

# Unemployment, public pensions and capital accumulation \*

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## Abstract

The present paper develops an overlapping generations model that interacts with a labor market characterized by equilibrium unemployment. This structure implies that young individuals can be in two different states, employed or unemployed. Hence, the social security system contains both old-age benefits and unemployment insurance. Including these features the model seeks to assess growth effects of three different pension systems: one unfunded and two funded, where it is separated between actuarial and non-actuarial funding strategies. It is shown that both funded systems generate higher growth than an unfunded system. Moreover, the actuarial system fosters higher growth than the non-actuarial.

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# 1 Introduction

Several OECD countries experience population ageing due to low fertility rates and higher life expectancies. At the same time many of these countries are plagued with high unemployment rates due to structural problems. A third observation is that many of these economies also suffer from a decline in productivity and slow economic growth. All of these issues are closely related to how the social security program is formed, and consequently social security reforms are highly prioritized on the policy agenda.

Since most of these countries have unfunded pension systems, population ageing will trigger increased tax burdens and/or reduced benefits in the future. This can affect savings. These issues have caused several countries to reform their social security systems. Typical changes have involved more funding strategies and changes in retirement incentives, in order to motivate workers to stay longer in the workforce. In most European economies these issues are accompanied by unemployment problems. High unemployment rates have been observed for a long time. Although there doesn't exist a consensus theory on this long-term unemployment, most economists agree on that the problems have some structural flavor and can be associated with institutional arrangements, such as the wage formation or the social security system and welfare programs (Ljungqvist and Sargent, 1998; Layard et al., 1991). As social security has an impact on both employment issues and saving decisions, social security must also be linked to economic growth.

In this paper I will analyze the relationship between different social security systems, unemployment and economic growth. To do this an overlapping generations model in discrete time is applied, where the young generation can be in two different states, either employed or unemployed. The extent of unemployment is related to the wage bargaining. The old generation is assumed to be non-working. As the model economy consists of two groups that are non-working, the social security system contains both unemployment insurance and old-age pension benefits. These characteristics and institutional features are relevant for European welfare states and labor markets.

Several papers have treated these issues separately, and the focus has often been on different aspects of the features mentioned above analyzed in isolation. The relation between pensions and growth is typically analyzed in intertemporal models as treated in Breyer (1989), Saint-Paul (1992) and Lambrecht et al. (2005). Motivated by the pension reform debate several authors have studied the transition from a non-funding pension scheme to a more or completely funded system (Verbon, 1989; Peters, 1991; Thøgersen, 2001). The shift is known to have a short-run cost, as the transition generation is constrained to pay twice, both for its own retirement and for the old part of the population through a pay-as-you go scheme. From a social welfare point of view the shift is therefore problematic. In Belan et al. (1998) however, the transition is studied in an endogenous growth model, and it is shown that a Pareto-improving social security reform is possible.

The link between social security and unemployment is however, ignored in this

literature, and usually studied in a separate class of models. Aghion and Howitt (1999) and Pissarides (2000) studies unemployment and growth when unemployment is due to search frictions and mismatch, while Bräuninger (2000) and Lingens (2003) study the same variables, but based on the presumption that unemployment is caused by the wage bargaining.

Corneo and Marquardt (2000) take a first step in integrating social security, unemployment and growth, where social security refers to the combination of public pensions and unemployment insurance programs. The labor market in their model is characterized by union wage setting, where the wage is set by a monopoly union. They find that unemployment does not affect growth and that the Pareto-improving pension reform studied in Belan et al. (1998) is maintained even when allowing for equilibrium unemployment. Bräuninger (2005) studies the same relations as Corneo and Marquardt, but he assumes that the wage is determined through wage bargaining. Bräuninger concludes that both of the insurance components in the social security program have a negative impact on growth. First of all the pension system has a direct negative effect since pensions crowd out savings, and therefore reduce capital accumulation and growth. Secondly, unemployment insurance has an indirect negative effect on economic growth by affecting unemployment. The unemployment benefits influence equilibrium unemployment through the wage bargaining.

To expand on this path of literature I model and compare how different pension systems affect capital accumulation and growth. To make the comparisons it is necessary to find analytical solutions for the growth factor of capital. I have therefore made some substantial changes to the set up by Bräuninger. Besides, not only a fully funded and a pay-as-you go (PAYGO) scheme are considered, but also a third alternative. This alternative is assumed to be similar to a PAYGO system with respect to the governmental intervention, but dissimilar with respect to the financing. Pension payments are here assumed to be financed by a pension fund governed by the government, and where the young individuals pays taxes to finance their own generation's old-age need. But, the system is also non-individualized and non-actuarial, hence there is no link between tax contribution and pension benefit. Thus it is not equivalent to an individual and fully funded system, which is perfectly actuarial.<sup>1</sup> I will elaborate on the different pension schemes in section 5.3 and in the conclusion. Addressing policy issues in a setting like this is often suppressed in the theoretical literature though it is a highly important issue for policy makers. The question I ask in this paper is therefore how different social security systems will influence savings, capital formation and growth, in an economy where wage bargaining leads to long-term unemployment, and the social security system accordingly comprise old-age pensions and unemployment insurance.

The rest of the paper is organized in the following way: Section 2 describes population in an overlapping generation setting. Special attention is given to the two states possible for a young worker. In section 3, households, lifecycle consumption

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<sup>1</sup>See Fehr and Thøgersen (Forthcoming, 2009) for a survey of different pension schemes and their effects on future generations.

and the individual savings function are modeled. Section 4 presents the production structure of the economy. Firms are assumed to act under monopolistic competition. This section also studies firms' interaction with trade unions and the wage bargaining. In section 5 I describe the government and the different social security systems. Then, section 6 assesses how capital accumulation and economic growth are affected by different funding strategies when capital is endogenous. Section 7 offers some concluding remarks.

## 2 Population and states

I consider a model with two cohorts. Each individual lives in two periods, and the ones who are young in period  $t$  are old in period  $t + 1$ . The number of the young in period  $t$  is  $N_t$ . Population grows at a constant rate  $n$ , and it follows that  $N_{t+1} = (1+n)N_t \Leftrightarrow n = (N_t/N_{t-1}) - 1$ , where  $N_{t-1}$  is the number of old in period  $t$ . Young individuals all supply one unit of labor, though they can be in two different states, either employed or unemployed. The proportion  $u \in (0, 1)$  is unemployed, so the total number unemployed and employed are respectively,  $uN_t$  and  $(1 - u)N_t$ . Each of the working individuals earns the wage  $w_t$ . The wage rate however, is taxed at a proportional rate  $\tau$ , in order to finance unemployment insurance ( $b$ ), and pensions. To which generation the pension is distributed depends on the pension system.

## 3 Households and lifecycle consumption

A representative young agent in generation  $t$  will choose a consumption path to maximize the following lifetime utility function,  $U_t = U(c_{1,t}, c_{2,t+1})$ , where  $U$  is strictly concave, increasing in each of its two arguments,  $c_{1,t}$  is consumption when young in period  $t$  and  $c_{2,t+1}$  is consumption when the same individual is old in the next period,  $t + 1$ . For analytical purposes I assume a logarithmic Cobb-Douglas description of the preference structure:

$$U_t = (1 - \delta) \log c_{1,t} + \delta \log c_{2,t+1}, \quad (1)$$

where  $\delta \in (0, 1)$  is the weight on consumption in the two periods and reflects the discount factor. When individuals are young they can either work and earn  $w_t(1 - \tau)$ , or they can be unemployed and receive unemployment benefits. The working part of the population will allocate income between consumption and saving ( $s_t$ ). All savings are allocated to investments which yield a positive rate of return. In the second period of life individuals are retired and receive a pension benefit along with a payoff from investments made when young. Pensioners consume all of their wealth, i.e. the model does not include bequests.

A crucial assumption in the current model is that taxes have distortionary effects. This assumption is necessary in order to distinguish between real effects of the two funded pension schemes. Due to the importance of this assumption it will be

further discussed in the last section. The modeling of the distortion can be done in several ways. One approach is to let labor supply be endogenous and include leisure in the utility function. However, such an approach in an overlapping generations model like the one applied here, would significantly complicate the capital market equilibrium. The dynamics of capital will then be characterized by a second order difference equation, that may involve multiple equilibrium paths and non-uniqueness. Accordingly, I have adopted the approach by Barro (1979) and Bohn (1992). They argue that taxation involves collection costs and/or misallocation costs that are imposed on the private economy. Hence, the tax on labor income will impose an excess burden on workers. This burden will not influence the unemployed since they do not pay taxes. The excess burden is denoted  $h(\tau)$ , where  $h'(\tau) > 0$  and  $h(0) = 0$ . Net labor income of a worker in period  $t$  is therefore  $w_t(1 - \tau - h(\tau))$ . This simplified modeling of the distortionary effect is not crucial for the qualitative results of the analysis.

In order to assess effects on economic growth it is convenient to derive an expression for aggregate savings. Aggregate savings is the sum of the savings made by the employed and the unemployed. The unemployed workers receive unemployment benefits while young, and pension benefits while retired. However, the unemployed workers also divides their income (transfers) between consumption and saving, and in the aggregate, their intertemporal choice is included. When analyzing growth implications, it is necessary to express both aggregate savings and potential wealth accumulation by the government. Whether the government can accumulate financial capital depends on the pension system. However, I start the exposition by setting up the intertemporal consumption decision of the working part of the population and thereby derive an expression for their individual saving. As the young and working individuals in period  $t$  will divide their net income between consumption and saving, consumption is given by:

$$c_{1,t} = w_t(1 - \tau - h(\tau)) - s_t. \quad (2)$$

Consumption of the old is given by their benefits and accumulated saving:

$$c_{2,t+1} = \theta w_t + s_t R_{t+1}, \quad (3)$$

where  $R_t$  denotes the interest factor, and  $\theta < 1$  denotes the constant pension ratio. From (2) and (3) one can derive the intertemporal budget constraint:

$$c_{1,t} + \frac{1}{R_{t+1}} c_{2,t+1} = w_t(1 - \tau - h(\tau)) + \frac{1}{R_{t+1}} \theta w_t =: \Lambda_t, \quad (4)$$

where  $\Lambda_t$  denotes net lifecycle income. Equation (4) expresses the net life income received in the first period, plus the discounted value of pensions received in the second period of life. The decision problem for young workers born in period  $t$  is to maximize lifetime utility (1) subject to the consolidated budget constraint in (4). To derive optimal individual savings, it is convenient to define the following savings

function:

$$s_t := \arg \max_{c_{1,t}, c_{2,t+1}} \{U(c_{1,t}, c_{2,t+1}) \mid c_{1,t} + (R_{t+1})^{-1}c_{2,t+1} = \Lambda_t\} . \quad (5)$$

Using the Cobb-Douglas specification of the utility function along with the budget constraints in period  $t$  gives optimal individual savings as:

$$s_t = \left[ \delta(1 - \tau - h(\tau)) - (1 - \delta) \frac{\theta}{R_{t+1}} \right] w_t . \quad (6)$$

Equation (6) states that individual savings depends on individual income during the working period, the tax distortion and the level of pensions. In section 6 I will include savings made by the unemployed in order to expand and derive an expression for total savings for all individuals.

## 4 Firms and wage bargaining

### 4.1 Technology and market structure

The firms act in a market characterized by monopolistic competition. It is therefore assumed to be a large number of firms, but all goods are imperfect substitutes such that there exists some profit to be shared between workers and firms. The sharing of these profits is a part of the bargaining. Each firm  $i$  makes use of two input factors, capital ( $K_{i,t}$ ) and labor ( $L_{i,t}$ ) in order to produce a variety of goods. The demand for these goods produced by firm  $i$ , is given by  $Y_{i,t} = \pi_{i,t}^{-\eta} Y_t$ , where  $\pi_{i,t}$  is the relative price of good  $i$ ,  $\eta$  is the price elasticity of demand and  $Y_t$  is an index of aggregate demand. Technology is given by a Cobb-Douglas production function with positive and diminishing marginal products of each input, constant returns to scale and labor-augmenting technology for firm  $i$ :

$$Y_{i,t} = K_{i,t}^\alpha (A_t L_{i,t})^\beta \quad \text{where} \quad \alpha + \beta = 1, \quad \alpha > 0 \quad \text{and} \quad \beta > 0 . \quad (7)$$

Effective labor is thus given by  $A_t L_{i,t}$ , where  $A_t$  measures labor efficiency which each single firm takes as given. Firms maximize profits  $\Pi_{i,t} = I_{i,t} - w_{i,t} L_{i,t} - R_{i,t} K_{i,t}$ , where  $I_{i,t}$  is revenue and given by  $\pi_{i,t} Y_{i,t}$ . It is assumed that capital fully depreciates each period. Insert the inverse demand function to obtain  $I_{i,t} = Y_t^{1/\eta} Y_{i,t}^\kappa$ , where  $\kappa := 1 - 1/\eta$ . Standard profit maximization implies that the marginal revenue of respectively capital and labor equals their input prices:

$$\frac{\partial I_{i,t}}{\partial L_{i,t}} = \frac{\beta \kappa I_{i,t}}{L_{i,t}} = w_{i,t} \quad \text{and} \quad \frac{\partial I_{i,t}}{\partial K_{i,t}} = \frac{\alpha \kappa I_{i,t}}{K_{i,t}} = R_{i,t} .$$

### 4.2 Aggregate production and technological spillovers

In order to assess growth effects in the aggregate economy, it is necessary to expand the profit maximizing framework above to include all the agents, employed and

unemployed. I assume that firms are symmetric hence, in the aggregate  $K_{i,t} = K_t$  and  $Y_{i,t} = Y_t$ . It is also assumed that the real wage is determined in the wage bargaining process as presented in section 4.3. The symmetry of firms imply that all prices are equal, hence the relative price is  $\pi_{i,t} = \pi = 1$ , for all  $t$ . The revenue of each firm will then be equal to output, i.e.  $I_{i,t} = Y_{i,t}$ . Furthermore, as all firms are assumed to be symmetric, the production function is equal for all firms, i.e.  $Y_t = K_t^\alpha (A_t L_t)^\beta$ . Since young people in generation  $t$  either are employed or unemployed,  $L_t = (1 - u)N_t$ , and aggregate production is thus given by:

$$Y_t = K_t^\alpha [A_t(1 - u)N_t]^\beta . \quad (8)$$

The profit maximizing first order conditions in section 4.1 is at the aggregate level given by  $\alpha\kappa Y_t/K_t = R_t$  and  $\beta\kappa Y_t/L_t = w_t$ . Inserting the production function in (8), gives the first order conditions as:

$$\alpha\kappa\hat{k}_t^{-\beta}(1 - u)^\beta = R_t \quad \text{and} \quad \frac{A_t\beta\kappa\hat{k}_t^\alpha}{(1 - u)^\alpha} = w_t , \quad (9)$$

where  $\hat{k}_t := K_t/A_t N_t$  denotes capital per unit of efficient labor. To endogenize the labor productivity index  $A_t$ , I follow the set up originally formulated by Arrow (1962), and the extensions by Romer (1986). Following this approach implies a technological spillover from the size of the aggregate capital stock on labor productivity in individual firms. This positive externality permits an endogenous growth process. In order to ensure the existence of a steady-state equilibrium, the technological spillover is assumed to be linear in the aggregate capital stock per young individual:

$$A_t = \frac{1}{a} \frac{K_t}{N_t} , \quad (10)$$

where  $a$  is a scaling productivity parameter reflecting the influence of capital intensity on labor productivity.<sup>2</sup> Moreover, the productivity of labor is decreasing in  $a$ . With respect to the production function, inserting (10) into (8) gives:

$$Y_t = K_t(1 - u)^\beta a^{-\beta} ,$$

i.e. the production function is of the  $AK$  type and linear in capital. Notice that unemployment reduces output. Such a relation is motivated by the empirical evidence of a negative effect of unemployment on growth.<sup>3</sup> In Bräuning (2005), it is shown that a convenient way to model this aspect is to let the technological spillover depend on young individuals in general, and not only on the employed individuals. This feature is captured in (10), and it follows that unemployment has a negative impact on output and economic growth. From (10) and the definition of  $\hat{k}_t$ , it follows that capital per effective unit of labor is constant:

$$\hat{k}_t := \frac{K_t}{A_t N_t} = a . \quad (11)$$

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<sup>2</sup>See Grossman and Yanagawa (1993), Wigger (2002) and Holler (2007) for similar approaches.

<sup>3</sup>See Daveri and Tabellini (2000), and Bräuning and Pannenberg (2002) for empirical studies.

It is straightforward from (9) to see that the interest factor  $R_t$ , and the wage per efficient unit  $w_t/A_t$ , are functions of  $\hat{k}_t$ . Substituting (11) into the first order conditions in (9) yields:

$$R_t = \alpha \kappa a^{-\beta} (1-u)^\beta = R \quad \text{for all } t, \quad (12)$$

$$w_t = \frac{A_t \beta \kappa a^\alpha}{(1-u)^\alpha}. \quad (13)$$

Thus, the interest factor is constant over time and the wage rate is proportional to the level of labor productivity, i.e. growing at the rate of growth of the technological spillover.

### 4.3 Unions, bargaining and equilibrium unemployment

In this subsection I present the wage setting and the equilibrium unemployment rate. This is done in order to analyze how unemployment will affect economic growth under different social security systems. Unemployment is the result of wage bargaining at firm level. This exposition of the labor market is conventional and can be found in Booth (1995), Layard et al. (1991) or Bräuninger (2005) among others.<sup>4</sup>

All workers, both employed and unemployed, are members of a trade union, and the total number of members are therefore  $N_i$ . The trade union is assumed to maximize the utility of a worker, given by net income. The workers can be either employed in firm  $i$  and earn  $(1-\tau)w_i$ , or have an alternative income  $m_i$  which either comes from employment in another firm or as unemployment benefit.<sup>5</sup> Each individual is sometimes unemployed, depending on the outcome of the bargaining and the exogenous fluctuations in the labor market. It is assumed that the outcome of the bargaining problem is given by maximization of the Nash product:  $\Phi = (V_i - \bar{V}_i)^\gamma (\Pi_i - \bar{\Pi}_i)^{1-\gamma}$ , where  $\gamma$  is the relative bargaining power of the union and  $V_i = N_i v_i$ , where  $v_i$  is the utility of a member.  $\bar{V}_i$  is the threat point of the union, and is given by the alternative income of a member in case of disagreement,  $m_i N_i$ . Likewise  $\bar{\Pi}_i$  is the threat point of the firm, and is given by the firm's payoff in case of disagreement,  $(-RK_i)$ . Using these expressions one obtains the following Nash product:  $\Phi = (L_i ((1-\tau)w_i - m_i))^\gamma (I_i - w_i L_i)^{1-\gamma}$ , and maximization implies:

$$(1-\tau)w_i = \mu m_i \quad \text{where} \quad \mu := 1 - \gamma + \frac{\gamma}{\beta \kappa}, \quad (14)$$

i.e. the wage is equal to the alternative income multiplied by a fixed mark-up  $\mu \geq 1$ . The higher the union power, the higher is the mark-up. If the union has no power,  $\gamma = 0 \Rightarrow \mu = 1$ , i.e. the firm offers a wage equal to the alternative income.

Union members not employed in firm  $i$  faces a probability  $(1-\phi u)$  of being employed in another firm and a probability  $\phi u$  of staying unemployed. The parameter  $\phi$  is exogenous and describes fluctuations in the labor market. These fluctuations

<sup>4</sup>The modeling of the wage setting follows Layard et al. (1991) and Bräuninger (2005), and the reader is encouraged to address these references for a further discussion.

<sup>5</sup>I drop time indexation in this part of the exposition.



take place at much higher frequency than the periods in the overlapping generations model, so they are not included formally in the model. By assuming that being employed in another firm gives the worker a net income of  $(1 - \tau)w$ , one can formulate the following relation for the alternative income:  $m_i = \phi ubw + (1 - \phi u)(1 - \tau)w$ . Inserting this equation into (14), one obtains:

$$(1 - \tau)w_i = \mu(\phi ubw + (1 - \phi u)(1 - \tau)w) . \quad (15)$$

The assumption of identical firms and homogenous workers, imply  $w_i = w$ . By inserting this into (15) and solve for the equilibrium unemployment rate yields:

$$u = \frac{(\mu - 1)(1 - \tau)}{\mu\phi(1 - \tau - b)} , \quad (16)$$

that is equivalent to Bräuninger (2005). The expression shows that equilibrium unemployment depends on taxes.

## 5 Government and social security

The government runs a social security system that consists of unemployment benefits, and old-age pension benefits. In order to finance unemployment insurance the government imposes a tax on the working individuals. This leads to an intragenerational transfer. Pension benefits are also financed by taxing the workers, but whether the transfer is intergenerational or intragenerational depends on the pension system. In the following, three different stylized systems are considered, one unfunded and two funded. While the funded schemes are intragenerational, the unfunded is intergenerational. The two funded systems differ with respect to the tax-benefit link, and whether they are individual or not. One system is assumed to be non-individual and non-actuarial, i.e. a weak tax-benefit link. The other funded system assumes individual and fully actuarial funding of the pension benefits. Under a perfect capital market this system is equivalent to a system without governmental interventions, with respect to total savings and capital accumulation (Blanchard and Fischer, 1989). Accordingly, it can be analyzed by assuming that pension payments and their earmarked taxes are zero. Hence, the social security system will only consist of unemployment insurance. As elaborated on in section 5.3, these distinctions are crucial to the formulation of the governmental budget restrictions.

In the non-actuarial funding system the government accumulates financial wealth. This is due to the time-lag between income from taxes and the payments to the old. This implies that the government under certain assumptions will contribute to the accumulation of national wealth. As will become clear in section 5.3, this is not the case with the other pension systems.

### 5.1 National wealth

It is assumed that the economy is closed, which implies that a country's national wealth ( $\Omega_t^n$ ) consists of the capital in the economy. Since capital can be accumulated

by the government in the case of a non-actuarial funding system, as well as by the private sector,  $\Omega_t^n := K_t = \Omega_t^P + \Omega_t^G$ , where  $\Omega_t^P$  and  $\Omega_t^G$  denotes the wealth by the households and the government respectively. The employed workers in the economy pay taxes to finance the two components of the social security system. The government distributes these revenues among the entitled ones. For expositional reasons it is distinguished between taxes paid to finance unemployment insurance ( $\tau_u$ ), and taxes paid to finance old-age pensions ( $\tau_p$ ). The government's wealth in the beginning of period  $t + 1$  is accordingly:

$$\Omega_{t+1}^G = R_{t+1}\Omega_t^G + \tau_p w_t N_t (1 - u) - P N_{t-1}.$$

Note that intragenerational transfers from the employed to the unemployed are excluded, due to their purely within generation distributional characterization. This part of the tax is not invested and can therefore not contribute to wealth accumulation. The government's wealth in per worker form is given by:

$$(1 + n)\omega_{t+1}^G = R_{t+1}\omega_t^G + \tau_p w_t (1 - u) - P(1 + n)^{-1}, \quad (17)$$

where  $\omega := \Omega/N$  and  $P = \theta w_t$ . The pension  $P$  received by the old part of the population is proportional to the current wage.

## 5.2 Unemployment insurance

The young and working individuals in period  $t$ ,  $(1 - u)N_t$ , finance unemployment benefits to the unemployed. These benefits ( $B$ ) are fixed in relation to wages,  $B = b w_t$ , where  $b < 1$  is the replacement ratio. Since the total number of unemployed are  $u N_t$ , total expenditures to the unemployed in period  $t$  is  $b u w_t N_t$ . Since expenditures must be equal to total taxes earmarked for unemployment benefits, the following budget restriction with respect to unemployment insurance must hold:

$$\tau_u w_t (1 - u) N_t = b u w_t N_t \iff \tau_u = \frac{b u}{(1 - u)}, \quad (18)$$

which implies that the tax rate with respect to unemployment insurance, is independent of the pension system.

## 5.3 Pension systems

As aforementioned I distinguish between three different stylized pension schemes: pay-as-you go (PAYGO), non-actuarial funding (NAF) and actuarial funding (AF).

### *PAYGO financing*

Within this regime it is assumed that taxes on labor income to the young part of the population are used to finance old-age pensions in the same period. This gives the following budget restriction with respect to pensions:

$$\tau_p w_t N_t (1 - u) = \theta w_t N_{t-1}. \quad (19)$$

As long as taxes to finance pensioners are paid out in the same period as they are received, the government can not accumulate wealth. This implies that  $\Omega_t^G = 0$ , for all  $t$ . One can then rewrite the restriction and solve for  $\tau_p$ :

$$\tau_p = \frac{\theta N_{t-1}}{(1-u)N_t} = \frac{\theta}{(1-u)(1+n)}.$$

The total tax levied on the worker under a PAYGO system can then be expressed as:

$$\tau^{PAYGO} = \tau_u + \tau_p = \frac{\theta + bu(1+n)}{(1-u)(1+n)}. \quad (20)$$

The tax is negatively related to  $n$  and positively related to the replacement ratio  $b$ , the pension ratio  $\theta$ , and the unemployment rate  $u$ .

#### *Non-actuarial funding*

This regime is also assumed to be non-individualized and the representative individual in each generation will pay taxes that are contributions to his own generation's pension fund. This means that the representative individual in the second period of life will receive a pension that equals his generations' earlier contributions plus accumulated interests. As the system is characterized by a non-actuarial relationship between each individual's contribution as young and his benefit received as old, the pension received by each individual does not necessarily reflect his contribution in particular. The government therefore distributes income on a generation basis, and not on an individual one.<sup>6</sup> In this system the government can contribute to the accumulation of national wealth. The governments financial wealth,  $\Omega_t^G$ , is pension taxes that the government receives in period  $t-1$ , and at the beginning of period  $t$ , the government has  $R_{t+1}\Omega_t^G$  at its disposal. This implies that  $\theta w_t N_{t-1} = R_{t+1}\Omega_t^G$ , and that  $R_{t+1}\omega_t^G = \theta w_t / (1+n)$  in per worker form. By inserting this into (17), it follows that:

$$(1+n)\omega_{t+1}^G = \tau_p w_t (1-u). \quad (21)$$

Equation (21) reveals that taxes paid by the employed in period  $t$ , gives the governmental wealth in period  $t+1$ . The budget restriction that characterizes the NAF strategy is accordingly:

$$\theta w_t N_t = R_{t+1} \tau_p w_t N_t (1-u) \iff \tau_p = \frac{\theta}{R_{t+1}(1-u)}. \quad (22)$$

Total tax under a NAF social security system is then given by:

$$\tau^{NAF} = \tau_p + \tau_u = \frac{\theta + buR_{t+1}}{(1-u)R_{t+1}}. \quad (23)$$

Equation (23) shows that the tax is now not directly affected by  $n$ . The growth in population will under a NAF regime not have a direct effect on the tax paid by

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<sup>6</sup>A similar approach can be found in Thøgersen (2001).

workers. From equation (9) however, one can see that population growth have an impact on the interest rate and therefore an indirect effect on taxes.

### *Actuarial funding*

This regime is also assumed to be individualized so the ordinary payments of the workers to social security is replaced with contributions to their own individual accounts. Workers save a mandated fraction of their labor income and invest it by themselves for their own old-age need. Provided that capital markets are perfect, an individualized and actuarial pension system is equivalent to absence of a pension system. Thus, the social security system only consist of an unemployment insurance scheme. Hence,  $\tau_u > 0$  and  $\tau_p = \theta = 0$ . Moreover, the government can not accumulate wealth, i.e.  $\omega_t^G = 0$ , for all  $t$ . Total taxes on labor income under a AF strategy is then:

$$\tau^{AF} = \tau_p + \tau_u = \frac{bu}{(1-u)}. \quad (24)$$

Hence, an actuarial funding strategy implies that taxes are independent of population growth, and the real interest factor. Notice also that taxes are lower in the AF system compared to the NAF system. Therefore, the excess burden of taxes are relatively higher in the NAF system. This feature illustrates an important difference between AF and NAF, and is important in the subsequent analysis.

The above analysis shows that the pension system affects the tax levied on workers.

## **6 Capital accumulation and alternative social security systems**

This section studies how capital accumulation and growth depend on the choice of the pension system. To incorporate different funding strategies the model uses the different governmental budget restrictions from section 5.3. The growth factor is determined by aggregate savings and capital accumulation. I will therefore derive an expression for aggregate savings where both the contributions from the employed and the unemployed are included. Moreover, the analysis of different pension systems makes it necessary to derive explicit solutions for the growth factor of capital in the different settings. The following exposition follows Bräuninger (2005), and expand his model by including tax distortions, different pension systems and comparative studies.

### **6.1 Aggregate savings**

Total savings in the economy is defined as the sum of the savings by the employed workers and the unemployed individuals. However, as the government can accumulate financial wealth if there is a time-lag between their tax income and social security payments, their contribution must also be included in the analysis. Thus, considering the equilibrium in the capital market, involves both total savings and

governmental savings. Total savings are given by expanding equation (6) to include all the employed and unemployed workers. The proportion of the employed is  $(1 - u)$  and their individual income is  $(1 - \tau)w_t$ . The proportion of the unemployed is  $u$ , and their income is  $bw_t$ . Aggregate savings are accordingly:

$$S_t = \delta(1 - \tau)(1 - u)w_tN_t - \delta h(\tau)(1 - u)w_tN_t - \frac{(1 - \delta)\theta w_t}{R_{t+1}}(1 - u)N_t + \delta buw_tN_t - \frac{(1 - \delta)\theta w_t}{R_{t+1}}uN_t.$$

Notice that the contributions from the employed must correspond to the benefits received by the unemployed in equilibrium. This means that  $\tau_u(1 - u)w_tN_t = buw_tN_t$ , and  $S_t$  can be simplified to:

$$S_t = \delta(1 - u)w_tN_t [1 - \tau_p - h(\tau)] - \frac{(1 - \delta)\theta w_t}{R_{t+1}}N_t. \quad (25)$$

Equation (25) shows that savings depend on the wage rate, the unemployment rate, the tax distortion and the pension ratio. Savings are however independent of the replacement ratio. The distortionary effects are in this expression related to the taxes paid to pensions. Due to this tax distortion aggregate savings are lower, than they would have been without it.

The pension ratio is important. How changes in the pension ratio will affect savings depends on the pension system under consideration.

## 6.2 Equilibrium conditions and capital accumulation

In period  $t$ , the equilibrium in the economy as a whole is defined by equilibrium in three markets: the labor market, the capital market and the final good market. In the labor market, equilibrium is given by  $L_t = (1 - u)N_t$ , which follows since a positive part of the population is at any time unemployed. The final good market equilibrium displays the resource constraint for the economy as a whole, and states that output can either be used for aggregate consumption  $C_t$ , or gross investment in period  $t$ :

$$Y_t = C_t + K_{t+1}, \quad (26)$$

where

$$C_t := N_t c_{1,t} + N_{t-1} c_{2,t}, \quad (27)$$

i.e. aggregate consumption is the sum of consumption by the young and the old individuals in period  $t$ .

As the depreciation rate of capital is assumed to be unity, capital evolves according to  $K_{t+1} = S_t$ . In the capital market the supply of capital comes from both private and governmental savings. The governments financial wealth is therefore essential in the consideration of the equilibrium condition for the capital market. The next period's capital is therefore given as

$$K_{t+1} = S_t + \Omega_{t+1}^G, \quad (28)$$

where aggregate savings by the private sector is given by equation (25). By dividing aggregate savings by  $N_t$ , one obtain savings per young individual, including both employed and unemployed. The dynamic behavior of capital per young individual is accordingly  $(1+n)k_{t+1} = S_t/N_t + (1+n)\omega_{t+1}^G$ , where  $k_t := K_t/N_t$  denotes capital per young individual. Thus, inserting (25) and (17) gives the dynamics of capital in the economy as:

$$(1+n)k_{t+1} = \delta(1-u)w_t[1-\tau_p-h(\tau)] - \frac{(1-\delta)\theta w_t}{R_{t+1}} + R_{t+1}\omega_t^G + \tau_p w_t(1-u) - \frac{\theta w_t}{1+n}. \quad (29)$$

The dynamic equilibrium in (29) is fundamental in the subsequent analysis of how social security and different public pension regimes affects the growth of capital in the economy. The long-run growth factor of capital in the economy is defined by:

$$g := \frac{k_{t+1}}{k_t}. \quad (30)$$

To obtain analytical expressions for the growth factor the following equations are necessary: the intertemporal equilibrium in (29), the first order conditions in (12) and (13), and to separate between different pension systems one need the governmental budget restrictions in (19), (22) and (24). It is also necessary to insert whether the government's wealth is equal to zero or not between two periods. Moreover, the following result turns out to be quite useful:

**Lemma 1** *The structure of the technological spillover applied in the production sector implies that  $w_t/k_t = \text{const.}$  for all  $t$ .*

**Proof.** Inserting (10) into (13) becomes:

$$\frac{w_t}{k_t} = \frac{A_t \beta \kappa a^\alpha}{k_t (1-u)^\alpha} = \frac{\beta \kappa}{(1-u)^\alpha a^\beta},$$

which is constant. ■

## 6.3 Comparing funding strategies

### 6.3.1 PAYGO financing

In a PAYGO system an increase in the pension ratio will affect savings in two ways. First, it increases the contributions of the young, so that their net income declines. Secondly, it affects the young generations motivation to save, since part of their consumption as old is financed by the next generations tax payments. Moreover, with a PAYGO pension scheme the government can not contribute to national wealth as all governmental transfers are intergenerational, and tax income obtained in one period are transferred to pensioners within the same period. Hence,  $\Omega_t^G = \omega_t^G = 0$ ,

for all  $t$ . To implement the governmental budget restriction into the growth factor defined in (30), it is convenient to solve equation (19) for the pension ratio  $\theta$ . This implies:

$$\theta = \tau_p(1 - u)(1 + n), \quad (31)$$

where  $(1+n)(1-u)$  is the ratio of employed workers in period  $t$ , relative to all workers in period  $t - 1$ . If population growth exceeds the ratio between the unemployment rate and the employment rate  $(1 - u)$ , then the pension ratio exceeds the pension tax.

Inserting (31) and  $\omega_{t+1}^G = 0$  into (29) gives:

$$(1 + n)k_{t+1} = \left\{ \delta [1 - \tau_p - h(\tau)] - \frac{(1 - \delta)\tau_p(1 + n)}{R_{t+1}} \right\} (1 - u)w_t, \quad (32)$$

which displays the dynamic behavior of capital with a PAYGO pension scheme. Inserting (12), (32) and Lemma 1 into (30), yields the following growth factor:

$$g^{PAYGO} = \left\{ \frac{\delta [1 - \tau_p - h(\tau)]}{1 + n} - \frac{(1 - \delta)\tau_p}{R} \right\} \frac{\beta\kappa(1 - u)^\beta}{a^\beta}. \quad (33)$$

Note that the growth factor is time invariant. It is straightforward to see that unemployment reduces the growth in capital. The reason for this is that unemployment leads to reduced output, lower aggregate income and therefore lower savings. Moreover, an increase in the pension tax, decrease savings by young individuals and consequently the growth factor. Formally:

$$\frac{\partial g^{PAYGO}}{\partial \tau_p} = - \left\{ \frac{[1 + h'(\tau)] \delta}{1 + n} + \frac{(1 - \delta)}{R} \right\} \frac{\beta\kappa(1 - u)^\beta}{a^\beta} < 0.$$

### 6.3.2 Non-actuarial funding

As aforementioned, the NAF funding strategy implies that the young working generation finances its own pension, but on a generation basis, and not on an individual one. The intragenerational characterization of the NAF system also implies that the pension ratio is independent of population growth. In section 5.3, it was shown that the government can contribute to the national wealth, due to the time-lag between tax income and pension payments. Hence, the government accumulates wealth according to (21). To derive the growth factor in an economy with a NAF pension scheme, it is necessary to solve the governmental budget restriction given in (22) for the pension ratio. This implies:

$$\theta = \tau_p(1 - u)R_{t+1}. \quad (34)$$

Using (21) and (34), the dynamic behavior of capital per young individual within this pension regime becomes:

$$(1 + n)k_{t+1} = [1 - h(\tau)] \delta(1 - u)w_t. \quad (35)$$

Consequently, the growth factor is derived by inserting (35) and Lemma 1 into (30):

$$g^{NAF} = \frac{\beta\delta\kappa[1-h(\tau)](1-u)^\beta}{(1+n)a^\beta}. \quad (36)$$

Hence, the growth factor is time invariant. As in the PAYGO program, unemployment and pension taxes reduce growth. The negative impact of the tax is due to the distortionary effect that remains, as long as the working generation pays taxes.

The following proposition compares the PAYGO pension scheme with the NAF scheme, with respect to impact on the growth factor of capital.

**Proposition 1** *Capital accumulation is higher in an economy with a NAF pension system, than in an economy with a PAYGO pension system, i.e.  $g^{NAF} > g^{PAYGO}$ .*

**Proof.** The proposition is proved by contradiction. By assuming that  $g^{PAYGO} \geq g^{NAF}$ , and utilizing the expressions in (33) and (36), it follows that:

$$\begin{aligned} g^{PAYGO} \geq g^{NAF} &\Rightarrow \\ \left\{ \frac{\delta[1-\tau_p-h(\tau)]}{1+n} - \frac{(1-\delta)\tau_p}{R} \right\} \frac{\beta\kappa(1-u)^\beta}{a^\beta} &\geq \frac{\beta\delta\kappa[1-h(\tau)](1-u)^\beta}{(1+n)a^\beta} \\ \Leftrightarrow \delta[1-\tau_p-h(\tau)] - \frac{(1+n)(1-\delta)\tau_p}{R} &\geq \delta[1-h(\tau)], \end{aligned}$$

which verifies a contradiction as  $\tau_p > 0$ , and the second term on the LHS is greater than zero. ■

The reason for this result lies in the government's contribution to the economy's total savings through a pension fund. Establishing a public social security fund and thereby accumulating financial wealth, the government indirectly stimulates capital accumulation. In this system the government accumulates financial wealth because of the time-lag between when taxes are received by the government and when it is transferred to the older generation. Hence, an important difference between this system and the PAYGO system is that the government now contributes to capital accumulation.

According to neoclassical growth theory capital accumulation will increase the growth rate temporary. In this model however we also get a permanent increase in growth, since accumulation of capital increases the stock of knowledge and positive spillovers stimulates perpetual growth.

### 6.3.3 Actuarial funding

The main point within this system is that individuals pay contributions to their own individual accounts. As noted in section 5.3, this implies that  $\tau_p = \theta = 0$ . But the employed workers still pay taxes in order to finance unemployment benefit. This implies that  $\tau_u > 0$ . Under a PAYGO and a NAF strategy, the tax consists of both  $\tau_u$  and  $\tau_p$ . Consequently, since  $\tau = \tau_u + \tau_p > \tau_u$  and  $h'(\tau) > 0$ , the following



inequality applies,  $h(\tau) > h(\tau_u)$ , i.e. the tax distortion is lower compared with the other social security programs.

The government has no opportunity to accumulate wealth in the AF system, as the only tax received is intragenerational, i.e.  $\omega_t^G = 0$ , for all  $t$ . By inserting these assumptions and corollaries into equation (29) one obtains the equilibrium dynamics in the capital market as:

$$(1+n)k_{t+1} = [1 - h(\tau_u)] \delta(1-u)w_t. \quad (37)$$

Comparing capital dynamics in the NAF system given by (35), and in the AF system, shows that the only feature that separates the social security programs is the size of the tax distortion  $h(\cdot)$ . Due to the lower tax in the AF program, the excess burden is higher in the NAF program.

Accordingly, the growth factor in the economy with an AF social security system is by inserting (37) and Lemma 1 into (30):

$$g^{AF} = \frac{\beta\delta\kappa [1 - h(\tau_u)] (1-u)^\beta}{(1+n)a^\beta}, \quad (38)$$

hence, the growth factor is time invariant. Inspection of (38) shows that the tax distortion still exists, but is now lower compared with the other pension systems. This follows as the total tax rate on wages is now lower.

The following proposition compares capital accumulation in an economy with a NAF pension program and an economy with an AF pension program.

**Proposition 2** *Capital accumulation is higher in an economy with an AF pension system, than in an economy with a NAF pension system, i.e.  $g^{AF} > g^{NAF}$ .*

**Proof.** The proposition is showed by contradiction. By assuming that  $g^{NAF} \geq g^{AF}$ , and utilizing the expressions in (33) and (38), it follows that:

$$\begin{aligned} g^{NAF} \geq g^{AF} &\Rightarrow \\ \frac{\beta\delta\kappa [1 - h(\tau)] (1-u)^\beta}{(1+n)a^\beta} &\geq \frac{\beta\delta\kappa [1 - h(\tau_u)] (1-u)^\beta}{(1+n)a^\beta} \\ \Leftrightarrow h(\tau) &\leq h(\tau_u). \end{aligned}$$

Since  $h(\tau) > h(\tau_u)$ , the inequality is not fulfilled and the proof is complete. ■

Individual net income is higher under an AF pension system than under a NAF system. This is due to both smaller taxes and smaller tax distortions. The government can not contribute to national wealth in an AF program as the unemployment tax is intergenerational and distributed within the cohort. However, the individuals in the economy compensate for this element by saving a larger proportion of their income. And as the tax and the collection costs are lower, the net income is higher. Therefore, total savings capital accumulation are greater in an economy with an AF social security scheme.

Another feature on individual and actuarial pension schemes, is that the motivation to save is higher within this system, than under a non-individualized and non-actuarial pension system. This is due to the one-to-one connection between individuals payments and received pensions (Sørensen et al., 2006).

## 7 Conclusion

In several European countries, population ageing, unemployment and economic growth are much debated issues among professional economists and politicians. These issues are highly related through the social security system.

Corneo and Marquardt (2000) consider a model where individuals live for two periods. In the first period, individuals can be either employed or unemployed, and in the second period they are all retired. Due to this structure the social security system refers to a combination of public pensions and unemployment insurance programs. A main point in Corneo and Marquardt is that unemployment is caused by the union wage setting. In their model the labor market is characterized by a monopoly union that determines the wage.

Bräuninger (2005) expands the model of Corneo and Marquardt to include wage bargaining at an intermediate level. The labor market is therefore characterized by a Nash bargaining solution, rather than a monopoly trade union.

The current paper contributes to this theoretical literature by expanding Bräuninger's model to include three different pension schemes. Moreover, the set up of the model makes it possible to compare the different pension schemes with respect to growth implications. The PAYGO and the AF system are fairly standard in the literature, except for the inclusion of long-term unemployment. However, the NAF system is rarely considered, and represents the counterpart to a fully funded system, regarding the tax-benefit link and the degree of individualism.

To distinguish real effects of the two funded schemes, it is necessary to assume some sort of distortion in the economy. In the current set up, this is done via the excess burden on workers, due to tax payments. If  $h(\tau) = 0$ , the NAF and the AF schemes would be equivalent with respect to capital accumulation and output.

The second novel part of the paper lies in the combination of the modeling of the pension systems and the endogenous growth framework, that permits an analytical solution of the growth factor of capital. To be able to express the growth factor explicit, the technological spillover is assumed to be linear in capital intensity. Moreover, to simplify the growth model compared with some of the earlier literature, I model the growth factor in terms of per young individual. These simplifications makes it possible to do a comparative analysis of the different pension systems.

It is shown that growth is higher in an economy with a NAF system rather than a PAYGO system. The result depends on the governmental contribution to national wealth and thereby capital accumulation, within the NAF scheme. Comparing the two funding strategies, reveals that an individual and actuarial pension scheme, foster higher growth than the non-individualized and non-actuarial pension scheme.

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