perfect inertial dragging

In 1966 D. R. Brill and J. M. Cohen [8] presented a calculation of the ZAMO angular velocity inside a rotating shell valid for arbitrarily strong gravitational fields, but still restricted to slow rotation, giving the expression (1). For weak fields, i.e. for $r_0 >> R_S$, this expression reduces to that of Thirring. But if the shell has a radius equal to its own Schwarzschild radius, $r_0 = R_S$, the expression above gives $\Omega_Z = \omega$. Then there is *perfect dragging*. In this case the inertial properties of space inside the shell no longer depend on the properties of the ZAMO at infinity, but are completely determined by the shell itself.

Brill and Cohen further write that a shell of matter with radius equal to its Schwarzschild radius together with the space inside it can be taken as an idealized cosmological model, and proceeds: “Our result shows that in such a model there cannot be a rotation of the local inertial frame in the center relative to the large masses in the universe. In this sense our result explains why the “fixed stars” are indeed fixed in our inertial frame.

The problem of the induction of a correct centrifugal force and Coriolis force by rotating masses was solved to order ω^2 by H. Pfister and K. H. Braun in 1985 [18]. They took into account two important facts: (a) Any physically realistic, rotating body will suffer a centrifugal deformation and cannot be expected to keep its spherical shape. (b) They noted that in order to realize corrects expressions for Coriolis and centrifugal forces – and no other forces – the spacetime inside the mass shell has to be flat. Hence, according to Pfister, the problem of inducing the same inertial properties inside a rotating shell as those in a rotating frame inside a static shell boils down to the question of whether it is possible to connect a “rotating” flat inerior metric through a mass shell (with, to begin with, unknown geometrical and material properties) to the non-flat but asymptotically flat exterior metric of a rotating body.

In 1995 H. Pfister [19] wrote that whether there exists an exact solution of Einstein’s field equations with flat spacetime and correct expressions for the centrifugal- and Coriolis acceleration inside a rotating shell of matter, was still not known. However, permitting singular shells such a solution certainly exists, as will now be made clear.

6.3. A source of the Kerr metric with perfect inertial dragging

In 1981 C. A. Lopez [20] found a source of the Kerr spacetime. A few years later Ø. Grøn [21] gave a much simpler deduction of this source and discussed some of its physical properties. The source is a shell with radius r_0 rotating with an angular velocity

$$\omega = \frac{ac}{a^2 + r_0^2} \tag{23}$$

The radius of the exterior horizon in the Kerr metric is

$$r_+ = m + \sqrt{m^2 - a^2} \tag{24}$$

Hence, if the radius of the shell is equal to the horizon radius $r_0 = r_+$, the ZAMO angular velocity just outside the shell is equal to the angular velocity of the shell,

$$\Omega_Z(r_+) = \omega(r_+) = \frac{ac}{2mr_+}$$

(25)

Demanding continuity of the dragging angular velocity at the shell it follows that the inertial frames in the Minkowski spacetime inside the shell are co-moving with the shell. There is perfect dragging of the inertial frames inside the shell. The properties of the shell, and of spacetime outside and inside the shell, solve Einstein's field equations without needing the assumptions of week fields and slow rotation. The inertial properties of space inside the shell, such as the Coriolis acceleration, do not depend on any property of an asymptotic far away region, only on the state of motion of the reference frame relative to the shell.

In the flat spacetime inside the cosmic shell there is a non-tidal gravitational field. With co-moving cylindrical coordinates in a reference frame in which the cosmic shell is at rest, the line element takes the form

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 d\theta^2 + dz^2 \tag{26}$$

and the non-vanishing Christoffel symbols are

$$\Gamma^r_{\theta\theta} = -r \ , \quad \Gamma^\theta_{r\theta} = \Gamma^\theta_{\theta r} = \frac{1}{r} \tag{27}$$

Inserting these into Eq.(15) gives a vanishing field strength of the non-tidal gravitational field. This is as expected since this frame is inertial, and a non-tidal gravitational field only exists in a non-inertial frame.

In a reference frame in which the cosmic shell is observed to rotate with an angular velocity ω the line element has the form

$$ds^2 = -\left(1 - r^2\omega^2 / c^2\right) c^2 dt^2 + dr^2 + r^2 d\theta^2 + dz^2 + 2r^2 \omega \, d\theta \, dt \tag{28}$$

The additional non-vanishing Christoffel symbols are

$$\Gamma^r_{tt} = -r\omega^2 \ , \quad \Gamma^r_{t\theta} = \Gamma^r_{\theta t} = -r\omega \ , \quad \Gamma^\theta_{tr} = \Gamma^\theta_{rt} = \frac{\omega}{r} \tag{29}$$

Then Eq.(15) shows that the non-tidal gravitational field at the point P has the components

$$g^r_{NT} = r\omega^2 + \omega^2 n^r \ , \quad g^\theta_{NT} = \omega^2 n^\theta \tag{30}$$

The term $r\omega^2$ is the "centrifugal" acceleration at the point P_0, and the other terms are due to the inhomogeneity of this field.

7. Is there perfect dragging in our universe?

The distance that light and the effect of gravity have moved since the Big Bang is called the lookback distance, $R_0 = ct_0$, where t_0 is the age of the universe. WMAP-measurements have shown that the age of the ΛCDM-model of our universe is close to its Hubble-age, $t_H = 1/H_0$, namely that $t_0 = 0{,}996 t_H$, and that the universe is flat, i.e. that it has critical density

$$\rho_{cr} = 3H_0^2 / 8\pi G \tag{31}$$

It follows that

$$8\pi G \rho_{cr} / 3c^2 = \left(H_0 / c\right)^2 \approx 1 / R_0^2$$

(32)

The Schwarzschild radius of the cosmic mass inside the lookback distance is

$$R_S = 2GM / c^2 = \left(8\pi G\rho_{cr} / 3c^2\right)R_0^3 \approx R_0 \tag{33}$$

Hence in our universe the Schwarzschild radius of the mass within the lookback distance is approximately equal to the lookback distance. It follows that the condition for perfect dragging may be fulfilled in our universe.

The question of perfect dragging in our universe has been considered from a different point of view by C. Schmid [22, 23]. By introducing a rotational perturbation in a realistic FRW-model he has shown that the ZAMO angular velocity in the perturbed FRW universe is equal to the average angular velocity of the cosmic mass distribution. Hence perfect dragging explains why the swinging plane of the Foucault pendulum rotates with the "starry sky". In Newtonian gravity where there is no dragging, this is a consequence of the absolute character of rotation. One says that the swinging plane of the Foucault pendulum is at rest relative to the starry sky because neither of them rotates. Hence the pendulum is in a room with an absolute rotation.

8. An extended model of Minkowski spacetime

In the general theory of relativity the significance of the Minkowski spacetime is that it is used as the asymptotic metric outside a localized mass distribution, for example in the Kerr spacetime. This means that absolute rotation is introduced into the general theory of relativity through this choice of boundary condition when solving Einstein's field equations.

This may have been noted already by H. Thirring when he worked on his 1918-paper on inertial dragging [17]. Pfister [16] writes: "Presumably, Thirring has realized that the rotating sphere and the rotating mass shell with their asymptotically Minkowskian boundary conditions do not answer the Machian question concerning a static Newton bucket inside a rotating celestial sphere."

S. Bhattacharya and A. Lahiri [24] have recently pointed out that when one has a cosmological event horizon one cannot reach the spatial infinity, and therefore the boundary conditions which must then be set at the cosmological event horizon may be very different from those of the asymptotically flat and empty spacetime.

Even without a cosmological event horizon one must choose a boundary condition to determine a solution of Einstein's field equations, and this has consequences for the status of accelerated and rotational motion in the theory of relativity, whether they are absolute or relative. It may also have significance for the quantization of the Minkowski spacetime.

A meaningful boundary condition for flat spacetime is to introduce a massive shell that represents the cosmic mass inside the shell. As shown in the previous section the mass inside the lookback distance of our universe has a Schwarzschild radius equal to the lookback distance. Hence, it is natural to impose the boundary condition that the asymptotically empty spacetime is replaced by the boundary condition that there is a mass shell at the lookback distance with radius equal to its own Schwarzschild radius and mass equal to the cosmic mass inside the lookback distance.

The extended model of Minkowski spacetime is also relevant in connection with a point made several years ago by C. Møller [25]. He wrote that when one solves Einstein's field equations in a rotating reference frame it is necessary to take account of the far away cosmic masses. However there was an exception for globally or asymptotic Minkowski spacetime, where there was no cosmic masses. In the extended model the Minkowski spacetime must be treated in the same way as any other spacetime.

In the spacetime inside the shell a centrifugal gravitational field appears in a reference frame rotating relative to the shell. An observer in a frame R rotating relative to the shell can maintain that the frame R does not rotate, and that it is the shell that rotates. His calculations would show that there is perfect dragging inside the rotating shell, and that this causes the centrifugal gravitational field. With this model of the Minkowski spacetime rotational motion is relative. Without the shell rotation is absolute.

Pfister [16] notes that the work of Einstein, Thirring and others, which conserved some aspects of the relativity of rotation in the model class of rotating mass shells, was often criticized for the asymptotic flatness of the exterior solution, instead of using cosmological boundary conditions. They should rather have been criticized for the asymptotic emptiness. In a model with flat space and cosmic mass producing perfect dragging, the mass must be represented by a cosmic shell, and this is a boundary condition making any exterior to the shell irrelevant.

Finally it may be mentioned that translational inertial dragging inside an accelerating shell has been investigated in the weak field approximation by Ø. Grøn and E. Eriksen [26]. They found that the inertial acceleration inside a shell with acceleration g, Schwarzschild radius R_S and radius R is

$$a = \frac{11}{6}\frac{R_S}{R} g \tag{34}$$

Hence, according to this approximate calculation there is perfect translational dragging inside a shell with radius $R = (11/6)R_S$.

9. Conclusion

The difference between an active and passive rotation is illustrated in Figure 1 (from Wikipedia).

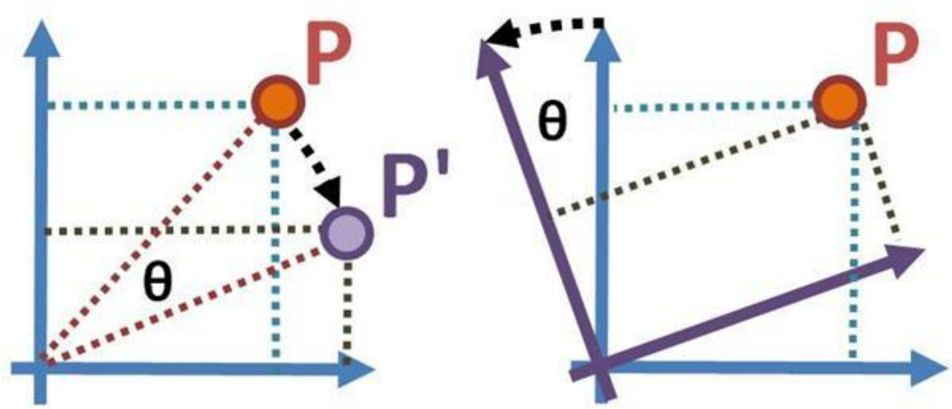

Figure 1. In the active transformation (left), point P moves relative to the coordinate frame to location P', while the coordinate frame remains unchanged, while in a passive transformation (right), point P is observed in two different coordinate frames.

An active rotation of a bucket with water makes the surface of the water curved, while a passive rotation (of the reference frame) makes no difference to the surface. Does the difference between an active and a passive transformation mean that rotation is absolute? With the globally empty Minkowski spacetime it does. But with the extended model including a cosmic shell of matter, the answer is different. Rotation of the water relative to the cosmic shell makes the surface of the water curved. In the rest frame of the water this is due to perfect inertial dragging caused by the rotating cosmic shell. An active transformation changes the rotation of the water relative to the cosmic shell, while a passive transformation does not. Hence it is possible to decide by a local experiment whether a bucket with water rotates relative to the cosmic shell or not. But this is a relative acceleration. One is perfectly free to consider either the water as rotating and the cosmic shell as at rest, or the water as at rest and the cosmic shell as rotating. But without the cosmic shell one must consider the water with a curved surface as rotating.

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