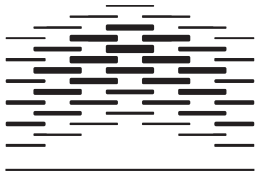


OSLO AND AKERSHUS
UNIVERSITY COLLEGE
OF APPLIED SCIENCES

Purchasing Power Parity in Scandinavian Countries: Multi-Period Approach

Adnan Qasim *

May 27, 2016

Master Thesis in ØAMAS 5900

Oslo Business School

*student number: s236918

Abstract

This paper investigate the idea of PPP for Scandinavian countries for the period 1946-2015. The dynamic properties of PPP is examined by graphical analysis and ADF test. Johansen test is applied to asses the long-run relationship between exchange rates and relative price levels. This paper finds partial support for the PPP hypothesis. That is, for both currency pairs, the PPP is supported only in one sub-sample while not in other sub-samples.

Sammendrag

Denne oppgaven tar for seg hypotesen om kjøpekraftsparitet mellom nordiske land for perioden 1946-2015. De dynamiske egenskapene til PPP er studert ved grafiske analyser og ADF test. Johansen test er anvendt for å studere den langsiktige relasjonene mellom valutakurser og relative priser. Denne oppgaven konkluderer at kjøpekratspariteten holder for begge valuta-par men kun i en periode.

Oslo Business School

OSLO 2016

1 INTRODUCTION

The idea of Purchasing Power Parity (PPP) requires that prices in different countries should be equal when expressed in common currency. Further, PPP implies that the nominal exchange rate between two countries should equal the relative price differences between the countries. Therefore, the long-run exchange rate movements should largely be determined by the difference between domestic and foreign prices. A more elaborate discussion of theoretical foundations of the PPP is provided the section 3.

PPP is an important concept in macroeconomics and is also a central part of the academic curriculum. Furthermore, PPP has great implications for households, corporations and economic policy makers. For example, PPP incorporate relative inflation (relative prices) to the exchange rate equation. Inflation has great consequences for nations such as it happened in Germany during the inter-war period and most recently in Zimbabwe.

Previous studies have not found uniform results regarding the validity of the PPP. Very few studies have rationalized on the probable causes for why there are long-term discrepancies between nominal exchange rate and the relative price differences. Studies that has supported or partially supported the PPP hypothesis are Abauf and Jorion 1990, Cheung and Lai 1993, Anoruo, Braha and Ahmad 2002, Norrbin and Smallwood 2010, Pappel 1997 and Doanlar, Bal and Ozmen 2009. Studies that did not find support for the PPP are Taylor 1988, Enders 1988, Patel 1990, Copper 1994 and Wang 2000. Section 2 contains an elaborate review of the previous studies.

This paper examine PPP between Norway, Sweden and Denmark for roughly (small variations in periods exist due to availability of the data series) the period between 1946 to 2014. The data series are then devided into sub-samples. For examination of the data series, graphical analysis, ADF test and Johansen test is applied. To the authors knowledge no previous studies have applied this approach for the data series mentioned here.

The remaining outline of the paper is as follows. Section 2 and 3 contain literature review and theory, respectively. Data and stationarity analysis are presented in section 4 and section 5 contains co-integration analysis. Section 6 contains the conclusion and the section 7 contains literature list.

[0]**Acknowledgment;** I would like to thank Professor Haakon Vennemo for his constructive feedback and advise.

2 LITERATURE REVIEW

Previous research for the most part do reject the PPP hypothesis in the short-term. Though, mixed results are presented when the data series are collected for a relatively longer period of time. As it is below very few studies have rationalized on probable causes for why PPP does not hold in the short- and long-run.

2.1 Long term PPP

Taylor (1988) finds no long relationship between the five major exchange rates[1] even when after allowance for some measurement error or transportation costs. The author does not provide any rational for inclusion of transportation costs. But it reasonable to assume that transportation costs increase the cost of purchasing goods or services abroad.

For example, if an identical pair of shoes cost NOK 100 in Norway and SEK 100 in Sweden and the SEK/NOK is 0,90 (price of 1 SEK) then it make sense to buy in Sweden. Because, from a Norwegian consumer's standpoint it only cost NOK 90 (100*0,90). But if it costs NOK 10 to transport the shoes to Norway then the Norwegian consumer should be indifferent, all else equal. If the transportations costs are greater then NOK 10 then it is more attractive to consume in the home country. On the contrary, results from the study suggest that prices do deviate from the PPP infinitely. Taylor does not discuss any causes for these results.

Enders (1988) test the idea of PPP using real exchange rates for US and its major trading partner (Germany, Canada and Japan) both under Bretton Wood and flexible exchange rate periods. Unit root or co-integration[2] tests finds no support for PPP. Though, Error Correction Model (ECM)[3], suggested that prices in the US trading partners did responded to a certain level in the case of a deviation from PPP (but not the US prices). Implying that the smaller countries (economies) are more reactive to prices changes then the larger ones. He does not provide any rationale for why PPP did not held.

[1] Relative prices and nominal exchange rates against US dollar for the UK, West Germany, France, Canada and Japan.

[2] Means that despite being individually non-stationary, a linear combination of two or more time series can be stationary. The Engle-Granger (EG) and the augmented Engel-Granger (AEG) testes can be used to find out if two or more time series are co-integrated. Co-integration of two (or more) time series suggests that there is long-run, or equilibrium, relationship between them. (Gujarati and Porter 2009:769)

[3] ECM developed by Engle end Granger is a means of reconciling the short-run behavior of an economic variable with its long-run behavior. (Gujarati and Porter 2009:769)

Patel (1990) does find support for long run PPP. However, he argues that exogenous shocks (shocks not due to fiscal or monetary policy) may cause structural changes (permanent difference in price levels) in traded and non-traded goods. Thus causing a deviation from PPP. The author does not suggest any theoretical reason for why a structural shock might cause a deviation from PPP[4]. Patel uses data for a short period of time. Furthermore, Marston (1986) argues that the breakdown of the PPP for the US-Japan maybe due to real productivity differentials.

Cooper (1994) rejects the hypothesis of PPP for all currencies under study. Though he argues that the critical reason for rejection of PPP is that it does not take into account capital flows.

The author does not go into the details of why capital inflows might cause a deviation from parity. Furthermore, he does not suggest any particular measure for such capital flows or how to take them into account in order to test the hypothesis. Perhaps capital in/ and outflows increase or decrease demand for a given currency. If this continues for a long enough period of time. The currency being purchased will appreciate, all else equal. This will lead to a divergence between the real exchange rate and the relative rate of inflation.

Papell (1997) using data from 1973-1994 construct panels or groups[5]. There are 8 panels, the largest with 20 pairs while the smallest with 6 pairs, tested with Augmented Dickey Fuller (ADF)[6]. These pairs are then categorized in monthly and quarterly frequency. The paper finds that the there is stronger support for PPP for larger panels then the smaller. Furthermore, the evidence in favor PPP is greater when monthly data is used rather then quarterly. Another aspect of this paper is the use of German mark as the base currency.

The author does not provide any reason for the choice of base currency. Perhaps DEM has a relative lower volatility against other European currency compared to USD. All else equal, the relative lower volatility in magnitude and in sign will increase the likelihood of no co-integration between exchange rates and the relative price levels. Another reason can be the status of the USD as the world reserve currency. This mean increased demand (strong/appreciation) for USD, all else equal.

He borrows the explanation for use of German mark as the base currency from Jorion and Sweeny (1996). That is the relative lower volatility of the real exchange

[4]Perhaps the author assumes that structural shocks change exchanges rates to new equilibrium. And relative prices will lag or do not respond to such a short term shock.

[5]He construct panels of 20 (quarterly) and 17 (monthly) real exchange rates with the USD nominal exchange rates and national CPIs.

[6]A test of stationarity or non-stationarity. (Gujarati and Porter 2009:754)

rates to German mark compared to US dollar (most of the pairs under study are European). The second reason is close geographic proximity to Germany relative to USA. Another explanation is the substantial appreciation and depreciation of the US dollar in 1980s. Papell finds evidence for the half-life of PPP deviations to be 2.5 years (expected number of years for a PPP deviation to decay by 50 percent).

Wang (2000) employs Johansen[7] co-integration test, with and without restrictions, on seven Asian countries (Indonesia, Japan, Malaysia, the Philippines, Singapore, South Korea and Thailand). He finds that the exchange rate, domestic and foreign prices are co-integrated but restrictions of symmetry and proportionality are rejected in all cases, which means that the exchange rates do not move one by one with relative prices of two countries. Therefore, the long-run PPP does not appear to hold.

The implications of these findings are that the real exchange rate is non-stationary. Main reason for that could be deviations in productivity differentials, countries in the Pacific Basin area have experienced a fast rate of growth over the last decade.

The author indicates that there are large sustained disproportional changes in real exchange rates and relative prices, i.e inflation. And Balassa-Samuelson hypothesis may exist in this example, which violates the validity of PPP and causes the permanent changes in the real exchange rate[8].

Cuddington and Liang (2000) used the same data (annual data spanning two centuries (1791-1990) for dollar-sterling and franc-sterling real exchange rates) as Lothian and Taylor (1996). Contrary to Lothian and Taylor (1996), they reject the stationarity (rejection of the PPP) of the USD-GBP data series.They explained two rationales for that; Augmented Dickey Fuller (ADF) test is sensitive to the choice of lag length. And the presence of the significant time trend[9], not considered by Taylor and Lothian.

Doganlar, Bal and Ozmen (2009), use monthly data for emerging markets[10] from 1995 to 2005. They find only support for Mexico and Peru. The authors suggest failure of the PPP due to following factors; difference in preferences, technology and productivity, existence of non tradable goods, speculative capital movements,

[7] A test approach that test co-integration. A group of non-stationary $I(1)$ time series is said to have co-integration relationship if a certain linear combination of these time series is stationary. (Wang 2003:18)

[8] Non-traded goods and productivity.

[9] An explanator in a time series regression model that makes the expected value of the dependent variable a function of the observation‘s specific time period. (Murray 2006:915)

[10] Brazil, India, Indonesia, South Korea, Mexico, Pakistan, Peru, the Philippines, South Africa and Turkey.

different economic structures, expectations of different inflation rates, transportation costs and trade barriers. The authors do not provide any rationale or theoretical foundation for why and how these factors can lead to a violation of PPP.

Chang, Tang, Liu and Lee (2010), used monthly data over the period of 1994 to 2008. This study empirically tests whether PPP holds among 15 COMESA (Common Market for Eastern and Southern Africa) and SADC (Southern African Development Community) countries. The results from the univariate unit root and panel-based unit root tests all fail to support the PPP throughout all 15 countries.

2.2 Support or partial support for PPP

Abuaf and Jorion (1990) use data from 1900-1987 for 10 developed countries[11]. They suggest that long term PPP might hold, but short-term deviations from PPP. Moreover, nominal rates are non-stationary and shocks to the real exchange rates seem to slowly cancel out over time. In floating exchange rate period (1973-1987) with monthly data, a 50% deviation in relation to PPP would take between 3 and 5 years to decay in half. Similarly, analyzing annual data over the period 1900-1972 reveals that a period of 3 years is needed for such a reversal.

Cheung and Lai (1993) test long-run PPP during the recent float[12] (1974-1989) by co-integration devised by Johansen for developed countries. They find significant evidence in favor of long-run PPP. From Autoregressive Model[13], it takes about 6.6 months to reduce a given deviation from equilibrium by 50 %. A 75 % correction to PPP equilibrium takes about 13.2 months. The implied speed of adjustment is quite fast. The residual-based tests have very low power in rejecting the no co-integration hypothesis even when an equilibrium relationship in fact holds in the long-run with a reasonable speed at which deviations from equilibrium are corrected.

Edison, Gagnon and Melick (1997) argue that the empirical failure of PPP in post Bretton Woods data is largely due to the low power tests employed[14]. They used quarterly data for developed countries and applied Horvath and Watson procedure. As a result, moderately stronger evidence for PPP is obtained compared to the

[11]Belgium, Britain, Canada, France, Germany, Italy, Japan, Netherlands, Norway and Switzerland.

[12]In February 1973, the official boundaries for the more widely traded currencies were eliminated and the floating exchange rate system came into effect. The gold standard became obsolete and the values of a range of currencies were to be determined by the market (Wang 2010:24)

[13]An Autoregressive Model is one where the current value of a variable, y, depends upon only the values that the variable took in previous periods plus an error term (Brooks 2008:215)

[14]The methodology employed fail to reject the random walk assumption even in situations where the real exchange rate exhibits slow reversals to PPP values.

Johansen test. The Hovarth and Watson procedure can be seen as an extension of the Johansen test. In the Johansen test an co-integrating vector can be imposed, but it is not initially enforced in the model. But this is the case for the Horvath and Watson procedure.

Furthermore, multivariate techniques claim to find significant evidence of co-integration between exchange rates and price levels, but the estimated co-integration vectors usually reject the restrictions of symmetry and proportionality implied by PPP. This paper shows that these results are partly due to the use of inappropriate critical values in small samples. The problem with small samples is that results or parameters estimates from the small sample cannot be generalized to the underlying population.

Salehizadeh and Taylor (1999) examined PPP for a large group of emerging economies against US for the post Bretton Woods era. Two key results are obtained from this study. First, the application of Johansens co-integration test provides relatively strong confirmation across 14 cases. Second, additional tests on real exchange rates yield these empirical findings: a rejection of symmetry and proportionality[15] as implied by PPP; the conclusion that departures from true exchange rate values can last for several years; and that a priori restrictions (symmetry and proportionality) on the co-integrating vector can lead to a false rejection of long-run PPP. Therefore, the restriction of proportionality in the co-integrating should be loosen up.

Anoruo, Braha and Ahmad (2002) examine PPP for 11 developing countries[16]. The sample covers the period 1961-99, with quarterly observations for all countries except Nigeria, of entire sample spans from 1961-98. Two important findings are obtained here also. First, PPP holds for the developing countries. Second, the use of conventional unit root tests in earlier studies failed to detect evidence of PPP because of the imposition of stringent restrictions.

They suggest the application of dynamic error-correction model to other developing countries to validate its applicability and robustness in detecting PPP for countries with nascent economies.

Alba and Park (2003) study PPP for real exchange rates of developing countries during the current floating period with USD as the base currency. They create 10-year moving panels based on country characteristics influencing; openness, inflation,

[15]One to one relationship between prices and exchange rates is proportionality and same relationship across currencies is symmetry.

[16]Argentina, Bolivia, Colombia, Cote d'Ivoire, Ecuador, Guatemala, Kenya, Nigeria, Peru, South Africa, and Venezuelan.

and level of and GDP growth per capita.

The authors do not provide any rational for inclusion of these variables. The sample is moving 10-year periods from 1976-85 up to 1990-99. First period: 1976-1985, second period: 1977-1986 and so on. Thus creating 15 moving periods and 210 panels. The rational for moving periods is that shocks that cause (their rational for deviation) deviation from PPP is likely to occur at irregular intervals.

They find support for PPP in only 14 out of 210 cases. Furthermore, the support for PPP is stronger in later periods of the sample compared to the earlier. The study suggests growing trade liberalization in developing countries along with rapid growth of global trade as a probable reason for that.

Achy (2003) examines PPP for middle-income countries[17], using data from 1973 to 1998. Achy provides two main findings: first, real exchange rate behavior is well described by a fractionally integrated model[18] in 28 cases. The second, in most cases, the random walk component is not as substantial as the ADF test tends to suggest. He concludes that correction to the parity tends to be faster in high inflation countries compared to low inflation countries and productivity improvement slower this process.

Bahmani-Oskooee and Hegerty (2010) examined bounds testing approach[19] of Pesaran et al. (2001) co-integration methods and PPP for 123 countries. They find strong support for co-integration between the nominal exchange rate and the relative price level in the majority of the cases. But they do not conclude in favor of PPP. Because, they require a stricter conditions to be met in order for PPP to hold. This approach is known to be as the strict approach for testing the validity of the PPP. Since it does not allow short-term deviations from PPP.

Norrbin and Smallwood (2010) examine mean reversion[20] of the real exchange rates. The US pairs, Canada, Japan and UK are tested using sample period from 1974 to 2001, except Germany for which the data series ends December 1998. They

[17]Bulgaria, Croatia, Czech Republic, Hungary and Poland, Brazil, Colombia, Ecuador, Guatemala, Paraguay and Uruguay.

[18]'Fractionally integrated processes are described as long-memory processes because they exhibit significant correlations even between observations widely separated in time. A shock to the series persists for a long time even though it eventually dissipates. Standard Autoregressive Moving Average (ARMA) processes cannot capture the long-term dependence as they are designed only to describe the short-run behavior of a time series.' Achy (2003)

[19]'A new approach to the problem of testing the existence of a level relationship between a dependent variable and a set of regressors, when it is not known with certainty whether the underlying regressors are trend- or first-difference stationary. 'Pesaran et al. (2001)

[20]A property of stationary time series variables; the expected value of such a variable at $(t+r)$, conditional on the variable's values prior to t, always approaches the variable's unconditional expected values as r grows large. (Murray 2006:908)

support a weak version of PPP with cyclical correction to PPP. They suggest that these cyclical corrections can take very long time. Furthermore, they also conclude that even though mean reversion exist it is so slow that it has little economical or theoretical value.

Arize, Malindretos and Nam (2010) investigate PPP for African countries[21] by applying multivariate error-correction model. The sample period used in the study ranges from 1973-2007. The find strong support for PPP and conclude that the half-life of PPP deviations is between 1.98 and 2.45 years.

No macroeconomic or fundamental reasons are discussed by the author as the probable cause for such results. Though, the author did suggest that the previous studies on African currencies applied statistical methods that had weak power in capturing the true nature of the co-integration between the African currencies. They have conducted their research on these currencies by applying Johansen test.

This finding is contrary to Rogoff (1996) who suggest half-life to be between 3-5 years. Furthermore, estimates of the mean and median time lags indicate that, on the average, exchange rates adjust faster to changes in domestic prices than to changes in foreign prices.

Rogoff (1996) raises a critical question; how do we reconcile the extreme short-term deviations of the real exchange rate with the slow rate of depreciation of those shocks. He provides the following rational, measurement problems and international versus domestic price volatility.

Rogoff argues that there is no appropriate measure of the right PPP. Because there is no clear way to decide which consumer price index, CPI, to use. This problem is further exacerbated when new goods are introduced or the weights in the CPI are changed. When constructing CPI, the choice of base years is critical. Furthermore, the problem with International Comparison Program (ICP)[22] is that it is gathered infrequently, at 5 years interval and does have a lagging problem. Further, measurement problems and frictions provide enough buffer for short-run deviations.

Domestic goods markets are more integrated (tendency to move together) then the international. The difference between the prices was a function of distance between the locations of the goods. This is the so-called border- effect. Possible

[21]Burkina Faso, Cameroon, Cote d'Ivoire, Ethiopia, Ghana, Kenya, Madagascar, Mauritius, Morocco, Niger, Rwanda, Senegal, South Africa and Togo.

[22]I am discussing here that indices such as CIP and other alternatives are not that simple as commonly understood. When PPP is studied, it is reasonable to know aspects of data series under study.

frictions (transportation costs, tariffs, non-tariffs barriers, pricing to market). He argues that transportation costs can vary among countries but the difference can be as large as 10 %.

The author does not provide any rationale for that. But the it is reasonable to assume that transportation costs increase the cost of purchasing goods or services abroad. The same example can be mentioned here.

For example, if an identical pair of shoes cost NOK 100 in Norway and SEK 100 in Sweden and the SEK/NOK is 0,90 (price of 1 SEK) then it make sense to buy in Sweden. Because, from a Norwegian consumer's standpoint it only cost NOK 90 (100*0,90). But if it costs NOK 10 to transport the shoes to Norway then the Norwegian consumer should be indifferent, all else equal. If the transportations costs are greater then NOK 10 then it is more attractive to consume in the home country.

Even traded goods exhibit non-traded components. For example bananas in the super market will reflect the cost of building rental, insurance, labor unions wedges, transport costs etc. Furthermore, tariffs and non tariffs due to restrictions on food import, inspections and food spoilage, all these factors will be included in the final price of the bananas. Some classes of goods make it difficult or impossible to arbitrage. For example the arbitrage in the case for automobiles and electronics (left handed vs right handed, different voltages), can not be realized due to difference in national infrastructure.

2.3 Violations of the law of one price

The law of one price is an important assumption underlying the PPP, identical goods or services should have the same price at different locations (discussed more elaborately in the theory section). If PPP is expected to hold for a basket of goods and service then it should also hold for individual goods and services (both traded and non-treaded).

Knetter (1989, 1993) shows that there were large volatile differential in the price of German beer exported to US versus UK (same good different price). Rogoff (1996) suggests that changes in relative nominal prices are far less then the exchange rate. A puzzling finding since the prices should effect exchange rates and not the other way around.

Hypothesis of Balassa and Samuelson (1964) is relevant to discuss here. That the law of price will hold more for traded goods versus non-traded goods. For example, Big Mac versus Gold. Some inputs of the Big Mac burgers are traded while other

or not. Disparities between Big Mac prices are much greater then the gold prices. This is perhaps due to labor costs, restaurant space and different or non-existing VAT rates. Violation of the law of one price, suggests that it is apparent that the PPP based on aggregate price will be violated. This lead us to assumption (identical basket of goods) underlying the PPP to be violated and conclusions derived from tests on PPP to not be valid.

2.4 Long horizon and cross-sectional data - Random walk

A data series process that follow a 'random walk' exhibits a structure that is contrary to PPP. Therefore, a failure to reject random walk implies a rejection of PPP.

Frankel (1986, 1990), argued that previous studies failed to reject the random walk hypothesis due to short horizon data. He was able to reject the random walk hypothesis using data from 1869-1984. He also suggested half-life to be 4.6 years (the expected numbers of years for PPP deviations to decay by 50 percent is 4.6 years). Later Abuaf and Jorion (1990) also rejected the random walk hypothesis. The consensus estimate for half-life of PPP deviations is 3-5 years, Rogoff (1996) and Abuaf and Jorion (1990).

Similar results for rejection of random walk hypothesis are obtained when using cross sectional data. Frankel and Rose (1996) using data from 1948-1992 for 150 countries reject the random walk hypothesis. Both in the long-run perspective and cross sectional perspective and suggest average rate of decay to be 15 % per annum.

2.5 Modification to PPP

Balassa and Samuelson hypothesized that rich countries tended to have relatively higher prices in the non-traded sector. Kravis and Lipsey (1983), Bhagwati (1984) supports the hypothesis but they suggested that the higher productivity was due to higher capita-labor ratios (big tools not big hands). Rogoff (1996) found mix result for the Balassa and Samuelson hypothesis (Balassa and Samuelson hypothesis is discussed more elaborately in section 3.2.2). Showing that the JPY/USD showed strong support for the hypothesis. Results are also in favor when rich countries are tested against poor. Though, it performs poorly when applied to exchange rates of industrialized countries.

Cumulated current account deficits and long run real exchange rate depreciation approach suggest that sustained deficits on current account are associated with long run depreciation of real exchange rates. Obstfeld and Rogoff (1995) found large and

significant correlation between the current account deficits and the real exchange for 15 OECD (Organisation for European Economic Co-operation) countries. But they did not suggested any causation.

2.6 Government spending and the real exchange rate

De Gregorio, Giovannini and Wolf (1994) argue that government spending usually falls heavily on the non-traded sectors. And this does rise the real exchange rate (increase in demand). But this will be a temporary affect since the capital and labor is not perfectly mobile across sectors. Because in the long run only productivity and supply factors should determine the real exchange rate.

2.7 Convergence based on Multivariate VAR

Clarida and Gali (1994) found that monetary and fiscal shocks account for roughly 45 percent of the forecast error variance (variability) between USD/DEM. Rogers (1995) using 130 years of data from UK and US find that roughly 50 % of the variance is due to monetary and fiscal shocks.

3 THEORY

The origins of PPP goes back to 1800s with the writing of Wheatly and Riccardo. Later Cassel updated these ideas and stressed that the nominal exchange rate, E, is the relative price of two currencies. Because, $1/P$ is the purchasing power of home currency and the $1/P^*$ is the purchasing power of foreign currency (Nelson C. Mark 2000:80-88). In equilibrium following equation must be satisfied (otherwise arbitrage opportunities exist)

$$\frac{1}{P} = \frac{1}{P*} \Leftrightarrow \frac{P}{P*} = 1 \tag{1}$$

Equation 1 requires same price for goods and services at different locations. For example, same price of a chocolate bar in two different grocery stores in the same town. If we extend the example to two countries, then the prices may differ for the same chocolate bar. This will violate equation 1, because if prices are indeed different at two locations then their quotient will differ from 1.

These relative prices differences must be captured by the bilateral nominal exchange rate, E. By replacing 1 in the equation 1 with E, gives:

$$E = \frac{P}{P^*} \tag{2}$$

The equation requires same price levels across countries when expressed in a common currency. For example, if a pair of shoes in Norway costs NOK 100 and in Sweden SEK 70, then the exchange rate should be 100/70 = SEK/NOK 1,43. This should be the price of one Swedish krone. The same pair of shoes costs same in Norway and Sweden when converted to a common currency. Because if you convert NOK 100, you will get SEK 70 (100/1.43).

3.1 Absolute and Relative PPP

PPP can take two forms[23], absolute and relative PPP. The absolute PPP requires that exchange rate should change proportionally to the changes the price levels in two countries i.e. the real exchange rate should be equal to 1, equation 3.

Relative PPP allows the real exchange rate to deviate from 1 but requires that it should stay constant in the long run[24] . That is, the relationship between exchange rate and the price levels should stay constant over time. The two variants of the PPP theory can be formally stated as:

3.1.1 Absolute PPP

$$R_t = \frac{E_t P_t^*}{P_t} = 1 \tag{3}$$

Where,

E_t: nominal exchange rate

P_t^*: foreign price level

P_t: home price level

R_t: real exchange rate

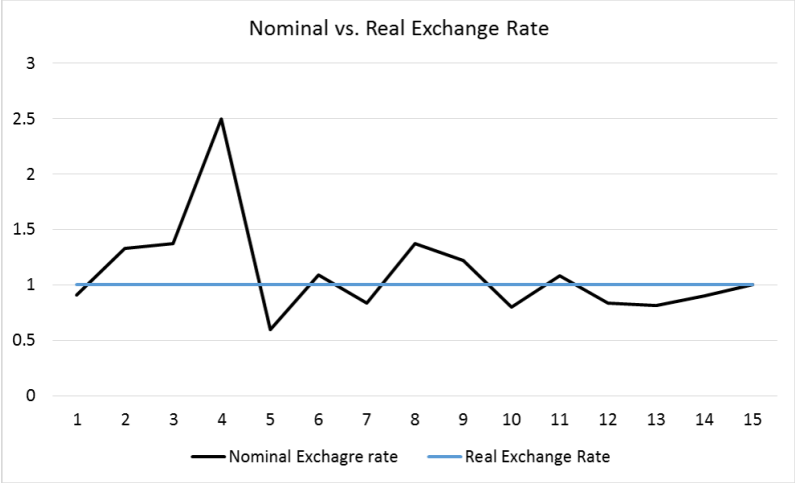
3.1.2 An illustrative example

Figure 1 presents a graphical example of simulated values of the price levels and the exchange rate to demonstrate the difference between equation 2 and equation 3. Equation 2 (the black line) requires a *proportional* relationship between the relative price levels and the nominal exchange rate. While equation 3 (the blue line) requires *a constant* relationship between relative price levels and the nominal exchange rate i.e. real exchange rate.

[23]Even though this paper is concerned with relative PPP, an overview of the subject matter does contribute to understanding.

[24]There is no unique definition of the short and long term. Though, a general consensus is that the short run can be up to 3-5 years and the long run can be from years to several decades.

Figure 1: Difference between equation 2 and equation 3



3.1.3 Relative PPP

$$R_t = \frac{E_t P_t^*}{P_t} = k \tag{4}$$

Equation 4 requires the real exchange rate, R_t to be constant, k, over the long run. But it is allowed to deviate from 1 in the short run.

Note that absolute PPP (equation 3) does not accept short run deviations. Since, changes in price levels are expected to be offset by the exchange rate immediately. While relative PPP (equation 4) does accept short run deviations. In the short run changes in price levels are partially offset by exchange rate and partially by the constant, k. By taking logarithms of equation 4 and rearranging gives:

$$r_t = e_t + p_t^* - p_t \Leftrightarrow e_t - p_t + p_t^* = r_t \tag{5}$$

Where r_t is the logarithms og the real exchange rate, e_t is the logarithms of the nominal exchange rate, p_t^* is the logarithms of the price levels abroad and p_t is the price level at home. The right hand side of the equivalence sign is tested in this paper.

PPP is considered to be more precise than the exchange rate since it is suppose to reflect the true prices of the goods and services. For example GDP per capita measured on PPP is assumed to be more precise GDP per capita. Because GDP denominated in local currency can be artificially high due to inflation. PPP can eliminate this bias.

By comparing basket of identical goods it should be theoretically possible to determine if the prices are too high or too low in a country. Because If prices are low in a country. The money from country with higher prices will flow into the

country with lower prices. If prices are higher in a country, money will flow out of the country to the country with lower prices.

The process described here is not necessarily referred to capital account but how the exchange rate will adjust to demand and supply for a given currency. And how this demand and supply is influenced by relative price levels.

3.1.4 An illustrative example

The example below illustrates a plausible outcome in a case of a disequilibrium in exchange rate, relative prices or both. Assume 1 kg Salmon in Norway costs NOK 70, in Germany it costs EUR 10 and the bilateral exchange rate is EUR/NOK is 10 (1 EUR = 10 NOK).

From German standpoint the cost of 1 kg Salmon is 7 EURs in Norway. Now since the Salmon is cheaper in Norway, the demand for NOK will increase and/or the demand for EUR will decrease. As a result of increased demand for NOK the price in Norway will/should increase, NOK will appreciate (NOK is bought and EUR is sold). Because of lower demand in Germany the prices of Salmon will fall and the EUR will depreciate[25].

A single good/service has little effect if any on exchange rates. But if the overall prices of goods and services (i.e. basket) are cheaper in one country then it is logically to assume that demand for its goods will increase and this country will have trade surplus. Since foreign currency is converted for this country it currency should appreciate in terms of other currency. And the country with higher prices will see a trade deficit and thereby see value of their currency depreciate.

Let us consider the same example in terms of real exchange rates and what it implies. Applying the information above gives real exchange rate: 10*10/70 = 1,43. That implies 1 kg of Solomon is 43% more expensive in Germany. The rate in terms of PPP:

$$PPP * 10/70 = 1 \Leftrightarrow PPP = 7(EUR/NOK)$$

Thus the NOK should appreciate. If the Norwegians and the Germans have the same nominal wage expressed in EUR, but they consumed in their respective countries, then the Norwegians would had substantial more purchasing power then

[25]Here I am not suggesting a relationship between a trade deficits but how changes in demand and supply for a given currency due to different price levels will or should converge to equilibrium.

their German counterparts. If this were true for all goods and services then the Norwegian would have higher real wages.

3.2 Perspectives on PPP

A valid question and perhaps quite intricate is what is the appropriate price level? The Casselian approach suggests the general price level, regardless of traded or non-traded goods. As a result consumer price index, CPI, is widely used in the empirical research of PPP. This view is also motivated by Frankel (Nelson C. Mark 2000:88) as he says:

'Some people believe that Purchasing Power Parities should be calculated exclusively on price indices for such commodities as for the subject of trade between the two countries. This is a misinterpretation of the theory . . .The whole theory of purchasing power parity essentially refers to the internal value of the currencies concerned, and variations in this value can be measured only by general index figures representing as far as possible the whole mass of commodities marketed in the country . . .'

3.2.1 The Casselian Approach

Casselian approach is viewed to be a long run relationship between exchange rates and relative price levels. Since in the long-run the two series (nominal exchange rates and the CPI based PPPs) tend to revert to each other. Samuelson on the other hand believes in the law of one price for internationally traded goods. Thus using commodity arbitrage approach, Producer Price Index, PPI, is a better choice for empirical research on PPP. The PPI is more heavily weighted toward traded goods then the CPI. (Nelson C. Mark 2001:62)

3.2.2 The Balassa-Samuelson hypothesis

B-S hypothesis has been briefly mentioned in the previous section. In this section B-S will be discussed more elaborately. B-S hypothesized that rich countries tended to have relatively higher prices in the non-traded sector. Since non-traded goods are more service intensive there is little room for technological superiority. This is clear from the historical data that technological progress in service (education, health, insurance etc.) intensive sector has been much slower then the manufacture. The reason for this is that in higher productivity in the traded goods sector will lead to higher wages. Thus the non-traded sector will raise their prices to match the

higher wages in the traded sectors (labor unions perhaps may force higher wages). Therefore one component of CPI is constant while other rising. This will lead to relatively higher prices.

3.2.3 Probable Reasons for Rejection of PPP Hypothesis

Even with a lot of research no solid evidence is presented for why the PPP is violated in the short-run. A short summary is presented below for the probable reasons for why the PPP is violated.

- There are no identical baskets, even within the basket the goods are weighted differently from country to country. People in different countries consume different goods and services. Though difficult to determine because of different habits (like vs dislike) in two nations, difference in quality of goods, and (structural) differences in two countries economies.
- There are a vast variety of factors affecting the real purchasing power of a currency. PPP is a long-term theory; it can take several years for exchange rates to reflect the difference in prices. Thus there will be deviations from PPP in short term. (Sticky prices from the monetary model)
- PPP assumes free trade, no taxes, tariffs and quotas etc. that is not true in reality.
- Exchange rates are influenced by government policies (trade or other restrictions), interest rates, speculation, hedging.

One might ask why is PPP considered in academia and why other macroeconomic models are inspired by this. The first argument is that we don't have alternatives to reconcile international prices with exchange rates. The second reason is that theories involve abstraction of reality that is not true. As Freidman (1953) said: 'we should not judge a theory about realism of its assumptions but by the quality of its predictions'.

3.3 PPP related Macroeconomic Models

Several macroeconomic models (variations of PPP) are closely related to the PPP. These models incorporate additional variables such as income levels, inflation levels, cost of living, etc. Here the goal is just not to compare the basket of goods in two countries in real conversion rates but what it costs compared to what public

(consumers) can afford to buy. The quantity theory of money and interest rate parity are worthwhile mentioning.

3.3.1 Quantity theory of money

The quantity theory money, QTM, suggests a proportional relationship between money supply and price level (Steigum 2007:206-208). It can be formally stated as:

$$MV = PY \Leftrightarrow P = \left(\frac{V}{Y}\right) M \tag{6}$$

Where,

M: money supply (money in circulation)

V: velocity of money (institutions etc.)

PY: nominal GDP, GDP-deflater*Real GDP

It is clear from the equation 6 that if V and Y are then there is a proportional relationship between money supply and prices. That implies for example a 5% change in money supply, M, cause a 5 % change in price level, P. It is important to note the relationship suggests its is the money supply that increases prices and not the other way around.

The proportionality between money supply and price level is a long-term relationship that arises after the price level has been given time to adapt to the money supply. In practice, price level is relatively fixed in the short-term, so changes in P normally come gradually.

Assumption that Y is exogenous, implies that the cyclical fluctuations in Y are not considered and a long-term trend in GDP is determined by the supply of labor and the development in total factor productivity (development of more efficient technology and production methods).

Since equation (6) applies to all periods, we can study the relationship of its components over time by stating the equation in growth form:

$$\pi = g_M - g_Y + g_V \tag{7}$$

Where, π: inflation rate

g_M: growth rate money supply

g_Y: growth rate in real GDP

g_V: growth rate of velocity of money

Earlier we assumed that changes in M was not affected by Y and V, $g_Y = g_V = 0$. In that case the growth in inflation rate must be equal to growth in the money supply. However, in the long-run, normally both Y and V vary in addition to the growth money supply.

For example, growth in GDP gives positive g_Y. This will reduce g_M in equation 7, all else equal. It is because the volume of transactions in the economy increases during economic growth. If $g_v = 0$ and GDP grows by 5 percent per year then the money supply must increase by 5 percent per year to offset the positive inflation pressure ($\pi = 0$).

3.3.2 QTM & Exchange rate

In this section the relationship between money supply, inflation and the exchange rate discussed.

Assume in equation 7, g_Y and g_V is constant in the long-run so that the growth in inflation (π) equals the growth in money supply ($\pi = g_M$). Further, from equation 2 assume that the monetary policy in home country influences price level (P) and do not affect the foreign price level (P^*). By setting equation 2 in growth form, we get the following relationship between growth in exchange rate (g_E), inflation rates at home (π) and abroad ($\pi*$):

$$\pi = g_E + \pi^* \Leftrightarrow g_E = \pi - \pi^* \tag{8}$$

The relationship in equation 8 requires for example that if inflation in Norway is 4% per year and inflation in Denmark is 2% per year, then the NOK should depreciate against DKK by 2%. Putting the QTM together implies that if growth in money supply is not offset by growth in GDP and/or V, this will increase inflation pressure (equation 7). Equation 8 implies that growth in exchange rate should equal interest rate differentials.

3.3.3 Interest rate parity

The Interest Rate Parity (IRP), relates exchange rates to interest rates. IRP helps determining the spot exchange rate. The IRP in its approximate form can be stated as:

$$i = i^* + \frac{\Delta E_t(S_{t+k})}{S_t} \tag{9}$$

where,

i: home interest rate

i^*: foreign interest rate

$\Delta Et(S_{t+k})$: is the change in expected future spot exchange rate

$\frac{\Delta E_t(S_{t+k})}{S_t}$: is the expected rate of depreciation of the home currency

The IRP relation in equation 9 requires that the domestic interest rate is equal to foreign interest rate plus a depreciation (change) of home currency. International capital flows will cause the same rate of return across countries.

For example if the interest rate in domestic currency is higher then foreign currency, then it can be borrowed in the foreign currency and invested in the domestic currency. This will increase the demand for foreign currency and decrease demand for domestic currency. Thus, the foreign currency will or should appreciate and domestic currency depreciate. In equilibrium the return from interest rate differential will be offset by adverse changes in future spot exchange rate. (Moffett, Stonehill and Eiteman 2010:175).

4 Data & Stationarity Analysis

Economic variables necessary to conduct research on the PPP hypothesis are bilateral exchange rate, consumer price index (or any other general price level index) for home - and abroad country. This paper focus on PPP between Norway and Sweden (SEK-NOK pair), and Norway and Denmark (DKK-NOK pair). Thus, requiring 3 data series for each pair. The test equation is presented below:

$$e_t - p_t + p_t^* = r_t$$

The data series are obtained from Norges bank, Statistics Sweden and Statistics Denmark. This gives the data series a high level of credibility. All data series starts in 1946 and ends in 2015 with both annual and monthly frequency. Further, all data series are transformed using logarithms.

There are two reasons to investigate PPP between Norway, Sweden and Denmark. First, there is a substantial amount of trade between these countries, not just at national or corporate level but also between corporations and consumers. For example the Norwegian-Swedish border trade and Norway-Denmark ferry travels. The second reason is the geographical proximity of these nations to each other. This is important because all else equal, transportations costs should be low between these countries since they are very close to each other.

For each pair the data series are divided into different sub-samples. The idea behind sub-samples is to study how the relationship between exchange rates and the relative price levels has varied *through* and *across* time, if at all.

For example, is it the case that it will take several decades before PPP return to some kind of equilibrium. So it is appropriate to use data over several decades. Another perspective is to look at the relationship across different time periods. For example is it the case that the relationship between exchange rates and the relative prices will be stronger under the 'free float' exchange rate policy compared to 'fixed exchange' rate policy.

The first sample period is the entire period (1946-2015). The second sample starts around the inception of the Bretton Woods in 1946 and ends at its collapse around 1971. The third sample period ranges from 1972 to around 1990, so that the post Bretton Woods activity is captured. Availability of the frequency has also impacted the start-end points of the sub-samples.

For Sweden the longest sample period, 1955-2015 has monthly frequency, 2178 observations. The second sample period is 1946-1972 (Bretton Woods period) with annual frequency, 81 observations. The third period is 1971-1992 (post Bretton Woods) with monthly frequency, 792 observation. The final period, 1992-2015 ('free float period') with monthly frequency, 810 observations.

Similarly, for Denmark the longest sample period is 1946-2014 with annual frequency, 207 observations. The second period is 1946-1979 with annual frequency, 102 observations. And finally 1980-2015 (fixed exchange rate period, first to German Mark than to Euro) with monthly frequency, 1272 observations.

Note that the choice of sub-periods is also influenced by the availability of the data. For example, for Denmark data with monthly frequencies was only available from 1980. The frequency of the data series is not supposed to influence results but the sample size can. Generally, large sample compared to small sample has a greater probability of replicating the true values of the population.

The primary statistical software used to conduct research on PPP is R Studio. Some miner data preparations are performed in Microsoft Excel so that the data is compatible with R Studio. These preparations include changing data format and naming columns.

4.1 Stationarity

Before we can formally test the relationship between exchange rate and the relative price levels. The statistical method, Johansen test, requires that the data series to satisfy certain characteristics, namely non-stationarity and the same level of integration. Thus, it is necessary that the data series demonstrate the structure required by the statistical method in order to make valid statistical inference.

A process is said to be stationary if it has constant mean, variance and autocovariance at each lag. But financial time series are normally integrated of order 1. That is, they are not stationary in there level forms but becomes stationary with first difference. A formal test for stationarity is the Augmented Dicky Fuller (ADF) test for unit root. With p-lags of the dependent variable the ADF test can be presented as:

$$\Delta y_t = \psi y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + u_t \tag{10}$$

Here the null hypothesis is carried under the null that the $\psi = 0$. That is, series contains a unit root. Against the alternative that the $\psi < 0$. That is the series is stationary. Since the test is one sided, ψ, cannot be positive. The test statistic can be calculated as follows:

$$\text{test statistics} = \frac{\hat{\psi}}{\widehat{SE}(\hat{\psi})} \tag{11}$$

Once test statistics is completed it compared to Dicky fuller critical value. If the test statistic is more negative then DF critical value, the null is rejected and concluded that the series is stationary. (Brooks 2008:328)

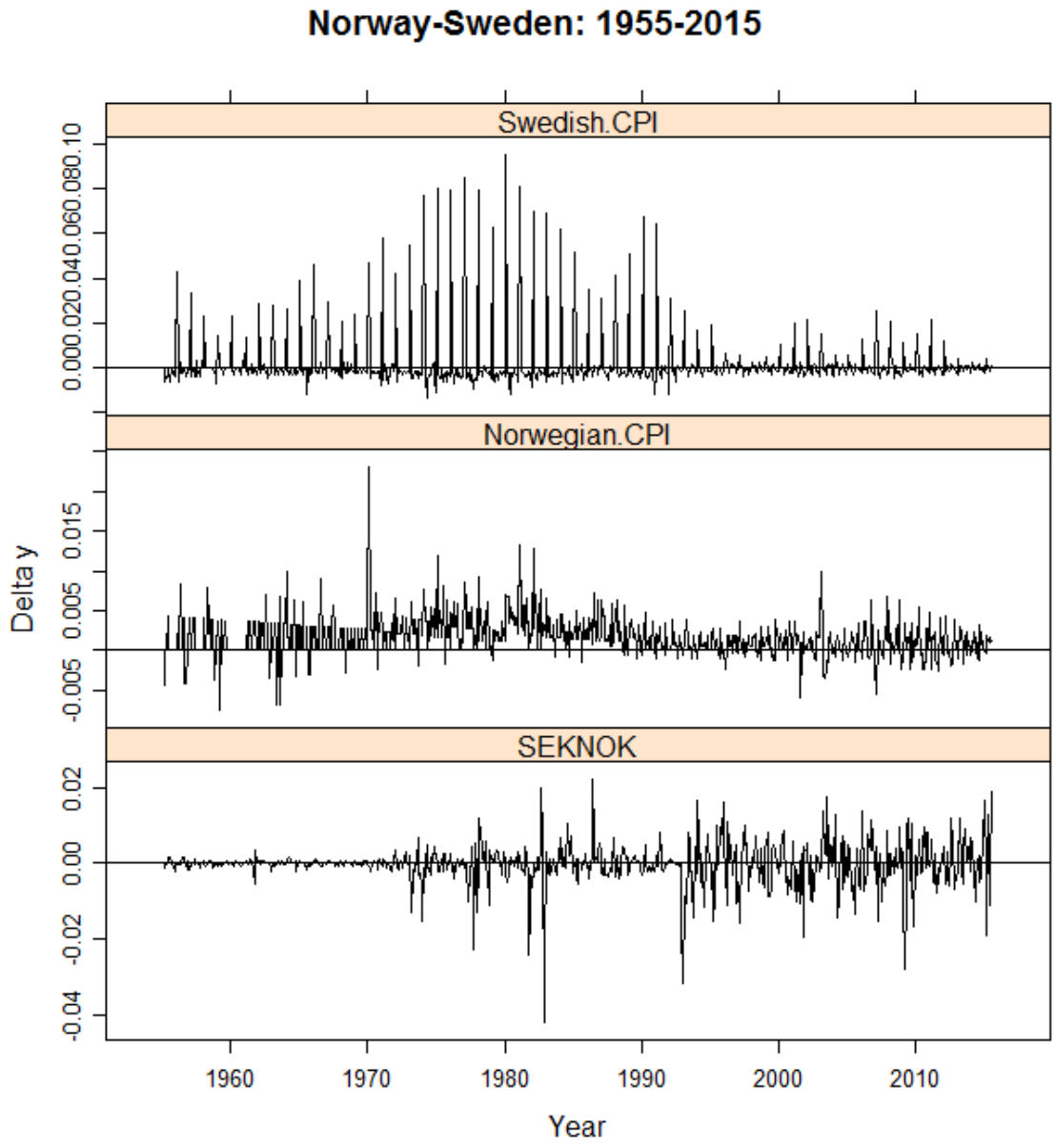
4.2 Graphical analysis of stationary

Figure 2 through 8 present first differenced data series for various time periods. The focus here is to observe the tendency of the observations above and below zero line ($y = 0$). Note that in Δy_t in equation 10 can deviate from $y = 0$ but is supposed to be constant around the zero line.

Large spikes can be seen as shocks. These shocks both historical and future, are supposed to decay gradually. If they don't, this can result in a non-stationary process.

In Figure 2 the Swedish CPI does vary around the zero line must of the time. But does have some large spikes, especially from 1970s to 1990. After 1990 Swedish CPI is relatively calm.

Figure 2: From top Swedish CPI, Norwegian CPI and SEK/NOK exchange rate.

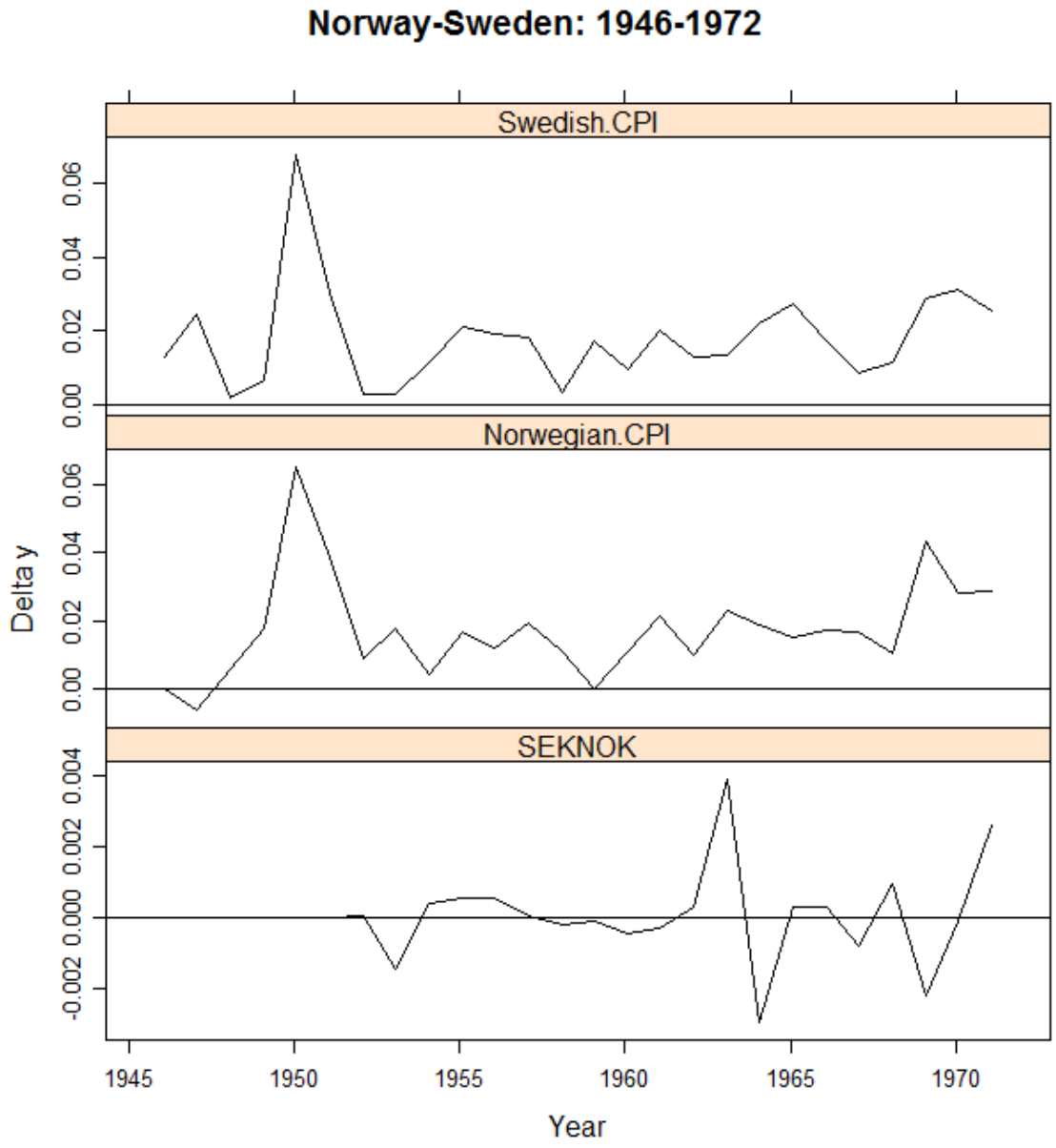


The Norwegian CPI compared to Swedish CPI has greater variability around zero line but less large spikes. For the exchange rate, NOK/SEK, is quite stable around the zero line. Though, from 1955 up to 1980 the variability around the zero line was quite small, large spikes in the early 1980s and the early 1990s. After the early 1990s the exchange rate experienced a larges variability around the zero line but fewer large spikes.

Figure 3 presents series for Bretton Woods period (1946-1972). It is apparent from the chart of Swedish CPI that it has a been above the zero during the entire period.

The same is the case for the Norwegian CPI. Both series experienced a large

Figure 3: From top Swedish CPI, Norwegian CPI and SEK/NOK exchange rate.



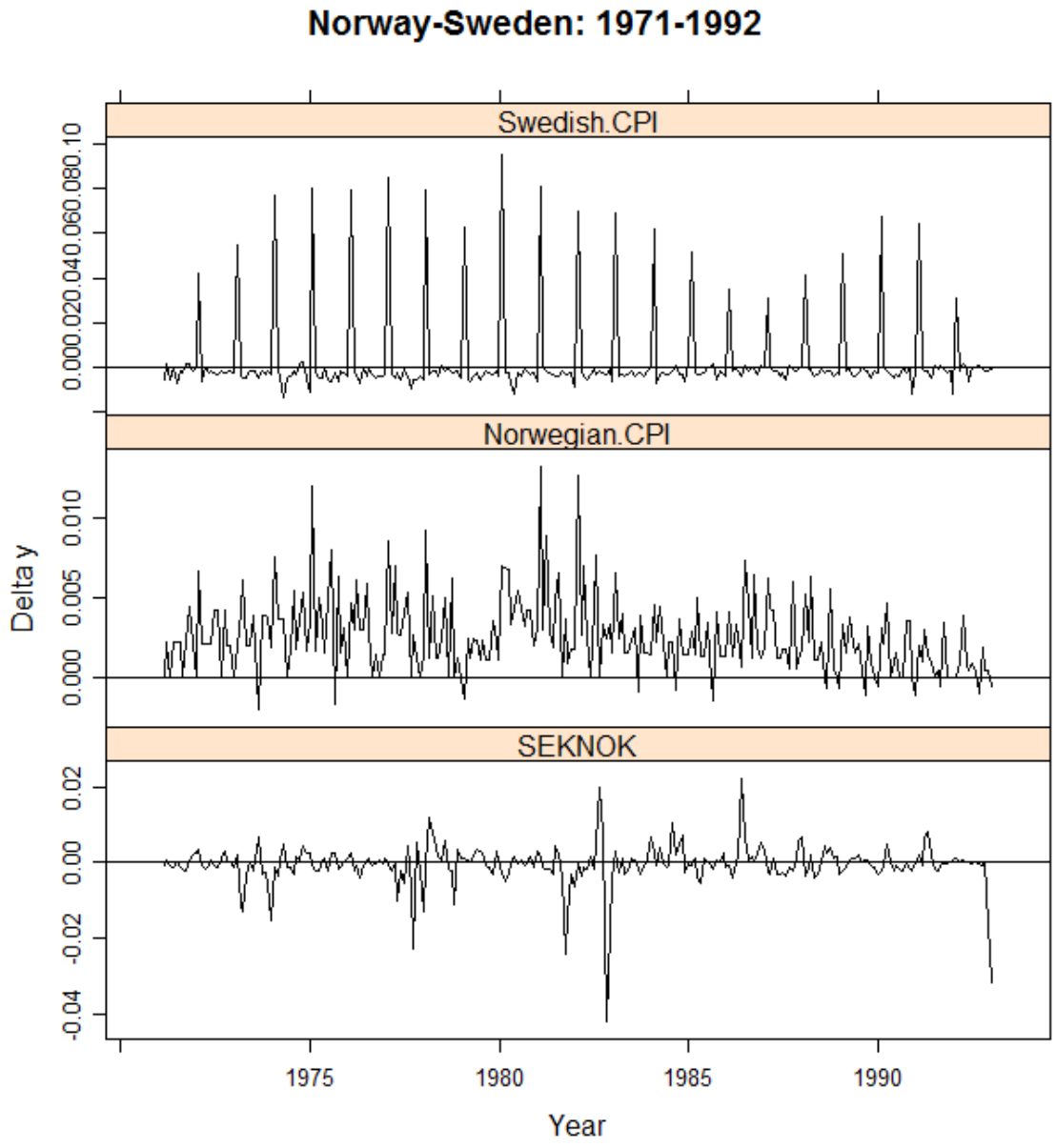
spike in the early 1950s. This is perhaps due to a controlled depreciation of 30,5 percent in 1949 against the USD and later fixing the exchange rate the US dollar in 1951[26]. For the entire period (1946-1972) the CPIs for both Sweden and Norway have been increasing at a increasing rate. The exchange rate quite stable around the zero line except for a large spike around 1964 (might be due to a controlled depreciation). And also in the early 1970s. This can be due to the Nixon shock (when US rejected the convertibility of US dollar to gold).

Figure 4 presents data series for the post Bretten Wood Period. The Swedish

[26]A controlled depreciation of 30.5 % against the USD on September 19, 1949. (1 USD = 5.17 SEK). Membership of the International Monetary Fund and part of the Bretton Woods system on August 31, 1951.(Wikipedia)

CPI does vary around the zero line but does have a pattern og regular spikes. This is perhaps due to a lot of controlled depreciations and a couple of appreciation in the period[27].

Figure 4: From top Swedish CPI, Norwegian CPI and SEK/NOK exchange rate.



[27]A controlled depreciation of 1.0 % against gold and a 7.5 % appreciation against the USD on December 21, 1971. A controlled depreciation of 5.0 % against gold and a 5.6 % appreciation against the USD on February 16, 1973. Membership in the European "currency snake" in March 1973. Adjustment of the exchange rates within the "snake"; a controlled 3 % depreciation against the DEM on October 18, 1976. Adjustment of the exchange rates within the "snake"; a controlled 6 % depreciation against the DEM on April 4, 1977. Sweden leaves the "snake". A controlled 10 % depreciation against a trade based "currency basket" on August 29, 1977. A controlled depreciation of 10 % against the "currency basket" on September 14, 1981. A controlled depreciation of 16 % against the "currency basket" on October 8, 1982. A tie to the European Currency Unit is introduced unilaterally on May 17, 1991. (1 ECU = 7.40 SEK). Floating exchange rate on November 19, 1992.(Wikipedia)

Norwegian CPI has also been quite volatile for the entire period, mostly above the zero line. But does not contain regular spikes in the CPI as the case was for the Swedish CPI. The exchange rate has been quite stable around the zero line except for some spikes in the late 1970s and in the early 1980s. A large spike is also apparent in the early 1990s, which might be due to 'free float' of the Swedish krone in 1992.

Figure 5 presents data series for the 'free float' period. All series are quite stable around the zero line, except for Swedish CPI, which has a few large spikes. These data series compared to previous periods have a relatively higher degree of volatility.

Figure 5: From top Swedish CPI, Norwegian CPI and SEK/NOK exchange rate.

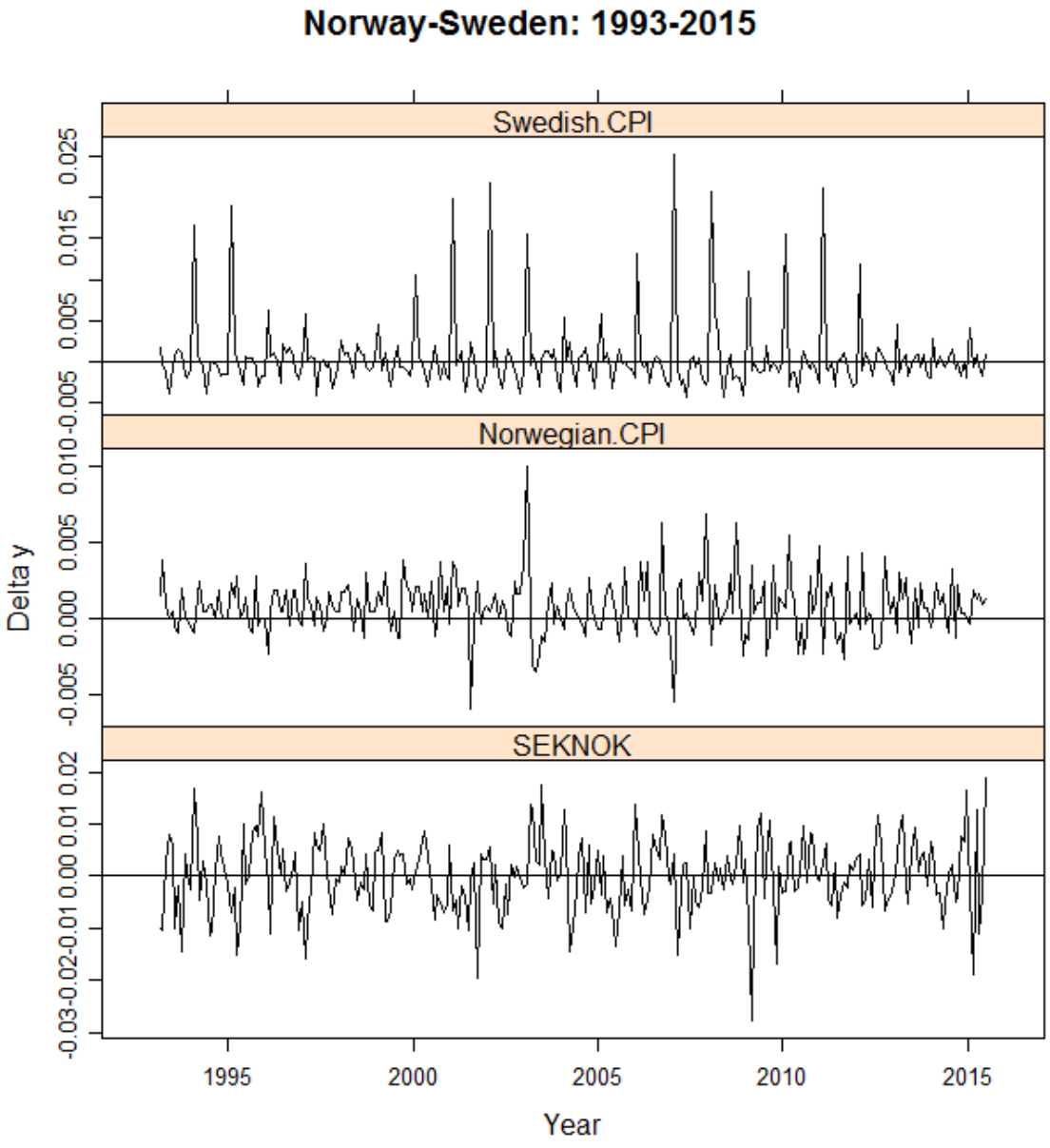
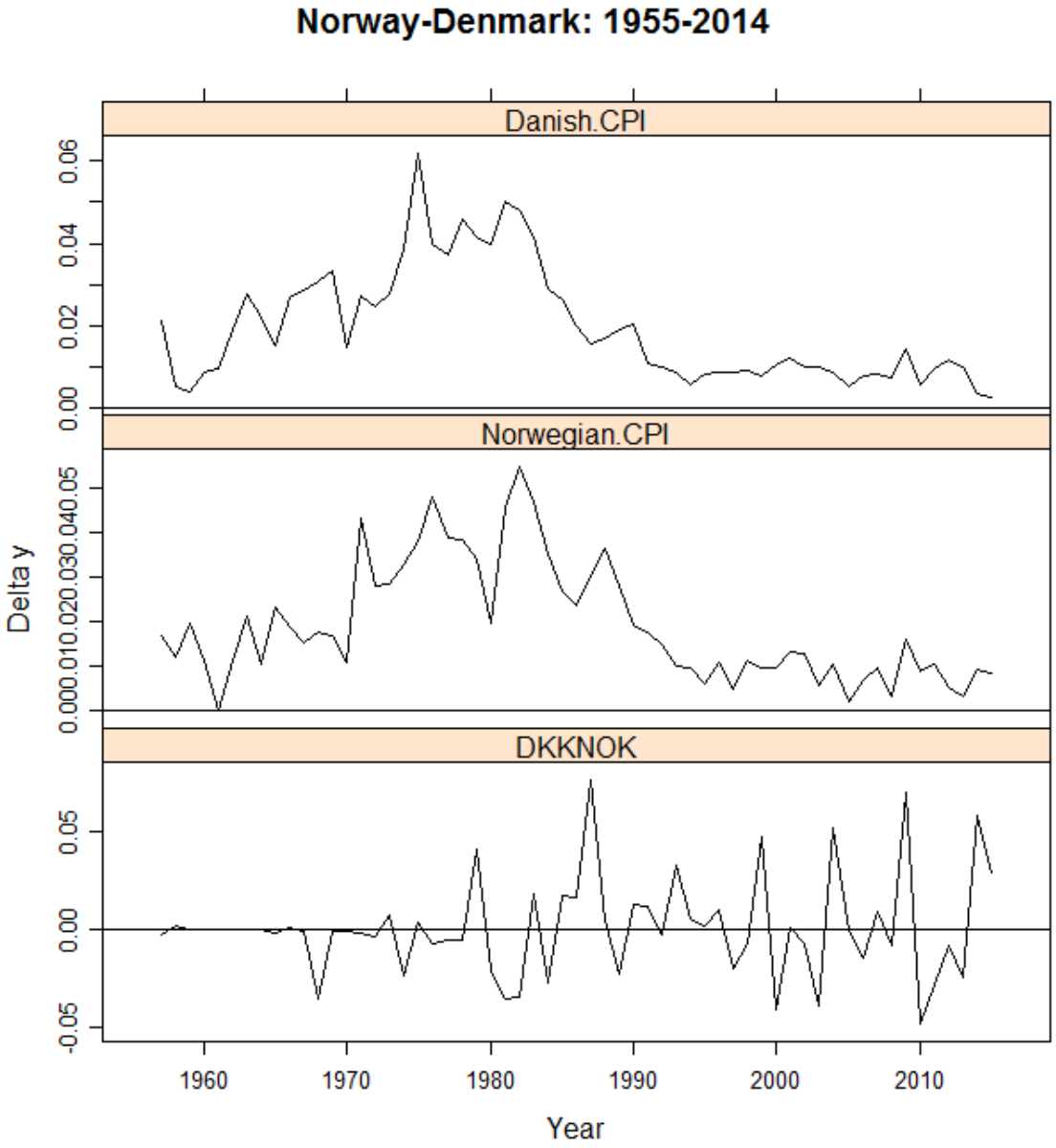


Figure 6 presents annual data series for Denmark and Norway. CPI for Denmark has clearly been above the zero line, must of the time, except for a small dip in the middle of the 1950s.

Figure 6: From top Danish CPI, Norwegian CPI and DKK/NOK exchange rate.



Danish CPI increased at an accelerating rate up to the middle of the 1970s and then gradually decreased rest of the period. Similar pattern is apparent for the Norwegian CPI except it increased up to early 1980s and then decreased for rest of the period.

The large increases can be due to controlled depreciations as it was the case for Sweden. The exchange rate has been quit calm up to 1970s but later very volatile for the entire period. It is important to note that the Danish krone has been tied

to German Mark up to 1999 and then Euro. The Danish Central bank intervenes regularly in the currency exchange market to hold the Danish krone in a constant level (Danish Central Bank).

Figure 7 shows that the CPI for both Denmark and Norway has been increasing at an accelerating rate for the almost entire period. Except for the early 1975 where it declined slightly. The exchange rate did not moved much up to 1970 and then some variability around the zero line is to be seen.

Figure 7: From top Danish CPI, Norwegian CPI and DKK/NOK exchange rate.

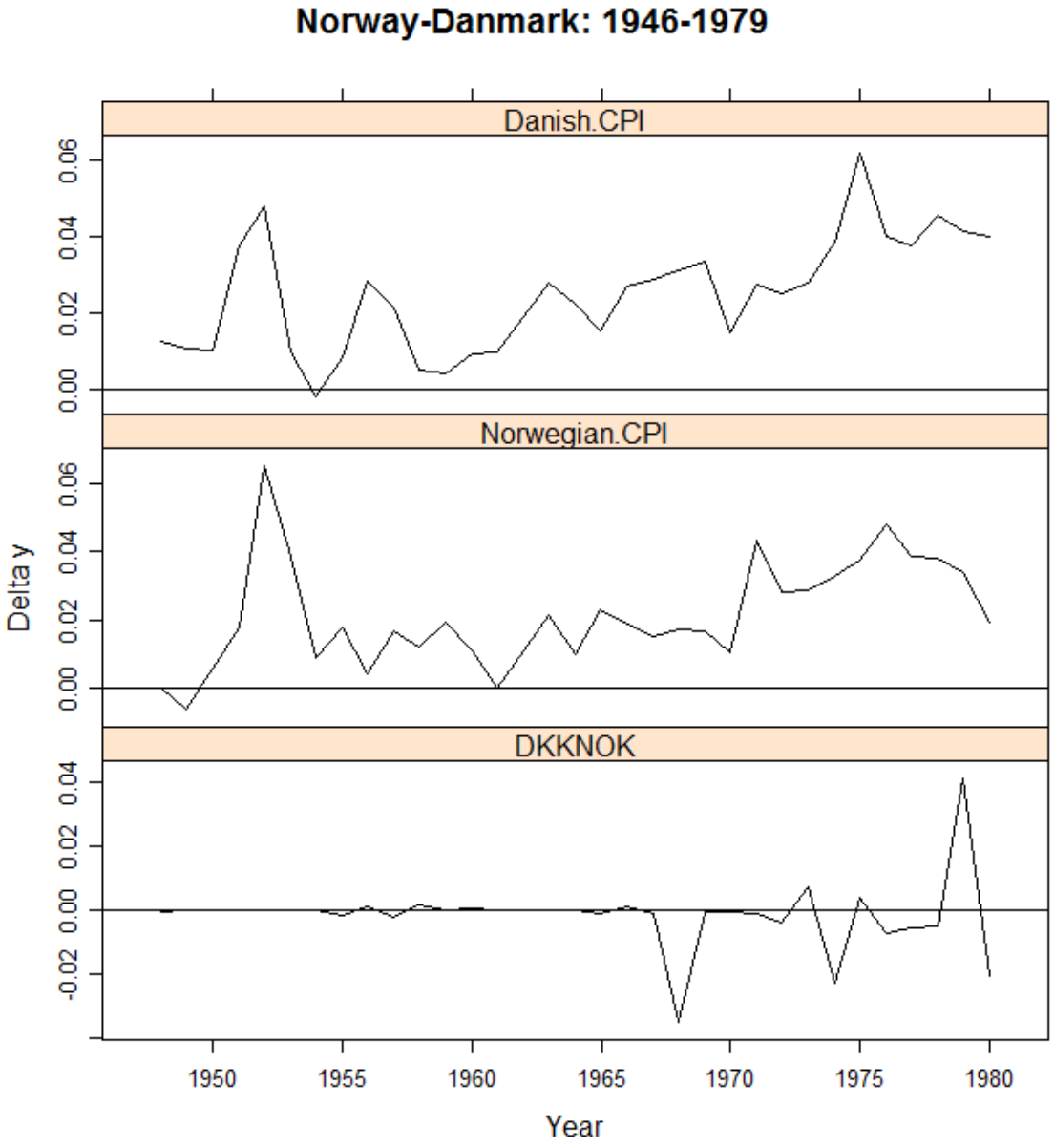
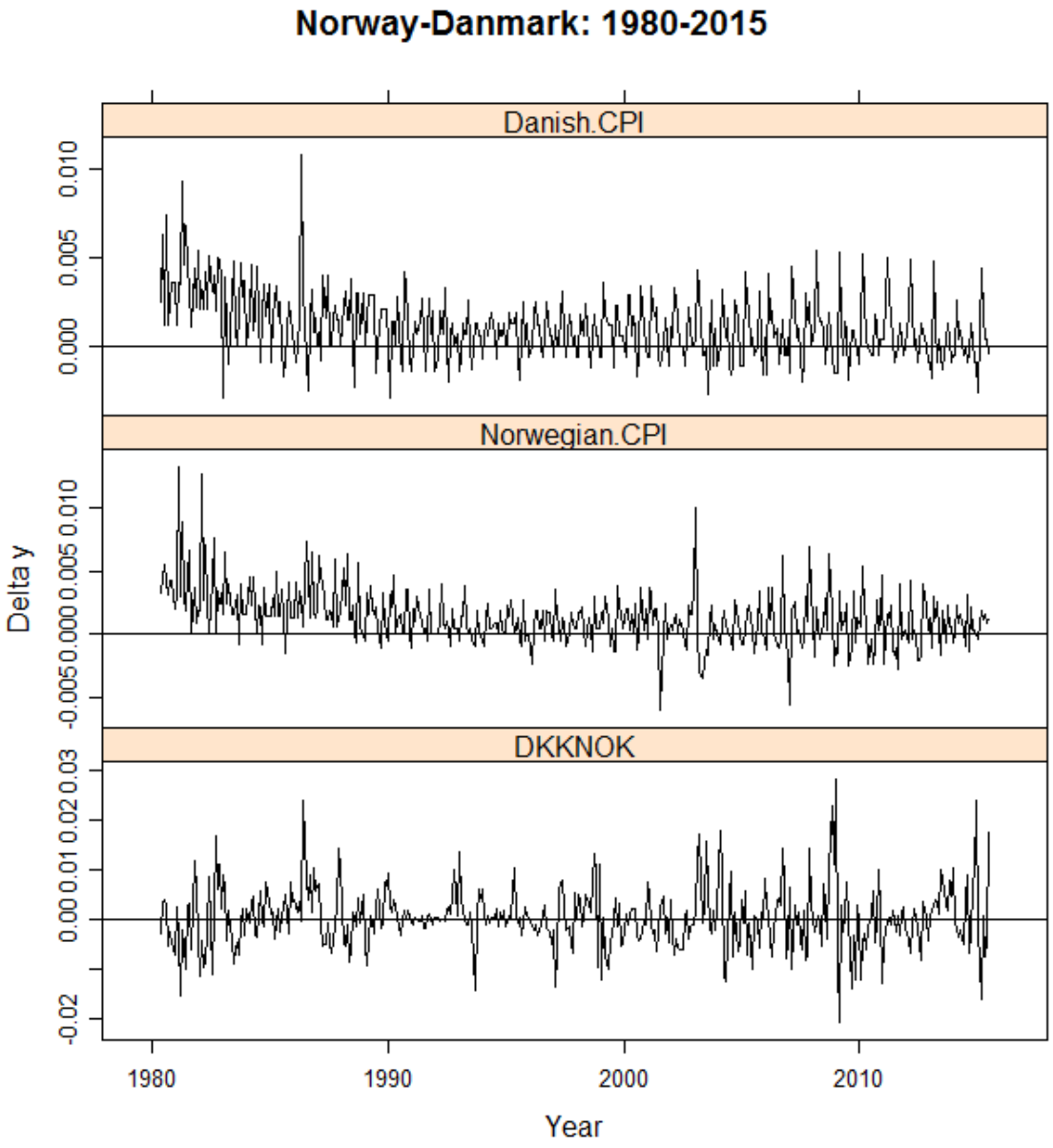


Figure 8 presents data series for the period 1980-2015. All series have varied quite stable around the zero line, most of the time. Only few large spikes are visible from the figure.

Figure 8: From top Danish CPI, Norwegian CPI and DKK/NOK exchange rate.



4.3 Augmented Dicky Fuller (ADF) test

In this section an ADF unit root test is performed to test stationarity. The task here is to determine whether the variables under study are stationary and if so of what integrated order. Table 1 contains results for SEK-NOK pair while table 2 contains results for the DKK-NOK pair.

For the SEK-NOK pair all data periods except for the 1946-1972 where difference stationary. That is, they are integrated of order 1. Since they have the same integration order, it is suitable to perform Johansen test to examine a probable co-integration. These results were also indicated from the graphical analysis in the

previous section. The test values in the table clearly shows that the Norwegian and Swedish CPI are integrated of order 2 for the period 1946-1972. For this period the Johansen test will not be performed.

Table 1: Augmented Dicky Fuller test for the SEK-NOK pair

Variable	Deterministic terms	Lags	Test value
Panel:1955-2015			
SEK/NOK	none	3	-1.57
CPI_{NOK}	.	3	7.62
CPI_{SEK}	.	3	4.00
ΔSEK/NOK	.	3	-18.62***
ΔCPI_{NOK}	.	3	-11.22***
ΔCPI_{SEK}	.	3	-21.04***
Panel: 1946-1972			
SEK/NOK	.	3	0.01
CPI_{NOK}	.	3	3.29
CPI_{SEK}	.	3	3.50
ΔSEK/NOK	.	3	-5.60***
ΔCPI_{NOK}	.	3	-1.26
ΔCPI_{SEK}	.	3	-1.78*
$\Delta\Delta CPI_{NOK}$	.	3	-4.36***
$\Delta\Delta CPI_{SEK}$	none	3	-6.35***
Panel: 1971-1992			
SEK/NOK	drift	3	-1.42
CPI_{NOK}	none	3	5.44
CPI_{SEK}	.	3	2.97
ΔSEK/NOK	.	3	-10.49***
ΔCPI_{NOK}	.	3	-4.66***
ΔCPI_{SEK}	.	3	-12.56***
Panel: 1993-2014			
SEK/NOK	.	3	-1.25
CPI_{NOK}	.	3	5.82
CPI_{SEK}	.	3	2.08
ΔSEK/NOK	.	3	-11.76***
ΔCPI_{NOK}	.	3	-10.18***
ΔCPI_{SEK}	none	3	-13.21***

Number of lags are obtained using the Akaike Information Criterion (AIC)| ***, ** and * denote significance at 1 %, 5 % and 10 % levels, respectively. The 5 % level is used as a benchmark in this paper.

Table 2 presents results for DKK-NOK pair. Only for the period 1980-2015 the variables are integrated of the same order. While for both periods 1955-2014 and 1946-1979 the Danish and Norwegian CPIs are integrated of order 2. Thus, Johansen test will be performed only for the period 1980-2015.

For Sweden one period did not qualified for a Johansen test, while for Denmark

Table 2: Augmented Dicky Fuller test for the DKK-NOK pair

Variable	Deterministic terms	Lags	Test value
Panel:1955-2014			
DKK/NOK	none	3	-1.37
CPI_{NOK}	.	3	0.74
CPI_{DKK}	.	3	0.48
ΔDKK/NOK	.	3	-5.94***
ΔCPI_{NOK}	.	3	-1.12
ΔCPI_{DKK}	.	3	-0.98
$\Delta\Delta CPI_{NOK}$	.	3	-7.47***
$\Delta\Delta CPI_{DKK}$	.	3	-7.12***
Panel: 1946-1979			
DKK/NOK	.	3	-0.76
CPI_{NOK}	.	3	2.94
CPI_{DKK}	.	3	2.16
ΔDKK/NOK	.	3	-3.92***
ΔCPI_{NOK}	.	3	-1.21
ΔCPI_{DKK}	.	3	-0.93
$\Delta\Delta CPI_{NOK}$	.	3	-4.89***
$\Delta\Delta CPI_{DKK}$	.	3	-6.68***
Panel: 1980-2015			
DKK/NOK	.	3	-1.52
CPI_{NOK}	.	3	5.48
CPI_{DKK}	.	3	6.51
ΔDKK/NOK	.	3	-12.96***
ΔCPI_{NOK}	.	3	-8.51***
ΔCPI_{DKK}	none	3	-9.94***

Number of lags are obtained using the Akaike Information Criterion (AIC)| ***, ** and * denote significance at 1 %, 5 % and 10 % levels, respectively. The 5 % level is used as a benchmark in this paper.

2 periods did not. This is interpreted as a case against the PPP. Because, for both pairs SEK-NOK (1946-1972) and DKK-NOK (1946-1979) graphical analysis reveals increasing inflation (CPI) at an increasing rate. This must interpreted as shocks to CPI (perhaps due to controlled depreciations) do not gradually die away in the sample period. This result is not surprising given the number and size of depreciations during the post Bretton Woods era.

For DKK-NOK during 1946-2014, the inflation was increasing at an increasing rate up to 1980s. While from early 1980s and onward has been decreasing toward the zero line. Considering the entire period, the shocks up to 1980s are still persist in the data series. The Danish krone has practically been fixed to other currencies throughout the entire period. This is perhaps the reasons for the long life of the shocks up to 1980s.

The results for the shorter period or fixed rate periods suggest a less likelihood of finding a relationship between exchange rate and the relative price levels.

5 Method & Co-Integration Analysis

Results from the stationarity test suggest that it is appropriate to apply an co-integration test on those sub-samples that qualified for Johansen test from the ADF test. Two or more non-stationary variables are said to be co-integrated if a linear combinations of the variables is stationary. It is a common phenomenon that series that are individually non-stationary but do have a strong tendency to 'move together' over time.

Even though, this is common for financial data series to moved together over time, it is not always the case. This is apparent from ADF tables from the previous section. Also, it is not reasonable to just assume stationary variables or some order of integration and perform test of co-integration. Therefore, even if the data series are assumed to move together over time, test for stationarity should be performed beforehand.

This can be due to economical mechanisms such as market force so that the series are bound by some relationship in the long run (Brooks 2008:336). This is also the main focus of this study, to investigate the long run relationship between exchange rate and the relative price levels. Since we have theoretical reason to assume a co-integration (long relationship) between exchange rate and the price levels, a Vector Error Correction Model (VECM) based on Vector Autoregressive Model (VAR) can be set up of the form:

$$\Delta y_t = \Pi y_{t-k} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + ... + \Gamma_{k-1} \Delta y_{t-(k-1)} + u_t \tag{12}$$

Where $\Pi = (\sum_{i=1}^{k} \beta_i)$ - I_g and $\Gamma_i = (\sum_{j=1}^{i} \beta_j)$ - I_g

The VECM contains first differenced form of the variables on both the LHS and the right hand side of the equation. The right hand side of the equation 12 contains $k-1$ lags of the dependent variables with Γ coefficient matrix. The central part of the Johansen test is concerned with the study of the Π matrix. The Π matrix can be viewed as the ling run coefficient matrix. Because in equilibrium all changes in variables in the LHS Δy_{t-1} will be zero and setting u_t to zero. This will give $\Pi y_{t-k} = 0$

Note that the VECM is comparable to ADF but is not the same. In the VECM y_t is k-dimensional, i.e. contains k variables on the LHS of the equation 12. In our

case 3 equations, one for each variable E, P and P^*.

The first term on the RHS of equation is referring to sum of the lagged values of the variable on the LHS, say E. While, Γ_i, are referred to lagged values of the other variables in the model, P and P^*.

For the second equation, P the first term of the RHS will be refereed to the sum of the lagged values of P. While, Γ_i, will contain the sum of the lagged values of the E and P^*. The same is true for equation 3, P^*.

This can be interpreted as a variables is a linear combination of its previous values and the values of other variables in the model. The first challenge with ADF test is its low power due to size distortion. This is especially true if the series exhibits a substantial moving average component (Maddala and Kim 1998:100). As a result the underlying distribution of the data series under study will significantly be different from DF distribution.

The test for co-integration is performed by looking at the rank, r, of the Π matrix via its eigenvalues, λ_i. The rank of the Π matrix is equal to its eigenvalues (roots) that are different from zero. The λ_i must be less then 1 in absolute value and positive. For non co-integrated variables the rank of the Π matrix will not be significantly different form zero, i.e $\lambda \approx 0 \; \forall \; i$.

If the rank of the $\Pi = 1$, (the linear combination is stationary and there is one long run relationship between the variables in the model, elaborately discussed in section 5.1) then $ln(1 - \lambda_1)$ will be negative and $ln(1 - \lambda_i) = 0 \; \forall i > 1$. This condition requires that for Π matrix to have a rank of 1, the largest eigenvalue must be significantly non-zero, while other will not be different from zero (Brooks 2008:351). To test the eigenvalues of the Π matrix a trace test of the following form is performed:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^{n} ln(1 - \hat{\lambda_i}) \tag{13}$$

Lambda trace is a joint test where the null is that the co-integrating vector is less than or equal to r, against the alternative that it is more/greater than r (r is referred to the rank of the Π matrix) if the test statistic is greater then critical values from Johansen tables, reject null and conclude either $r+1$ or more co-integrating relations are present. Continue until the null cannot be rejected.

Note that the rank of Π matrix cannot be greater then $y - 1$. Where y is the number of variables. The results from the trace test are presented in next section.

5.1 Co-integration analysis

Table 3 and 4 contains result from the trace test for the Swedish and Danish data series. The null of a zero rank Π matrix is rejected at 5 percent level for the SEK-NOK pair for the period 1955-2015. And the alternative hypothesis that there is one or more rank is accepted. That is, one co-integration is present for the period 1955-2015.

Implying that there is long relationship between the exchange rate and the relative prices. This can be interpreted as relationship between exchange rates and relative prices is indeed influenced by market forces. This can also be interpreted as an equilibrium relationship. It is possible that the variables may deviate from their relationship in the short run, but their relationship will return in the long run.

For other two periods, 1971-1992 and 1993-2014, there is no evidence for any co-integration. Since the null of a zero rank Π matrix cannot be rejected for neither the period 1971-1992 nor 1993-2015.

Table 3: Johansen trace statistic SEK-NOK

Null hypothesis	Alternative	Test value	Critical value		
$\lambda(k = 10)$			10 %	5 %	1 %
Period:1955-2015					
$r \leq 2$	$r > 2$	1.22	6.50	8.18	11.65
$r \leq 1$	$r > 1$	13.24	15.66	17.95	23.52
$r = 0$	$r > 0$	33.63**	28.71	31.52	37.22
Period:1971-1992					
$r \leq 2$	$r > 2$	2.85	6.50	8.18	11.65
$r \leq 1$	$r > 1$	9.11	15.66	17.95	23.52
$r = 0$	$r > 0$	23.18	28.71	31.52	37.22
Period:1993-2014					
$r \leq 2$	$r > 2$	0.32	6.50	8.18	11.65
$r \leq 1$	$r > 1$	4.48	15.66	17.95	23.52
$r = 0$	$r > 0$	20.46	28.71	31.52	37.22

The null for a zero rank and rank 1 Π matrix must be rejected for the DKK-NOK for the period 1980-2015. Thus, for the DKK-NOK pair there are two co-integrating relationship. Implying that not just a long run relationship between the exchange rate and the relative prices, but also a long run relationship within the Danish and Norwegian CPI.

Table 4: Johansen trace statistic DKK-NOK

Null hypothesis	Alernative	Test value	Critical value		
$\lambda(k=3)$			10 %	5 %	1 %
Period:1980-2015					
$r \leq 2$	$r > 2$	1.64	6.50	8.18	11.65
$r \leq 1$	$r > 1$	25.59***	15.66	17.95	23.52
$r = 0$	$r > 0$	105.62***	28.71	31.52	37.22

5.2 Short and long run parameters from VECM

The Π matrix which is used to determine the co-integrating relationship in Johansen test is based on short and long-run parameters. The composition of the Π matrix can be stated as:

$$\Pi = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} \end{pmatrix} \begin{pmatrix} \Delta E_t \\ \Delta P_t \\ \Delta P_t^* \end{pmatrix}_{t-k}$$

The short and long run parameters from the VECM estimation are presented in table 5. Since this paper does not impose restriction on the co-integrating vector (does not test for absolute PPP) the parameters in table 5 are not uniquely identified. They are presented here because of convention and not interpretation.

Table 5: VECM estimates for SEK-NOK and DEK-NOK

Panel:1955-2015	SEK/NOK	CPI_{NOK}	CPI_{SEK}
$\hat{\alpha}$	-0.02	0.01	0.05
	(-2.57)	(2.68)	(3.37)
$\hat{\beta}$	1	0.62	-0.44
		(1.85)	(-1.35)
Panel: 1980-2015	DKK/NOK	CPI_{NOK}	CPI_{DKK}
$\hat{\alpha}$	-0.003	0.008	0.005
	(-1.26)	(8.55)	(6.21)
$\hat{\beta}$	1	-5.34	4.80
		(-6.30)	(4.93)

6 CONCLUSION

Previous studies have rationalized on a variety of reasons for the failure of the PPP. But not much attention os given to the LOP. CPIs that are assumed to be the same not just across borders but also through time, in reality are not true for either case. Further, the base year used to calculate the CPI is crucial and can effect inference from the analysis in either direction. This is especially true if the base year for the CPI is close to major macroeconomic events or financial crisis. From graphical analysis substantial spikes were present around inception and collapse of Bretton Woods. The spikes were also present in the event of monetary interventions in the post Bretton Woods era.

Analysis from graphical analysis suggests that consumer prices and and the exchange rate can rise or fall for sustained period of the time. This was especially case for the SEK-NOK pair for the period 1946-72 and the Danish and Norwegian CPIs for the period 1946-1979. Both periods are mostly related to the Bretton Woods period. As discussed in the graphical section, there were substantial interventions from the central bank, both in terms of frequency and magnitude. Generally, large spikes in CPIs were also apparent in the charts.

The results from the ADF test confirmed the visual examination of the data series. It was apparent from the ADF parameters that CPIs can increases at accelerating rate while changes in exchange rate are for the most part stationary.

The Johansen test supported the PPP hypothesis for both SEK-NOK pair and DKK-NOK pair for one period. The support was initially assumed to be present through all period, but it was not the case. Further, even though the test does support for one period for both pairs it does not suggest any strength of the relationship.

Considering graphical analysis, results from the ADF test and finally from the Johansen test, it must be concluded that the there is a mixed support for the PPP theory. There is a evidence for long relationship for some periods but not for other periods.

I would suggest further studies to focus on the monetary aspect. That is to examine how radical monetary policy, dramatic increase or decreases in money supply, controlled depreciations or appreciation or changes in interest rates can effect equilibrium in the exchange market and the price levels. And for how long can effect of monetary interventions persist in the exchange rates and price levels.

7 LITERATURE

1. Abuaf Niso and Jorion Philippe 1990. ≪ Purchasing Power Parity in the Long Run.≫ *The Journal of Finance*, Vol. 45, No. 1, pp. 157-174

2. Achy L. 2003. ≪ Parity reversion persistence in real exchange rates: middle income country case.≫ *Applied Economics*, Vol. 35, pp. 541-553

3. Alba D. Joseph and Park Donghyun 2003. ≪Purchasing Power Parity in Developing Countries: Multi-Period Evidence Under the Current Float.≫ *World Development*,Vol. 31, No. 12, pp. 2049-2060

4. Anoruo Emmanuel, Braha Habtu and Ahmad Yusuf 2002 ≪ Purchasing Power Parity: Evidence from Developing Countries.≫ *International Advances in Economic Research*,Vol. 8, No. 2, pp. 85-96

5. Arize C. Augustine, Malindretoes John and Ghosh Dilip 2015. ≪ Purchasing power parity-symmetry and proportionality: Evidence from 116 countries. ≫ *International Review of Economics and Finance*,Vol. 37, pp. 69-85

6. Arize C. Augustine, Malindretos John and Nam Kiseok 2010 .≪Cointegration, dynamic structure, and the validity of purchasing power parity in African countries .≫ *International Review of Economics and Finance*,Vol. 19, pp. 755-568

7. Bahmani-Oskooee Mohsen and Hegerty W. Scott 2010 .≪ Bounds testing cointegration methods and PPP: Evidence from 123 Countries.≫ *Applied Economics Letters*,Vol. 17, pp. 13351340

8. Balassa Bela 1964.≪ The Purchasing-Power Parity Doctrine: A Reappraisal.≫ *Journal of Political Economy*,Vol. 72, No. 6, pp. 584-596

9. Bhagwati N. Jagdish 1984.≪Why are Services Cheaper in the Poor Countries .≫ *The Economic Journal*,Vol. 94, No. 374, pp. 279-286

10. Brooks Chris 2008. ≪ Introductory Econometrics for Finance.≫ Second Edition. Cambridge University Press

11. Chang Tsangyao, Tang De-Piao, Liu Wen-Chi and Lee Chia-Hao. 2010 .≪Purchasing power parity for 15 COMESA and SADC countries: evidence based on panel SURADF tests .≫ *Applied Economics Letters*,Vol. 17, pp. 1721-1727

12. Cheung Yin-Wong and Lai S. Kon. 1993 .≪Long-run purchasing power parity during the recent float .≫ *Journal of International Economies*,Vol. 34, pp. 181-192

13. Clarida Richard and Gali Jordi 1994 .≪Sources of Real Exchange Rate Fluctuations: How Important Are Nominal Shocks? ≫ Center for Economic Policy Research, Discussion Paper, No. 951

14. Cooper, J. C. B. 1994 .≪Purchasing power parity: a cointegration analysis of the Australian, New Zealand and Singaporean currencies .≫ *Applied Economics Letters*,Vol.1, pp. 167-171

15. Cuddington T. John and Liang Hong. 2000.≪Purchasing power parity over two centuries? ≫ *Journal of International Money and Finance*, Vol. 19, pp. 753-757

16. De Gregorio Jos, Giovannini Alberto and Wolf C. Holger 1994 .≪International evidence on tradables and nontradables inflation.≫ *European Economic Review*,Vol. 38, pp. 1225-1244

17. Doanlar Murat, Bal Harun and zmen Mehmet. 2009 .≪Testing long-run validity of purchasing power parity for selected emerging market economies .≫ *Applied Economics Letters*,Vol. 16, pp. 1443-1448

18. Edison J. Hali, Gagnon E. Joseph and Melick R. William 1997 .≪Understanding the empirical literature on purchasing power parity: the post-Bretton Woods era .≫ *Journal of International Money and Finance*,Vol. 16, No. 1, pp. 1-17

19. Enders, W. 1988.≪ARIMA and Cointegration Tests of PPP under Fixed and Flexible Exchange Rate Regimes.≫ *The Review of Economics and Statistics*,Vol.70, pp. 504-508

20. Frankel A. Jeffery 1986 .≪International Capital Mobility and Crowding-out in the U.S. Economy: Imperfect Integration of Financial Markets or Goods Markets?≫ in *How open is the U.S economy?* Ed.: Rik W. Haffer. Lexington: Lexington Books, pp. 33-67

21. Frankel A. Jeffery 1990 .≪Zen and the Art of Modern Macroeconomics: A Commentary ≫ in *Monetary policy for a volatile global economy*, Ed.: William

S. Haraf and Thomas D. Willett. Washington, DC: American Enterprise Institute for Public Policy Research, pp. 117-122

22. Frankel A. Jeffery and Rose K. Andrew 1996 .≪A Panel Project on Purchasing Power Parity: Mean Reversion Within and Between Countries.≫ *Journal of International Economics*,Vol. 40, pp. 209-224

23. Friedman Milton 1953.≪Essay in Positive Economics .≫ First Edition. University of Chicago Press

24. Gujarati N. Damodar and Porter C. Dawn 2009.≪Basic Econometrics.≫ International Edition. McGraw-Hill/Irwin

25. Jorion P. and Sweeney J. R. 1996 .≪Mean reversion in real exchange rates: evidence and implications for forecasting.≫ *Journal of International Money and Finance*,Vol. 15, No. 4, pp. 535-550

26. Knetter M. Michael 1989 .≪Price Discrimination by U.S. and German Exporters.≫ *The American Economic Review*,Vol. 79, No. 1, pp. 198-210

27. Kravis B. Irving and Lipsey E. Robert 1983 .≪Toward an explanation of national price levels.≫ *Princeton Studies in International Finance*,No. 52

28. Lothian R. James and Taylor P. Mark. 1996 .≪Real Exchange rate Behavior: The Recent Float from the Perspective of the Past Two Centuries.≫ *The Journal of Political Economy*,Vol. 104, No. 3, pp. 488-509

29. Maddala G. S. and Kim In-Moo 1998. ≪ Unit Roots, Cointegration, and Structural Change. ≫ Cambridge University Press

30. Marston, R. C. 1986 . ≪ Real exchange rates and productivity growth in United States and Japan. ≫ National Bureau of Economic Research. Working paper 1922

31. Moffett H Michael, Stonehill I. Arthur and Eiteman K. David 2010. ≪ Multinational Business Finance.≫ Twelfth Edition. Pearson Education

32. Murray, Michael P. 2006. ≪ Econometrics: A modern introduction.≫ Pearson International Edition. Pearson Addison Wesley

33. Nelson C. Mark 2001.≪ International Macroeconomics and Finance: Theory and Empirical Methods.≫ First Edition. Blackwell Publishers

34. Norrbin C. Stefan and Smallwood D. Aaron 2010 .≪ Generalized long memory and mean reversion of the real exchange rate.≫ *Applied Economics*,Vol. 42, pp. 1377-1386

35. Obstfeld Maurice and Rogoff Kenneth 1995.≪The Intertemporal Approach to the Current Account.≫ In *Handbook of International Economics*,Gene Grossman and Kenneth Rogoff, eds., Vol 3, pp. 1731-1799

36. Papell, D. H. 1997.≪Searching for stationarity: Purchasing power parity under the current float.≫ *Journal of International Economics*,Vol. 43, pp. 313-332

37. Patel, J. 1990.≪Purchasing Power Parity as a long-run relation.≫ *Journal of applied econometrics*,Vol.5, pp 367-379

38. Pesaran M. H., Shin Y., Smith R.J. 2001.≪Bounds testing approaches to the analysis of level relationships.≫ *Journal of Applied Econometrics*,Vol.16, pp. 289-326

39. Rogers H. John 1995.≪Real Shocks and Real Exchanges in Really Long-Term Data.≫ Board of Governors of the Federal Reserve System, International Finance Discussion Paper, No. 493

40. Rogoff Kenneth 1996.≪ The Purchasing Power Parity Puzzle.≫ *Journal of Economic Literature*,Vol. 34, No. 2, pp. 647-668

41. Salehizadeh Mehdi and Taylor Robert 1999 .≪ A test of purchasing power parity for emerging economies.≫ *Journal of International Financial Markets*,pp. 535-550

42. Samuelson A. Paul 1964.≪ Theoretical Notes on Trade Problems.≫ *The Review of Economics and Statistics*,Vol. 46, No. 2, pp. 145-154

43. Steigum Erling 2007.≪ Moderne makrokonomi.≫ First Edition. Gyldendal

44. Taylor, M. P. 1988.≪ An empirical examination of long-run purchasing power parity using cointegration techniques.≫ *Applied Economics*,Vol. 20, pp. 1369-1381

45. Wang Peijie 2003.≪ Financial Econometrics: Methods and models.≫ Routledge

46. Wang Peijie 2010.≪ The Economics of Foreign Exchange and Global Finance.≫ Second Edition. Springer

47. Wang Ping 2000.≪ Testing PPP for Asian economies during the recent floating period.≫ *Applied Economics Letters*, Vol. 7, pp. 545-548

7.1 Websites

1. Data from Norges Bank, Exchange rates and Norwegian CPI:
http://www.norges-bank.no/en/Statistics/Historical-monetary-statistics/Consumer-price-indices/

2. Data from Statistics Sweden, Swedish CPI:
$http://www.scb.se/sv_/Hitta-statistik/Statistik-efter-amne/Priser-och-konsumtion/Konsumentprisindex/Konsumentprisindex-KPI/33772/33779/Konsumentprisindex-KPI/33895/$

3. Data from Statistics Denmark, Danish CPI:
http://www.statistikbanken.dk/10072

4. Wikipedia:
$https://en.wikipedia.org/wiki/Monetary\_policy\_of\_Sweden$