The growth of self-efficacy in teaching mathematics in pre-service teachers: developing educational purpose

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Abstract

This thesis consists of four papers and a comprehensive summary of the work. Its overarching aim is to gain an understanding of how teacher education fosters the development of future mathematics teachers in terms of its contribution to pre-service teachers' (PSTs') reflections on the mathematics teacher they not only *want* to be, but *can* be. Taking self-efficacy in teaching mathematics (SETM) and its operationalisation in novice PSTs as its point of departure, this thesis focuses on subject matter knowledge as a core component and a key construct in their development.

The contribution to knowledge is built across the four papers of this thesis. Paper 1 reports on an instrument designed to measure SETM in novice PSTs. Paper 2 creates a research-based picture of different 'types' of novice PSTs in terms of their pre-programme identities, SETM and self-efficacy in mathematics. Paper 3 sets out to investigate the ways in which PSTs describe their experiences of success and failure at university and in school placement as sources of their developing SETM. It reports on how PSTs perceive their own subject matter knowledge and its role in teaching, how they reflect on this knowledge, and how these perceptions and reflections become part of their developing identities as mathematics teachers. Finally, Paper 4 measures to what extent PSTs develop SETM during teacher education, and reports on elements of the nature of this development. Taken together, a major contribution of this thesis is the connection between subject matter knowledge, self-efficacy and identity in PSTs: perceptions of subject matter knowledge and SETM are brought together as a means of investigating how they contribute to PSTs' developing identities as mathematics teachers.

The overall theoretical perspective of this thesis connects PSTs' developing identities as mathematics teachers with educational purpose, in Biesta's sense. It argues that teacher development is not just a question of acquiring subject matter knowledge, but is also a matter of how PSTs *perceive* the role of that knowledge, and how these perceptions influence their actions as a consequence. This approach comes into play through an investigation of the role of PSTs' reflections on their subject matter knowledge as a characteristic that enables them to take agency in making judgements in relation to educational purposes.

The project described in this thesis takes a mixed methodological approach, and comprises two main means of gathering data: survey and semi-structured interviews. PSTs were enrolled in a four-year programme for primary school teachers in Norway (grades 1-7, ages 6-13), which includes a compulsory course in mathematics methods spanning the first two years. The cohort

of 2013 completed the survey on two occasions, at the beginning of the compulsory mathematics methods course (N = 191), and at the end of this course (N = 102). 10 case study PSTs were interviewed six times in the intervening period, before and after each of their first three school placements. Rasch analyses were conducted on survey data, and interview data were analysed primarily in terms of operationalisations of Biesta's framing of multidimensional educational purposes.

Five major findings within the papers have implications for teacher education. First, the new instrument reveals elements of the nature of PSTs' developing SETM, with potential for use as an intervention with novice PSTs which can draw attention to the role and nature of subject matter knowledge in teaching mathematics. Second, the findings from both survey and interviews contribute to our understanding of the role of subject matter knowledge in the different sources of self-efficacy that PSTs draw on. Together with the absence of accounts of feedback on their subject matter knowledge (or lack of it) in PSTs' narratives of school placement and teaching at UC, the research described here suggests that this is needed. Moreover, third, the findings suggest a need for a variety of sources in different communities, and propose that there are ways to draw more on the most powerful sources during teacher education. Fourth, PSTs' narratives revealed that reflection on subject matter knowledge was an important personal characteristic. A focus on PSTs' reflection on their own subject matter knowledge as a form of agency, and as a key means by which they can approach their possible future teacher selves, highlights its importance in their developing ideas of 'the teacher I can be' as opposed to less reflected ideas of 'the teacher I want to be'. Hence, this research suggests that teacher education should provide more opportunities to reflect on one's own subject matter knowledge, both in school placement and at University College. Finally, fifth, a longitudinal reading of the four papers shows how each offers a new and more positive understanding of "weak" PSTs who might often be seen as "hopeless cases".

Alongside the specific findings of the four papers, this thesis aims to contribute to an understanding of issues which are recognisable to others involved in educating future mathematics teachers. In pursuing this aim, it adds to the body of research whose constant concern is to improve teacher education.

Table of Contents

List of Papers

Paper 1

Bjerke, A. H., & Eriksen, E. (2016). Measuring pre-service teachers' self-efficacy in tutoring children in primary mathematics: an instrument. *Research in Mathematics Education*, *18*(1), 61-79.

Paper 2

Bjerke, A.H. (2014). Self-efficacy in mathematics and teaching mathematics in novice elementary pre-service teachers. In Østern, et al. (Eds.), *Once a teacher – Always a teacher?NAFOL Year Book 2014* (pp. 195 – 215). Trondheim: Akademika Publishing.

Paper 3

Bjerke, A.H., & Solomon, Y. (submitted). 'The mathematics teacher I want to be': Self-efficacy and development of judgement in pre-service teachers. *Educational Studies in Mathematics.*

Paper 4

Bjerke, A.H. (accepted). The development of self-efficacy in teaching mathematics in pre-service teachers. *Nordisk matematikkdidaktikk,* *x*(y), pp-pp.

List of abbreviations

The following abbreviations are used in the thesis:

MSE	Mathematics self-efficacy
NESH	The National Committee for Research Ethics in the Social Sciences and Humanities
NSD	Norwegian Social Science Data Services
PST	Pre-service teacher
RSM	Rasch Rating Scale Model
RQ	Research question
SETcPM	Self-efficacy in tutoring children in primary mathematics
SETM	Self-efficacy in teaching mathematics
UC	University College

List of Figures

List of Tables

1 Introduction

The title of this thesis, ‘The growth of self-efficacy in teaching mathematics in pre-service teachers: developing educational purpose’, may in itself create certain ideas and expectations about what will be presented here. It is certainly about the development of self-efficacy in teaching mathematics (SETM) in pre-service teachers (PSTs). It certainly addresses SETM in connection to how PSTs experience different educational purposes, in the sense of developing a vision of their goals as teachers. What the title does not reveal, however, is the role of subject matter knowledge in this developmental picture.

In his influential contribution, “Those who understand: Knowledge growth in teaching”, Shulman (1986) raises important questions regarding what he calls ‘the missing paradigm’. His comparison of examinations from 1875, when theories and methods of teaching were important but played a secondary role to that of subject knowledge in teacher qualification, with those taken 100 years later where the focus had shifted from content to methods of teaching, made him ask “Where did the subject matter go? What happened to the content?” (Shulman, 1986, p. 5). More than 30 years later, in 2017, there is an ever-growing body of research investigating what teachers need to know in order to teach (Adler & Sfard, 2016; Hoover, Mosvold, Ball, & Lai, 2016; Rowland & Ruthven, 2011). Nevertheless, in my experience as a teacher educator, there is still a need to address content and Shulman’s important questions. This is the point of departure for the work in this thesis. Unlike most research on subject matter knowledge in the context of teaching mathematics, this thesis is concerned with how PSTs experience the need for subject matter knowledge as they develop identities as future mathematics teachers. Rather than investigating what knowledge is needed, this thesis gives PSTs a voice in the matter, allowing me to explore how *they* perceive the role of, and the need for, subject matter knowledge as they develop their ideas about the teacher they not only *want* to be, but *can* be.

1.1 Background and motivation

Improving teacher education is a constant concern. In an editorial, Cochran-Smith (2004) identified three periods in the history of US teacher education distinguished by changes in the focus of such concern. From 1950 to 1980, teacher education took a *training focus* (Cochran-Smith, 2004), that advocated preparing PSTs “with the skills to apply a fixed set of techniques” (Smith, 2016, p. 406). However, teacher education was critiqued for its superficiality, as it did not engage PSTs in making professional decisions (Smith, 2016). The following period, from approximately 1980 to 2000, showed a *learning focus* with an emphasis on how PSTs thought and learned in pre-service programs and schools and the multiple conditions and contexts that shaped their learning (Cochran-Smith, 2004, p. 296); reflection was seen as a key to learning from experience (Smith, 2016). The learning approach to teacher education was extensively critiqued, with the most damning comment being that "it focused on teachers' knowledge, skills, and beliefs without adequate attention to pupils' learning" (Cochran-Smith, 2004, p. 297). Thus, the link between teachers' knowledge and beliefs, and pupils' measurable learning outcomes was not established (Cochran-Smith, 2004).

As a response to this critique, the third period, beginning around 2000 (and extending to the present (Smith, 2016)), is what Cochran-Smith (2004) labelled *policy focused*, a period where international assessments such as TIMSS (Trends in International Mathematics and Science Study) and PISA (The Programme for International Student Assessment) have made it possible to compare performance across institutions and countries. The main goal has been to raise student achievement through an emphasis on the 'efficiency'[1] of teaching (Smith, 2016). These periods of changing focus also occurred in the European and Norwegian context, where educational systems have not escaped the resulting pressure towards accountability. Smith (2016) describes these shifting foci as a pendulum swinging from skills and techniques to reflection, which now appears to be swinging back to skills again.

Teacher education is a major target in evaluation of the results of TIMSS and PISA, being a key component of the educational system (Darling-Hammond, 2012; OECD, 2005). When student achievement on international and national tests is seen to be unsatisfactory, teacher education is often blamed, and reform is likely to follow shortly after a "PISA shock"; Norway is no exception (Darling-Hammond, 2012; Smith, 2009). Indeed, Smith (2011) connects the most recent reforms in Norway to its mediocre rankings on OECD's PISA report, combined with an extensive report carried out by NOKUT (2006) (the Norwegian Agency for Quality Assurance in Education), which critically evaluated Norwegian education at both national and institutional levels. Following on these publications, a new teacher education programme was implemented in 2010, offering national curriculum regulations (Ministry of Education and

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[1] What efficiency of teaching constitutes has been a longstanding focus in research on mathematics teaching (i.e. Askew, Brown, Rhodes, Wiliam, and Johnson (1997)). In this study, I understand effective mathematics teaching as teaching that promote student learning and understanding.

Research, 2010a) for differentiated[2] primary and lower secondary teacher education programmes for years 1 – 7 and years 5 – 10[3]. Based on NOKUT's recommendations, the new teacher education should focus on strengthening PSTs' subject matter knowledge, improving relationships between the field of practice (schools) and academic institutions, and increasing the focus on research (NOKUT, 2006).

The 'what' and 'how' of teacher education have become more centrally controlled, however, and policy decisions and reforms are rarely supported by a strong research rationale (Smith, 2016). As Smith (2016) points out, a recent general trend in Norway is the push to strengthen teachers' content knowledge and to expand periods of school placement in teacher education (Ministry of Education and Research, 2013). However, it is well documented that students experience a disconnect between school placements and the theoretical input from university college (Bjerke, Eriksen, Rodal, Smestad, & Solomon, 2013a; Gainsburg, 2012; Nolan, 2008), and it is hard to see that such expansion on its own could bridge the theory-practice divide. Working on the possibility that bringing some clinical experiences into the UC setting might help, Grossman, Hammerness, and McDonald (2009) propose that teacher education be organised around a set of core practices. Such core practices can take place at UC, where PSTs can be supported in developing the knowledge, skills, and an emerging professional identity around these practices. In this approach, avoiding 'drilled techniques', the relative unpredictability of teaching is acknowledged (Grossman et al., 2009). Focusing on subject matter knowledge in different teaching situations and on context-specific learning goals for students gives purpose to the work of teaching (Forzani, 2014). Thus, Grossman et al. (2009) propose core practices as a way of avoiding the dichotomous view of theory and practice, in which principles for teaching and academic knowledge are presented at UC, followed by observation and enactment of related strategies during school placements.

In Norway, National curriculum regulations are operationalised into subject-specific National curriculum guidelines (Ministry of Education and Research, 2010b), with bulleted lists of the knowledge, skills and competences that new teachers must achieve, and be measured against. The pressure of accountability on both international and national levels means that "teacher education is in danger of becoming an education responding to a checklist of pre-decided

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[2] In Norway, it is common that teachers move with their students as they progress through the school grades. Previous to this reform, Norway had one teacher education programme for grades 1 – 10 (ages 6 – 16).

[3] The primary teacher education programme (grades 1 – 7, ages 6 – 13) educates generalist teachers prepared to teach all subjects in grades 1 – 7. The secondary teacher education programme (grades 5 – 10, ages 10 – 16) educates specialists each choosing a few subjects.

knowledge and goals that newly qualified teachers are to achieve and document" (Smith, 2011, p. 347). In the context of mathematics, introducing future teachers to 'the problematic nature of teaching' (Smith, 2011) adds a new perspective: it is no longer just about doing mathematics for oneself; it is also about helping others to do and understand mathematics. This added dimension is hard to articulate in a bulleted list or 'a checklist of pre-decided knowledge'. In line with the ideas around core practices, subject matter needs to be a critical component of the goals and activities that constitute the professional curriculum (Forzani, 2014, p. 359). In this way, teacher education transforms mathematics into a subject where judgement is needed.

This leads me to focus on PSTs' perceptions of their own subject matter knowledge and its role in teaching, rather than measuring their actual subject matter knowledge. Support for this stance comes from Kagan (1992), who noted that PSTs' perceptions lie at the heart of teaching, and Pajares'(1992) comparison of 16 studies, concluding that PSTs' perceptions play a pivotal role in the way they acquire knowledge during pedagogical training.

One way of studying PSTs' perceptions of their own subject matter knowledge and its role in teaching, is by paying attention to their SETM. Teacher efficacy is considered one of the key motivation beliefs influencing teachers' professional behaviours and school student learning (Klassen, Tze, Betts, & Gordon, 2011). Because of the situatedness of teacher efficacy, there is a need for more attention to domain-specific explorations (Klassen et al., 2011), and mathematics is an especially interesting context, since PSTs often express doubt about their own self-efficacy in mathematics (Gresham, 2007). Moreover, research indicating that teacher efficacy develops mainly during teacher education (Hoy & Spero, 2005) underlines the importance of investigating how and to what extent SETM develops in PSTs. In this thesis, this is done in two ways, by a quantification of SETM through instrumentation, and through interviews focusing on PSTs' perceptions of their own subject matter knowledge and the need for such.

In order to investigate how PSTs perceive the role of subject matter knowledge and develop SETM in practice, I turn to Biesta's (2012a, 2012b, 2014) conceptualisation of educational purpose (outlined in Paper 3, p.6). Following this line, my aim is to contribute to a redirection of the pendulum back towards reflection, and in doing so to produce results applicable for practice in order to improve teacher education.

1.2 Context of the study

The research reported in this thesis was conducted within the generalist primary teacher education programme for grades 1 – 7 (ages 6 – 13). Data were collected at a University College (UC) in Norway, which admitted 200 new PSTs in 2013 for training in primary teacher education, constituting 26% of the 768 primary PSTs nationwide that year (NSD, 2017).

In this programme, PSTs must take a minimum of 30 ECTS in mathematics, where the mathematics teacher educators promote and model inquiry-based and connectionist approaches (Bjerke, Eriksen, Rodal, Smestad, & Solomon, 2013b). At this particular UC, the course spans the first two years of a four-year programme involving four periods of school placement. I situate my work in the tension between these two components of teacher education: teaching at UC and school placement.

Tensions between inquiry-based approaches at university level and instrumentalism in schools are often described in research on teacher education (Gainsburg, 2012; Nolan, 2008; Nolan, 2012). Barnes, Cockerham, Hanley, and Solomon (2013) report that both pre-service and in-service teachers struggle to put the reform approaches advocated in teacher education into practice; policy-driven pressure to produce easily measured evidence of pupils' progress in short time frames makes it easier in the short term to focus on rote-learned algorithms and 'teaching to the test'. Nolan (2012) points not just to the role of accountability and assessment in schools, but also to the force of students' educational habitus, which is firmly embedded within experiences at a very young age and highly resistant to change. Similarly, Arvold (2005) found that PSTs experience and interpret their teacher education programmes through the lens of prior experience and the beliefs that go with that experience.

The MAPO group of researchers[4] (to which I belong) has examined the issue of tensions in the Norwegian context, through action research conducted on PSTs in primary teacher educations[5], finding that many PSTs in a first-year sample seemed to 'miss the point' of much of the UC's theoretical input. They favoured school placement above UC learning, valuing the practice-based learning they gained from their teacher-mentors (Bjerke et al., 2013b). Investigating the possibility that this was a product of the inquiry-based/instrumentalist divide (Gainsburg, 2012; Nolan, 2008), PST focus group data indicated that mentors did not necessarily enact reform approaches in practice, even though they subscribed to these in theory (Bjerke et al., 2013a).

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[4] MAPO (Matematikk i Praksisopplæringen) - The interface between theory and practice: supporting the development of mathematics pedagogy - a four year long research project from 2012 – 2016.

[5] The setting of the MAPO research parallels the setting in my project, but the cohorts of PSTs are different.

Taken together, these findings suggest that "differences between universities and schools have more to do with culture or with historically embedded practice than with explicitly formulated views of mathematics teaching" (Solomon, Eriksen, Smestad, Rodal, & Bjerke, 2017, p. 143). This provides important information on the setting for PSTs in my study: many mentors in their school placements are unlikely to oppose inquiry-based approaches to teaching mathematics; nevertheless, many are equally unlikely to promote and exemplify such approaches.

1.3 Research questions

Learning to teach is not only about acquiring professional knowledge and skills, it can also be regarded as developing teacher identity (Haniford, 2010). In this sense, my study aims to explore PSTs' developing identities as mathematics teachers by focusing on the nature of their developing SETM, and on how they perceive the role of, and the need for, subject matter knowledge in this developmental picture. Hence, four research questions have underpinned my work, each leading to a paper in this thesis.

First, there was a need to measure PSTs' developing SETM. A review of the literature (see Chapter 2) revealed the need for a new instrument suitable for novice PSTs, leading to the following research question:

> What are the necessary features of an instrument designed to measure the core of SETM in the population of novice PSTs? (Paper 1)

The first implementation of this instrument marked the start of my project, while a second implementation marked its end. Focusing on SETM development led to a further research question:

> To what extent does PSTs' SETM develop during a mathematics methods course in primary teacher education, and what is the nature of this development? (Paper 4)

In the intervening period between these two papers, my work focused on understanding what contributes to developing SETM, exploring how PSTs perceive the role of subject matter knowledge, and understanding more about the importance of SETM for their developing identities as mathematics teachers. Thus, I also asked the following two questions:

> What are the connections between novice PSTs' perceptions of their own subject knowledge and their self-efficacy as a potential teacher in mathematics? What are the implications for the identity work these PSTs need to do? (Paper 2)

> How do PSTs perceive the nature of mathematics and mathematics learning/teaching, and what influences their perception of their own subject matter knowledge and SETM? (Paper 3)

As these research questions indicate, I have chosen a mixed methodological approach (see Section 3.2). As I will show in Section 2.1, addressing the qualitative elements of these questions led to a choice of shifting theoretical lenses as the project developed in line with ongoing findings.

1.4 The concept of self-efficacy

As indicated above, one concept in particular permeates my work, and I clarify here how the concept of *self-efficacy in teaching mathematics* is understood in this thesis. I have adopted Albert Bandura's social-cognitive construct of self-efficacy. This is concerned with performance in that it predicts the goals people set for themselves and their performance attainments; that is, it is concerned with judgements of personal capability (Bandura, 1997). According to Bandura (1997), many people confuse this concept with that of self-esteem, which is concerned with judgements of self-worth and has nothing to do with personal goals or performance.

Bandura started out as a behaviourist, but his realisation that behaviour changes when we observe others perform adds vicarious learning, something a pure behaviourist would reject. For Bandura, to be an agent is to influence ones' life conditions intentionally, and no mechanism of human agency is more central than peoples' beliefs in their causative capabilities. Bandura calls this belief self-efficacy. "Unless people believe they can produce desired effects by their actions, they have little incentive to act" (Bandura, 1977, p. 2). Such intentional acts constitute what Bandura refers to as agency. Put in other words: If one does not believe one has the power to produce results, there is no need to attempt to make things happen.

Self-efficacy is defined as a person's judgement of his or her abilities to execute successfully a course of action (Bandura, 1997), a future-oriented belief about the level of competence one expects to show in a specific situation. It has two components: a personal belief about one's own ability to cope with a task, a personal self-efficacy, and judgments about the outcomes that are likely to flow from such performances, an outcome expectancy (Bandura, 2006, p. 309).

In this thesis, teacher efficacy is defined as a measure of "the extent to which teachers believe their efforts will have a positive effect on student achievement" (Ross, 1994, p. 4), the 'personal self-efficacy' component of Bandura's theory. Moreover, in the context of my work, SETM is understood to be the component of teacher efficacy corresponding to Bandura's concept of personal self-efficacy, seen in the subject-specific situation of teaching mathematics. In this way, teacher efficacy, and SETM in particular, is a belief sub-construct[6] (Pajares, 1992).The reader is referred to Paper 1 (p. 63) for a detailed account of why SETM has to be treated separately from self-efficacy in teaching in general, and likewise differently from self-efficacy in teaching other subjects.

1.5 Outline of the thesis

This thesis consists of four papers and what in Norway is called a 'Kappe'. A 'Kappe' is a scientific text intended to give a comprehensive overview of the connections between the papers that comprise the thesis, and to bring together the problems, results and conclusions presented in them. This 'Kappe' consists of five chapters.

Following this introduction in *Chapter 1*, the theoretical landscape is outlined in *Chapter 2* where I position my work in the existing body of research, and present the theoretical perspectives that frame the four papers and this 'Kappe'. *Chapter 3* argues for a mixed methodological approach where I combine what looks like an objective measure with a narrative approach. I elaborate on the methodological considerations arising from the way in which this thesis blends different theoretical perspectives.

The results of each of the four papers are presented in *Chapter 4*. In order to underline the connections between the papers and their contribution to the overall project, I tell the story of one particular PST who can be tracked through all the papers. In *Chapter 5*, I focus on and discuss three major findings that appear across the four papers, and I discuss the implications for teacher education and the contribution of my work to the field of research on teacher education.

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[6] Unfortunately, this is sometimes unclear in the papers included in this thesis. In Paper 1, p. 63, we say: «Research shows that SETM is influenced by the teachers' own mathematics self-efficacy, their mathematical beliefs (Briley, 2012), and their past experiences as learners of mathematics (Brown, 2012)». Moreover, in Paper 2, p. 200, I write: «This newly added dimension of how to teach mathematics is captured in the concept of SETM which is influenced by beliefs, previous experience as learners, and MSE». This can be read as if self-efficacy is something other than a belief sub-construct, which is misleading.

2 Theoretical landscape

Two major concerns in mathematics teacher education research are the role of subject matter knowledge (Adler & Sfard, 2016; Hoover et al., 2016; Rowland & Ruthven, 2011) and the development of SETM in PSTs (Klassen et al., 2011; Philippou & Pantziara, 2015; Wheatley, 2005). These two bodies of research are normally not brought together, but I do so in this thesis by developing a measure of PSTs' SETM and exploring its sources; I investigate how they perceive their own subject matter knowledge and its role in teaching, how they reflect on their own subject matter knowledge, and how these perceptions and reflections become part of their developing identities as mathematics teachers. In this chapter, I outline and position the theories I have utilised in my work that have enabled me to bring together these very different ways of looking at mathematics teacher development.

In an editorial, da Ponte (2011) makes an important distinction between practicing teachers and PSTs. While the primary responsibility of practicing teachers is "to promote the mathematics learning of their students, in the frame of broader social and educational goals", the primary responsibility of PSTs is "to develop themselves to become competent mathematics teachers" (da Ponte, 2011, p. 415). There exists a wide variety of definitions of competence, from the very narrow to those "so broad that it might be difficult to see what is not included in the idea of competence" (Biesta, 2013, p. 122). As Biesta (2013) goes on to note, regardless of how it is defined, the notion of 'competent' and 'competence' is frequently driven by policy, as in the European Commission's 'Common European Principles for Teacher Competences and Qualifications' which lists the key competences teachers should hold. Consequently, although 'competence' is an interesting notion with some potential, the problem is the that "the idea of competence is beginning to monopolise the discourse about teaching and teacher education" (Biesta, 2013, p. 122), with a tendency towards uniformity. Biesta (2013) offers the example of how PISA and similar systems (see Section 1.1) seem to create an illusion that a wide range of different educational practices are comparable and therefore ought to be comparable. Thus, in the world of education policy, 'competence' can be seen as part of the language of audit and accountability (Day, 2017).

In my project, given the wide variety of definitions, it is equally important to say how I am inspired by Biesta's ideas on what competence is *not*. Going back to the key competencies in 'Common European Principles for Teacher Competences and Qualifications', Biesta (2013) observes that, in this text, education is predominantly described in terms of learning.

Deliberately employing an ugly word, Biesta (2012b) refers to this as the 'learnification' of educational discourse. This leads to his major point on the problem of the focus on competences: the current emphasis on measurable competences in education is problematic in that 'learning' is a process term saying very little about relationship, purpose and direction. Biesta (2012b) underlines the problem of the focus on the accumulation of disconnected competences without any sense of an overall future purpose; "Without a sense of purpose, there may be learning but not education" (Biesta, 2014, p. 3). This is not to say that definitions of competence never address judgement in some sense, and indeed there are examples of this[7]. Nevertheless, judgement about what needs to be done always needs to be made with reference to the purpose of education (Biesta, 2013).

Biesta proposes a conceptual framework comprising three domains of educational purpose: *qualification*, *socialisation* and *subjectification*[8] (Biesta, 2009). The question of purpose needs to be raised in each of these domains (Biesta, 2012b), making it a multidimensional question (Biesta, 2013). Consequently, judgement also needs to be multidimensional: there are trade-offs between the different domains whereby a gain in one domain might lead to a loss in another (Biesta, 2013). Thus, for Biesta, the capacity for judgement is different from having competences, since it is more value-laden, having the quality of what Aristotle would call *virtue*, being described as a practical wisdom that "denotes what we might call a holistic quality, something that permeates and characterises the whole person" (Biesta, 2013, p. 134).

This thesis is guided by what Biesta calls a virtue-based approach to teacher education where competence is seen as a necessary, but not sufficient, condition (Biesta, 2014) since it needs to be accompanied by judgements that "always need to be made with reference to the purpose of education" (Biesta, 2012b, p. 15).

Instead of seeing the primary responsibility of PSTs as becoming competent teachers (in its narrowest sense), 'competent' is replaced in this thesis by 'good', resulting in a crucial

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[7] One example is the definition of mathematics competence given in the Danish KOM-project: «…knowledge of, understanding, doing and using mathematics and having a well-founded opinion about it, in a variety of situations and contexts where mathematics plays or can play a role» (Niss, 2004, p. 183).

[8] Originally developed as concepts that can be used to consider the aims and purpose of teacher education, they are equally useful when investigating how PSTs view their progress and their potential as future teachers in mathematics able to *practice judgement*. In Paper 3 (p. 6), these domains are operationalised in the context of PSTs attending a mathematics methods course during elementary teacher education. Paper 3 explores how PSTs make judgements about how to balance the different domains, and at the same time articulate their priorities in relation to each domain.

rephrasing of da Ponte's statement – 'to develop themselves to become *good* mathematics teachers'. Good mathematics teachers have competence, and additionally an ability to make multidimensional judgements in the different domains of educational purpose (Biesta, 2014). Pursuing Biesta's stance on what is needed to become a good teacher, in this thesis I argue that teacher development is not just a question of acquiring competence (where I focus on knowledge), but is also a matter of how PSTs perceive the role of that knowledge, and how these perceptions influence their actions. In my work, this approach comes into play through an investigation of the role of subject matter knowledge in PSTs' developing SETM and their developing identities as future mathematics teachers.

In this chapter, I first elaborate on the view of knowledge taken in this thesis (2.1). I then focus on three central issues which are explored in the papers that form the central body of this thesis: PSTs' developing identities as mathematics teachers (2.2); The role of mathematics knowledge in teacher identities (2.3); and Sources of self-efficacy in teaching mathematics (2.4). I end this chapter with an overview of how the different theories have together informed and contributed to the development of this thesis (2.5).

2.1 Theories of knowledge and learning theories

Different theories of knowledge, and consequently learning theories, are based on specific assumptions about knowledge, learning, and reality. In this section, I outline the assumptions underpinning my research on PSTs' developing identities as mathematics teachers during their teacher education, and give an account of how I have brought very different epistemological and ontological approaches to knowledge together in this thesis. In Section 2.1.1, I describe the way I understand the concept of knowledge in this thesis. Next, in Section 2.1.2, I give an account of how different theoretical approaches and different views on learning have informed my work at different stages in the research process. Assumptions about reality and how knowledge can be measured and studied are discussed in Chapter 3, on methodology.

2.1.1 The concept of knowledge

As Murphy, Alexander, and Muis (2012) point out, there are many ways to interpret both the noun *knowledge* and the verb to *know*. The difficulties go way back: "… is it not shameless when we do not know what knowledge is, to be explaining the verb "to know"?" (Plato, *Theaetetus*, trans. 2006). Referring to Plato's dialogue Theaetetus, Gustavsson (2000) notes that Plato (427-347 bc) defined knowledge to be *true, justified beliefs*. This definition is captured in Aristotle's *episteme*, denoting scientific-theoretical knowledge and the highest form of knowledge for Aristotle. He adds two more forms of knowledge, *techne* and *phronesis*, where *techne* or 'skills' are described as practical and productive knowledge rooted in pragmatism, and *phronesis* or 'wisdom' as a kind of personal practical knowledge which is about developing judgment, ethics and moderation (which adds to the concept of *virtue* discussed in the introduction to this chapter). These different paths have developed through history and provided us with different ways of looking at and talking about knowledge (Gustavsson, 2000).

Aristotle's forms of knowledge underpin my work. *Episteme* is in practical terms the point of departure for my research, and is essential throughout in the form of my focus on subject matter knowledge in both SETM and developing identities. Following Biesta (2014), I see *techne* as the application of this knowledge as PSTs gain experience and gradually are introduced to the subject matter knowledge needed in order to teach. Finally, *phronesis* and *virtue* are embedded in my work in terms of their role in addressing judgement and educational purpose, and seeing it as an integrated whole. In the next section, I show how these various understandings influenced the development of my exploration of PSTs' developing identities as mathematics teachers.

2.1.2 Theorising knowledge within the individual and the social

Murphy et al. (2012) suggest that there are two important dimensions to consider, or more precisely, two 'epistemic vectors' to be drawn when positioning one's research. One vector is concerned with the sources of knowledge, and the other with where knowledge resides. In Figure 1, these two vectors are drawn as axes: the horizontal axis spans a continuum from a view of knowledge as purely individually formed to one where knowledge is perceived as purely socially derived; and the vertical axis contrasts the extremes of knowledge viewed as residing entirely in the mind versus entirely in the environment.

While papers 1 and 4 focus on the development and use of an instrument that investigates SETM in PSTs, papers 2 and 3 focus on PSTs' developing identities as mathematics teachers. I find it helpful to first position my research in Murphy et al.'s two-dimensional space, and it is possible to do this be tracking my work with papers 2 and 3 as moving around in the landscape shown in Figure 1. I do that here by addressing the theoretical frameworks utilised in Paper 2 and Paper 3 and the shifts between them. The very different theoretical underpinnings of papers 1 and 4, and the ways in which I finally combined these differing approaches, will be focused in sections 2.2, 2.3 and 2.4.

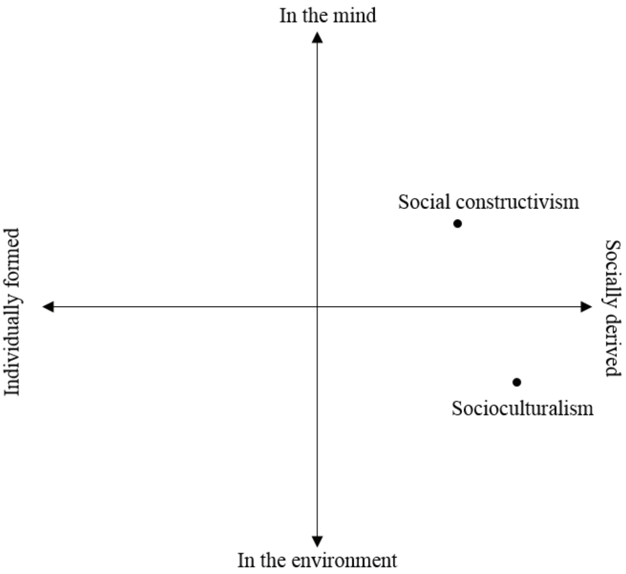

Figure 1. Sources of knowledge (horizontal) and where knowledge resides (vertical) (Murphy et al., 2012, p. 211, simplified version)

The original point of departure for my work was an investigation of the role of placement experiences in teacher development; I was concerned to understand how PSTs worked across the intersecting practices of UC and school, particularly with regard to assumptions about what sort of knowledge teachers needed to be successful. This focus on practice lent itself to exploration through a socio-cultural lens (see Figure 1, 4th quadrant), and consequently Paper 2 was framed within Wenger's social learning theory (Wenger, 1998) which builds on the idea that learning is a social action. In this approach, learning occurs in the interaction *between* people, not *in* people. In order to understand the complexity of being a novice PST in an unknown constellation of practices, and to make sense of the processes of identity formation in becoming mathematics teachers, it is useful to consider what Wenger (1998) characterised as three distinct modes of belonging, and later as modes of identification (Wenger, 2012): engagement, imagination and alignment. *Engagement* is active involvement in practice, while *imagination* involves standing back from the world and seeing oneself in it as a part of the whole picture (Wenger, 1998). *Alignment* is all about doing what it takes to play a part in the practice.

Investigating PSTs' different modes of identification was useful for making sense of their individual trajectories and their early development as teachers of mathematics in the making. Wenger's (1998) focus on the ways in which we identify ourselves by what we can do and understand - our developing competences[9] in a community of practice - presented a means of investigating PST's developing identities in relation to their perceptions of their competence in teaching mathematics. Taking this contribution from Wenger enabled me to theorise how individual trajectories and perceived competences in mathematics teaching connected with different modes of belonging. In particular, this lens enabled me to operationalise what this meant in the context of future identity work where reflection emerged as an essential personal personal characteristic. In Wenger's (2010) account, "identities become personalized reflections of the landscape of practices" (p.6), but he addresses the issue of reflection in relation to the mode of imagination in particular, in the sense that taking a distance enables us to become aware of the multiple ways in which we can interpret our lives. Although spending time reflecting can detract from engagement, "the combination of engagement and imagination results in a reflective practice" (Wenger, 1998, p. 217).

The mixed analysis reported in Paper 2 revealed the need for a reflective view not only on one's participation in new communities of practice, but also on one's own subject matter knowledge. It is not enough to see reflection purely as an aspect of imagination, however. Additionally, there is an emergent need to reflect on one's own knowledge, or lack of it, in order to develop active involvement in a practice (as in the mode of engagement). This observation drew attention to the importance of investigating how PSTs perceive the role of subject matter knowledge and its role in teaching as an important contributor in their developing identities.

The theoretical framework for Paper 2 highlighted the importance of reflection in PSTs' developing identities as mathematics teachers, where reflection turned out to be something other than just an aspect of imagination, and something other than yet another competence. The conclusion of Paper 2 led to a new focus on reflection as a form of agency, and to the work of Biesta (2012a). In the rest of my work, reflection connotes a private internal process, and can be understood as the conscious consideration of one's experiences (Hiebert, 1992). In this way, reflection can be used to establish relationships between ideas or actions, as it involves thinking back on one's experiences and taking them as objects of thought (Hiebert, 1992). In my work, the role of subject matter knowledge in these experiences is the focus.

The emergent need for a theoretical framework addressing the topic of reflection on subject matter knowledge led me to Biesta's work. (Biesta, 2012a) provided additional insights

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[9] Wenger (2010) talks about a "regime of competence," a set of criteria and expectations by which one recognizes membership in a community of practice. In this way, in Paper 2, when talking about competences, knowledge and skills were part of it.

concerning ‘the good teacher’ versus ‘the competent teacher’ and the relationship between competences and PSTs’ developing practice as teachers in terms of their growing awareness of how to make judgements in relation to educational purposes. As he says, “a teacher who possesses all the competences teachers need but who is unable to judge which competence needs to be deployed and when, is a useless teacher” (Biesta, 2012a, p. 42). In my work, I operationalise PST’s reflection on their own subject matter knowledge and its role in teaching as a characteristic that help PSTs making judgement in relation to educational purposes.

The move from socio-culturalism (in the 4th quadrant) to social constructivism (in the 1st quadrant) in the 2-dimensional landscape in Figure 1 suggests how the focus on *reflection* (and its renewed operationalisation and role), which emerged in Paper 2[10], directed my work. It positions the virtue-based approach that guides the work of Paper 3 and the framing of this thesis. Additionally, it highlights the need to investigate how PSTs reflected on and interpreted sources of SETM; and finally, it positions the overarching ideas this thesis draws on: the importance of exploring how PSTs perceive the role of subject matter knowledge in their possible and future selves.

Even though both social constructivism and socio-culturalism arise from the work of Soviet scholars, most particularly Vygotsky, they differ significantly in terms of the assumptions they make about the nature and development of knowledge. In socio-cultural theory, knowledge is possessed by the collective and by the knower as part of the community, and is therefore more group oriented (Murphy et al., 2012). The process ontology that underlies socio-cultural theory, and which suggests that learning involves reproducing the social and cultural structures through participation in practices (Murphy et al., 2012), makes it different from the social constructivist understanding of the individual and their construction of knowledge. Despite the prominent role in social constructivism of social life and participation in communities of practices in the acquisition of knowledge, knowledge remains a uniquely personal construction (Murphy et al., 2012). The need for reflection that emerged in Paper 2 made this personal construction of knowledge essential in my work. Drawing on Ernest’s (2010) understanding of constructivism and especially social constructivism, PSTs can be said to construct their own knowledge and understandings based on personal interpretation of their experiences and their pre-existing knowledge. The findings in Paper 2 regarding the importance of reflection on individual PSTs’ knowledge and its role in becoming a teacher led to a shift in my stance on where knowledge

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[10] In Paper 2, I do not fully clarify on how I operationalise reflection, which gives a rationale for devoting more room for it here.

resides. This can best be explained as a vertical move in Figure 1, closer to viewing knowledge as residing in the mind. At the same time, it is important to add that individual constructions are not isolated from, but are largely dependent on, social interaction, which explains why I still locate my research in the right half of Murphy et al.'s diagram.

The emphasis on reflection on one's own perceptions of subject matter knowledge that arose in Paper 2 provides the point of departure for Paper 3. In order to frame this now social constructivist approach, I use Biesta's concept of domains of educational purposes as a tool for exploring how PSTs understand and act on their subject matter knowledge in their developing identities as (future) mathematics teachers. A focus on these perceptions and reflections can work as a response to Biesta's critique of current educational practice - that it prioritises learning without a purpose, as in 'learnification'. In this way, Biesta's virtue-based approach not only guides the work of Paper 3, but the thesis as a whole. The added focus on judgement and educational purpose enables me to make connections in Paper 3 between novice PSTs' ideas about 'the mathematics teacher I *want to* be' and their later more experienced, reflected and future-oriented ideas about 'the mathematics teacher I *can* be'. I now turn to how these ideas of 'the mathematics teacher I can be' are treated in the literature on identity addressing *possible selves*.

2.2 Pre-service teachers' developing identities as mathematics teachers

As already noted in the introduction (Section 1.3), learning to teach can be regarded as developing teacher identity (Haniford, 2010). Teachers are engaged in their practice with all their being, not only on the basis of what they know but also who they are, how they see themselves as teachers, and how they reflect on and identify with the profession (da Ponte & Chapman, 2008). Beijaard, Meijer, and Verloop (2004) identified three categories of research on teachers' professional identity. The mixed methodological approach in this thesis enabled me to work across these categories and situate my work within research on (1) professional identity formation, (2) teachers' perceptions on aspects of their professional identity, and (3) professional identity represented by teachers' narratives. Teacher identity is prominent in this thesis in a developmental perspective; it connects PSTs' current identities to both their past experiences and their future imagined trajectories as 'the mathematics teacher I can be' (category 1). These developmental perspectives are storied in the PSTs' narratives (category 3), where their perceptions and reflections on subject matter knowledge and its role in teaching come through (category 2).

In Section 2.2.1, I position the work on identity in my thesis within a sociological framework, where I adopt the notion of *possible selves* and underline the importance of this concept in my work. Having established these overarching reference points, in Section 2.2.2, I link the different ways of addressing identity that appear in Paper 2 and Paper 3, where I draw on Wenger (1998) and Biesta (2013) respectively. Section 2.2.3 connects identity and the fundamental role of subject matter knowledge and self-efficacy in my work and highlights how Paper 1 and Paper 4 contribute to an exploration of identity.

2.2.1 Possible selves

As is the case with research into identity in general, identity research in mathematics education draws on two distinct paradigms: a sociological approach, which conceptualises identity as a fluid action (or a process), and a psychological approach which conceptualises it as a stable acquisition (something we have inside of ourselves). I situate the research presented in this thesis within the sociological framework derived from Mead (1934) and his description of identity as multiple, sometimes contradictory, and performative.

One of the most complex issues in the determination of what identity is revolves around the notion of self and its relationship to identity (Beauchamp & Thomas, 2009). The idea of the self is associated with continuity and coherence, and in this way the self can be seen as the meaning *maker* (or the teller of stories), while identities are the stories told or the meaning *made* (Rodgers & Scott, 2008). Within literature specific to teaching, a number of authors consider an understanding of self as a key component of teacher development, and therefore of the shaping of identity (Beauchamp & Thomas, 2009). In my work, the self is indirectly investigated through my focus on shifting identities as PSTs story themselves as future mathematics teachers, informed by both the past and current performance in placement. I thus gain a 'snapshot' insight into PSTs' self-positioning as novices at the time of their first interview (Paper 2), followed by a more developmental picture drawn from their accounts of being and becoming a mathematics teacher over time (Paper 3). Similarly to Lutovac & Kaasila's (2014) work on PSTs' narrative mathematical identity, I draw attention to the role of reflection in PSTs' accounts of their past experiences with mathematics and their accumulated experiences as they progress through teacher education, and articulate a sense of their future selves as mathematics teachers-to-be.

In Paper 3 these narratives are referred to as 'developmental stories', told through semi-structured interviews where I listen to PSTs' voices at various points as they go through teacher education. A particular interest is the presence of gaps in their stories; these include gaps between what they are able to do, and what they want to be able to do, with particular respect to their accounts of the role of more profound understanding of subject matter knowledge and their lack of it (see 'Findings' in Paper 3). Applying the notion of *possible selves* proposed by Markus and Nurius (1986), Lutovac and Kaasila (2014) suggest that the presence of a gap is of key importance for evoking the teacher (and hence PST) change process. While this notion is not used in Paper 3, it enables further theorisation of its findings in terms of a broader examination of PSTs' anticipations of the future: possible selves can be defined in terms of what one *might* become, what one *would like* to become, and what one is *afraid* of becoming in the future (Markus & Nurius, 1986). In my work these ideas come to play in the crucial distinction between PSTs' early ideas on 'what kind of mathematics teacher do I want to be' and the slight but important rewording in 'what kind of mathematics teacher can I be'. This distinction captures how identities change over time and are constantly under (re)construction, something that is possible to understand as the PSTs draw on ever new experiences which at some point enable them to be forward-looking and more reflective on their future mathematics teacher selves.

2.2.2 Understanding the role of reflection

Although the way in which my work developed led me to use different theoretical approaches in Paper 2 and Paper 3 (see Section 2.1), the term 'identity' is central to both, and is anchored in a view of identity as an action and a process that fits within a sociological framework. In this section, I will outline how drawing on the work of Wenger (1998, 2000, 2009) and Biesta (2012a, 2013, 2014) together has enabled me to investigate important aspects of PSTs' developing identities as mathematics teachers, and in particular, the role of reflection.

For Lave and Wenger (1991), learning and identity are inseparable: learning can be seen as involving the construction of identity, and this can be applied to learning to teach. In order to investigate the connections between novice PSTs' perceptions of their own subject matter knowledge and self-efficacy as a potential teacher in mathematics and implications for identity work, Paper 2 investigates PSTs' modes of identification (Wenger, 1998) and their initial thoughts on being a PST. For Wenger, identities exist both in us and in our relations with others. He sees identity as developed through "negotiated experiences of self" (p.150), "not an object, but a constant becoming" (pp.153 – 4), where our perceptions of ourselves and others, and others' perceptions of us are crucial. This fits within a Meadian view (Mead, 1934), where Wenger links identity closely to practice (including experience and knowledge), describing engagement, imagination and alignment as different modes of belonging (or identification) in relation to practice. As explained in Section 2.1.2, I thus used Wenger's theory of communities of practice to explore PSTs' understandings of their early experiences of teaching mathematics. Investigating the intersection of their different modes of belonging and their pre-program identities as learners of mathematics in terms of SETM and mathematics self-efficacy in Paper 2 revealed the central role of reflection in their trajectories as becoming mathematics teachers. Reflection is a key means by which teachers can approach their *possible selves,* and the new operationalisation of reflection outlined in Section 2.1.2 directed me towards Biesta's work. I wanted to uncover PSTs' reflections on their own subject matter knowledge and its role in teaching coming through in their narratives, and see how these reflections connected with their sense of agency.

Sutherland, Howard, and Markauskaite (2010) note that the nature of PSTs' experiences in typical teacher education programs impacts on the outcome of their reflections on teaching and being a teacher. Reflection can be more than looking back, however, and can also be linked to future teaching actions (Lauriala, Kukkonen, Denicolo, & Kompf, 2005). This leads me to note an important shift from Paper 2 to Paper 3: while Paper 2 is more concerned with PSTs' perceptions of their past experiences and backgrounds, Paper 3 is more future-oriented. PSTs can take agency in terms of how they position themselves in relation to their teacher education programmes and input from their teacher-mentors (Haniford, 2010). Moreover, as they gain more experience, as related in Paper 3, it is possible to address identity in relation to their agency and their ongoing negotiation of identity and its reshaping within experience.

This shift in focus from Paper 2 to Paper 3 resulted in another way of addressing identity in my work. Biesta (2013) tends to avoid the notion of identity because he sees it as how we identify with existing orders and traditions. Instead, he talks about *socialisation* and *subjectification*. *Socialisation* is about the ways in which, through education, individuals become parts of existing orders and traditions, while *subjectification* is about ways of being that are not entirely determined by these existing orders and traditions (Biesta, 2013). In this way, subjectification expresses a particular interest, where those being educated are "seen as subjects in their own right; subjects of action and responsibility" (Biesta, 2013, p. 18). In my work, I continue to use identity as a way of talking about both *socialisation* and *subjectification,* but also about *qualification*[11], with a particular focus on PSTs' future-oriented ideas of 'the mathematics teacher I can be'. Thus da Ponte and Chapman's (2008) way of considering pre-service mathematics teachers' identity is descriptive of the approach I take in Paper 3: "…it is not only about what it means for one to know, do, learn, and teach mathematics but what it means to view oneself as a professional teacher and how one sees one's ongoing development as a teacher of mathematics" (da Ponte & Chapman, 2008, p. 242). In this way, identity is articulated through PSTs' developmental stories (or, to use Biesta's (2013) term 'emergence of subjectivity'), where past experiences influence those in the future, and where reflections are a factor in their developing identities as mathematics teachers, linked to future actions. In Paper 3, the primary focus of PSTs' reflections is the nature and role of their subject matter knowledge in their developing ideas of 'the teacher I can be'. Hence, a major contribution of this thesis is the connection between subject matter knowledge, self-efficacy and identity. In the following section, I will outline and explain these connections in more depth.

2.2.3 Connecting identity with knowledge and self-efficacy

Meaney and Lange (2012) report that PSTs' knowledge of mathematics[12] (or lack of it) affects professional identities, and further, that PSTs' reflections on their own learning of mathematics affect their perceptions of the kind of person they are becoming (Radford, 2008). Reviewing recent international studies, Brown and McNamara (2011) found that reflection in relation to mathematics teaching is easier if PSTs are less troubled by the mathematics itself (as in knowledge *of* mathematics, and hence subject matter knowledge, see Footnote 12). Drawing a line backward, this means that possessing profound subject matter knowledge (to be defined in Section 2.3) makes it easier to reflect on one's subject matter knowledge and its role in teaching, and being able to reflect is of major concern in developing identities as mathematics teachers (findings in Paper 2, revisited in Section 2.2.2).

Having established the connection between subject matter knowledge and identity, there is yet another connection to make clear, that between self-efficacy and identity. Various researchers make this connection in educational research in general. For instance, Beauchamp and Thomas

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[11] There are instances where I use Biesta's terms directly, as in the operationalisation of Research question 3 in Paper 3 (see Section 3.2).

[12] Ball (1990) distinguishes "knowledge *about* mathematics" from "knowledge *of* mathematics". "Knowledge *about* mathematics" highlights the nature of knowledge in the discipline: where it comes from, how it changes, and how truth is established. "Knowledge *of* mathematics" is knowledge of concepts and procedures, and is "what others most easily recognize as "subject matter knowledge" (Ball, 1990, p. 458), which is how I understand it when reading Meany and Lange's (2012) paper.

(2009) note a strong connection between agency, self-concept and self-efficacy, stressing that it cannot be ignored in a discussion of identity. Grootenboer, Smith, and Lowrie (2006) see identity as a unifying concept that brings together and potentially connects interrelated elements including beliefs, attitudes, emotions, cognitive capacity and life histories, explicitly listing self-efficacy as one locus of identity. In my work, the role of subject matter knowledge makes the connection even clearer. Subject matter knowledge is essential in my operationalisation of SETM in novices (see Section 3.3.1), as measured by a Self-efficacy in tutoring children in Primary Mathematics (SETcPM) instrument (reported in Paper 1). The SETcPM-instrument accesses PSTs' perceptions of their own subject matter knowledge in terms of their perceived ability to teach it, providing an insight into this element of their developing identities as mathematics teachers as explored further via pre- and post-test comparisons in Paper 4.

In the next two sections, I explain how subject matter knowledge and self-efficacy are theorised in this thesis.

2.3 Mathematics knowledge in teacher education

Biesta's virtue-based approach to teacher education encourages us to pay attention to what one needs to *know* (Biesta, 2014). In the context of developing competences, PSTs need to know mathematics in order to become good mathematics teachers. In my work, this 'knowledge-package' is mainly concerned with subject matter knowledge. This section provides an overview of how subject matter knowledge comes into play in different forms in the papers in this thesis. It is never concerned with measuring PSTs 'actual' subject matter knowledge, however; rather, it is always concerned with their perceptions of their subject matter knowledge and of the role of such knowledge in their developing practice.

In Section 2.3.1, I connect subject matter knowledge to Aristotle's types of knowledge, followed by an overview in Section 2.3.2 on how I theorise and address subject matter knowledge in the papers that make up this thesis. In Section 2.3.3, the role of subject matter knowledge is theorised and reviewed with a special emphasis on how PSTs perceive their own subject matter knowledge and the role of such knowledge.

2.3.1 Different conceptualisations of subject matter knowledge

Aristotle's types of knowledge, *episteme* (science), *techne* (skills) and *phronesis* (wisdom), capture some of the complexity of the conceptualisation of knowledge, and are therefore also an interesting starting point when discussing what kind of knowledge a teacher in mathematics should possess. As already pointed out in Section 2.1.1, in the context of my research, the way in which episteme is conceptualised is important. Elaborating on Aristotle's episteme, techne and phronesis in the case of teacher education, Kemmis and Smith (2008) argue that episteme is to be understood as the attainment of knowledge or truth, and the development of logical thinking. To understand what this is in the context of teaching mathematics, I find it helpful to take a closer look at Shulman's classification as a way of elaborating how *episteme* is to be understood in my work, a classification that fits with Kemmis and Smith's (2008) description.

Shulman's (1986) paper "Those who understand: Knowledge growth in teaching" is one of the most influential contributions to research in mathematics education (Ball, Thames, & Phelps, 2008). Shulman has probed the complexities of teacher understanding and transmission of content knowledge, and investigated the need for a more coherent theoretical framework and a focus on 'the missing paradigm' – 'subject content knowledge' (Shulman, 1986). He divided subject content knowledge into three categories: content knowledge, pedagogical content knowledge and curricular knowledge (Shulman, 1986). The first category, content knowledge, is what Shulman called subject matter knowledge, and is where I situate the focus in my work.

Several frameworks for mathematics teachers' knowledge have arisen from Shulman's (1986) contribution, such as Fennema and Franke's (1992) *Teacher knowledge: developing in context*, Ball et al.'s (2008) *Mathematical Knowledge for Teaching*, and Rowland et al.'s (2009) *Knowledge Quartet*. Ball et al.'s framework builds directly on Shulman's work, by setting out to clarify the distinction between subject matter knowledge and pedagogical content knowledge. They suggest that subject matter knowledge can be divided into three categories: common content knowledge, specialised knowledge and horizon knowledge (Ball et al., 2008). Fennema and Franke's model builds on and modifies Shulman's framework by suggesting that the knowledge needed in teaching is a context-specific knowledge that is interactive and dynamic in nature. They suggest four components of mathematical knowledge for teaching: knowledge of the content, knowledge of pedagogy, knowledge of student's cognition and teachers' beliefs (Fennema & Franke, 1992). The Knowledge Quartet (Rowland et al., 2009) can be seen as a response to Fennema and Franke's (1992) call "to develop studies that focus on the identification of a framework for thinking about the ways in which different components of teachers' knowledge are integrated and come into play in the classroom" (Petrou & Goulding, 2011, p. 19). It acknowledge parallels to Shulman's knowledge taxonomy, but it does not seek to refine that model. "Rather, it is designed to provide a guide to mathematical knowledge-in-use that is well suited to supporting teachers' professional reflection and learning" (Ruthven, 2011, p. 85). Rowland et al.'s (2009) framework classify four broad categories - foundation, transformation, connection and contingency - of situations in which mathematics teachers' knowledge comes into play, and may serve as a tool for lesson observations (Petrou & Goulding, 2011).

Even though these researchers have stressed different domains of teacher knowledge, and their frameworks define subject matter knowledge differently, there are similarities where all three frameworks "focus on the importance of seeing the content to be taught as an important part of teaching" (Petrou & Goulding, 2011, p. 20). Comparing the frameworks, Ruthven (2011) describes four lines of thought regarding the conceptualisation of mathematical knowledge for teaching, where the first, *Subject knowledge differentiated,* is of special interest for my argument. It asserts that "expert teaching requires more than what would ordinarily constitute expert knowledge of a subject" (Ruthven, 2011, p. 83), underlining the need to focus on content and subject matter knowledge in teacher education. Pursuing this line of thought in my own conceptualisation of subject matter knowledge, I turn to Liping Ma (1999), a student of Deborah Ball.

Comparing the knowledge bases of teachers in the US and in China and noting major differences, Ma developed the concept of a *profound understanding of fundamental mathematics* (PUFM) as the most complex and important form of subject matter knowledge for teachers to hold (Ma, 1999). In Ma's PUFM, the notion of *fundamental* mathematics captures its qualities of being elementary (giving basic ideas and its procedures), foundational (providing a foundation for future mathematics learning) and primary (in the sense that it contains the rudiments of more advanced concepts). *Profound understanding* involves depth (connecting a topic to more conceptually powerful ideas), breadth (connecting topics to other concepts) and thoroughness (connecting topics into a coherent whole). For Ma, PUFM emphasises those aspects of knowledge most likely to contribute to a teacher's ability to teach and explain important mathematical ideas. Following from the large body of research examining the depth, importance, nature and categorisation of the knowledge needed to teach mathematics (such as those reviewed above), there seems to be a general agreement that teachers need solid subject matter knowledge (as in PUFM) in order to teach.

2.3.2 Subject matter knowledge in this thesis

As already outlined (in Section 2.2.3), subject matter knowledge plays a crucial role in my research addressing PSTs' developing identities as mathematics teachers. Due to the fact that the different papers "live their own lives", and partly due to the moves discussed in Section 2.1, subject matter knowledge comes into play in different packaging in the papers in this thesis. In this section, I will give an overview over the ways I address and talk about subject matter knowledge in the papers and on how these approaches connect with each other.

Shulman's subject matter knowledge involves two kinds of understanding - knowing *that* and knowing *why*, emphasising that a teacher should not only understand that something is so, but additionally also understand *why* it is so. This is a crucial distinction (Ball et al., 2008; Skemp, 1976), and as I will show, it guided the development of the instrument reported in Paper 1, and revisited in Paper 4. In both these two papers, subject matter knowledge has a central place because of its role as the key pillar in the SETcPM-instrument. The SETcPM-instrument consists of 20 items, each item asking the respondent how confident they are helping a child to solve a mathematics task. 10 tasks focus on *rules* and procedures in mathematics, and 10 focus on *reasoning*. In this way, the SETcPM-instrument places helping pupils with mathematics tasks at the core of teaching mathematics, demanding that respondents consider their own subject matter knowledge. The rules-items are strictly algorithmic or "find the right answer"-items, often involving the verb *calculate*, requiring PSTs to describe *how* procedures work. The reasoning-items focus on understanding, involving the verb *explain*, as in explaining *why* procedures work. In Paper 2 this distinction between the two types of tasks is related to Skemp's (1976) *instrumental understanding* - 'rules without reasoning', later referred to as 'Rules', versus *relational understanding*, requiring "knowing both what to do and why" (Skemp, 1976, p. 20), later referred to as 'Reasoning'. In Paper 3 the notion *conceptual understanding* is used alongside relational understanding.

Paper 2 also addresses an instrument consisting of 21 statements relating to beliefs about mathematics, where 10 items tap instrumental understanding and more *transmission teaching* beliefs ('Rules'), and 11 tap relational understanding and more *connectionist approaches* ('Reasoning'). These notions can also be associated with how I have chosen to follow Shulman (1986) and Skemp (1976) in the way I address subject matter knowledge in my work. Table 1 gives an overview of how the labels utilised in my work correspond and collectively contribute to address the same underlying distinction rooted in different ways of understanding mathematics and mathematics teaching.

Knowing *that*		Knowing *why*
Rules	—	Reasoning
Calculate	—	Explain
How	—	Why
Instrumental understanding	—	Relational and conceptual understanding
Transmission teaching	—	Connectionist and inquiry-based approaches

Table 1. Two ways of understanding mathematics and mathematics teaching

Paper 3 investigates how PSTs perceive subject matter knowledge and its role in teaching, and how reflections on subject matter knowledge contribute to their developing ideas of 'the mathematics teacher I can be'. The future-orientation in this approach, and the overarching virtue-based approach in this thesis (see the introduction to this chapter), led to a focus on Ma's (1999) conceptualisation of mathematics teaching because of its detailed treatment of the nature and role of deep mathematical understanding in teachers' practices and its focus on connected knowledge. Its detailed treatment of the nature and role of deep mathematical understanding in teachers' practices and its focus on connected knowledge made it possible to see parallels with what is embedded in the second column of Table 1. Additionally, the way PUFM is presented by Ma suggests a 'goal' for PSTs that I connect with the idea of educational purpose in Biesta's virtue-based approach.

2.3.3 The perceived role of subject matter knowledge

There are commonly accepted correlations between the quality of teachers' mathematical knowledge and the quality of learning opportunities for students (Goulding, Rowland, & Barber, 2002; Ma, 1999; Stylianides & Ball, 2008). Mathematical knowledge for teaching is found to be directly related to quality teaching (Ball, Lubienski, & Mewborn, 2001) and students' learning (Hill, Rowan, & Ball, 2005). Despite their findings, Ball, Hill and colleagues are aware of the limitations of these results and the need for caution in their interpretation, such as problems with generalisability due to the nature of the population and the possible impact of other types of knowledge (Hill et al., 2005). Nevertheless, alongside later findings from Hill and colleagues that also report on corresponding correlations (Hill, Umland, Litke, & Kapitula, 2012), these results indicate the importance of subject matter knowledge, or preferably PUFM in future mathematics teachers. Even though Skemp (1976) underlines that to have strong knowledge of mathematics does not guarantee 'success' as a mathematics teacher, he adds that teachers who do not possess such knowledge are likely to be limited in their ability to help students develop relational and conceptual understanding. However, as noted by da Ponte and Chapman (2008), the nature of this knowledge (discussed Section 2.3.2, knowing that/ knowing why) is a critical factor in this relationship, and, I would add, the nature of how PSTs perceive this knowledge. In my work the emphasis is on how PSTs perceive the role of subject matter knowledge, how they perceive the need for such knowledge and how such knowledge (or lack of it) influences them and colours their interpretation of their experiences as they progress through teacher education. In this way, my work intends to add another layer to the existing body of research. This comes into play in papers 1 and 4 when investigating PSTs' initial SETM, and their later developing SETM. Further, in papers 2 and 3 this additional layer plays a role in the investigation of how their perception of subject matter knowledge influences their developing identities as mathematics teachers.

2. 4 Pre-service teachers' self-efficacy in teaching mathematics

One way of investigating PSTs' perceptions of their subject matter knowledge and its role in teaching is by investigating their SETM: "the extent to which teachers believe their efforts will have a positive effect on student achievement" (Ross, 1994, p. 4) in the subject-specific situation of teaching mathematics. SETM is already positioned as a key construct in this thesis. This section builds on the definition and presentation of the construct given in Section 1.4, and focuses on the role of SETM in PSTs' developing identities as mathematics teachers.

Following his review of teacher efficacy research, Wyatt (2014) argued that poor conceptualisations of the role of knowledge have obscured understandings of how teachers' self-efficacy beliefs develops. Moreover, Morris, Usher, and Chen (2016), offering the first review that focuses *exclusively* on the sources of self-efficacy in teaching, add that it is clear that teachers' knowledge, and their beliefs about that knowledge, can play an important role in their development of self-efficacy. Building on this, the choice of connecting SETM closely to subject matter knowledge in my work is visible throughout the papers included in this thesis. First, the centrality of subject matter knowledge is reflected in the title of the instrument reported in Paper 1 and revisited in papers 2 and 4 – the SETcPM-instrument –, which, I propose, concentrates on the core of teaching mathematics: confidence in helping a child to solve mathematics tasks. Second, subject matter knowledge is central in my operationalisation of Biesta's educational purposes; qualification, socialisation and subjectification in Paper 3 (p.6). Consequently, the sources that PSTs draw upon in order to develop SETM are closely related to their perceptions of their own subject matter knowledge and its role in teaching.

Albert Bandura (1997) describes four sources of information that may contribute to the formation of efficacy beliefs: *Mastery Experiences, Vicarious Experiences, Verbal Persuasion,* and *Physiological Responses*. Investigation of how PSTs draw on these sources enables me to add another perspective on their developing identities as mathematics teachers. Tschannen-Moran, Hoy, and Hoy's (1998) proposal that one's interpretation of efficacy-relevant information influences self-efficacy, caused me to revisit the need for reflection identified in Paper 2 in terms of how PSTs interpret the sources of SETM in Paper 3. According to Morris et al. (2016), in order to understand the factors that contribute to self-efficacy development, researchers must not only identify the sources, but also the ways that individuals *reflect* on their experiences. These sources, which are not necessarily equally effective (Bandura, 1997), are only briefly described in Paper 3, and I offer a more detailed description in Section 2.4.1.

2.4.1 Sources of self-efficacy

Instead of viewing PSTs' perception of knowledge as a source of self-efficacy as does Palmer (2011) for example (with several researchers following his lead), I follow Bandura (1997) and Wyatt (2014), who noted that knowledge is not a source of self-efficacy in itself. However, as noted by Klassen et al. (2011), there is a need to better understand the role of knowledge in the development of teacher efficacy. In my work, I do this by investigating the role of subject matter knowledge in the sources.

Mastery Experiences are constituted by previous perceived success in performing a particular task (Bandura, 1997). For PSTs (and teachers) this is taken to refer to the performance of actual classroom teaching, and is an important source of efficacy information because it is only in situations of actual teaching that individuals can accurately assess their capability (Tschannen-Moran et al., 1998). Bandura (1997) rates mastery experiences as the most powerful source of efficacy information. But mastery experiences can be elusive in complex tasks such as teaching, because it is not always easy to identify when one has been successful (Palmer, 2011); Skaalvik and Skaalvik (2007) emphasised that it is not success per se that provides efficacy information, but rather one's perception of success.

Because teaching lacks absolute measures of adequacy, teachers must appraise their capabilities in relation to the performance of others (Bandura, 1997). *Vicarious Experiences* are situations in which one watches another person successfully perform or model the behaviour one is contemplating (Bandura, 1997). For a PST this other person can be a peer, a mentor or even a mathematics teacher at UC. When watching another person perform a task, this can raise the self-efficacy of the observer in feeling that "if she can – then I can". Observing peers with the same level of ability and experience is most effective for enhancing self-efficacy (Palmer, 2011).

Verbal Persuasion involves verbal input from others with the intention of enhancing a person's belief that they have the capability to perform a given task at a certain level. It is easier to sustain a sense of efficacy when receiving positive feedback and encouragement from significant others, than when significant others convey doubts (Bandura, 1997). Verbal persuasion "is likely to be effective when it is received from a highly competent individual who is perceived as an expert in the field" (Palmer, 2011, p. 580), such as a PST's mentor in school placement. Verbal persuasion alone may be limited in its power to create an enduring increase in teacher efficacy, but may work together with other sources to provide teachers with encouragement to strengthen their teaching skills (Tschannen-Moran & McMaster, 2009, pp. 229,230). Bandura (1997) viewed verbal persuasion as a comparatively weak source.

Physiological responses and affective states can be a source of efficacy information. People are often aware of their physiological and affective arousal, providing indirect information about their capability to deal with challenging situations (Palmer, 2011, p. 580). Debilitating factors such as stress and fear can give a feeling of ineptitude, but in a small, controllable amount such debilitating factors can improve performance by focusing attention to the task (Palmer, 2011, p. 580). Bandura (1997) viewed this particular source of efficacy information as the least effective source as they were not reliably diagnostic of one's capability.

In addition to explaining the development and use of an instrument for measuring PSTs' SETM, the conclusion of Paper 1 and the findings of Paper 2 draw attention to the importance of sources of SETM. Morris, Usher, and Chen's (2016) extensive review of 82 empirical studies focusing on measuring and conceptualising sources of teacher efficacy reveals that mastery experiences (n = 73) were the most commonly assessed source across the 82 studies, followed by vicarious experiences (n = 58), social persuasion (n = 56) and physiological and affective states (n = 43). The predominance of attention given to mastery experience is not surprising when taking into account that Bandura (1997) rates this as the most powerful source of efficacy information. Extending the view, Morris et al. (2016) raise an interesting issue: Early student teaching experiences tend to raise PSTs' teacher efficacy, while the first year as a classroom teacher is reported to decrease teacher efficacy (Hoy & Spero, 2005). To illustrate their point, Morris et al. (2016) draw on an example from Morris and Usher's (2011) study of 12 teaching award-winning professors who were asked how they developed a sense of teacher efficacy. Their answers were easily interpreted as mastery experiences, but when pressed to elaborate on how they knew they had done well, their teacher efficacy was informed by social persuasion rather than mastery experience. Therefore, what initially seemed like mastery experiences in the form of perceptions of one's past teaching experience were derived from a variety of sources (Morris et al., 2016). Taken together, these results collectively illustrate that teachers reflect on many different sources when providing such general appraisals of their past teaching performances (Morris et al., 2016).

When narrowing the context and focusing on sources of SETM in PSTs, the literature review in Paper 3 draws attention to contradictory results of research on sources of self-efficacy in PSTs in the context of teaching mathematics, pointing out that the research in this area is sparse and inconclusive.

2.5 An overview: The role of theory in this thesis

The work described in this thesis is based on a range of theoretical drivers, chosen for different purposes and at different stages. I will end this chapter by summing up diagrammatically how theory has contributed to guide and move my work forward. When reading Figure 2, it is important to bear in mind the role of subject matter knowledge and self-efficacy theory throughout my work.

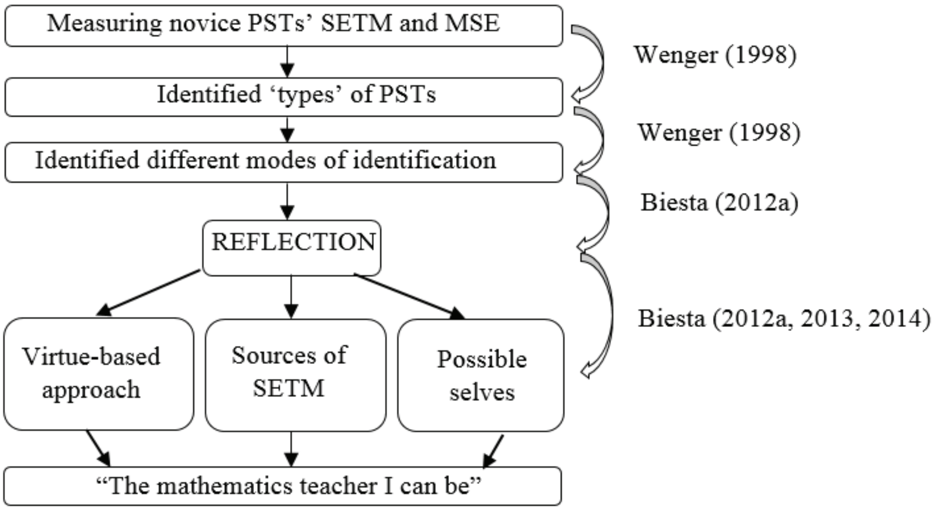

Figure 2. The main theories utilised in this thesis

3 Methodology

In this chapter, I will make the case for the mixed methodological approach utilised in this thesis, and, in particular, the combination of what looks like an objective measure with a narrative approach. I begin in Section 3.1 with a brief account of the ongoing debate around mixed methods, and I elaborate on the methodological considerations arising from the way in which this thesis blends different theoretical perspectives (outlined in Chapter 2). Next, in Section 3.2, I present the explanatory sequential research design of the study. In sections 3.3 and 3.4, I describe the data collection and analysis strategy in relation to each of the papers included in this thesis. The chapter concludes with a discussion of ethical considerations with a special focus on legitimation in Section 3.5.

3.1 Mixed methods research – a third way

Debates in Western philosophy about how we can view the world date back to the ancient Greeks. While Socrates and Plato argued for a singular or universal truth in describing the world, later associated with quantitative research, the Sophists argued for multiple or relative truths as reflected in qualitative research (Johnson, Onwuegbuzie, & Turner, 2007). However, Aristotle argued for a balance or mix of extremes (Johnson et al., 2007), and in his categories of thought we find that both quantity and quality are included, making him something of a proto mixed methods thinker (Johnson, 2016). Nevertheless, the ongoing "knowledge war" (Burke Johnson's (2016) labelling) gave birth to mixed methods research as a way of overcoming the long standing perception of a fundamental incompatibility between objectivist epistemology and mechanistic ontology on the one side, and subjectivist epistemology and social ontology on the other. This long standing tension is known as the 'incompatibility thesis' (Biesta, 2010).

In stating a mixed research problem, the emphasis is often on a combination of *understanding* insider perspectives or exploration of some process (the qualitative part) and *explaining* through prediction, correlation and statistical description (the quantitative part) (Johnson, 2016). The distinction between explanation and understanding maps onto the distinction between a mechanistic and a social ontology (Biesta, 2010), and consequently, it is important to position my purpose within the tradition of research that seeks to *explain* or within that of research that seeks to *understand* (Biesta, 2010). In this thesis, I aim to *understand* the role of PSTs' perceptions of their own subject matter knowledge and the need for it, and the reflections on their developing identities as future mathematics teachers. Likewise, I aim to *explain* elements of the nature and development of SETM.

For Biesta (2010), pragmatism presents a framework for the justification and development of mixed approaches which may help to overcome the 'incompatibility thesis' by breaking down 'alleged epistemological hierarchies' between different methods and methodologies (p. 96), and clarifying what it means to claim knowledge on the basis of mixed methods designs and approaches. As outlined in Chapter 2, I settled on a social-constructivist learning theory, which characterises knowledge in terms of relationships between agency and reflection in individuals. This fits well with an epistemological stance grounded in Dewey's pragmatism (Dewey, 1904), which offers an understanding of knowing that starts with indefinite interactions where the key concept is experience (Biesta, 2010). Pragmatism claims that knowledge is always about relationships between actions and consequences, or in other words, the combination of reflection and action leads to knowledge (Biesta, 2010). In this way, knowledge is both a temporal and transactional process (Johnson, Onwuegbuzie, de Waal, Stefurak, & Hildebrand, 2017) in which knowing can enable us to take more control over our actions. This view on what it means to know maps onto the role of reflection on subject matter knowledge and experiences in my work.

While Dewey's pragmatism primarily contributes in the domain of epistemology (Biesta, 2010), in this work, I take a critical realist position on ontology, that is, that the social world is reproduced and transformed in daily life. Drawing on Maxwell and Mittapalli's (2010) account, this ontology has various implications for my work. First, since critical realism recognises the explanatory importance of the context of the phenomena studied, it relies on an understanding of the process by which an event or situation occurs, which in turn recognises that process-based approaches are as legitimate as variance-based causality. This enables me for instance both to measure SETM-development and to explore the nature of this development. Further, as a critical realist, I recognise that there are multiple valid perspectives on the world, a stance that underlines the reality and importance of meaning. Since I gather knowledge through PSTs' perceptions of reality, there is no possibility of attaining a single, "correct" understanding of the world in my project. Hence, critical realism provides a framework for a better understanding of the relationship between PSTs' perspectives (as in their perceptions of subject matter knowledge and its role in teaching) and their actual actions.

My experience of over a decade working as a teacher educator and my interest in pure mathematics inform the ontological, epistemological and methodological premises for this work. Furthermore, I find Dewey's axiology descriptive of my beliefs about the role of values and ethics in conducting research (outlined in Section 3.5). Taken together, these ontological, epistemological and axiological positions enable a research design with a combination of quantitative and qualitative methods and analyses. In the next section, I will outline and argue for the chosen design of my research.

3.2 Research design

The purpose of a study not only determines the nature of the approach (Husén, 1988), it also provides the framing for specific research questions (Biesta, 2010). I revisit the original research questions here, in order to provide an overview (see Table 2) of their wording and operationalisation in each of the papers, and to elucidate the inductive qualitative drive across the thesis.

Research question 1 Paper 1	What are necessary features of an instrument designed to measure the core of SETM in the population of novice PSTs?
Research question 2 Paper 2	What are the connections between novice PSTs' perceptions of their own subject knowledge and self-efficacy as a potential teacher in mathematics? What are the implications for the identity work these PSTs need to do?
Research question 3	How do PSTs perceive the nature of mathematics and mathematics learning/teaching, and what influences their perception of their own subject matter knowledge and SETM?
Operationalised in Paper 3 as three sub-questions:	How do PSTs perceive the role of subject matter knowledge in their development as teachers of mathematics in the domains of University College and school placement?
	What role does subject matter knowledge play in their accounts of success and failure, and how do these experiences contribute to their developing SETM?
	How do PSTs reflect on and value sources of self-efficacy in balancing qualification, socialisation and subjectification as they develop a sense of 'the teacher they not only *want* to be, but *can* be'?
Research question 4 Paper 4	To what extent does PSTs' SETM develop during a mathematics methods course in primary teacher education, and what is the nature of this development?

Table 2. List of research questions

Closer inspection of each research question (RQ) reveals the need for a combination of research strategies, and in what follows, I present two categories of research questions in this thesis: mixed and inductive. Both RQ1, RQ2 and RQ4 have a mixed nature (as understood by Plano

Clark and Badiee (2010)). RQ1 leads me to use theory as a point of departure when developing an instrument to be utilised in real life, but the study conducted in order to answer the question combines both deductive construction of items and inductive reflection and analysis that leads to a set of items that in theory have some validity in defining SETM.

The first part of RQ2 asks for descriptions of trends in the quantitative data, investigating connections between how novice PSTs perceive their own subject matter knowledge and SETM. This is more deductive as it investigates connections such as comparisons and correlations. The second part has a more inductive nature, and seeks answers through analyses of interview data. In the same way, the first part of RQ4 seeks a description of a developmental trend and identification of differences by measuring outcomes at different points of time, testing whether SETM (or more precisely, SETcPM) develops during teacher education on a cohort-level. The theory and literature review suggest that it will, so the nature of this approach is deductive. Furthermore, the second part of the question demands more in-depth investigation of the nature of this development as far as the SETcPM-instrument allows. Having made this shift, answering this particular question is more inductive and contributes to theory building.

RQ3 is more inductive and descriptive, generating a need for qualitative methods that enable in-depth exploration, searching for nuances in relatively few cases. Tracking cases over time enabled a more holistic perspective. In utilising these methods, I seek not to generalise, but to be transparent enough for my cases to be recognisable for others in my research community. Summing up, my research has an inductive, qualitative theoretical drive (Johnson, 2016).

I chose to label my qualitatively driven research as *interventionalist* (Biesta, 2010) with an *explanatory sequential design* (Johnson, 2016), but it should be noted that this research is not situated around an intervention implemented and staged by me as a researcher. In line with pragmatism, which denies that knowledge can be gained in any other way than through interventions (Biesta, 2010), I look at my involvement in the PSTs' life as a type of intervention because of the way both the qualitative and quantitative data collection intervened in their lives. I endeavoured to understand the meaning they ascribed to their reality, knowledge and experience. In order to understand PSTs' developing identities as mathematics teachers, rather than talk about cause and effect, I refer to causes as means and effects as consequences (as proposed by Dewey, presented in Biesta (2010)). Identifying correlations between events is central to Dewey's transactional theory of knowing, which focuses on relationships between actions and consequences. As such, there are several possible descriptions and reflections connected to situated experiences (both at UC and in school placement). These arise in the interviews presented in Paper 3. My task as a researcher is to focus how different PSTs experience teacher education, and the similarities and differences in their perceptions.

Figure 3 indicates that my research is not just a set of quantitative and qualitative mini-studies in one overall research study. On the contrary, my data, analyses and findings are integrated into an overall study with *explanatory sequential design* (Creswell, 2014). The arrows in Figure 3 indicate links between the sequences in my data collection and their integration into the overall design.

QUAL →	quan →	QUAL →	quan
Qualitative piloting and development of survey	Pre-test and analysis	Interviews and analysis	Post-test and analysis

Figure 3. Research design
“QUAL” stands for qualitative (capital letters to indicate that the research is qualitatively driven); “quan” stands for quantitative; “→” stands for sequential.

Qualitative data collected at the beginning of the project resulted (the first arrow) in the SETcPM-instrument that was one of three instruments in the pre-test. The second arrow indicates that the quantitative data from the pre-test, and its analysis, informed the series of interviews taking place in the second qualitative data collection phase. This is especially apparent in the first interviews, where my analysis of each of the 10 interviewees’ responses on the pre-test were discussed with each respondent. As the project developed towards an increasing focus on SETM, subject matter knowledge and PSTs’ developing identities as mathematics teachers, the period of interviews resulted in a decision to only use the SETcPM-instrument in the post-test implementation, indicated by the third arrow in Figure 3 (this is further explained in Section 3.3.1).

A more detailed description of how data collection, analyses and findings are integrated is presented in Section 3.4.3, while Section 4.6 focuses on the integration of the findings of the four papers.

3.3 Data collection

At this particular UC, the compulsory mathematics methods course spans the first two of four years. During these first two years, there are four periods of school placement, with one period of 3 – 5 weeks in each semester, meaning that the PSTs undertake 30 days of placement in each of the first two years in teacher education. At the beginning of the project, I held an information meeting for the cohort of 2013 in which I explained my research and data collection plan, and asked them to consider whether they were willing to let me interview them repeatedly during their first two years of teacher education. At a national level, this cohort (200 PSTs) represented approximately 25% of the total of PSTs in elementary teacher education in 2013 in Norway. A total of 191 PSTs (95% of the cohort, average age of 22.5 years, and about 20 % men) completed the pre-test, in August 2013. The same cohort completed the post-test in May 2015, with 104 PSTs participating (52% of the cohort)[13].

The pre-test included a question on their willingness to participate in a series of interviews taking place before and after three consecutive periods of school-placement. While many indicated willingness by ticking the box, fifteen of these PSTs wrote additional comments to highlight their interest. Since retention over the course of six interviews was important, these PSTs were invited to participate. Of these fifteen, ten eventually attended for the interviews, three men and seven women, all in their twenties. While the need to recruit students who would participate over the entire study motivated me to choose these especially interested PSTs, it probably led me to a group who are different from those who did not make extra comments. I recognise this, and take it into account when drawing conclusions based on the interview data. Table 3 illustrates the data collection pattern across the project, and the focus of each interview.

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[13] Teaching at UC is not compuslory, and the number of PSTs attending classes varies. Hoping to reach as many PSTs as possible, I chose a class that ought to have been important to them in that it was set up to prepare for the upcoming exam. Nevertheless, I recruited fewer students than I hoped, which may have skewed the results.

Method		DP	Data collection	Focus	N
Qualitative	Spring 2013	0	Pilot work: Interviews, test-items, discussions with colleagues	Instrument development	 4 94
Quantitative	Autumn 2013	1	Questionnaire: pre-test	Self-efficacy Subject matter knowledge	191
Qualitative	Autumn 2013	2	Interview 1	Their mathematics story Self-efficacy Expectations	10
			1st School placement		
		3	Interview 2	Their first school placement	10
	Spring 2014	4	Interview 3	What is a mathematics teacher?	10
			2nd school placement		
		5	Interview 4	Experiences from school placement	10
	Autumn 2014	6	Interview 5	Their developing identities as mathematics teachers	9
			3rd school placement		
		7	Interview 6	Their developing identities as mathematics teachers Looking forward	9
Quantitative	Spring 2015	8	Questionnaire: post-test	Self-efficacy Subject matter knowledge	104

Table 3. Design of the data collection

3.3.1 The questionnaire

The first quantitative data collection point involved a three-part Likert-scale questionnaire that I planned to re-administer at the end of the PSTs' mathematics method course. The questionnaire was designed to capture novice PSTs' mathematics self-efficacy (MSE), SETM, and beliefs about mathematics.

Part I: Self-Efficacy in Tutoring children in Primary Mathematics[14]

The instrument developed as a crucial part of this study requires respondents to state how confident they are helping a child with 20 different mathematics tasks, 10 focusing on *rules* and

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[14] In Paper 2, this instrument is labelled 'SETM', but based on the argumentation in Paper 1, on the core role of subject matter knowledge when evaluating SETM, this instrument was later called *Self-Efficacy in Tutoring children in Primary Mathematics* (SETcPM). Paper 2 was published before Paper 1. This explains the inconsistent labelling of the instrument in papers 1 and 2.

procedures in mathematics, and 10 focusing on *reasoning* (see Appendix A_1 (Norwegian version) and Appendix A_2 (English version)). Paper 1 presents a detailed description of the instrument's structure, content, underlying ideas and validation. However, Paper 1 does not describe the process of piloting the instrument, and I make space for it here in order to explain how data gathered in data point 0 informed the pre-test implementation (see Table 3). The first two steps involved PSTs from the cohort of 2012, while the third coincided with the first data collection from the 2013 cohort.

When developing an instrument, the process of piloting is crucial. It is necessary to explore how different respondents interpret the items in the scale, and to investigate the nature of each item to see if they contribute to what the scale aims to measure – in this case, SETcPM. As a first step, individual semi-structured interviews were conducted on how four PSTs interpreted and acted upon seven test items in an early version of the instrument (see Appendix A_3). The interviews began with each PST completing test items without interruptions. Retrospective reflections where participants explain their thoughts when responding to items are a way of providing evidence for the substantive aspect of validity (Wolfe & Smith Jr, 2007b). For this reason, consistency between the PSTs' answers on the test items and their reflections in interview, alongside later discussions with colleagues, suggested that the test items were eliciting the intended distinctions. At this stage, the items did not fit thematically into 'number sense and early arithmetic operations', which eventually turned out to be the thematic headline for the items in the instrument (see Paper 1, pp.66 – 67 for the argument for this decision).

In the second step, the instrument was expanded to 20 items where each rules-item was thematically paired with one reasoning-item, i.e. the rules-item "*Calculate 23 × 0,7*" and the reasoning-item "*Explain why you can expect the result of 31× 0.5 to be less than 31.*" This version was piloted on 94 PSTs, and the Rasch Rating Scale model (RSM) was used to analyse the data (see Section 3.4.1 and Paper 1 for more detailed descriptions of Rasch analysis). The main goal at this stage was to investigate whether the requirements for RSM held, and hence explore whether the instrument measured what it was designed to measure. All but three of the 20 items contributed to measuring the underlying construct (SETcPM), and detailed Rasch analysis provided guidance on how to reword those poorly fitting items. Adjustments resulted in 20 items on a unidimensional scale, based on data collected from 191 PSTs in the third step. This third step coincides with the first implementation of the instrument for the 2013 cohort.

Part II: Beliefs about mathematics

Part II consists of 21 statements relating to beliefs about mathematics, 10 tapping instrumental understanding and more transmission-teaching beliefs (‘Rules’), and 11 tapping relational understanding and more connectionist approaches (‘Reasoning’) (see Appendix B1 (Norwegian version) and Appendix B2 (English version)). Responses options use a four-point Likert-scale with categories “Disagree entirely”, “Disagree somewhat”, “Agree somewhat” and “Agree entirely”. The belief element of the questionnaire and its two underlying constructs ‘Rules’ and ‘Reasoning’ were developed and validated by Drageset (2010, 2012) using 365 in-service Norwegian elementary teachers. In contrast to Drageset (2010, 2012) who concluded that these 21 items tapped two different constructs, Rasch analysis (see Section 3.4.1) showed no evidence of two constructs in my data from novice PSTs[15]. As a result of the way in which my project later developed towards an increased focus on SETM and the role of subject matter knowledge in PSTs’ developing identities as mathematics teachers, this part of the three-fold questionnaire is only referred to within the interview analysis in Paper 2 and is not utilised in any of the remaining papers[16].

Part III: Mathematics Self-Efficacy

The Mathematics Self-Efficacy (MSE) instrument is an adaptation of an instrument originally developed by Pampaka, Kleanthous, Hutcheson, and Wake (2011)[17]. It requires respondents to state how confident they would be using mathematics to solve 30 different problems, using a four-point Likert scale with answer categories “Not confident at all”, “Not very confident”, “Fairly confident” and “Very confident”. *They were not asked to actually solve the problems.* The tasks in the original English instrument (see Appendix C2) were designed to measure MSE as a learning outcome of post-compulsory mathematics education in the pre-university phase (Pampaka et al., 2011). For the current study, they were translated into Norwegian (see Appendix C1), and mapped onto the Norwegian upper secondary school curriculum in order to ensure that novice PSTs should be able to relate to them. Rasch analysis of the MSE-instrument showed a unidimensional underlying construct.

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[15] This may be in part due to changes I made to the original instrument in order to make all items fit under one rather than three headings, and to avoid using the word ‘pupils’. My reasoning was that novice PSTs might identify themselves as pupils (in school) as they had done until recently. Retrospectively, I realise this decision could have influenced the results.

[16] Although my data provided no evidence for two different constructs for the cohort of 2013, the analysis still yielded information that provided interesting inputs to the semi-structured interviews.

[17] I contacted the authors who gave me permission to use the instrument.

The rationale for involving this instrument as part of the questionnaire is that PSTs often express doubt about their own self-efficacy in mathematics (Gresham, 2007), and, moreover, that the concepts of self-efficacy in mathematics and self-efficacy in *teaching* mathematics appear to be closely related (Briley, 2012; Phelps, 2010). Hence, the analysis of PSTs' responses to both this part of the questionnaire and the SETcPM played a crucial role in Paper 2 when identifying different 'types' of PSTs (see Paper 2 for a detailed account). Additionally, its analysis was important in informing Interview 1.

To summarise, parts II and III of the three-part instrument used for the first quantitative data collection point were not revisited in the post-test (see Table 3). Part III was not relevant at this point, since the MSE-instrument relates to mathematics learned in upper secondary school, and the mathematics method course does not focus on this level of mathematics. Moreover, an increased focus in the project on SETM and subject matter knowledge in PSTs' developing identities as mathematics teachers led to a need for the SETcPM-instrument alone at pre-test.

3.3.2 Semi-structured interviews

The interviews were conducted as individual, in-depth semi-structured interviews (Mason, 2014), with a shifting focus and length as the study progressed in order to fit with the PSTs' current situation and the experiences they drew on. The interview guides were developed in close correspondence with the research questions, and also on the basis of notes on my thoughts and ideas for further questions immediately after each interview. Due to the narrative aspect and the focus on their 'developmental stories' (see Paper 3 for an outline), I made space for individual questions in all the interviews. In addition, with their prior agreement, I sent the interviewees an email during their second school placement with two questions. The first asked them to tell me about an experience during school placement where they had experienced success, and the second asked for an account of an experience where things did not go as planned. The rationale was to gather these stories while they still were fresh in their minds. Their answers were not treated as data, but as background information for Interview 4.

Before the first interview took place, the interviewees signed an informed consent form (see Appendix D_1 (Norwegian version) and Appendix D_2 (English version)[18]). The interviews took place in my office and were recorded. An interview typically lasted for about 15 – 25 minutes,

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[18] The consent form is not updated to the current title and rationale for my project. It is representative of the title and rationale as it was at the time it was signed.

but at times as much as 30 – 35 minutes, particularly as time went by and both parties gained confidence in the interview setting.

All 10 participants were involved in the first four interviews, but only nine in the last two (one PST withdrew after four interviews). The interviews were conducted before and after each of the three first school placements, as illustrated in Table 3. The overall aim was for PSTs to tell their story, from their very first thoughts as a novice anticipating their first placement, through their first-year experience of repeated placements and the role of the UC in their preparation for teaching, to the start of the second semester of their second year where they had considerable placement experience and were ready to look ahead and reflect on the mathematics teacher they can be.

The first interview was informed by preliminary analysis of the questionnaire. The point of departure was how their answers to the questionnaire had informed me about their background, their view on mathematics as a school subject, and their thoughts on becoming an elementary mathematics teacher (reported in Paper 2). The longitudinal approach enabled the early interviews to inform the later ones, both in terms of stimulated recall, and in terms of how some aspects became increasingly evident and for that reason influenced how the interview guides developed. This list gives the key topics for particular interviews (for detailed interview guides, see see Appendix E_1 (Norwegian version) and Appendix E_2 (English version)):

- what is their relationship to mathematics [interviews 1 and 3]
- what are their thoughts on becoming a mathematics teacher [Interview 1]
- what does it mean to be a mathematics teacher [interviews 3, 4 and 6]
- what have they learned in school placement [interviews 2, 4 and 6]
- what have they learned at UC, and how does this connect to school placement (if it connects) [interviews 3 and 5]
- what is their focus as PSTs at UC and in placement [interviews 4 and 5]
- how has their relation to mathematics developed or changed [Interview 5]
- how have they developed as a mathematics teacher and what has contributed to this development [interviews 5 and 6]
- looking forward: what kind of mathematics teacher do they want to be [Interview 6]

3.4 Analysis of data

My analyses of quantitative and qualitative data aimed to build explanations and arguments about how SETM has developed in PSTs, the role and nature of subject matter knowledge in this development, and PSTs' developing identities as mathematics teachers. Analysing both questionnaires and semi-structured interviews, and relating findings and trends from the quantitative analysis to the qualitative, can provide a detailed, contextual and multi-layered understanding of processes and experiences. In this section, I outline the quantitative and qualitative data analysis process (in sections 3.4.1 and 3.4.2, respectively). I end this section with a commentary on how the analyses and findings are integrated and reported on in the papers constituting this thesis (Section 3.4.3).

3.4.1 Quantitative data analysis

In the search for an appropriate tool for analysing my quantitative data, I was introduced to Rasch analysis through the work of Pampaka et al. (2011). Rasch measurement is based on an equation developed by George Rasch, and provides a way to examine responses on items that can be considered to work together to measure a single underlying construct. Describing our work on validating the SETcPM-instrument in Paper 1, my co-author and I took the same approach as Pampaka et al. (2011) when developing their MSE-instrument, and followed guidelines (Wolfe & Smith Jr, 2007a, 2007b) which are themselves based on Messick's (1989) validity definition. The reader is referred to Paper 1 for descriptions of technicalities and the actual analyses performed in the developmental process of the SETcPM-instrument, and to both papers 1 and 4 for how Rasch analysis is used to read and interpret my data. Paper 2 gives an account of the analysis of the MSE-instrument. In this section, I have added an important feature that I could not find room for in the papers (except for a short paragraph in Paper 4, p.4): the way in which RSM enables a transformation from Likert-scale numbers to a continuous scale; this explains further why Rasch analysis is employed in my work.

Simply adding up a PST's responses on a Likert-scale and using this raw score to denote their level of SETcPM is problematic. Such raw scores tend to clump students around the mean scores and do not adequately contrast the results of the more (in my case) confident PSTs with those of less confident ones (Bond & Fox, 2007). Some items in the instrument are found harder to endorse, and these harder-to-endorse-items should be reflected in a PST's SETcPM-measure by assigning more confidence to those reporting that they are 'Very confident' on such an item. This is not reflected in a PST's SETcPM-score when Likert-scale numbers are presented as raw scores. To illustrate this point, Figure 4 shows the nonlinear relationship between raw scores and logits (the units used in Rasch measurement to express endorsability) for the first SETcPM-implementation. If we compare a PST who scored 59 to one who scored 58, in terms of raw scores, these two PSTs differ by 1 point, which corresponds to a difference of approximately 1.0 logits. If we next compare two PSTs, one with a score of 10, and one with a score of 9, the difference is still 1 point (in terms of raw score), but now the difference is approximately 0.3 logits. This contrast shows the benefit of using the conversion of raw scores to logits using RSM: Logits calculated from RSM allow us to avoid using non-equal interval values in parametric analyses that assume linearity (Boone & Scantlebury, 2006). In this way, the aim of a Rasch analysis is analogous to helping construct a ruler, but with the data of a test or questionnaire.

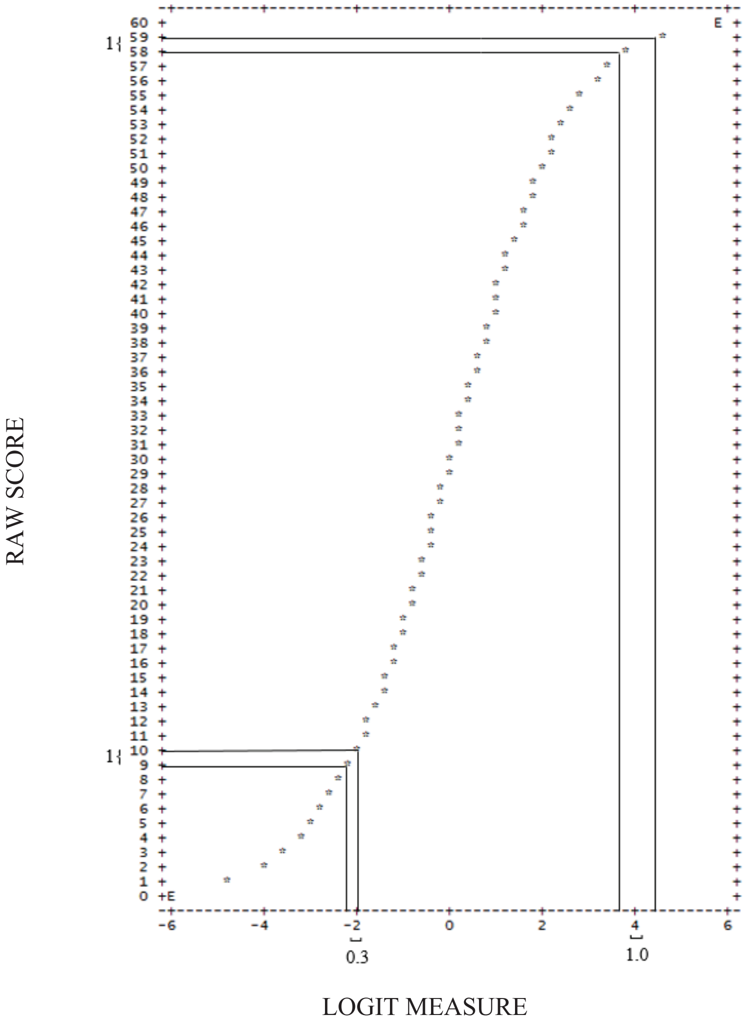

Figure 4. The relationship between raw score and logit measure

Additionally, there is a rationale for choosing RSM over other item response theory models. The emphasis is normally on finding a model that best characterises the given data. However, in RSM, the emphasis is in identifying and studying anomalies in the data disclosed by RSM (RUMM2020, 2016). This was my point of departure, as I used RSM model to ensure that the included items contributed to measuring the underlying construct, which in my case is SETcPM. RSM helped me to very quickly diagnose misfitting items, and from this perspective it allowed me to take control of the analysis and to be able to follow the evidence to see where the responses may be invalid.

3.4.2 Qualitative data analysis

The 58 semi-structured interviews resulted in 13.5 hours of recordings that were transcribed (in Norwegian). Due to the large amount of time consumed on such a big task, the interviews were forwarded to a transcription service (in anonymised versions). The analysis of the transcripts consisted of several approaches. First, a reading, alongside listening to the audiotaped interviews, resulted in some corrections to the transcriptions[19] and a decision to reduce the data both in terms of leaving out short interruptions destroying the flow (e.g. the interviewer's affirmative responses, like 'yes' and 'mm'), but also personal references, and off-topic 'chat'. Since I undertook the data reduction and coded the Norwegian versions of the interviews alone, I have made the original (anonymised) transcripts available for inspection.

I listened to the audio-taped interviews during several stages of the research process, but the transcriptions formed the basis for the coding. In the remainder of this section, I will describe how the reduced data set was cross-case analysed at two different points in time and for two different purposes. A cross-case analysis involves the simultaneous analysis of data yielded by multiple cases (Onwuegbuzie & Combs, 2010), where I take each PST's narrative as one case.

Noting the two different theoretical frameworks presented in Chapter 2, I make a distinction here between the analyses reported in Paper 2 and Paper 3. Interview 1 was analysed before the total series of interviews was completed in order to elaborate, support and explain findings in the quantitative analysis of the first iteration of the questionnaire (Paper 2). Later, interviews 1 – 6 were analysed holistically (and reported in Paper 3).

Analysing Interview 1

The first round of semi-structured interviews took place six weeks after the cohort of 2013 completed the pre-test. As described in more detail in Paper 2, the first interviews of five of the 10 PSTs' were selected in order to capture a range of positions represented on a scatterplot

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[19] Since transcriptions do not provide 'objective records' of interviews, this can be seen as a first important step in data analysis (Mason, 2014, p. 77). Therefore, this was this step important.

generated by different combinations of MSE and SETcPM-scores. In the consequent narrative analysis (as understood by Onwuegbuzie and Combs (2010)), I treated the data as stories which in turn enabled me to investigate the PSTs' own perceptions and reflections. Interviews with the five chosen PSTs were coded inductively, where the data were examined to identify the meaning units in their narratives. The coded extracts were subsequently mapped on to Wenger's theory of modes of belonging (Wenger, 1998). This first interview was designed to capture their initial thoughts on being PSTs, their relationship to mathematics and their retrospective thoughts on their responses to the questionnaire and my interpretation of their MSE and SETcPM-scores. The latter enabled a more detailed investigation of the reasoning behind their initial MSE- and SETcPM-responses, and provided an opportunity to explore further their beliefs about mathematics and their relation to their prior experience as mathematics learners. Combined with the questionnaire scores, these data provided an opportunity for triangulation, and enabled further insights into the complex relationship between their beliefs about mathematics, MSE and SETM, and the range of PSTs' different starting points.

Analysing Interviews 1 – 6

Due to the considerable amount of data collected from the interviews, this second round of qualitative analysis was much more comprehensive than before. My analysis process is best described as consisting of three steps: first, a process of abductive coding (an interactive process of deductive coding based on theory and an inductive coding contributing to theory-building); second, a cross-sectional indexing; and third, a more contextual bounded, holistic case study approach.

In line with Mason's suggestions, when reading the reduced set of data, it was important to use a system of coding that was consistent across the whole data set (Mason, 2014). In order to establish this set of consistent codes, I started out by analysing Interview 2 across the whole group (I did not start out by analysing Interview 1 because I did not want the codes to be influenced by my earlier analysis for Paper 2). I was mostly concerned with what could be seen as the interviewees' perceptions and understandings, and their versions of how teacher education contributes to their developing thoughts on 'the mathematics teacher I want to be', and later 'the mathematics teacher I can be' (as reported in Paper 3). In order to explain their view, I needed to stay focused on their interpretations while recognising my role in seeing their thoughts through the selected theoretical lens utilised in this analysis. Interview 2 was coded in accordance with Biesta's (2012a) three domains of educational purpose: *qualification,* *socialisation* and *subjectification*, and his concept of *judgement*, a set of codes obtained from the literature and theory (see Chapter 2). This coding was based on a pre-understanding of what these domains are all about, based on Biesta's own theorisation of them. While coding each of the second interviews according to these three domains, additional notes and keywords were written in the margin, resulting in finer-tuned codes.

After this initial coding of Interview 2, the coded slices were reorganised according to how it was indexed. Following Mason (2014), I looked at the slices of indexed data as unfinished sources, and every slice of indexed data was gathered under the appropriate heading coded with the same educational purpose. I kept the additional comments and subordinate codes capturing descriptions of the role of subject matter knowledge in different learning contexts and in experiences of success and failure; accounts of making educational judgements and justification of judgement; and accounts of being and becoming a mathematics teacher in light of Bandura's (1997) four sources of self-efficacy.

A detailed reading of this organisation of Interview 2 gave me further insight into how each of the three domains were expressed in my data and resulted in some fine-tuning of the more theory-driven coding initially used. This new understanding of how to code the interviews according to the three domains in the setting of mathematics in teacher education was used when coding Interview 3. Instead of reorganising the codes according to the three domains, the emphasis on context (whether the PSTs talked about experiences from UC or school placement) made me reorganise Interview 3 more thematically according to the key topics for this particular interview (see the list given in Section 3.3.2). This enabled a more holistic reading of Interview 3 while I managed to keep track of the three domains by using colours. When all six sets of interviews had been through the steps of coding and thematic reorganisation according to their key topics (a re-coding of Interview 2 was necessary), I arrived at a list of operationalisations of Biesta's (2012a) domains of educational purposes and judgement (see Paper 3, p.6).

In order to explore developmental patterns in the occurrence of these references, the third step of the analysis entailed a longitudinal reading of each of the PSTs' six interviews. Such reading of holistic 'units' is helpful when trying to produce an explanation of processes, experiences and practices that characterises that unit (Mason, 2014). This approach enabled me to note common trends across the group, reported in Paper 3. Additionally, it enabled me to explore one PST who acted as a foil to the presentation of the data from the other nine participants (also reported in Paper 3, and in Section 4.5).

3.4.3 Data integration

I have discussed integration at various points in this chapter, especially in Section 3.2 where I talked about integration in terms of how the sequences of data collection are mixed in my study with *explanatory sequential design* (Creswell, 2014). In this section, I will give a more detailed description of how data collection, analyses and findings are integrated.

Figure 5 shows which data points each paper draws on. Paper 1 reports analysis from data points 0 and 1, which collectively triangulate and complement each other in the process of developing an instrument. Paper 2 draws on the analysis collected in data points 1 and 2, enabling identification of different 'types' of PSTs. Further, there was a process of on-going re-readings of interviews throughout the period of conducting interviews in data points 2 – 7. In this way, "findings" from one interview influenced the next, reflecting the narrative aspect of the interviews. Figure 5 shows that the whole series of interviews feeds into the analysis reported in Paper 3. In the same way that Paper 1 bookmarks the beginning of my data collection, Paper 4 bookmarks the end. Paper 4 draws on pre- and post-test data from data points 1 and 8.

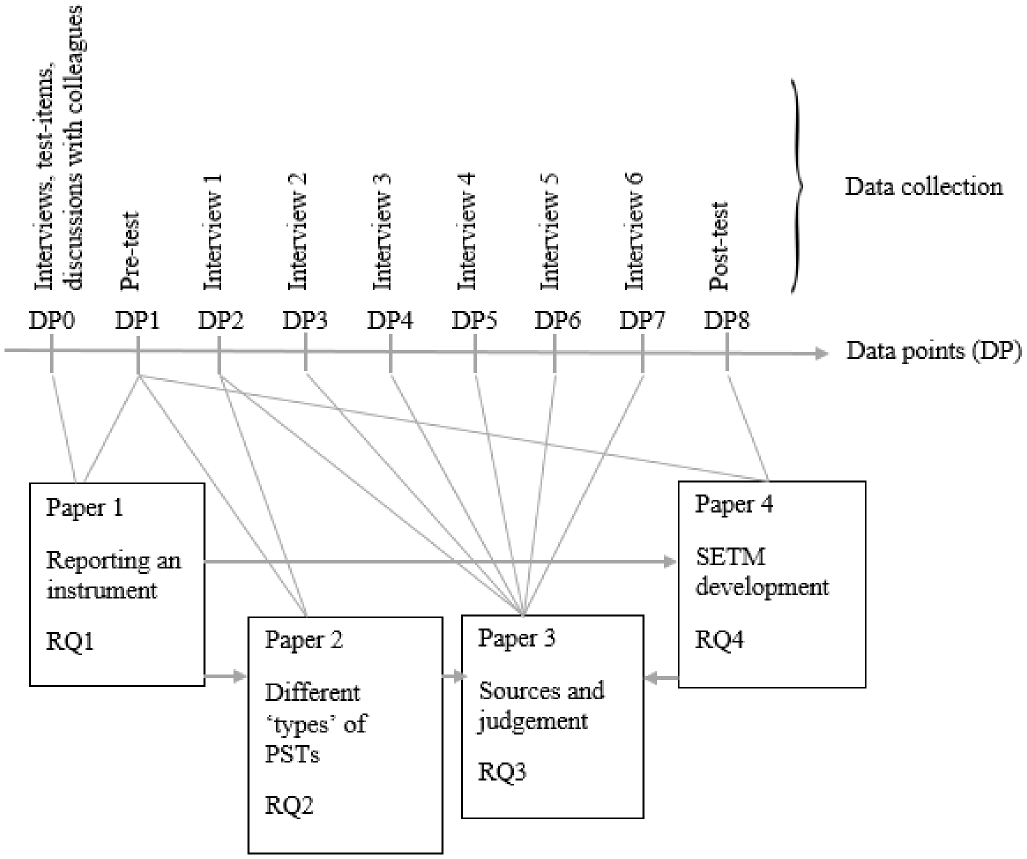


Figure 5. Integration of data, analyses and findings

The lower part of Figure 5 shows how the papers build on each other. Findings in Paper 1 are integrated into Paper 4, and also into Paper 2 in terms of the identification of ‘types’ of PSTs. Paper 2 raises the issue of reflection and the need to gain knowledge about sources of SETM, providing the point of departure for Paper 3. At the same time, Paper 4 addresses the need to bring together research on teacher efficacy and subject matter knowledge, which also leads on to Paper 3. The reader is referred to Section 4.6 for a more detailed outline of the integration of findings of each paper, and how they collectively contribute to different bodies of research.

3.5 Ethical considerations

This section addresses the ethical considerations I took into account while planning and conducting this research. The discussion focuses on legitimation in mixed methods research in general and in this study in particular.

The National Committee for Research Ethics in the Social Sciences and Humanities (NESH) in Norway gives directions worthy of careful consideration. NESH was established in 1990 when the Research Ethics Act provided a legal mandate for NESH as an impartial advisory body established to provide guidelines for research ethics to promote good and responsible research (NESH, 2016).

NESH emphasises the importance of a researcher’s “responsibility for research” (NESH, 2016, p. 11). This is important considering my background with more than a decade’s experience as a mathematics teacher in the teacher education program where my research is situated. As a researcher in my own institution, I have to distance myself, and set high standards for analytic perspectives which minimise the influence of preconceived notions and unconscious views:

> Great demands are placed on the justifications of the researchers for their choice of questions, methods and analytical perspectives, and also on the quality of the documentation used to support conclusions, so that preconceived notions and unwitting opinions have minimal influence on the research. (NESH, 2016, p. 11).

Consistent with NESH, Hammersley and Atkinson (2007) categorise five main ethical issues: informed consent, privacy, harm, exploitation, and consequences for future research. I will now comment on all these issues and show how they are played out in my work.

All participants in research should be informed that participation is voluntary. Before conducting the pre-test, I informed the PSTs about its purpose, and the time and the place for its implementation (at the start of their first mathematics class), allowing for those unwilling to drop the first 20 minutes of the class. I was aware of the power relationship between the PSTs and myself (even though I am not involved as a teacher for this cohort) and recognised that this might make it hard for some students to opt out of the pre-test. There was not a lot that could be done to address this power issue apart from recognising it and being sure to stress that the PSTs were free to choose whether to participate. They could choose anonymity by writing a code instead of their name on the questionnaire, only needing to give their name if they wanted to be followed through in my small sample. Those who ended up as participants in my interviews signed an informed consent form (see Appendix D_1 (Norwegian version) and Appendix D_2 (English version)).

Transgressions of the right to privacy in the name of research are not regarded as acceptable (Bryman, 2012). Even though my interviewees had signed an informed consent, I bore in mind that they were entitled to refuse to answer certain questions even when assured that their data and quotes would not be traceable back to them. I made every effort to ensure confidentiality, and ensured that the quotes used in my publications never involved any names of persons or places. The process of translating the quotes to English made statements more anonymous.

As a researcher conducting interviews, it was inevitable that I would receive information about other people in addition to those voluntarily participating in my study. I followed NESH's Guidelines of *important respect for third parties,* aiming to protect those third parties, ensuring that they were not identifiable in the data, and that information would not be traceable back to them.

Bryman (2012) refers to different professional associations when defining what harm is. He emphasises the Social Research Association's *Ethical Guidelines* requirement that the "social researcher should try to minimise disturbance both to subjects themselves and to the subjects' relationship with their environment" (Bryman, 2012, p. 136). I did not want to be perceived as intruding in the PSTs' learning or everyday lives, and I also needed to think about the potential impact of the research on PSTs' relationship to their peers and mentors. It was important that the mentors (who had not signed any informed consent) did not feel that their work was being evaluated (Hammersley & Atkinson, 2007).

People who supply information need something in return in order not to feel exploited, although their contributions cannot be measured on any absolute scale (Hammersley & Atkinson, 2007).

This most certainly applied to the PSTs I followed for more than 1.5 years. It is difficult to anticipate what will make people feel rewarded for their efforts, and there were limits to what I could give them back, but I offered some of my time to discuss their progress and try to help them in progressing further (alongside some smaller gifts expressing my gratitude).

Finally, when conducting research it is important to act in a way that makes future research within the same field with the same kind of informants possible. I hope my ethical considerations discussed here will ensure no negative consequences for future research.

Research projects involving electronic handling of sensitive personal information are obliged to apply to Norwegian Social Science Data Services (NSD) to ensure that plans for data storage comply with data protection legislation. This ensures that the names of my research participants and the location of my research are not identifiable, that only I have access to the interviews as long as they are not anonymised, and that the data will be kept according to the rules. This research project has received two approvals from NSD, one for the pilot work, and one for the main data collection. In addition to these important ethical considerations, I next address how my findings are validated throughout the work in this thesis.

3.5.1 Legitimation

Due to the nature of mixed research, assessing the validity of findings is particularly complex (Onwuegbuzie & Johnson, 2006). Each of the papers in this thesis addresses validity in some form, and its role and appearance may be most prominent in Paper 1 (pp. 68 – 69) which follows Wolf & Smith Jr's (2007a, 2007b) guidelines (many of the same validity issues are revisited in Paper 4). In the more qualitatively driven papers, such as papers 2 and 3, issues related to reliability (or dependability), credibility, transferability and confirmability are discussed.

Here, I choose to follow Onwuegbuzie and Johnson's (2006) suggestion, and term validity in my overall study as *legitimation*. In what follows, I will consider Johnson's (2016) typology of 10 types of legitimation for mixed methods research, which themselves are an expansion of the original nine proposed in Onwuegbuzie and Johnson (2006). *Philosophical legitimation* was outlined in Section 3.1, where I explained my epistemological and ontological stance. The same goes for *integration legitimation* or *meta-interference*. This is commented upon in the outline of my research design in Section 3.2 and more in depth in Section 3.4.3. Integration is a constant concern in mixed methods research, and as already argued, I have been committed to integrating my research on several levels. Nevertheless, retrospective reflections reveal what one might call a ‘beginner’s mistake’: I started out collecting too much data. The Beliefs about mathematics part of the questionnaire in the pre-test could have served me better as simply inspiration for the SETcPM-instrument (‘rules’ and ‘reasoning’), and as a backdrop for the first semi-structured interviews. I continue to aim for meta-interference when drawing my conclusions in Chapter 5. C*onversion legitimation* is only relevant in studies where data are converted. In my study, this is only apparent when I convert the non-continuous portions of the questionnaires to a continuous scale. This process of conversion is outlined in Section 3.4.1. *Socio-political legitimation* is harder to address in a project conducted by a single individual rather than a research team with multiple perspectives on a problem. However, by advocating pluralism of perspectives in my research, I aim to make my study arguable to multiple stakeholders, while it is possible to look at how my study results are perceived by those who work with PSTs in the future. I comment on the six remaining types of legitimation in the subsequent paragraphs.

Inside-outside legitimation. I strive to give the emic viewpoint of the PSTs as insiders, and to be clear when my epic viewpoint as an outsider is given. In my study this legitimation is compromised by my role as a mathematics teacher on the programme (not for the particular cohort involved in my study), which makes it easier to get involved and ‘go native’ (as Onwuegbuzie and Johnson (2006) put it). It is particularly important to address this type of legitimation when combining inferences from the qualitative and quantitative phases of a study (Onwuegbuzie & Johnson, 2006). My strategy for obtaining a justified etic viewpoint was to involve my supervisor and co-author in Paper 3 as another outsider who examined the interpretations being made, and the conceptualisations and the relationship between data and conclusions.

Commensurability approximation legitimation. This type of legitimation demands that I rest my case on considerations of both qualitative and quantitative thinking, going back and forth between qualitative and quantitative lenses. Being a quantitative and qualitative researcher demands experience and training (Onwuegbuzie & Johnson, 2006) which is hard to lay claim to. I have tried to compensate for lack of experience by paying close attention to the demands of both approaches when it comes to how the data are collected and analysed, and to focus on the Gestalt switch (Onwuegbuzie & Johnson, 2006) in order to see how I can achieve more by combining approaches. The added value of mixing methods and methodologies is most apparent in Paper 2 and in Chapter 5 of this thesis.

Briefly revisiting *weakness minimisation legitimation*, in Section 3.2, I outlined how I planned my research design with different data collection methods utilised in different phases, and how they build on each other. I repeat some of this here in order to connect it to legitimation. I found it impossible to develop a quantitative instrument that would give me the same information that in-depth interviews would; at the same time, interviews could not supply the same information as an instrument. The interviews following the pre-test implementation enabled me to ask questions which shed light on why they answered the pre-test as they did. And the other way around, the results from the pre-test made me ask questions during interviews that I probably would not have thought of otherwise. The pre- and post-test design offers a way to measure development, while the interviews add more about the nature of that development.

Sequential legitimation. If I was asked whether my research would have led to different results if I had switched the order in which I conducted the quantitative and qualitative data collection and analysis, I would probably have to agree, since the results build on each other as explained in the previous section. However, I have to add that by 'different' I do not mean 'contradictory' – rather, the depth in which the findings would have been elaborated would have differed. This is only speculation of course, as I have only conducted this research in one particular way and some parts of my research could not have been done in a different order (for example the two quantitative papers that bookmark the beginning and end of data collection), but nevertheless, it is interesting to think this through.

Sample integration legitimation. In my research, the interviewees are a smaller subset of the cohort of 2013 which provided the sample for the quantitative data collection, and it is clearly problematic to generalise from qualitative conclusions to the whole cohort. I use the interviewees' narratives to put forward examples of different 'kinds of PSTs' and different 'developmental trajectories', and the aim of my research is not to generalise, but to present research that is recognisable. The latter point applies at two levels; I do not intend to generalise my quantitative findings to all teacher education in Norway; and I do not intend to generalise my qualitative findings to all PSTs in the cohort of 2013.

The final point to be made is about *paradigmatic legitimation*. Section 3.1 gave an outline of the methodological approaches underlying this research, where I tried to make my epistemological, ontological, axiological and methodological beliefs explicit. I have strived to blend these beliefs into a usable package, and conduct research that fits with these assumptions. In the next two chapters, I will present the findings from the papers constituting this thesis (Chapter 4) and offer discussions of these findings and their contributions (Chapter 5).

4 Findings

This chapter first presents a short account of each of the papers included in this thesis, summarising their research questions, theoretical stance and findings (sections 4.1 – 4.4). In Section 4.5 I present ‘the case of Maia’ as a means of showing how the papers comprising this thesis jointly contribute to an overall story of PST development. While different fragments of her story appear in each paper, I tell it here as a single narrative across the papers. Focusing on this one PST not only illustrates one way in which the four papers are connected and integrated, but, as I will show in Chapter 5, Maia’s story contributes to a new understanding of “weak” PSTs who are often seen as “hopeless cases”. This chapter closes with an integration of the results describing how the different papers contribute to central aspects of PSTs’ developing identities as mathematics teachers (Section 4.6).

4.1 Paper 1: Developing an instrument

> Bjerke, A. H., & Eriksen, E. (2016). Measuring pre-service teachers’ self-efficacy in tutoring children in primary mathematics: an instrument. *Research in Mathematics Education*, *18*(1), 61-79.

The main purpose of this paper was to report and validate an instrument developed to measure SETM in novice PSTs. It addressed the question: “*What are necessary features of an instrument designed to measure the core of SETM in the population of novice PSTs?*” It is the term ‘core’ that connects the title of the paper and the research question: instead of measuring SETM, which is a comprehensive construct (see Section 1.4), the instrument reported in this paper more accurately measures self-efficacy in tutoring children in primary mathematics (SETcPM). Grounded in this specification, the paper arrives at two proposals. First, it proposes that SETcPM is a central part of SETM, and the SETcPM-instrument consequently operationalises the idea of SETM in novice PSTs. Second, it proposes that a better understanding of SETcPM can contribute to a better understanding of the development of SETM.

Paper 1 offers a review of existing instruments with the potential for measuring SETM in novice PSTs, but concludes that no instrument can adequately do this. In response, it reports on the development of an instrument which targets SETM in light of the core activity of teaching mathematics: helping a generic child with mathematics tasks. The instrument takes subject matter knowledge and Skemp's (1976) two ways of understanding mathematics as its point of departure (see Section 2.3.2).

In making sense of SETM and its measurement in novice PSTs, this new instrument adds to the existing collection of instruments measuring SETM at different career stages (see review of existing instruments in Paper 1, p.64). Additionally, in response to literature that suggests that much of today's teacher efficacy research is hard to use in teacher education (Klassen et al., 2011; Wheatley, 2005), my co-author and I propose ways in which the SETcPM-instrument can be used as a resource for teacher educators in their day-to-day interactions with PSTs. In addition to informing teacher educators about their students' SETM, by providing them with an insight into their confidence and hence an opportunity to address this aspect of their development explicitly, the SETcPM-instrument can be used to encourage metacognitive activity in PSTs themselves. I discuss this further in Section 5.1, where the findings of Paper 1 are connected to those of the other papers (see also Figure 5 in Section 3.4.3).

In the particular administration of the instrument described in Paper 1, seven items emerged as important for teacher educators, indicating issues that need to be addressed in order for PSTs to gain confidence in SETcPM. The paper offers examples of how the analysis of SETcPM-results may be operationalised.

4.2 Paper 2: Self-efficacy in novice pre-service teachers

> Bjerke, A.H. (2014). Self-efficacy in mathematics and teaching mathematics in novice elementary pre-service teachers. In Østern, et al. (Eds.), *Once a teacher – Always a teacher?NAFOL Year Book 2014* (pp. 195 – 215). Trondheim: Akademika Publishing.

The point of departure of this mixed-methods paper is an investigation of novice PSTs' SETM (as measured by the SETcPM-instrument reported in Paper 1) and MSE. Rasch analysis of the cohort-level SETcPM-implementation reported in Paper 1, combined with Rasch analysis of an MSE-instrument developed by Pampaka et al. (2011) and implemented with the same cohort of PSTs, offered a way of identifying different 'types' of PSTs. These 'types' were classified according to PSTs' combined levels of SETcPM and MSE, for example those with high SETcPM but low MSE, those with high scores on both, those with low scores on both, and so on. In Paper 2, the 10 case-study interviewees in my overall study are identified in terms of their placing within this pattern, and five of the 10 are analysed in detail since they collectively cover a range of different combinations, and hence represent different 'types' of PSTs.

The two-fold research question guiding the work in this paper is “*What are the connections between novice PSTs’ perceptions of their own subject knowledge and self-efficacy as a potential teacher in mathematics? What are the implications for the identity work these PSTs need to do?*” ‘Perception of their own subject knowledge’ was measured by the MSE-instrument and ‘self-efficacy as a potential teacher in mathematics’ by the SETcPM-instrument. In order to explain the patterns of the five PSTs’ scores on these two measures in relation to their identity work, I analysed their first interviews through the lens of Wenger’s (1998) theory on modes of belonging (or identification).

I used the contribution from Wenger outlined in Section 2.1.2 to theorise how individual trajectories and perceived competences in mathematics teaching connected with different modes of belonging. The analysis revealed that PSTs present a diverse range of identities and trajectories, where none of the five PSTs tended to take a mode of engagement as novices. For Wenger, opportunities for engagement are very important (detailed in Section 2.1.2), and employing this lens highlighted the role of *reflection* in future teachers’ identity work.

These findings in Paper 2 indicated the need for a theoretical framework that would encompass reflection on competences, and subject matter knowledge in particular. Biesta (2012a) provided additional insights; thus, Paper 2 not only identified different ‘types’ of PSTs in terms of SETM, MSE and identity work, but it also led to the role of Biesta’s work in my later analysis, with implications for Paper 3 and for the rest of the work within this thesis (see Figure 5 in Section 3.4.3).

4.3 Paper 3: Self-efficacy and judgement in pre-service teachers

> Bjerke, A.H., & Solomon, Y. (submitted). ‘The mathematics teacher I want to be’: Self-efficacy and development of judgement in pre-service teachers. *Educational Studies in Mathematics.*

This paper follows up on future directions indicated in both Paper 2 and Paper 4. It aims to investigate the ways in which PSTs describe their experiences of success and failure at UC and in school placement as sources of their developing SETM in terms of ‘what kind of mathematics teacher can I be?’, a question with connections to their perception of the role of subject matter knowledge and its role in teaching.

The overarching research question is *"What are PSTs' perceptions of the nature of mathematics and mathematics learning/teaching, and what influences PSTs' perception of their own subject knowledge and SETM?*". This question is broken down into three subsequent questions: "*How do PSTs perceive the role of subject matter knowledge in their development as teachers of mathematics in the domains of University College and school placement*"; "*What role does subject matter knowledge play in their accounts of success and failure, and how do these experiences contribute to their developing SETM?"*; and "*How do PSTs reflect on and value sources of self-efficacy in balancing qualification, socialisation and subjectification as they develop a sense of the teacher they not only <u>want</u> to be, but <u>can</u> be?*"

To address these questions, the narratives of 10 PSTs gathered from repeated interviews (described in Section 3.3.2) were analysed (see Section 3.4.2) in terms of Biesta's (2012a) account of educational purpose (outlined in Section 2.1.2) and sources of SETM (see Section 2.4.1). Thus, Paper 3 brings Biesta's work on educational purpose together with Bandura's construct of self-efficacy and its sources. This novel (as far as I know) combination offers a way to explore the contribution of experience to PSTs' developing identities as mathematics teachers.

Paper 3 identified three consecutive periods in PSTs' development (novice, more experienced and forward looking) where we see evidence for different roles for Bandura's four sources, and their links to perceptions of the role of subject matter knowledge in the different domains of educational purpose. Novice PSTs acknowledge the importance of subject matter knowledge in a 'competence-based' way, without connecting it to their experiences of success and failure in teaching. Sources related to subject matter knowledge and 'how to do it' skills were predominantly physiological and affective.

Having gained more experience, the PSTs tended to emphasise the role of school placement in their development, leading them to prioritise 'real teaching' in school in contrast to 'mere' theorisation of it at UC as part of what they saw as a list of competences (see the introduction on Chapter 2). They evaluated their placement learning as successful or not in accordance with mentor feedback, and their new skills contributed to a sense of increasing self-efficacy. In contrast, they saw UC learning as a question of fulfilling arbitrary checklists, but also as contributing to their emergent understanding of the role of subject matter knowledge in teaching.

Moving to a later stage of looking forward to the teacher they can be, the PSTs came to recognise the roles of *both* UC *and* school placement in their development as mathematics teachers in terms of the connection between knowledge and educational judgement. They saw subject matter knowledge as important for underpinning educational judgement, and their reflections revealed that both conflicts and synergies between the domains of educational purpose were manageable and developmental. A sense of self-efficacy based purely in practice was replaced by a more critical stance and a desire for *constructive* verbal persuasion and mastery experiences that were associated with UC subject matter knowledge.

Hence, Paper 3's findings add to Bandura's emphasis on mastery experience as the most powerful source of self-efficacy the need for teachers to reflect on what actually constitutes mastery in a particular context. Looking at how sources are played out in the different domains of education, Paper 3 shows that PSTs' perceptions of subject matter knowledge and the need for a profound understanding of subject matter knowledge directs their onward trajectory in the landscape of educational domains.

In addition to identifying sources of SETM in relation to educational purposes, the analysis reported in Paper 3 contributes to a deeper understanding of lower-performing PSTs who may be seen as "hopeless cases" because "they don't know any maths". A major section of Paper 3 focuses on one PST, PST9 (Maia, in Chapter 4.5), who found the teacher education at UC programme challenging, because of the focus on connected knowledge (as in PUFM). While PST9's perception of her own subject matter knowledge and the role of such is indeed an issue, her reflection of this situation, and how she dealt with it, is equally important. A detailed description of PST9 is offered in Section 4.5, and discussed in Section 5.5.

4.4 Paper 4: Pre-service teachers' developing self-efficacy in teaching mathematics

> Bjerke, A.H. (accepted). The development of self-efficacy in teaching mathematics in pre-service teachers. *Nordisk matematikkdidaktikk*, *x*(y), pp-pp.

This paper builds on Paper 1 in two ways: its point of departure is the SETcPM-implementation reported in Paper 1, and further, it builds on its analysis and results. The paper addresses the following research question: "*To what extent does PSTs' SETM develop during a mathematics methods course in primary teacher education, and what is the nature of this development?*"

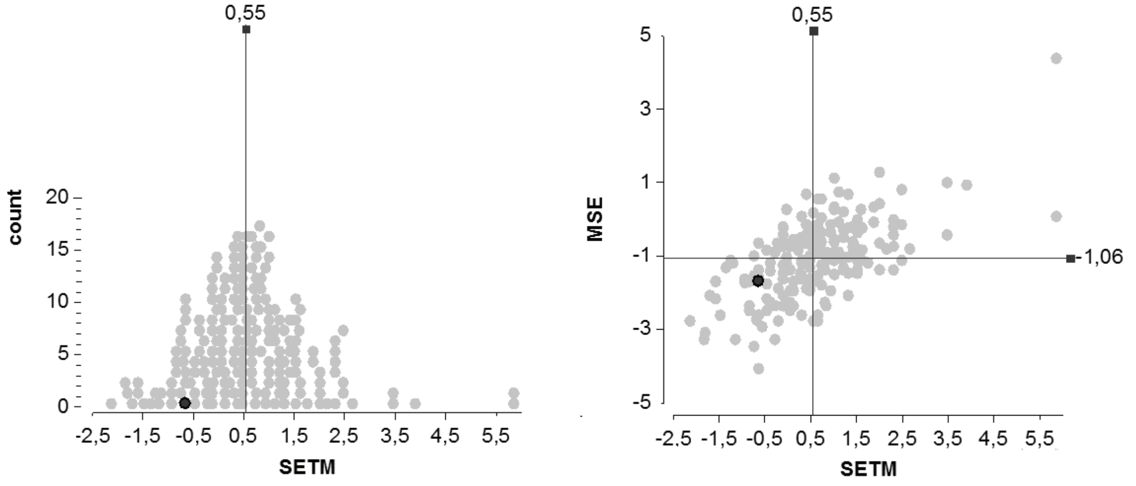
Paper 1 adds to the body of research showing that teacher efficacy, and SETM in particular, develops during teacher education. Its comparisons of pre- and post-test responses inform teacher education programmes on the nature of the development of SETcPM as a core component of SETM. In particular, the Rasch analysis conducted for Paper 4 shows that PSTs tend to be less confident when it comes to being able to explain why and how things work in mathematics. Moreover, it reveals that on two items 2nd year PSTs were less confident at the end of the course than expected based on their perception at the start of the course. A possible explanation for this is that teacher education might have made the 2nd year PSTs aware of the challenges of being able to help children knowing both what to do and why, as in Skemp's relational understanding.

In addition to illustrating positive trends in the group as a whole, Paper 4 shows some of the advantages of looking at individual trajectories and the insights they provide into the complexity of the group. Teacher education has different effects on different PSTs. We need to understand more about what makes some lose confidence while others gain it, for example. An important thought emerges concerning those losing confidence: Is this always a bad thing? The discussion in Paper 4 suggests that it is not, since losing confidence in this setting can be seen as a more reflective view on one's own subject matter knowledge as they gain more experience. Hence, this paper argues that teacher education enables PSTs to gauge their confidence in their own subject matter knowledge more accurately as it prepares them for the demands of teaching mathematics. This raises another important issue for teacher education: how can it prepare PSTs for what teaching mathematics will demand of them in terms of their own subject matter knowledge?

4.5 Maia's story – "I will try to survive"

The presentations in sections 4.1 - 4.4 reveal some of the connections between the four papers in this thesis. In this section, I recount the case of Maia as a means of showing how the papers jointly tell an overall story of one PST's developing identity as a mathematics teacher, and how this work enables a new understanding of "weak" PSTs often seen as "hopeless cases".

While Paper 1 reports on common trends in novice PSTs' SETcPM, Paper 2 adds MSE-measures from the same 191 PSTs. Taken together, these measures of SETcPM and MSE are used to identify different 'types' of PSTs. One PST, let us call her Maia, is present in Paper 2 as 'PST 5'; a representative of a 'type' with MSE-measure just below the mean and with one of the lowest SETcPM[20]-measures in the cohort of 2013. Figure 6 places novice Maia amongst her peers, as reported in Paper 1 (left) and in Paper 2 (right).



Paper 1, Maia among her peers as measured by the SETcPM-instrument.

Paper 2, Maia among her peers when combining SETcPM- and MSE-measures.

Figure 6. Maia amongst her peers

From Paper 2 we learn that Maia likes mathematics when she is able to do it, but when she does not 'get it', she does not like it. This and related statements highlight the role of emotion in her novice story. In Wenger's terms, although her mode of belonging seems to be one of alignment at some points, at others she falls into a category of non-participation. Maia does not know if she likes the thought of becoming a mathematics teacher, and she has few ideas and thoughts on what to expect from teacher education; she simply hopes that it "fits" her way of doing mathematics. She seems insecure and passive, and in Paper 2, Wenger's lens suggests that Maia has considerable identity work ahead of her in becoming a mathematics teacher, if she does. Recalling that reflection emerged as an essential personal characteristic in Paper 2, we see that at this point, Maia does not reflect on her learning and appears unable to make judgements about what it takes to switch from an identity as a learner to that of a teacher.

In Paper 3, we meet Maia as PST 9, where she acts as a foil to our presentation of the data from the other nine participants. In this paper, in particular, Maia contributed to a deeper understanding of lower-performing PSTs who may be seen as "hopeless cases" because "they don't know any maths". While my earlier analyses might suggest that Maia is not of interest, the application of Biesta's concepts of educational purpose and judgment in Paper 3 shows that there is another way of understanding her. The analysis reveals that subject matter knowledge

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[20] Recall that the SETcPM-measures are reported as SETM-measures in Paper 2 (a consequence of Paper 2 being published before Paper 1).

is indeed an issue in her developmental story: she finds mathematics challenging at UC, because of the focus on connected knowledge (as in PUFM). She expresses awareness of, and concern about, her inability to explain mathematics, but she does not see UC as a potential source of support. The role of subject matter knowledge in sources of self-efficacy in the sense of understanding connections and underlying principles is strikingly absent, and it appears to contribute to her insecurity in general.

As she becomes more experienced, Maia's story is noticeably more positive. She puts major emphasis on the benefits in terms of *feeling* like a teacher, and it turns out that her experiences, her perception of them, and how she deals with them are equally important. The analysis reported in Paper 3 shows that by her own account, Maia's identity as a future mathematics teacher depends on the support and feedback that is offered in school placement. While she acknowledges the importance of subject matter knowledge, and the fact that she struggles with it and finds her UC study threatening, she deals with the demands of teaching by focusing her learning in school placement, trying to copy her mentor, and preparing in exhaustive and rigid detail. She finds a strong source of self-efficacy in her placement, but this prevents her from further development in terms of independent judgement.

Like Paper 1, Paper 4 enables a comparison of Maia's level of SETcPM with those of her peers. Figure 7 shows a copy of Figure 2 from Paper 4 where Maia's scores as both a novice and a 2nd year PST are marked with vertical dotted lines. Maia is more than 1 logit below the mean as a novice, but closer to the mean as a 2nd year PST (0.63 logits), revealing that her SETcPM has developed more during these two years compared to the average PST. By the end of the mathematics method course Maia is 'Confident' or 'Very confident' on 14 items.

Bringing the findings from papers 3 and 4 together, we see that her strategy of over-preparing in order to manage her mathematics teaching, and drawing on her placement as a strong source of mastery, has resulted in positive development in her SETM. This is the positive side of Maia's story. Unfortunately, there is a more negative side; when talking about UC, she says: "I'll try to survive the last years in teacher education and get through it". By the end of the mathematics methods course spanning the two first years of teacher education, she is still stuck in her strategy of 'learning how to do it'. She is unable to see what UC has to offer in terms of contributing to her developing an identity as a mathematics teacher.

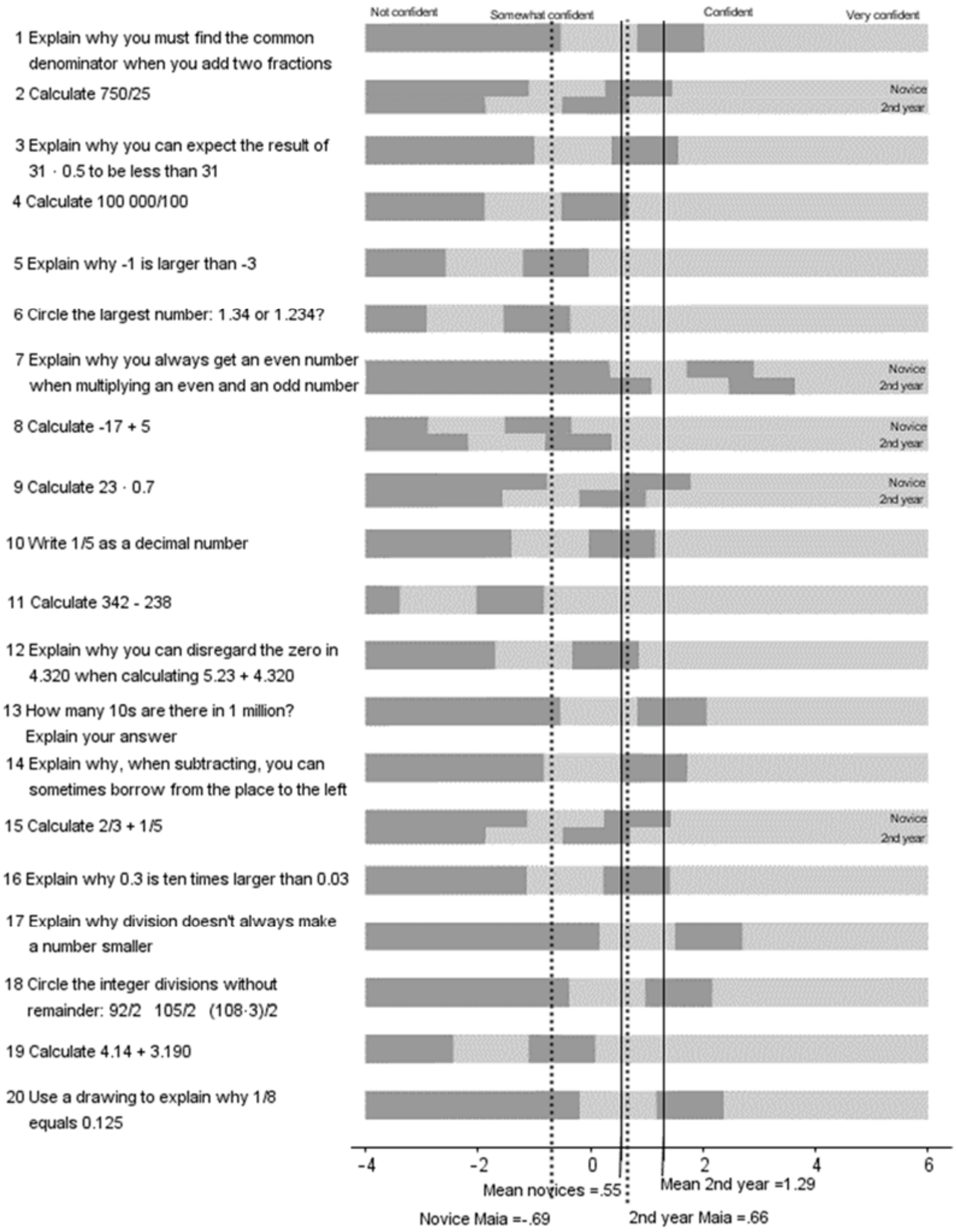


Figure 7. Ordinal Map showing Maia's measures

4.6 Integration of findings

In addition to contributing a new understanding of lower-performing PSTs, what is the overall picture when scrutinising the findings of the papers comprising this thesis? The overarching focus of this thesis is to investigate PSTs' developing identities as mathematics teachers in terms of SETM and the central role of subject matter knowledge, and Table 4 presents my work and findings in terms of their contribution to existing research. All four papers contribute to the body of research examining 'Learning to teach' or 'Developing identities' (theorised in Section 2.2). 'The perceived role of subject matter knowledge in PSTs' (theorised in Section 2.3, especially Section 2.3.3) is investigated through the 10 case study narratives, focusing on how PSTs see the need for subject matter knowledge in their future work as a mathematics teacher. The thesis makes a substantial contribution to the growing body of research on ‘PSTs developing SETM’ (theorised in Section 2.4 with a special focus on sources in Section 2.4.1). SETM is revisited in all the papers, and is one of the key concepts in the overall study design. Table 4 gives an overview of the papers, their focus, data, and their main findings, where the right-hand column shows the rationale for the integration of findings.

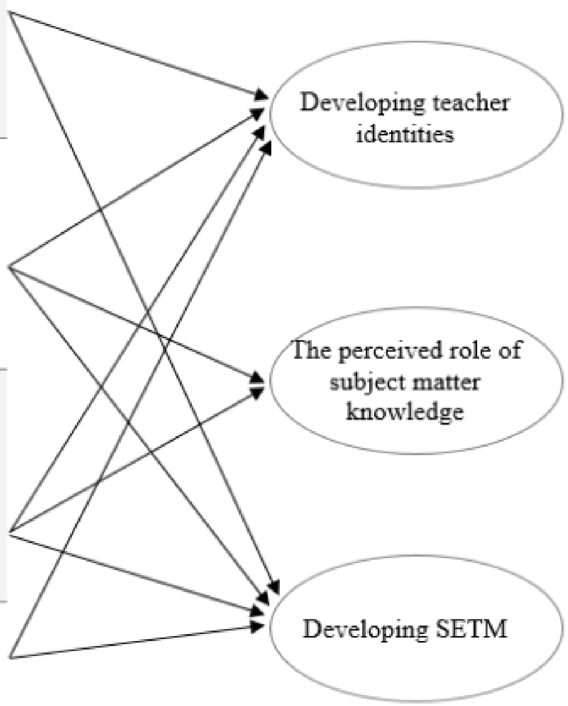
Paper	Findings	Integration of findings
1 Bjerke, A. H., & Eriksen, E. (2016). Measuring pre-service teachers' self-efficacy in tutoring children in primary mathematics: an instrument. *Research in Mathematics Education*, *18*(1), 61-79.	• A new instrument. • Operationalise the idea of SETM in novice PSTs. • Propose SETcPM is a central part of SETM. • A snapshot of novices in terms of SETcPM on a cohort level.	
2 Bjerke, A.H. (2014). Self-efficacy in mathematics and teaching mathematics in novice elementary pre-service teachers. In Østern, et al. (Eds.), *Once a teacher – Always a teacher?NAFOL Year Book 2014* (pp. 195 – 215). Trondheim: Akademika Publishing.	• Identified different novice trajectories based on PSTs perception of their own subject matter knowledge (as in MSE) and their SETM. • Identified the role of reflection and making judgements in developing identities as mathematics teachers.	
3 Bjerke, A.H., & Solomon, Y. (submitted). 'The mathematics teacher I want to be': Self-efficacy and development of judgement in pre-service teachers. *Educational Studies in Mathematics.*	• Operationalise the role of subject matter knowledge in PST's developing identities as mathematics teachers. • Identified how PSTs' perceptions of the role of subject matter knowledge changed with experience. • Explanations of how PSTs draw on sources of SETM in different domains of educational purposes. • A new understanding of the "weak" PST.	
4 Bjerke, A.H. (accepted). The development of self-efficacy in teaching mathematics in pre-service teachers. *Nordisk matematikkdidaktikk*, *x*(y), pp-pp.	• PSTs' developing of SETM, and the nature of this development. • The issue of over-confidence in novices.	

Table 4. Overview of the papers included in this thesis and their findings

5 Discussion and conclusion

In the previous chapter, I showed how each of the papers in this thesis answer the research questions guiding my work. In the current chapter, I focus on and discuss three major findings and results across the four papers and their contribution to the research field: the SETcPM-instrument and its practical implications (Section 5.1), PSTs' developing SETM during teacher education (Section 5.2), and the perceived role of subject matter knowledge in PSTs' developing mathematics teachers' identities (Section 5.3). In the latter section, I highlight the importance of PSTs' reflections on their own subject matter knowledge, and their need of it. In Section 5.4, I note some methodological considerations. I end this chapter, and thereby this thesis, with a concluding section (5.5), where I comment on implications of my study and suggest some directions and recommendations for further research.

5.1 A new instrument

Developing an instrument measuring self-efficacy in PSTs for educational purposes is a demanding task. Fortunately, there are lessons to be learned from previous work, both in terms of guidance concerning validity (Wolfe & Smith Jr, 2007a, 2007b), previous examples (e.g. the Rand studies, see Armor et al. (1976); Gibson and Dembo (1984); Riggs (1988); Enochs and Riggs (1990); and Huinker and Enochs (1995)), and Bandura's (2006) own guide for constructing self-efficacy scales. In addition to these extensive sources of guidance, the SETcPM-instrument reflects my experience of working for more than a decade as a teacher educator. This experience can be viewed as an in-depth study of practice, and it was this that gave me the idea of taking Skemp's (1976) two types of understanding mathematics as my point of departure when developing an instrument intended to be useful in teacher education.

An honest account by Wheatley (2005) goes like this:

> Before us sat a school principal...Speaking passionately, he said his school's students were eager to learn, and his teachers were committed to teaching. As a new principal trying to substantially improve his school, he was asking our teacher education faculty for help. I wondered, 'What unique guidance can I provide him, from my own area of research - teachers' efficacy beliefs?' I could think of none. (Wheatley, 2005, p. 747).

Moreover, he adds, "…why isn't it clearer how to use teacher efficacy research in teacher education?" (p.748). Building on a well-argued conviction that teacher efficacy is an important teacher characteristic (Woolfolk & Hoy, 1990) with many positive knock-ons (many of them summarised in Moulding, Stewart, and Dunmeyer (2014)), my initial idea and hope when developing this instrument was for it to have practical applications for teacher education. Wheatley's (2005) important question, and others like it, has been discussed by various researchers (for example Goddard, Hoy, and Hoy (2004); Henson (2002)), and more recently raised and summarised in Klassen et al.'s (2011) conclusions from their review of teacher efficacy research. Two of Klassen et al.'s (2011) six propositions are important when discussing how the SETcPM-instrument can contribute to making teacher efficacy research more applicable to practice. First, it addresses the issue of an optimal level of domain specificity, where "self-efficacy measures are most predictive of future behaviors when measures are narrowly defined, but they lose generalisability to other settings as specificity increases" (Klassen et al., 2011, p. 24). The chosen level of specificity is discussed in Paper 1 (p. 65). Second, Klassen et al.'s (2011) discuss the resolution of measurement problems, where greater attention to the congruence of measurement with theory has been demanded, reminding us that self-efficacy refers to judgements about *capabilities* which are conceptually distinct from other self-referent constructs (see Section 1.4 for more details). Paper 1 argues that the SETcPM-instrument reflects forward-looking capability. Taken together, I suggest that the way in which the SETcPM-instrument deals with these two propositions from Klassen et al. (2011) makes it potentially informative and useful for teacher education.

Wheatley (2005) notes that researchers do not normally design and intend a scale for in-program usage; rather, it has been more common to use such instruments when *evaluating* programmes. He stresses that administering teacher efficacy scales and computing scores is simple, but the problem lies in how to use the resulting data. It is easy to follow his argument; for instance, what does it mean that an average PST has a SETcPM-measure of 1.15 logits? These kind of results are of little practical use in a teacher education programme. A way to address the practicalities is to pay attention to the instrument's individual items (reported in Paper 1, and in Appendix A2), which can reveal some of the nature of SETM development (see Paper 4 for details). However, even at the level of focusing on the items in an instrument, Wheatley (2005) points out the problem of ambiguous wording and lack of subject matter context. This leads me to make some concrete suggestions. Following my assertion that the SETcPM-instrument was designed for in-program usage, and my constant focus on avoiding ambiguous wording (triangulated through piloting) and a strong subject matter context, I make some observations here about how to use the SETcPM-instrument and its items in teacher education courses, since this is only briefly discussed in the papers in this thesis.

I suggest that the instrument is of use in two ways: it can inform the mathematics teacher educator about their students, and equally importantly, it can be about involving PSTs in their own development. Presenting the test analysis to PSTs and discussing the individual items and overall focus on Skemp's (1976) two types of understanding mathematics can alert PSTs to the nature of their SETM and draw attention to important aspects of mathematics in teacher education. By focusing on the core activity in teaching mathematics, *helping children with mathematics tasks* (see Paper 1, p. 66, for an outline of this line of thought), the SETcPM-instrument captures the added dimension that teacher education introduces; the ability to *explain* mathematics, not just *do* mathematics. Paper 2 (or, more precisely, Figure 1, p. 204 in Paper 2) shows the distribution of the 20 items in the SETcPM-instrument based on how hard the items were to endorse for the pre-test-population in my research. The 10 items that focus on rules and procedures (teaching instrumental skills) tended to be easier to endorse than the remaining 10 that focus on *reasoning* (connectionist investigative approach). Recall that the 20 items are paired up thematically (outlined in Section 3.3.1), with each pair consisting of one rules-item addressing Skemp's (1976) instrumental understanding and one reasoning-item addressing relational understanding. By looking at the difference between the item-estimate for the reasoning-item and the item-estimate for the rules-item in each pair, this difference can offer useful information: are there any pairs in which the reasoning item is easier to endorse than the rules-item?; and in which cases are the differences between the pairs fairly big (more than one logit)? When discussing what subject matter knowledge *is* for a mathematics teacher, I propose that such analysis can work as an interesting and eye-opening approach.

Recall that PSTs' reflections on their own learning of mathematics affects their perceptions of the kind of person they are becoming (Radford, 2008), and that PSTs in my study tended to spend a considerable time in teacher education before reflecting on the role of subject matter knowledge in their development as mathematics teachers (Paper 3). Alongside the need for reflection identified in Paper 2, I suggest that the SETcPM-instrument can work as a tool for such reflection in novices. Additionally, results from Paper 4, which show that teacher education needs to focus even more on the ability to explain why and how things work in mathematics (underlined in the literature on core practices as in Forzani (2014) and Grossman et al. (2009)), suggest that an early implementation of the SETcPM-instrument can work as a positive intervention in novice PSTs' developing trajectories.

5.2 Developing self-efficacy in teaching mathematics during teacher education

Writing in 2017, I would argue, in line with Pfitzner-Eden (2016), that the *development* of teacher efficacy remains an under-researched area, especially in PSTs. When investigating teacher efficacy development, there seem to be two main approaches: the first is quantitative in the form of comparisons of pre- and post-test results, while the second is concerned with investigating sources and their effects. The latter approach uses both quantitative and qualitative methods. I situate my research within both camps.

While Paper 1 reports on the development of an instrument measuring a core component of SETM in novice PSTs (as in SETcPM) and pre-test results from a cohort of PSTs, Paper 4 reports post-test results from the same cohort, and analyses of their SETcPM-development. Pre- and post-test design has a longitudinal nature, and longitudinal approaches that shed light on the development of teacher efficacy are few (Henson, 2002; Klassen et al., 2011; Tschannen-Moran et al., 1998). As pointed out in the previous section, the focus of the SETcPM-instrument enables the analysis to reveal some of the nature of PSTs' development, such as the tendency for PSTs to be less confident as novices when it comes to tasks requiring them to explain why (reported in papers 1 and 4). Additionally, when comparing pre- and post-test results, Paper 4 reports on signs of over-confidence in some novice PSTs. This suggests another important aspect of teacher education: preparing PSTs for what teaching mathematics demands of them when it comes to their own subject matter knowledge.

Most research investigating teacher efficacy development takes Bandura's (1997) four sources (mastery experiences, vicarious experiences, verbal persuasion, and physiological and affective states) as their point of departure, but very few tend to investigate sources of teacher efficacy in the context of teaching mathematics (see literature review in Section 2.4). I find this lack of domain-specific research particularly interesting, largely because of the general agreement that teacher efficacy is domain-specific (Klassen et al., 2011), and because mathematics teaching is an especially interesting context, since PSTs often express doubt about their self-efficacy in mathematics itself (Gresham, 2007). Paper 3 adds to this sparse body of research by investigating how PSTs reflect on and value sources of self-efficacy, and how their perception of their own subject matter knowledge is played out in these sources.

Paper 3 reports on the analysis of interviews with 10 case-study PSTs conducted over a period of 1.5 years, between the pre- and post-test implementations. This analysis adds to the body of research on sources of SETM (see Section 2.4.1). Instead of viewing PSTs' sense of knowledge (or in my words, perceptions of subject matter knowledge) as a source of self-efficacy as, for instance, Palmer (2006, 2011) did (several researchers followed his lead), I follow Bandura (1997) and Wyatt (2014) who noted that knowledge is not a source of self-efficacy in itself. Bandura (1997) saw knowledge as something derived from previously identified sources of self-efficacy, and Wyatt (2014)argued that poor conceptualisations of the role of knowledge have obscured understandings of how teachers' self-efficacy beliefs are formed. This is crucial in Paper 3. Adding to the results of Paper 2, the focus in Paper 3 is on how PSTs *reflect* on their subject matter knowledge and how subject matter knowledge is played out *in* the sources. In this way, Paper 3 conceptualises the role of subject matter knowledge in PSTs developing SETM.

Paper 3 describes three phases in PSTs' developing identities as mathematics teachers, as they move from pure novices to more experienced students of mathematics teaching, to a phase of looking forward to the teacher they can be. PSTs appeared to draw on different sources as they developed, beginning in the novice phase with primarily physiological/affective sources, where mastery experiences were embedded in emotional references to themselves as PSTs. As they gained experience, they drew more on mastery experiences, where verbal persuasion in form of (positive) feedback from mentors contributed a sense of self-efficacy. As they neared the end of the 1.5 years, they began to develop a sense of themselves as teachers based on mastery experiences judged within critical and constructive feedback from mentors.

Consistent with my findings, among the four sources, mastery experience generally has the strongest effect (e.g., Bandura (1997)). PSTs' field experiences, such as in-school placement, are the most likely opportunity for developing mastery by practicing the skills and actions of a teacher (Moulding et al., 2014). Previous research on teacher efficacy development has tended to focus on the teaching practicum (or school placement) because it provides opportunities to investigate mastery experiences (Pfitzner-Eden, 2016); despite the fact that teacher preparation varies widely across the world, field experiences, such as student teaching and placement, are a standard component of most programs worldwide (Darling-Hammond & Lieberman, 2013). My own study was designed so that the interviews took place immediately before and after three consecutive periods of school placements. This design made it easier to focus on experiences of success and failure taking place during school placement, but during interviews, I tried to make room for PSTs to recount experiences from UC. In paper 3, instead of connecting sources to the different communities (of school placement and UC), I used Biesta's (2012a) domains of educational purpose (see Section 2.1.2) as a way of framing how the sources were played out. In this way, sources were connected to PSTs' ideas about *how* they drew on particular sources and for which purposes, rather than only adding to the body of research emphasising the role of mastery experiences played out during school placement.

5.3 The perceived role of subject matter knowledge in developing identities as mathematics teachers

In the previous section, I argued for the importance of seeing sources of teacher efficacy in relation to mathematics as distinct from other school subjects. The central role of PSTs' subject matter knowledge in developing SETM leads to the emerging idea of investigating PSTs' perceptions of subject matter knowledge and its role in their teaching. In Paper 3, PSTs' early accounts of 'What kind of mathematics teacher do I *want* to be' later changed to a more experienced and reflective idea of 'What kind of mathematics teacher *can* I be'. This shift was influenced by their perception of the role of subject matter knowledge and its role in teaching.

In Paper 2, identity is linked closely to practice. Lave and Wenger (1991) see learning and identity as inseparable and thus learning to teach within a new community contributes to PSTs' developing identities as mathematics teachers. Using Wenger's (1998) account of identity, I found that novice PSTs' predominant modes of belonging were imagination or alignment, but an emergent finding was the importance of reflection and its role in engagement (Wenger's third mode of belonging). Noticing the importance of reflection in developing teacher identities is not new: in their comprehensive review of the literature aiming to clarify the meaning of teacher identity, Beauchamp and Thomas (2009) found that the role of reflection in exploring and shaping teacher identity is crucial. My study connects reflection and practice within a theoretical framework, highlighting the need for reflection in order to enter in to an engaged mode of belonging. The move from Wenger to Biesta outlined in Section 2.1.2 offers a way to explore Bandura's sources and how PSTs perceive the role of subject matter knowledge in the different domains of educational purpose. The PSTs' narratives reveal some clear connections, for instance: there are some early accounts of interplay between their perception of their own subject matter knowledge and physiological and affective responses in the domain of qualification; and their perception of the role of subject matter knowledge is essential in mastery experiences in the domain of socialisation. Paper 3 adds several more such connections, and offers explanations on how this interplay is situated.

In Paper 3 I found that PSTs' identities are shifting as they move through teacher education, and in Section 2.2.1, I adopted the notion of *possible selves* to capture their developing identities. PSTs' perceptions of their own subject matter knowledge (or lack of it) were essential in these shifting identities. The case of Maia in Section 4.5 illustrates how the papers describe and explain PSTs' constantly changing ideas of possible selves in terms of 'What kind of teacher can I be'. Focusing on the role of subject matter knowledge in possible selves - what one *might* become, what one *would like* to become, and what one is *afraid* of becoming (see Section 2.2.1) - provides new insights into PSTs' developing identities. Bringing together research on subject matter knowledge and on teacher efficacy enabled me to explain in more detail the complex role of subject matter knowledge in PSTs' developing identities as mathematics teachers.

Taken together, these findings show that PSTs' perceptions of their own subject matter knowledge and its role in teaching, and how they reflect on this knowledge are crucial in their developing identities as mathematics teachers.

5.4 Methodological considerations

In Chapter 3, I have argued for the choice of mixed methodologies in this study. While I do acknowledge criticisms which question the epistemological and ontological sense of mixing methods, fortunately, many (or most?) purists have now reached basic agreement on several major points of earlier philosophical disagreement (Johnson & Onwuegbuzie, 2004). My pragmatic and mixed position provides a foundation which combines the insights gained from qualitative and quantitative research into a workable solution. Nevertheless, Johnson and Onwuegbuzie (2004) point out some disadvantages with this approach, suggesting that it might be difficult for a single researcher to carry out both qualitative and quantitative research, in terms of learning about multiple methods and approaches and understanding how to mix them appropriately. I agree that mixing methodologies is a major task, but the advantages are substantial, enabling me not only to answer a broader and more complete range of research questions (Johnson & Onwuegbuzie, 2004) but also to provide stronger evidence for my conclusions through corroboration of findings. For example, in Paper 2 the use of narratives added meaning to PSTs' positions in a scatterplot, positions that were solely decided by measures resulted from quantitative data and analyses. In addition to providing insights that might have been missed if only a single method was used, this process enabled a move to *understand* more rather than just offer descriptions of positions in a scatterplot. Moreover, the other way around, Paper 1 underlines the role of qualitative data in developing a quantitative instrument. Qualitative methods informed the process of developing a quantitative instrument: The qualitative data ensured better and more informative quantitative data from SETcPM-implementations. This enabled me not only to measure SETM-development, but additionally to *explain* more about the nature of this development on the basis of the information provided by the instrument.

Methodology is also about choices of theoretical lenses and concepts. As a researcher choosing a theoretical lens entails a simultaneous acceptance of a range of preconditions. In my case, I find it important to comment on two preconditions regarding the theory of self-efficacy. First, I rest my case on assumptions which assert that it is possible to specify what self-efficacy is. Second, I assume that teacher education benefits from further quantifying and measures of self-efficacy. In Section 1.4 and Section 2.4 I have shown how I build on recent analyses of the construct that suggest a need for further research since it has been found to be a central teacher characteristic.

The final methodological consideration that I make space for here also concerns theory. The shift from Wenger's (1998) socio-cultural theory to a more social constructivist approach, in which I used Biesta's (2014) lens to investigate PSTs' perceptions and reflections, involves much more than a shift in tools for analysing my data. A retrospective look at this shift, driven by my findings in Paper 2, raises an interesting question: If I had chosen to stay with Wenger, would the outcome and results of my research have been different? I am sure they would. And perhaps, this can be left as an idea for the future – to analyse my data through Wenger's lens and compare the results with those I have presented here.

5.5 Conclusion

This study addresses the role of PSTs' perceptions of subject matter knowledge in their developing identities as future mathematics teachers, and the role of this knowledge in developing SETM. The point of departure was my own experience with PSTs, many of whom struggle with their difficult relationship with mathematics. I found that investigating their MSE and SETM in relation to subject matter knowledge, as in SETcPM, was an informative starting point for understanding more about how they see their own knowledge and its place in their teaching. In developing and implementing the SETcPM-instrument, I found a need to pay extra attention to how PSTs perceive the role of subject matter knowledge in their developing ideas of ‘what kind of teacher I can be’. I pursued this by investigating how they reflected on subject matter knowledge and its role in relation to the ends and purposes of teacher education, and the ways in which they drew on experiences of success and failure within each of Biesta’s (2012b) domains of qualification, socialisation and subjectification.

5.5.1 Implications

In many ways, Wheatley’s (2005) call to make research on teacher efficacy more applicable in teacher education has guided the approach taken in this thesis. In Section 5.1, I propose that an early implementation of the SETcPM-instrument can work as intervention in PSTs’ developing trajectories. Here, I suggest four major implications for teacher education that can be drawn from the work included in this thesis: recognising the agency of the “weak” PST, recognising the role of subject matter knowledge in sources of SETM, recognising the need for a variety of sources in different communities, and recognising the power of reflection on subject matter knowledge and the role of such knowledge in developing identities.

Recognising the agency of the “weak” PST. Bandura (1997) argues that self-efficacy is central to the exercise of human agency, and my investigation of this construct in PSTs from novice to more experienced and forward-looking, offers a new way of recognising the agency of “weak” PSTs. The story of Maia in Section 4.5 is an example of someone with relatively low MSE and low SETcPM, who seems unwilling to reflect on the role of subject matter knowledge, who is unwilling to engage in the ‘new dimension’ of mathematics - the ability to *explain* mathematics - and who sees teacher education as something to survive; she is easy to describe as a “lost cause”. Based on my own experience as a teacher educator, I suspect that there are many PSTs like Maia in different teacher education programmes, struggling to take in what teacher education has to offer. Maia wanted mathematics to be like it was in upper secondary school, which she saw as a set of rules to learn and use. Teacher education demands that she engage in *understanding* mathematics, and be able to explain *why* it makes sense to use those rules, not only *how* to use them. My findings suggest that such “hopeless cases” are not simply “those who cannot do any maths”, but rather, have low MSE and a hesitation to engage with subject matter knowledge in a new way. Applying these insights, I suggest that teacher education should focus more on the fact that, in many ways, mathematics in teacher education is a different subject from the mathematics they meet in upper secondary school. An early implementation of the SETcPM-instrument may present a way to engage novice PSTs with this issue.

Recognising the role of subject matter knowledge in sources of SETM. In line with Klassen et al.s' (2011) call for better understanding of the role of knowledge in the development of teacher efficacy, Paper 3 reports on the role of subject matter knowledge in the different sources of self-efficacy that PSTs draw on during teacher education. In addition to recognising the connections between subject matter knowledge and sources of SETM, it is important to notice sources where subject matter knowledge seems absent. In the analysis undertaken for Paper 3, there was no evidence of verbal persuasion that explicitly focused on PSTs' subject matter knowledge and its role in teaching. This has to be a problem, and based on my findings I am tempted to ask: How often do PSTs get feedback on their subject matter knowledge (or lack of it), at either UC or during school placement? My research suggests that this is essential for developing PSTs' SETM and for supporting their developing identities as mathematics teachers. Articulating the role of subject matter knowledge as part of verbal persuasion is important, and, as Bandura (1997) argues, verbal persuasion is more powerful when provided by a significant other with high credibility. Both teacher educators and placement mentors would do well to highlight the role of subject matter knowledge: "You did well *because* you really knew how to explain to the students how to add two fractions". I propose that this should be pointed out in future training for mentors, as they are perhaps the most significant others at times.

Recognising the need of a variety of sources in different communities. There was little evidence of vicarious experience in my interview data. This is of particular concern, since vicarious experience occurs when observing significant others succeeding at the target task (Bandura, 1997). When a PST has limited previous experience, vicarious experience has substantial influence on self-efficacy (Moulding et al., 2014). It is still unclear in the literature what role these vicarious experiences may play in the development of a teacher's sense of efficacy (Moulding et al., 2014), but vicarious experience where subject matter knowledge explicitly contributes to the success of a significant other is a source worth pursuing. Teacher education could aim to offer more vicarious experience where subject matter knowledge is present as a criteria of success.

Since mastery experience appears to be the most powerful source of efficacy beliefs (Bandura, 1997) and the most important source in improving teachers' knowledge (Morris et al., 2016), it is appropriate to ask if is it only possible to draw on this source during school placement. Turning to the body of research on core practices, this is less concerned with where PSTs' training takes place, and more with what PSTs are helped to learn and how to learn it (Forzani, 2014). Experiencing mastery in UC (in addition to those taking place during school placement), might raise PSTs' SETM and offer opportunities to reflect immediately after such experiences. I propose that core practices can offer such opportunities. For instance, Tschannen-Moran and McMaster (2009) described as especially effective an intervention including 'coaching sessions' to provide teachers with specific and individualised social persuasion. Here the focus was social persuasion, but including such 'coaching sessions' organised as core practices might provide mastery experiences. Giving PSTs teaching tasks for smaller groups of peers, presenting a new mathematics theme or an outline of a solution of a task or other similar activities, may be one way to situate mastery experiences in UC teaching.

Recognising the power of reflection in developing identities. The power and importance of reflection is well established. In the introduction (Section 1.1), reflection was highlighted as the key to learning from experience (Cochran-Smith, 2004), the focus of teacher education in the period from approximately 1980 – 2000 (Smith, 2016). Nevertheless, as Smith (2016) observes, today's policy focus means that the pendulum is swinging away from reflection. In my work, reflection emerged as an important personal characteristic, found to be something other than competence (see Section 2.1.2). Focusing on PSTs' reflection on their own subject matter knowledge and its role as a form of agency, and as a key means by which teachers can approach their *possible selves,* highlights its importance in PSTs' developing ideas of 'the teacher I can be' as opposed to less reflected ideas on 'the teacher I want to be'. This research has additionally highlighted the importance of subject matter knowledge as the focus of reflection on experience, which suggests that teacher education should provide more opportunities to reflect on one's own subject matter knowledge, both in school placement and at UC. Core practices can offer such opportunities to reflect, as they intend to take subject matter knowledge and context-specific learning goals for students as their point of departure (Forzani, 2014).

5.5.2 Recommendations for further research

Paper 1 reports that SETcPM is a part of SETM that can be measured in novice PSTs without any teaching experience, since it contains items representing tasks of teaching mathematics (Ball et al., 2008) that are imaginable for this population. Ball et al.'s (2008) list of 16 recurrent tasks of teaching mathematics presented possibilities for grounding the SETcPM-instrument in authentic situations. However, due to novices' lack of teaching experience, many of the tasks in this list were "poorly suited for this version of the SETcPM-instrument, since they are so far on the horizon that nPSTs [novice PSTs] will not be able to judge their own confidence in carrying them out" (Paper 1, p. 65). The analysis reported in Paper 1, particularly the results from its implementation with in-service teachers, and the follow-up analysis reported in Paper

4, suggest ways to expand the SETcPM-instrument based on the list of recurrent tasks of teaching mathematics (for details, see Paper 1, p. 74).

I propose that an expanded version of the SETcPM-instrument should be developed for new implementations with a pre- and post-test design, where the new version targets more experienced PSTs. The anchoring techniques (Boone, Staver, & Yale, 2014) allow different versions of an instrument to be implemented in longitudinal studies. With or without this new version of the SETcPM-instrument, new implementations will be helpful in investigating how its analysis can be used in teacher education (as discussed in Section 5.1). This way of gaining insight into how teacher education programs foster teacher efficacy is a neglected aspect in the existing research on teacher efficacy (Klassen et al., 2011).

In order to concentrate on measuring sources of teacher efficacy in the context of teaching mathematics, I decided to employ qualitative methods in line with Klassen et al.'s (2011) proposition. Recently, there has been progress in the field on how to address the sources using quantitative methods. Pfitzner-Eden's (2016) source inventory is designed to investigate Bandura's sources of teacher efficacy in PSTs after a practicum experience at a school. To my knowledge, only two additional quantitative measures are reported in peer-reviewed journals, a four-item measure designed by Heppner (1994) and Poulou's (2007) Teaching Efficacy Sources Inventory. Morris et al. (2016) evaluated the ways in which 82 empirical studies measured and conceptualised the sources of teaching self-efficacy. In their recommendations for future research, one thing struck me - the absence of a domain-specific focus. Given that self-efficacy is fundamentally situated (Klassen et al., 2011), there seems to be an agreement that research on teacher efficacy beliefs should be domain-specific (Bandura, 1997; Klassen et al., 2011). I propose that it follows that this also should apply to sources, especially when it comes to the nature and conceptualisation of the sources. None of the three instruments mentioned above are related to any specific subject, and for that reason, a source inventory for teaching mathematics could have the potential to inform the body of research investigating how SETM develops during teacher education.

Adding to this, and drawing on the implications in Section 5.5.1, my findings also suggest the need to investigate the possibilities of core-practices as scenes for mastery experiences. With mastery experiences being such a powerful source, there are reasons to believe that PSTs and teacher education could benefit from mastery experiences not being dependent upon expanded periods of school placement in teacher education (as discussed in Section 1.1).

In Section 4.5, I present Maia's story. Her story was, to be honest, an unexpected contribution to the field, made possible because Maia turned out to play an important role in both Paper 2 and Paper 3 (and was trackable in papers 1 and 4). There is a need to know more about PSTs like Maia, preferably through longitudinal approaches, in order to capture developmental aspects of their stories.

In this thesis, I have blended two bodies of research which are not normally brought together. By taking subject matter knowledge as my point of departure when investigating PSTs' developing SETM, and its role in the sources they draw on, and moreover, how they perceive subject matter knowledge and its role in teaching, I have gained an understanding of how, together, these contribute to a better understanding of PSTs' developing identities as mathematics teachers. Further research is needed in order to understand more about the role of subject matter knowledge in this developmental picture.

This piece of research has not provided definitive answers to Schuman's call 'Where did subject matter go?'. Rather, it has contributed to underlining the importance of subject matter knowledge in developing SETM, and moreover, in PSTs' developing reflected ideas on 'the mathematics teacher I can be'. And, hopefully, 'want to be'.

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Papers

Appendices

A Forestill deg at du skal hjelpe et barn med en hjemmelekse. Hvor trygg er du på at du kan hjelpe barnet med følgende oppgaver?

Der det står "forklar" skal du hjelpe barnet med å kunne forklare hvordan det fant svaret. Ellers skal du kun hjelpe barnet med å finne riktig svar.

		Ikke trygg	Litt trygg	Trygg	Veldig trygg
A1	«Forklar hvorfor vi må finne fellesnevner når vi skal legge sammen to brøker.»				
A2	«Regn ut $750: 25$»				
A3	«Forklar hvorfor du kan forvente at $31 \cdot 0{,}5$ gir et mindre resultat enn 31.»				
A4	«Regn ut $100\,000: 100$»				
A5	«Forklar at -1 er større enn -3»				
A6	«Sett ring rundt det største tallet: 1,34 eller 1,234?»				
A7	«Forklar hvorfor en alltid får et partall til svar når en multipliserer et partall med et oddetall.»				
A8	«Regn ut $-17 + 5$»				
A9	«Regn ut $23 \cdot 0{,}7$»				
A10	«Skriv $\frac{1}{5}$ som desimaltall.»				
A11	«Regn ut $342 - 238$»				
A12	«Forklar hvorfor vi kan se bort fra nullen i 4,320 når vi skal regne ut $5{,}23 + 4{,}320$»				
A13	«Hvor mange 10-ere er det i 1 million? Forklar svaret ditt.»				
A14	«Når du skal subtrahere, forklar hvorfor du kan låne fra plassen til venstre.»				
A15	«Regn ut $\frac{2}{3} + \frac{1}{5}$»				
A16	«Forklar at 0,3 er ti ganger større enn 0,03»				
A17	«Forklar at deling ikke alltid gjør mindre.»				
A18	«Sett ring rundt regnestykkene som ikke gir rest (går opp): $92: 2$ $105: 2$ $\frac{108\cdot 3}{2}$»				
A19	«Regn ut $4{,}14 + 3{,}190$»				
A20	«Bruk tegning til å forklare at $\frac{1}{8}$ er det samme som 0,125»				

Appendix A2 - The SETcPM-instrument, English version

A	**Imagine that you are going to help a child with their homework. How confident are you that you can help the child with the tasks listed below?** When the verb «explain» is used, you are asked to help the child to be able to explain. Otherwise you are just asked to help the child answer correctly.	Not confident	Somewhat confident	Confident	Very confident
A1	“Explain why you must find the common denominator when you add two fractions.”				
A2	“Calculate 750/25”				
A3	“Explain why you can expect the result of 31× 0.5 to be less than 31.”				
A4	“Calculate 100 000/100”				
A5	“Explain why -1 is larger than -3.”				
A6	“Circle the largest number: 1.34 or 1.234?”				
A7	“Explain why you always get an even number when multiplying an even and an odd number.”				
A8	“Calculate -17 + 5”				
A9	“Calculate 23 × 0,7”				
A10	“Write $\frac{1}{5}$ as a decimal number.”				
A11	“Calculate 342 – 238”				
A12	“Explain why you can disregard the zero in 4.320 when calculating 5.23+4.320”				
A13	“How many 10s are there in 1 million? Explain your answer. ”				
A14	“Explain why, when subtracting, you can sometimes borrow from the place to the left.”				
A15	“Calculate $\frac{2}{3}+\frac{1}{5}$”				
A16	“Explain why 0.3 is ten times larger than 0.03.”				
A17	“Explain why division doesn’t always make a number smaller.”				
A18	“Circle the integer divisions without remainder: 92/2 105/2 $(108 \cdot 3)/2$”				
A19	“Calculate 4.14 + 3.190”				
A20	“Use a drawing to explain why $\frac{1}{8}$ equals 0.125.”				

Norwegian version

Å undervise matematikk

		Ikke trygg i det hele tatt	Litt trygg	Trygg	Veldig trygg
	Noen elever jobber med oppgaver som listet opp under. Svar på hvor trygg du er på å skulle hjelpe elever som ber om din hjelp til disse oppgavene.				
C1	Calculate 4.14 + 3.190 =				
C2	Hvilket tall er størst; 1,34 or 1,234?				
C3	Forklar hvorfor 30 · 0,5 er mindre enn 30.				
C4	Forklar at deling ikke alltid gir større svar.				
C5	Regn ut omkretsen av et rektangel med sider på 2 m.				
C6	Hvor mange meter er det i en kilometer?				
C7	Forklar at 5 m^2 = 50 000 cm^2.				

English version

To teach mathematics

		Not confident at all	Somewhat confident	Confident	Very confident
	Some students are working with the tasks listed below. How confident are you helping students asking for your help?				
C1	Calculate 4.14 + 3.090 =				
C2	Which number is larger; 1.34 or 1.234?				
C3	Explain why you can expect the result of 30 × 0.5 to be less than 30.				
C4	Explain why division doesn't always make a number larger.				
C5	Calculate the circumference of a rectangle with sides equal 2 m.				
C6	How many meters are there in a kilometre?				
C7	Explain that 5 m^2 = 50 000 cm^2.				

B	Svar på hvor enig du er i påstandene under	Veldig uenig	Uenig	Enig	Veldig enig
B1	Det viktigste i matematikk er å kunne regler og klare å følge disse.				
B2	Det er viktig å kunne følge en annens resonnement.				
B3	Den beste måten å lære matematikk på er å se et eksempel på riktig løsningsmetode, enten på tavla eller i boka, for deretter å prøve å gjøre det samme selv.				
B4	Det er viktig å kunne forklare egne svar.				
B5	Hvis du pugger og øver nok blir du god i matematikk.				
B6	Matematikk bør læres som et sett av algoritmer og regler som dekker alle muligheter.				
B7	Det er viktig å kunne løse sammensatte oppgaver der en må bruke flere sider av matematikken.				
B8	Det du kan gjøre, forstår du.				
B9	Det er viktigere å forstå hvorfor en metode fungerer enn å lære regler utenat i matematikk.				
B10	En lærer mer matematikk av oppgaver som ikke har en gitt fremgangsmåte, der en heller må prøve seg frem og vurdere svar og fremgangsmåte underveis.				
B11	Det er viktig å kunne argumentere for at svaret er riktig.				
B12	Den som får rett svar, har forstått.				
B13	Å løse matematiske problemer innebærer ofte bruk av hypoteser, tilnærminger, testing og revurderinger.				
B14	En lærer av å se ulike måter å løse en oppgave på, enten ved at elevene får presentere sine måter, eller ved at læreren presenterer alternative løsningsmetoder.				
B15	Det er viktig å lære regler og metoder utenat.				
B16	Det er viktig å kunne forklare sitt eget resonnement.				
B17	Undervisningen må fokusere mest mulig på forståelse, slik at en kan begrunne metoder og sammenhenger.				
B18	Matematikk er å finne det riktige svaret på en oppgave.				
B19	Det er viktig å kunne vurdere andre framgangsmåter enn sin egen.				
B20	Det er viktig å kunne argumentere for egne fremgangsmåter og svar.				
B21	Det er viktig å lære formelle sider ved matematikken (som f.eks. riktig oppstilling) så tidlig som mulig.				

Appendix B_2 - The Belief about mathematics instrument, English version

B_2

B	Please indicate the extent to which you agree with the statements below	Disagree entirely	Disagree somewhat	Agree somewhat	Agree entirely
B1	The most important aspect of mathematics is to know the rules and to be able to follow them				
B2	It is important to be able to follow the reasoning of anothers				
B3	The best way to learn mathematics is to see an example of the correct method for solution, either on the blackboard or in the textbook, and then to try to do the same yourself				
B4	It is important to be able to explain your answers				
B5	If you cram and practice enough, you will get good at mathematics				
B6	Mathematics should be learned as a set of algorithms and rules that cover all possibilities				
B7	It is important to be able to solve complex problems where one must use several aspects of mathematics				
B8	What you are able to do you also understand				
B9	In mathematics, it is more important to understand why a method works than to learn rules by heart				
B10	The pupils learn more mathematics from problems that do not have a given procedure for solution, where instead they have to try out solutions and evaluate answers and procedures as they go				
B11	It is important to be able to argue for why the answer is correct				
B12	Those who get the right answer have understood				
B13	Solving mathematical problems often entails the use of hypotheses, approaches, tests, and re-evaluations				
B14	The pupils learn from seeing different ways to solve a problem, either by pupils presenting their solutions or by the teacher presenting alternative solutions				
B15	It is important to learn formal aspects of mathematics (e.g. the correct way to write out calculations) as early as possible				
B16	It is important to be able to explain one’s reasoning				
B17	Teaching must focus on understanding as much as possible in order to learn how to explain methods and connections				
B18	Mathematics means finding the correct answer to a problem				
B19	It is important to be able to evaluate other procedures than one’s own				
B20	It is important to be able to argue for one’s own procedures and answers				
B21	It is important to learn formal aspects of mathematics (e.g. the correct way to write out calculations) as early as possible				

C Om å bruke matematikk

I denne delen blir du spurt om hvor trygg du er på å bruke matematikk til å løse ulike problem. **Du blir ikke bedt om faktisk å løse problemene.**

Forestill deg at du har fått disse oppgavene i hjemmelekse. Du kan bruke for eksempel notatene dine, bøkene dine og kalkulator hvis nødvendig. Du blir bedt om å angi på en skala fra 1 til 4 hvor trygg du føler deg på å skulle kunne løse disse oppgavene, **uten faktisk å løse dem**.

Si at du for eksempel blir spurt om å si hvor trygg du er på å løse oppgaven nedenfor. Om du ikke føler deg veldig trygg, da ville du markert felt 2.

Eksempel

Løse praktiske problem som involverer penger ved hjelp av kalkulator, slik som:

Ikke trygg	[✓ Litt trygg]	Trygg	Veldig trygg
1	[✓ 2]	3	4

Beregn hvilket glass med pulverkaffe det lønner seg å kjøpe:

Cappuccino 160 g til 26 kr
Caffe latte 220 g til 33 kr

Hvor sikker du føler deg på at du skulle kunne løse hver av oppgavene under? Marker alternativet som best beskriver hvor sikker du føler deg.

Husk: Du er ikke bedt om å løse oppgavene.

C1

Løse praktiske problemer som involverer penger ved hjelp av kalkulator, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Hvilken bussreise har beste pris beregnet i kroner per kilometer:

Oslo – Trondheim, 500 km, koster kr 630,-
Oslo – Stockholm, 530 km, koster kr 680,-

C2

Beregne kostnader for et prosjekt som involverer penger og komplekse datatabeller fra Internett, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Bruk informasjonen under til å beregne feriekostandene for en gruppe på 6 personer. Det er to par og to single i reisefølget. De ønsker å starte ferien 5. august, og skal bo på hotell i 7 netter. Ett par og en person i singelrom ønsker å bestille sjøutsikt (sea view) og halvpensjon (half board).

2006 - Saturdays
per person in a twin room

Date	Price	Date	Price	Date	Price	Date	Price
Apr 8	£845	Jun 3	£875	Jul 29	£755	Sep 23	£865
Apr 15	£845	Jun 10	£875	Aug 5	£745	Sep 30	£865
Apr 22	£855	Jun 17	£855	Aug 12	£775	Oct 7	£875
Apr 29	£855	Jun 24	£855	Aug 19	£775	Oct 14	£855
May 6	£865	Jul 1	£745	Aug 26	£775	Oct 21	£845
May 13	£865	Jul 8	£745	Sep 2	£875		
May 20	£855	Jul 15	£735	Sep 9	£875		
May 27	£855	Jul 22	£735	Sep 16	£865		

Supplements per person

Supplement	Price
Single supplement	£170

Extension

7-nights Sorrento	Price
Grand Hotel Vesuvio	£375
(Aug - Oct)	£425
Single supplement	£70
Sea view	£75
Half board (lunch or dinner)	£50
Sea view	£75

Price Includes
Air travel, UK departure taxes, overseas airport taxes, all transportation, breakfast daily, dinner on days 4 & 7, itinerary as described, tour escort and official city guides, guidebook
Not Included

C3

Forstå og bruke metriske mål, enheter og notasjoner, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Tabellen gir en oversikt over lengden til elver målt i meter, men uttrykt på forskjellige måter. Sett elvene i synkende rekkefølge i henhold til lengde.

Elv	Lengde
Amazon	$6{,}39 \cdot 10^6$ meter
Yellow	$4{,}67 \cdot 10^9$ millimeter
Nile	6690000000 millimeter
Yangtze	6380 kilometer
Congo	4371000 meter
Mississippi	$6{,}27 \cdot 10^3$ kilometer

C4

Løse problem som krever beregninger med mål og enheter, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Finn ut hvor mange ekser med målene 50 cm x 40 cm x 40 cm som kan passe inn i en kontainer med målene 6 m x 2,5 m x 2,5 m.

C5

Løse lineære likninger med x på begge sider av likhetstegnet, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

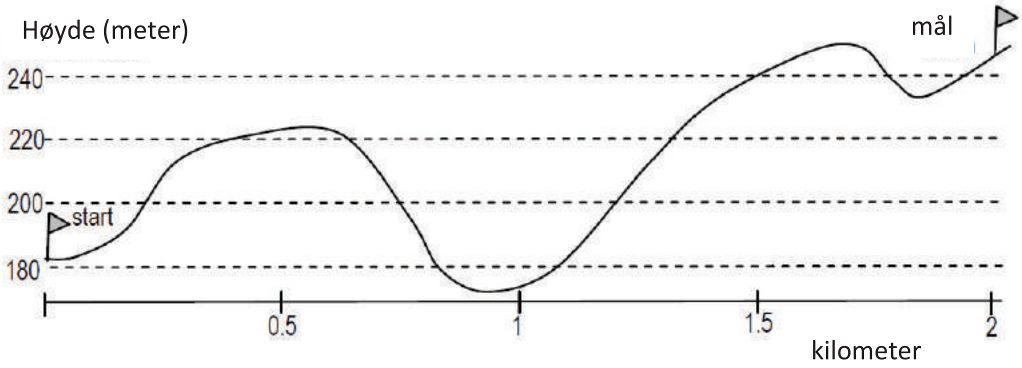
$15 - 2x = 3x + 25$

C6

Løse praktiske problem ved hjelp av grafiske fremstillinger, regning og skalaer, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Diagrammet under viser høyde/avstandsprofilen av en gåtur med høyde i meter og horisontal distanse i kilometer. Disse ulike skalaene tatt i betraktning, kan du ved hjelp av Pythagoras' setning beregne omtrentlig hvor langt du totalt gikk i nedoverbakke i løpet av gåturen?

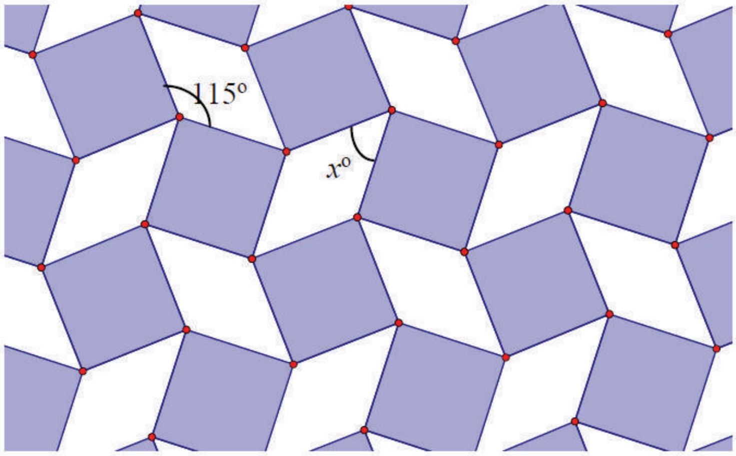


C7

Løse problem ved å bruke geometriske egenskaper, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Diagrammet under viser et flisemønster som er laget av kvadrat og parallellogram. Beregn størrelsen på vinkelen markert med x°.



C8

Løse praktiske problem som involverer andregradslikninger, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

En golfspiller treffer ballen slik at ballens høyde over bakken, h meter, er gitt ved $h = 20t - 5t^2$.

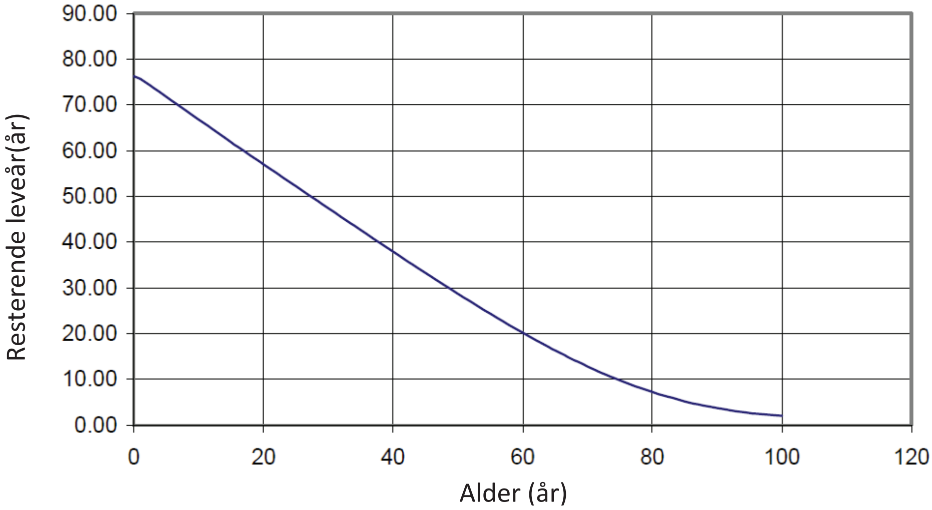
Finn ut når ballen er 5 meter over bakken ved å løse likningen $5 = 20t - 5t^2$.

C9

Finne en formel for å beskrive eksperimentelle resultat, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Grafen under vise hvordan resterende leveår for menn, E år, varierer med alder, A år. Finn en lineær formel som viser sammenhengen mellom E og A for menn mellom 0 og 60 år.

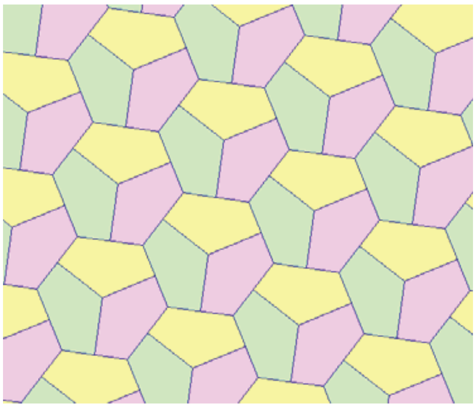


C10

Beregne areal til sammensatte figurer ved å bruke Pythagoras' setning eller trigonometri, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Diagrammet viser et flismønster. Beregn arealet til en av femkantene når du vet at sidelengdene i sekskantene er 6 cm.

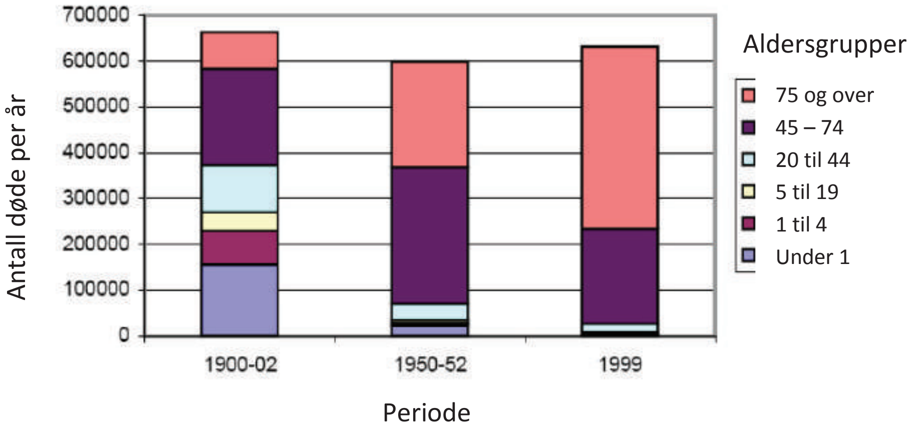


C11

Tolke komplekse eller uvanlige grafer eller tabeller, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Tolk grafen nedenfor slik at du kan beskrive hvordan antall omkomne i trafikken i ulike aldersgrupper har endret seg over tid.

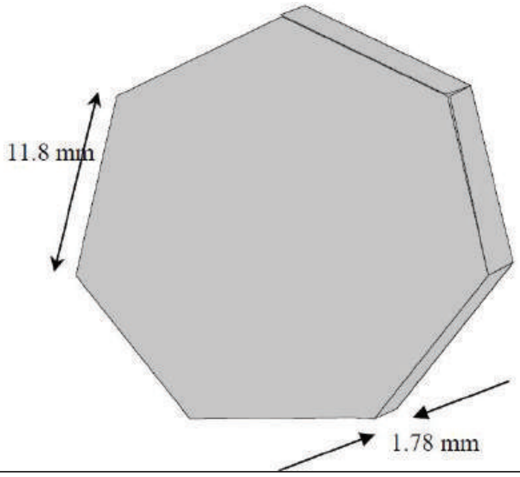


C12

Modellere eller løse problemer som går ut på å beregne volumet av mer komplekse former ved å bruke Pythagoras' setning eller trigonometri, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

En 50 pence mynt kan modelleres som et prisme med en regulær 7-kant som topp- og grunnflate. Finn volumet til en 50 pence ved å bruke målene oppgitt på illustrasjonen av 50 pencen under.

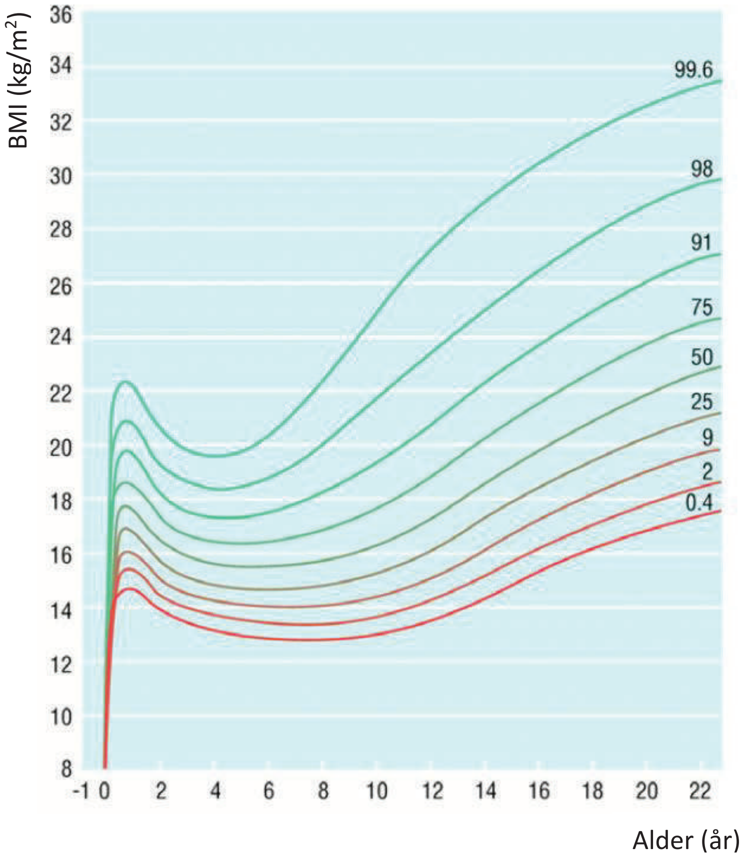


C13

Tolke komplekse eller uvanlige grafer eller tabeller, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Tolk grafen nedenfor og forklar hvordan *body mass index* (BMI) til menn tilhørende de nederste 2 prosentene er sammenlignet med BMI til menn i de øverste 2 prosentene.



C14

Løse et ligningssett som har to løsninger og består av en lineær og en andregradslikning, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

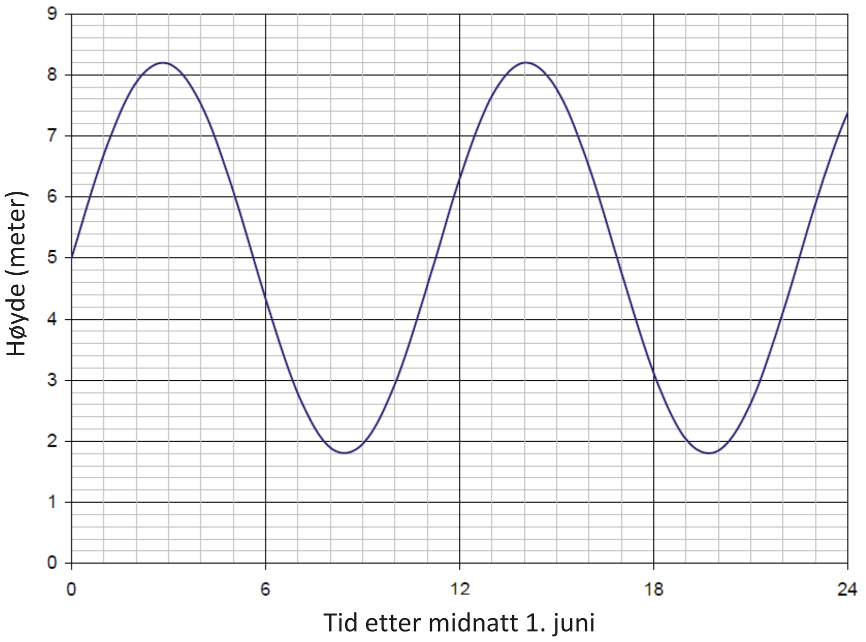
Finn de to skjæringspunktene til den rette linjen $y + x = 5$, og parabelen $y = x^2 - 2x + 1$.

C15

Lese og tolke data fra trigonometriske grafer og bruke dem til å løse praktiske problem, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Grafen nedenfor, som viser vannhøyden i løpet av et døgn på en tenkt kyststrekning, viser at tiden mellom høydevann ikke er 12 timer. Gjør overslag for tidspunkter for høy- og lavvann i løpet av uken som følger etter dette døgnet.



C16

Lage og bruke algebraiske formler for å løse problemer, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

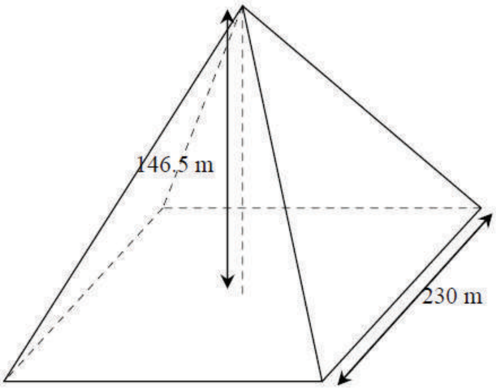
Skriv en formel som vil gi MVA betalt i T kr når den totale prisen på en vare er P kr.
Anta at MVA er 25 %.
Bruk din egen formel til å finne totalprisen på en bil når du har betalt kr 75 000,- i MVA.

C17

Løse 3-dimensjonale problemer ved å bruke Pythagoras' setning og trigonometri, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Den store pyramiden ved Giza i Egypt har målene som vist på tegningen nedenfor. Finn vinkelen som en trekantet sideflate danner med den (kvadratiske) grunnflaten.



C18

Lage og bruke algebraiske formler for å løse praktiske problem, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

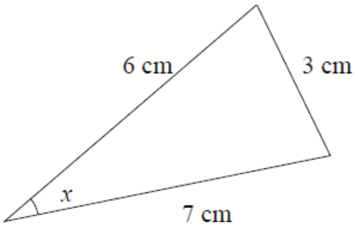
Anta at huspriser stiger med gjennomsnittlig 7 % hvert år.
Et hus har en verdi på V kr nå.
Ved å skrive en formel for V så kan du estimere fremtidig verdi på huset, $V(t)$, t år fram i tiden.
Finn forventet verdi på huset om 15 år hvis dagens verdi er satt til 4 750 000 kr.

C19

Løse oppgaver om trekanter som ikke er rettvinklet ved å bruke trigonometri, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Bruk cosinussetningen $a^2 = b^2 + c^2 - 2bc\cos A$ til å finne vinkelen som er markert med x.



C20

Finn formler ved å bruke kompliserte data, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Tabellen viser en kjørerute fra Warrington til Kendal i England ved å bruke ulike veityper. Finn formlene som har vært brukt til å beregne estimat for tid brukt på (i) sidegater (markert med gatenavn), (ii) hovedveier (markert med A) og (iii) motorveier (markert med M).

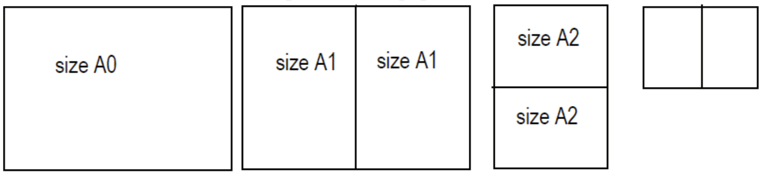
	Instruksjon	Avstand	Avstand så langt	Tid
1	Kjør fra Warrington	0,2 km	0 km	10:00
2	Ta til venstre inn på Legh Street	0,2 km	0,2 km	10:00
3	Ta til høyre inn på A57	0,4 km	0,2 km	10:00
4	Ta til venstre inn på Winwick Street	0,2 km	0,8 km	10:01
5	Hold til venstre på sideveien	0,2 km	1 km	10:01
6	Ta andre avkjørsel i rundkjøringen inn på Pinners Brow	0,2 km	1,2 km	10:02
7	Hold til venstre	0 km	1,4 km	10:02
8	Ta til venstre inn på Lythgoes Lane	1,8 km	1,4 km	10:02
9	Ta andre avkjørsel i rundkjøringen inn på A49	1 km	3,2 km	10:04
10	Ta første avkjørsel i rundkjøringen inn på A49	1,2 km	4 km	10:05
11	Ta første avkjørsel i rundkjøringen inn på A49	0,8 km	5,2 km	10:06
12	Ta tredje avkjørsel i rundkjøringen inn på A49	2,2 km	6 km	10:06
13	Ta første avkjørsel i rundkjøringen inn på M6-Jn22	0,4 km	8,4 km	10:08
14	Fortsett rett fram inn på M6	120,4 km	8,8 km	10:09
15	Ta av til venstre inn på M6-J36	0,4 km	129 km	11:04
16	Ta første avkjørsel i rundkjøringen inn på A590	6,6 km	129,4 km	11:05
17	Forsett rett fram inn på A591	4,2 km	136 km	11:09
18	Ta til venstre på sideveien	0,2 km	140,2 km	11:11
19	Ta til venstre inn på A6	4 km	140,2 km	11:11
20	Ta til venstre inn på Lowther Street	0 km	144,2 km	11:15
21	Du er framme i Kendal	0 km	144,2 km	11:15

C21

Løse praktiske problem ved å bruke forhold og proporsjoner, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Hvert av arkene som i figuren nedenfor er omtalt som A0-, A1-, A2-, A3-, A4-papir er laget slik at forholdet mellom lengden og bredden til hvert ark alltid er $\sqrt{2}: 1$.
A0-ark har et areal på 1 kvadratmeter.
Størrelsen A1 er laget slik at når en legger to A1-ark inntil hverandre så vil summen av breddene til disse to arkene være lik lengden til A0-arket.
Størrelsen A2 er laget slik at når en legger to A2-ark inntil hverandre så vil summen av breddene til disse to arkene være lik lengden til A1-arket, også videre.
Beregn dimensjonene til at A4-arket, med to desimaler i svaret.

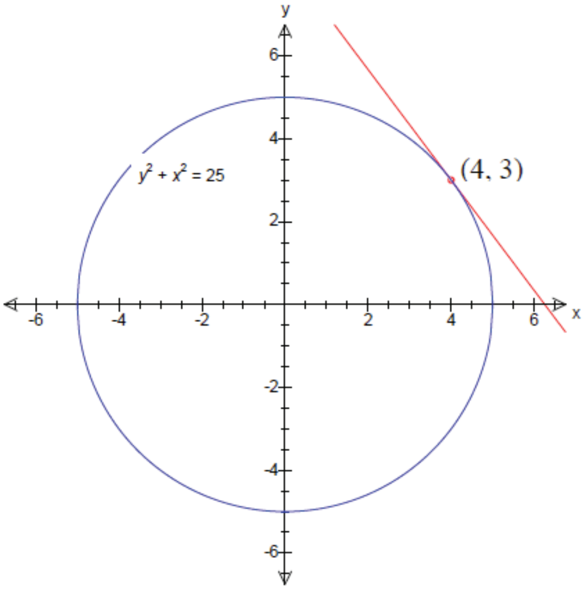


C22

Bruke egenskaper til stigningstall til linjer som står vinkelrett på hverandre til å løse geometriske problemer, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Finn stigningstallet til tangenten til sirkelen i punktet (4,3).

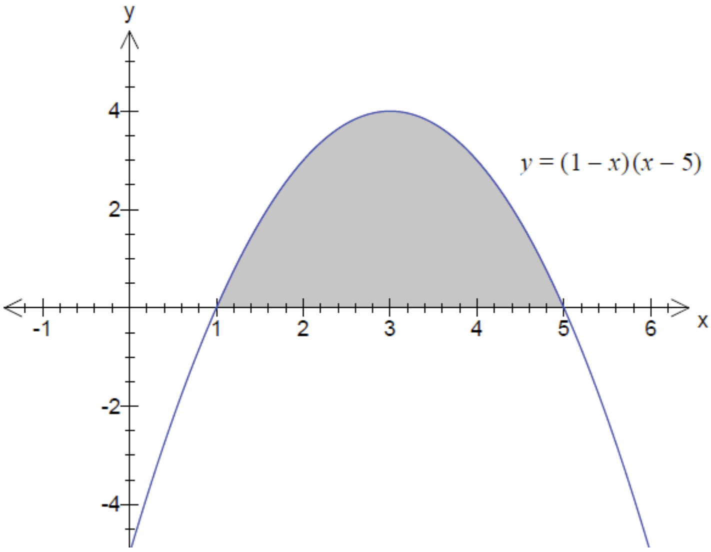


C23

Bruke algebra og integralregning til å beregne areal under kurver, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Beregn arealet til det skyggelagte arealet.



C24

Bruke komplekse algebraiske modeller til å beregne størrelser i problemløsning, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

I sterk vind kan temperaturen kjennes lavere enn den faktiske temperaturen, T (målt i °C). Vi snakker da om den effektive temperaturen.
Den effektive temperaturen, T_w°C, er gitt ved formelen:
$T_w \ = \ 13{,}112 + 0{,}6215T - 11{,}37V^{0{,}16} + 0{,}3965TV^{0{,}16}$, hvor V er vindens hastighet målt i km/t.

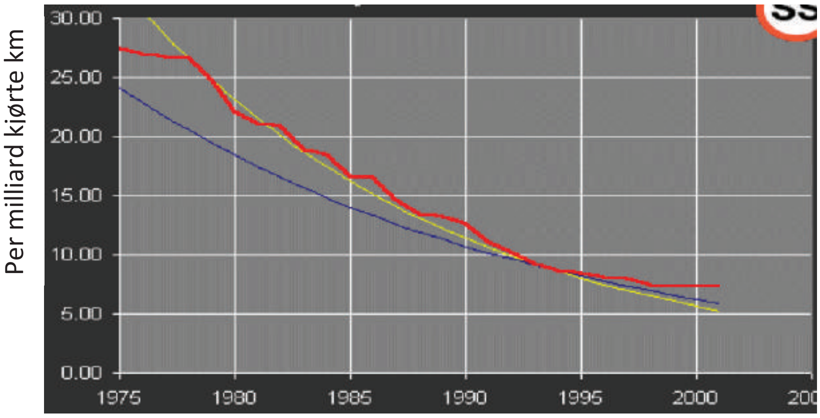
Bruk denne formelen til å finne den effektive temperaturen en dag den faktiske temperaturen er 5°C og vinden måles til 25 km/t.

C25

Finne en funksjon for å modellere data ved å plotte en rett linje ved å transformere data ved å bruke logaritme, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Transformer grafen under slik at den har en logaritmisk skala på den vertikale aksen for så å finne en eksponentialfunksjon som modellerer hvordan dødsfall på veiene går ned over tid.



C26

Bruke komplekse algebraiske modeller og faktiske data til å beregne størrelser i problemløsning, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Finn det totale varmetapet per time, Q(målt i Btu/t), i et typisk klasserom ved å anslå størrelsen på veggene, taket, gulvet, dører og vindu, og ved å bruke typiske temperaturer og informasjonen nedenfor.

Formelen $Q = UA(T_{inne} - T_{ute})$ gir varmetapet per time gjennom et materiale der $U(Btu/t - ft^2 - °\text{F})$ er varmeoverføringskoeffisienten. A er arealet, T_{inne} er innetemperaturen og T_{ute} er utetemperaturen.

Tabell for ulike U-verdier.		$Btu/t - ft^2 - °\text{F}$
Yttervegg	11'' isolert murstein	0,10
Innervegg	Gipsplater, 4'' mellomrom, gipsplate	0,32
Gulv	Tømmer	0,12
Tak	Flatt, 50 mm isolert	0,12
Vindu	Tre/ dobbelt glass	0,51
Dør	Ytterdør i solid tømmer	0,42
	innerdør	Anta samme som innervegg

C27

Gjennomføre komplekse finansielle beregninger, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Et lån på 500 000 kr har en månedlig rente på 0,00667 %. Du betaler 4000 kr i måneden. Hvor lang tid vil du bruke på å halvere lånet?

C28

Tolke kompliserte datasett, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

I tabellen nedenfor presenteres resultatene fra en standardisert test gitt ved en skole, ved å sammenlikne prestasjonen med det som kalles for benchmark, det vil si statistisk data over testresultatene til elever som gikk på tilsvarende type skoler i tidligere år. Den første tabellen viser hvor stor prosentandel av elevene på denne skolen som oppnår nivå 5 på testene i engelsk, matematikk og naturfag, og hva det betyr for skolens rangering sammenliknet med skolene som danner grunnlag for benchmarks. Den andre tabellen gjør det samme for nivå 6.

Eksempel: 60 % av skolens elever presterte på nivå 5 eller over i engelsk.

Tolk hvordan denne skolens elever presterte i hvert av fagene og på tvers av fag.

Sammenligning av elever som oppnår nivå 5

Prosentil	95.	Øvre kvartil	60.	40.		Nedre kvartil		5.
Engelsk	83	75	70	64	**60**	59		47
Matematikk	80	73	70	66		62	**62**	55
Naturfag	81	74	70	66		62	**58**	52

Sammenligning av elever som oppnår nivå 6

Prosentil	95.	Øvre kvartil	60.	40.		Nedre kvartil		5.
Engelsk	49	36	31	25		21	**20**	12
Matematikk	56	48	41	41	**39**	37		30
Naturfag	45	36	32	28	**28**	24		16

C29

Bruke beregninger som involverer volum og ulike mål til å løse problemer, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

Tabellen under viser hvordan omkretsen til et glass varierer med høyden. Ved å modellere glasset ved å bruke en serie konsentriske sylindere, finn høyden til dit du må fylle glasset for å få et halvt glass med øl.

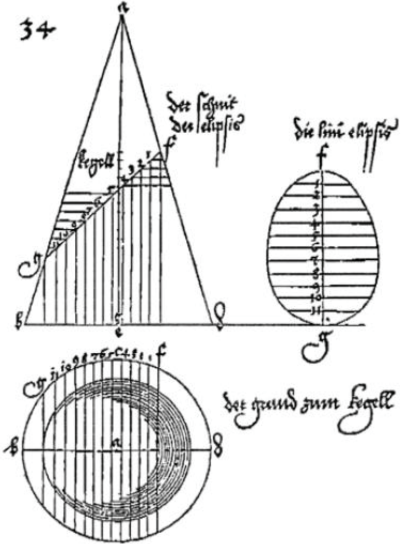
Høyde (cm)	Omkrets (cm)
0,0	0,0
0,3	8,7
1,0	16,6
2,0	22,6
3,0	26,4
4,0	29,8
5,0	32,0
6,0	33,3
7,0	32,0
8,0	30,0
8,7	27,3

B30

Forstå og tolke matematiske diagram, slik som:

Ikke trygg	Litt trygg	Trygg	Veldig trygg
1	2	3	4

En kjegle kan kuttes med et plan for å danne ulike figurer. Diagrammet nedenfor finnes i et arbeid utført av den tyske kunstneren og matematikeren Durer og viser hvordan et elliptisk snitt kan lages. Forklar hvordan ellipsen kan tegnes helt presist ved å ta målene på fronten på kjeglen og planfiguren vist nedenfor.



Section C Using mathematics

In this section you are asked to say how confident you would be at using mathematics to solve different problems. **You are not asked to actually solve the problems.**

Imagine that you have been given the following maths questions to do, perhaps for homework. You would be able to use your notes, textbook(s), calculator and so on when necessary. You are asked to rate how confident you are that you will be able to solve each problem, **without actually doing the problem**, using a scale from 1(=not confident at all) to 4(= very confident).

For example, suppose that you are asked to say how confident you are that you are able to solve the problem below, and that you felt not very confident, you would circle 2 in the box.

Example:
Solving practical problems involving money using calculations, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	[✓ 2]	3	4

Calculate which jar of coffee is better value.

Instant Cappuccinno Regular 160g costs £1.98
Medium Roast Granules 220g costs £2.50

How confident are you that you are able to solve problems of the kind given in each case? Circle the description that best describes your level of confidence.

Remember, you do not need to solve the problems.

C1
Solving practical problems involving money using calculations, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

Calculate which coach journey is better value in terms of pence per mile.

Birmingham – London, 110 miles, cost £14.50
Oxford – Leeds, 170 miles, cost £24.60

Remember, you do not need to solve the problems.

C2

Costing a project involving everyday arithmetic involving money and complex data tables from the internet, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

Use the information below to calculate the cost of a holiday for a group of six travellers. There are two couples and two single travellers in the group. They wish to start the main holiday on August 5th, with one couple and one single traveller extending the holiday for 7 nights in Sorrento in a room with a sea view and with half board.

2006 - Saturdays
per person in a twin room

Date	Price
Apr 8	£845
Apr 15	£845
Apr 22	£855
Apr 29	£855
May 6	£865
May 13	£865
May 20	£855
May 27	£855

Date	Price
Jun 3	£875
Jun 10	£875
Jun 17	£855
Jun 24	£855
Jul 1	£745
Jul 8	£745
Jul 15	£735
Jul 22	£735

Date	Price
Jul 29	£755
Aug 5	£745
Aug 12	£775
Aug 19	£775
Aug 26	£775
Sep 2	£875
Sep 9	£875
Sep 16	£865

Date	Price
Sep 23	£865
Sep 30	£865
Oct 7	£875
Oct 14	£855
Oct 21	£845

Supplements per person

Supplement	Price
Single supplement	£170

Extension

7-nights Sorrento	Price
Grand Hotel Vesuvio	£375
(Aug - Oct)	£425
Single supplement	£70
Sea view	£75
Half board (lunch or dinner)	£50
Sea view	£75

Price Includes
Air travel, UK departure taxes, overseas airport taxes, all transportation, breakfast daily, dinner on days 4 & 7, itinerary as described, tour escort and official city guides, guidebook
Not Included

CP/1

C3

Understanding and using metric measures, units and notation, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

The table gives the lengths of rivers in metres but expressed in different ways.

Put these rivers in decreasing order of length.

River	Length
Amazon	6.39×10^6 metres
Yellow	4.67×10^9 millimetres
Nile	6690000000 millimetres
Yangtze	6380 kilometres
Congo	4371000 metres
Mississippi	6.27×10^3 kilometres

M/1

Remember, you do not need to solve the problems.

C4
Solving problems involving calculations with measures and units, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

Find how many boxes with dimensions 50 cm by 40 cm by 40 cm could fit into a shipping container 6 metres by 2.5 metres by 2.5 metres

P/1

C5
Solving linear equations with x on both sides of the equation, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

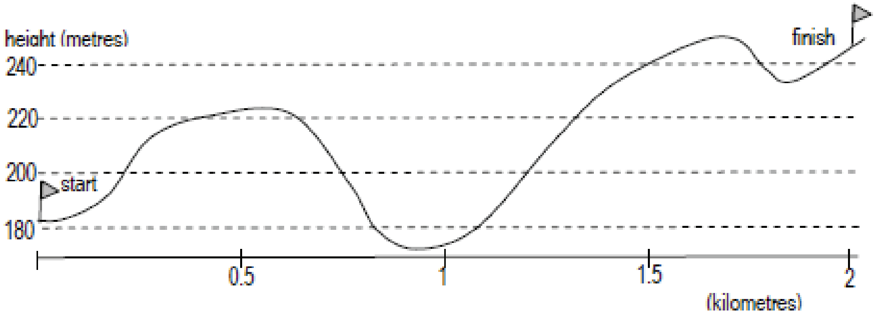
Solve for x:
$15 - 2x = 3x + 25$

A/1

C6
Solving practical problems using charts, arithmetic and scales, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

The diagram below shows the height / distance profile of a walk in Derbyshire with height in metres and horizontal distance in kilometres.
Taking into account these different scales estimate as accurately as you can, using Pythagoras' Theorem, the distance walked down hill.



MD/2

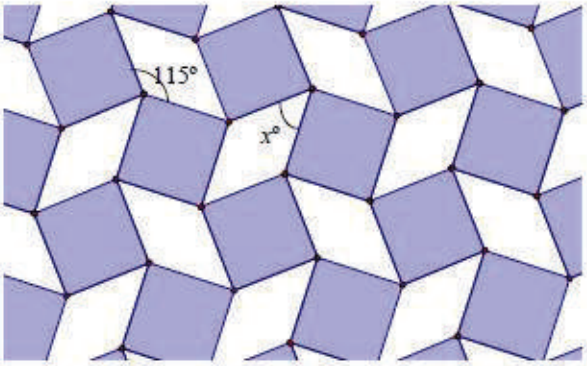
Remember, you do not need to solve the problems.

C7

Solving problems using geometrical properties, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

The diagram shows a tiling pattern formed by tessellating squares and parallelograms. Find the angle marked $x°$.



MD/1

C8

Solving practical problems involving quadratic equations, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

A golfer hits a ball so that its height, h metres, above horizontal ground is given by $h = 20t - 5t^2$. Find when the ball is 5 metres above the ground by solving $5 = 20t - 5t^2$.

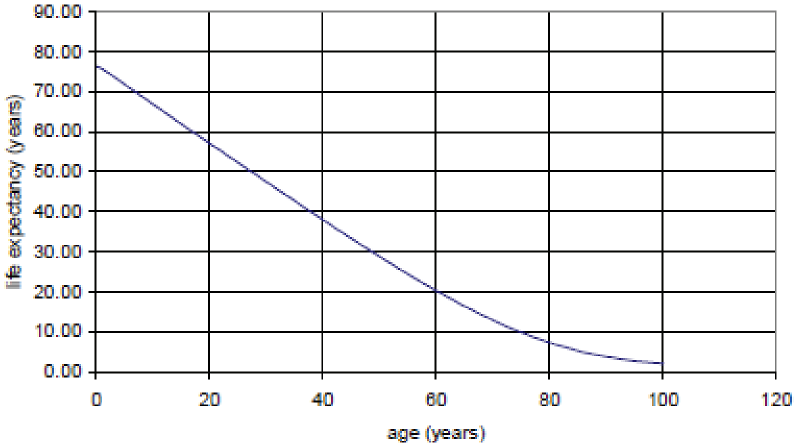
A/2

Remember, you do not need to solve the problems.

C9
Finding a formula to describe experimental results, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

The graph below shows how male life expectancy, *E* years, varies with age, *A* years. Find a linear formula connecting *E* and *A* for males aged between 0 and 60 years.

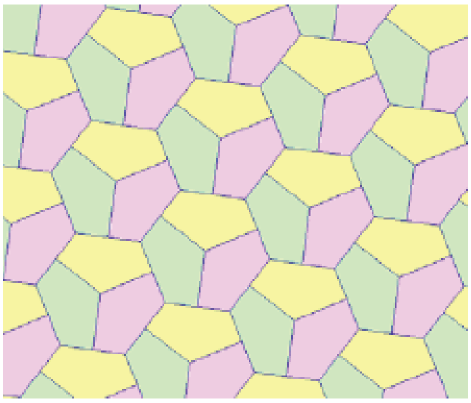


G/1

C10
Calculating the areas of complex shapes using Pythagoras' theorem or trigonometry, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

The diagram shows a tiling pattern. Calculate the area of one of the pentagons if you know the length of the edge of one of the hexagons is 6 centimetres.



M/2

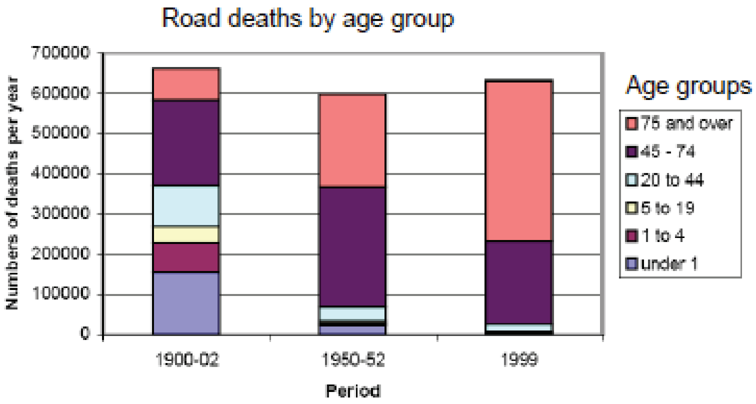
Remember, you do not need to solve the problems.

C11

Interpreting complex or unfamiliar graphs and charts, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

Interpret the graph below to describe how road casualties of some different age groups have changed over time.



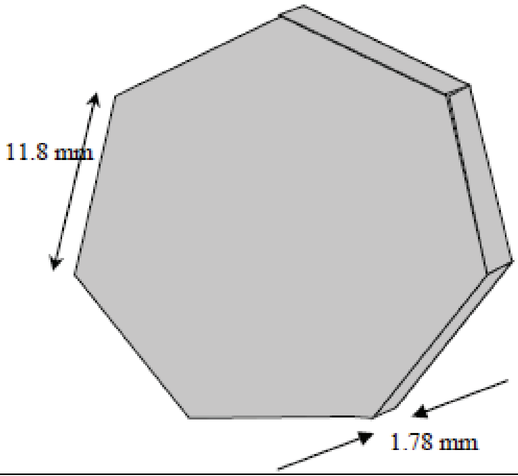
D/1

C12

Modelling or solving problems involving calculating the volumes of more complex shapes using Pythagoras' theorem or trigonometry, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

A 50 pence piece can be modelled as a prism with a regular seven sided figure as its cross section. Using the dimensions in the diagram find the volume of metal in a 50 pence piece.



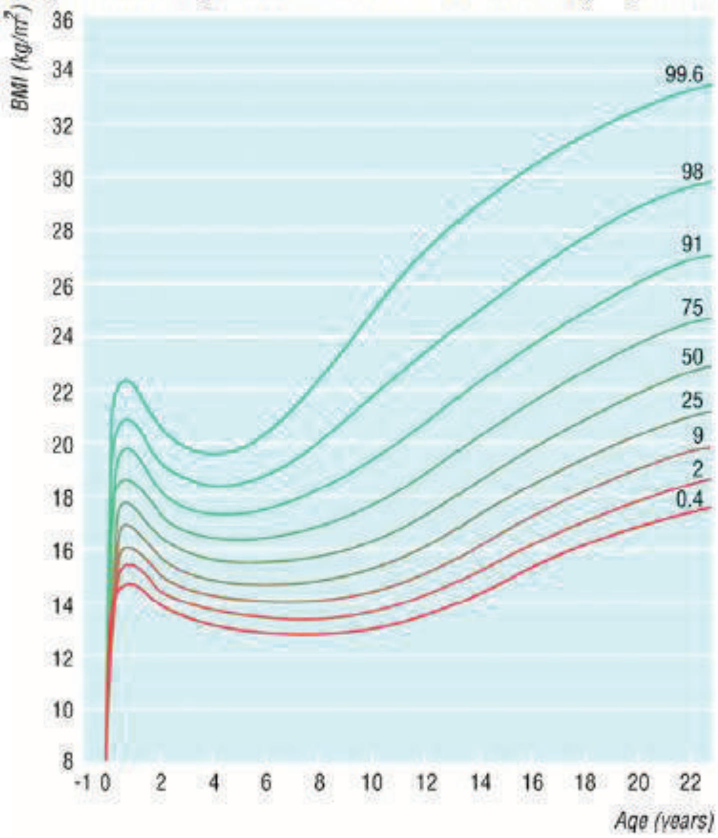
M/2

Remember, you do not need to solve the problems.

C13
Interpreting complex or unfamiliar graphs and charts, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

Interpret the graph below to explaining how the body mass index (BMI) of males in the bottom 2 per cent compares with that of those in the top 2 per cent.



D/2

C14
Solving a linear and a quadratic equation simultaneously when there are two different solutions, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

Find the two points of intersection of the straight line, $y + x = 5$, with the parabola, $y = x^2 - 2x + 1$.

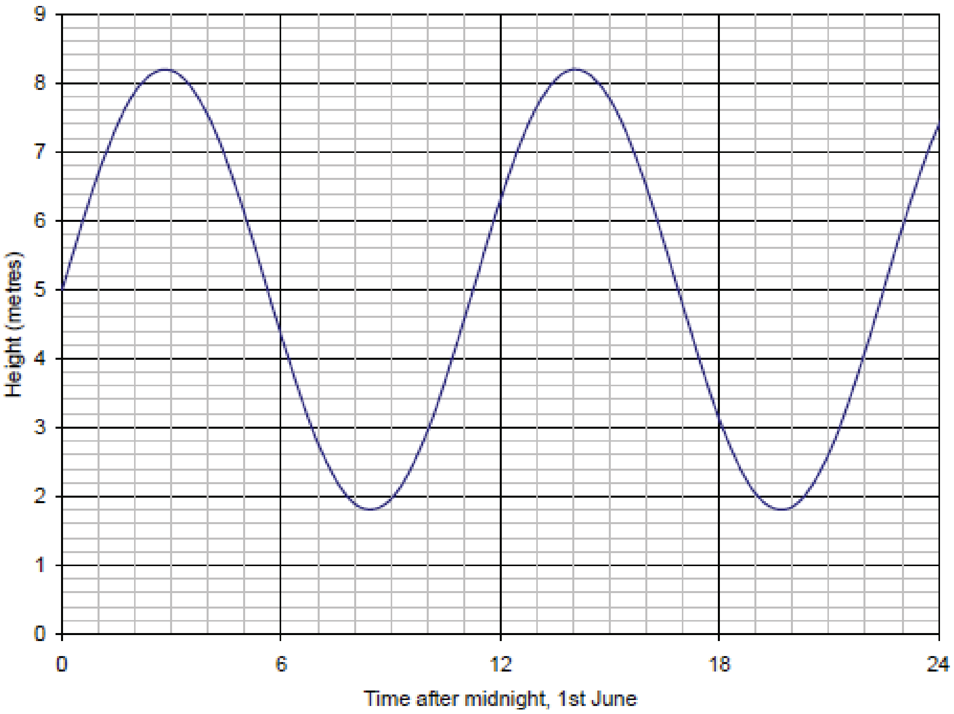
A/2

Remember, you do not need to solve the problems.

C15
Reading and interpreting data from trigonometric graphs and using these to solve practical problems, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

The graph below of the height of water during one complete day at Fleetwood, on the Lancashire coast, shows that the time between high tides is not twelve hours.
Estimate the **times** of high and low tides one week after the day shown on this graph.



G/2

C16
Making and using algebraic formulae to solve problems, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

Write down a formula that will give the VAT (tax) paid, £*T*, if the total cost of an item is £*C*. Assume that the rate of VAT is 17.5%.
Use your formula to find the total cost of a car if the tax paid is £750.

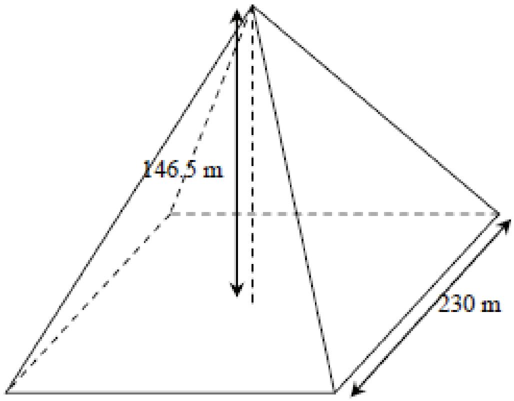
F/1

Remember, you do not need to solve the problems.

C17
Solving 3-dimensional problems using Pythagoras' theorem and trigonometry, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

The Great Pyramid at Giza in Egypt has dimensions as shown in the diagram.
Find the angle that a triangular face makes with its base which may be assumed to be square.



M/2

C18
Making and using algebraic formulae to solve practical problems, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

Assume that on average house prices rise 7% every year.
A house is valued at £V now.
By writing down a formula, in terms of V, that will allow you to estimate the future value, £$V(t)$, of a house in t years time, find the expected value of a house in 15 years time if it is valued at £175000 now

F/2

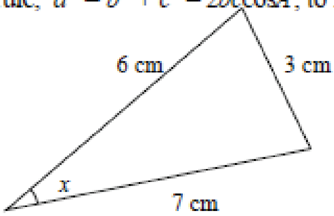
Remember, you do not need to solve the problems.

C19

Solving triangles that are not right angled using trigonometry, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

Use the cosine rule, $a^2 = b^2 + c^2 - 2bc\cos A$, to find the angle marked x.



A/1

C20

Finding formulas using complex data, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

The table shows a route from Warrington to Kendal using different types of roads. Find the formulae that have been used to calculate the estimates of times on (i) side streets, (ii) A roads and (iii) motorways.

	Instruction	Distance	Distance so far	Time
1	Depart Warrington	0.1 miles	0 miles	10:00
2	Turn left into Legh Street	0.1 miles	0.1 miles	10:00
3	Turn right onto A57 Midland Way	0.2 miles	0.2 miles	10:00
4	Turn left into Winwick Street	0.1 miles	0.4 miles	10:01
5	Bear left	0.1 miles	0.5 miles	10:01
6	At the roundabout, take the second exit into Pinners Brow	0.1 miles	0.6 miles	10:02
7	Turn left onto slip road	0 miles	0.7 miles	10:02
8	Turn left onto Lythgoes Lane	0.9 miles	0.7 miles	10:02
9	At the roundabout, take the second exit onto A49	0.5 miles	1.6 miles	10:04
10	At the roundabout, take the first exit onto A49	0.6 miles	2 miles	10:05
11	At the roundabout, take the first exit onto A49	0.4 miles	2.6 miles	10:06
12	At the roundabout, take the third exit onto A49	1.1 miles	3 miles	10:06
13	At the roundabout, take the first exit onto M6 Jn 22	0.2 miles	4.2 miles	10:08
14	Continue straight ahead onto M6	60.2 miles	4.4 miles	10:09
15	Exit to the left onto M6 J36	0.2 miles	64.5 miles	11:04
16	At the roundabout, take the first exit onto A590	3.3 miles	64.7 miles	11:05
17	Continue straight ahead onto A591	2.1 miles	68 miles	11:09
18	Turn left onto slip road	0.1 miles	70.1 miles	11:11
19	Bear left onto A6	2 miles	70.1 miles	11:11
20	Turn left onto Lowther Street	0 miles	72.1 miles	11:15
21	Arrive Kendal	0 miles	72.1 miles	11:15

F/2

Remember, you do not need to solve the problems.

C21

Solving practical problems using ratio and proportion.

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

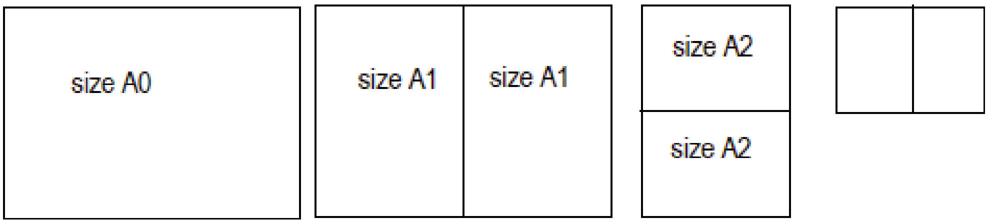
The A0, A1, A2, A3, A4 series of paper sizes is such that the ratio of length to width of each sheet is always $\sqrt{2}$: 1.

The A0 paper has an area of 1 square metre.

Size A1 is formed by aligning two sheets so that the sum of their widths is the length of size A0.

Size A2 is formed by aligning two sheets so that the sum of their widths is the length of size A1, and so on.

Calculate the dimensions of a piece of A4 paper, correct to the nearest millimetre.



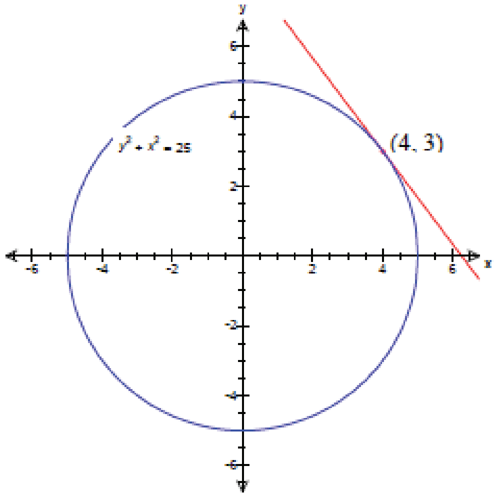
P/3

C22

Using properties of gradients of perpendicular lines to solve problems in geometry, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

Find the gradient of the tangent to the circle at the point (4, 3).



A/3

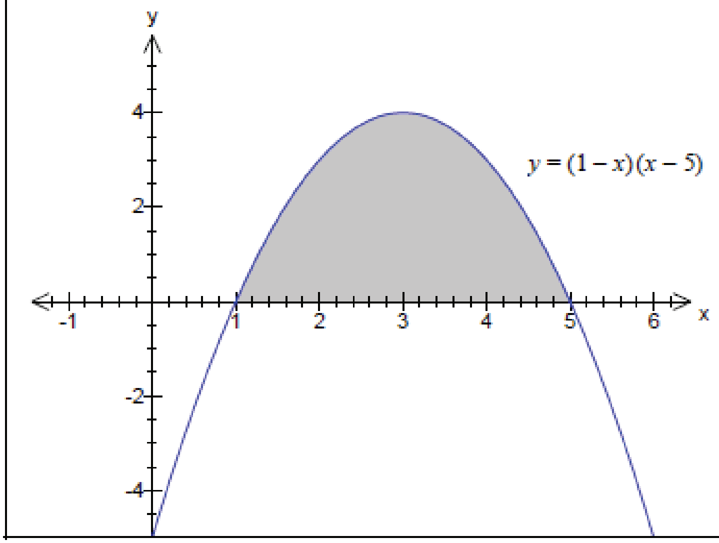
Remember, you do not need to solve the problems.

C23

Using algebra and integral calculus to calculate areas under curves, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

Calculate the shaded area.



A/3

C24

Using complex algebraic models to calculate quantities in solving problems, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

In high winds the temperature appears colder than the actual temperature, T°C. The apparent temperature, T_w°C, is given by the formula:

$$T_w = 13.112+0.6215T - 11.37V^{0.16} + 0.3965TV^{0.16}$$

where V is the wind speed in kilometres per hour.

Use this to find the apparent temperature on a day when the actual temperature is 5°C but a wind is blowing at 25 kilometres per hour.

F/3

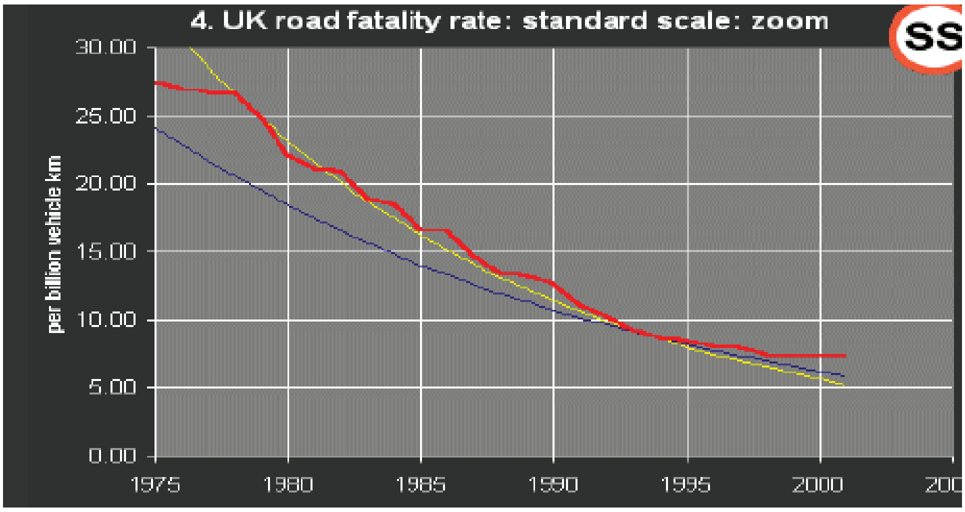
Remember, you do not need to solve the problems.

C25

Finding a function to model data by plotting a straight line graph by transforming data using logarithms, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

Transform the graph below so that it has a logarithmic scale on the vertical axis to find an exponential function that models how road fatalities are falling with time.



G/3

C26

Using complex algebraic models and real data to calculate quantities in solving problems, such as

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

Find the total hourly heat loss, Q (Btu/hr), from a typical classroom by estimating the sizes of walls, ceiling, floor, doors and windows, and values for typical temperatures and using the information below.

The formula $Q = UA(T_{in} - T_{out})$ gives the heat loss through a material where U (Btu/hr-ft^2-°F) is its heat transfer coefficient, A its area, T_{in} the inside temperature and T_{out} the outside temperature for the room.

U values for typical materials.

		Btu/hr-ft^2-°F
Wall (outer)	11˝ brick-block cavity insulated	0.10
Wall (internal)	plasterboard, 4˝ studding, plasterboard	0.32
Floor (ground)	suspended timber	0.12
Ceiling	flat, 50 mm insulation	0.12
Window	wooden/upvc frame double glazed	0.51
Door	external solid timber	0.42
	internal	assume the same as internal wall

C/3

Remember, you do not need to solve the problems.

C27
Carrying out complex financial calculations, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

A loan for £50,000 is charged interest at 0.00667% per month. You pay £400 per month. After how long will the amount remaining to be paid be half of the original loan, i.e. £25,000

CP/3

C28
Interpreting complex data sets, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

The tables below show how a school's pupils perform in tests compared with schools containing pupils who performed at a similar level in previous tests.
The school's percentages performing at level 5 or above and level 6 or above in English, Mathematics and Science are shown in bold.
For example, 60% of the school's pupils achieved level 5 or above in English and this was below the 40^{th} percentile and above the lower quartile for similar schools.
Interpret how this school's pupils performed in each subject and across subjects.

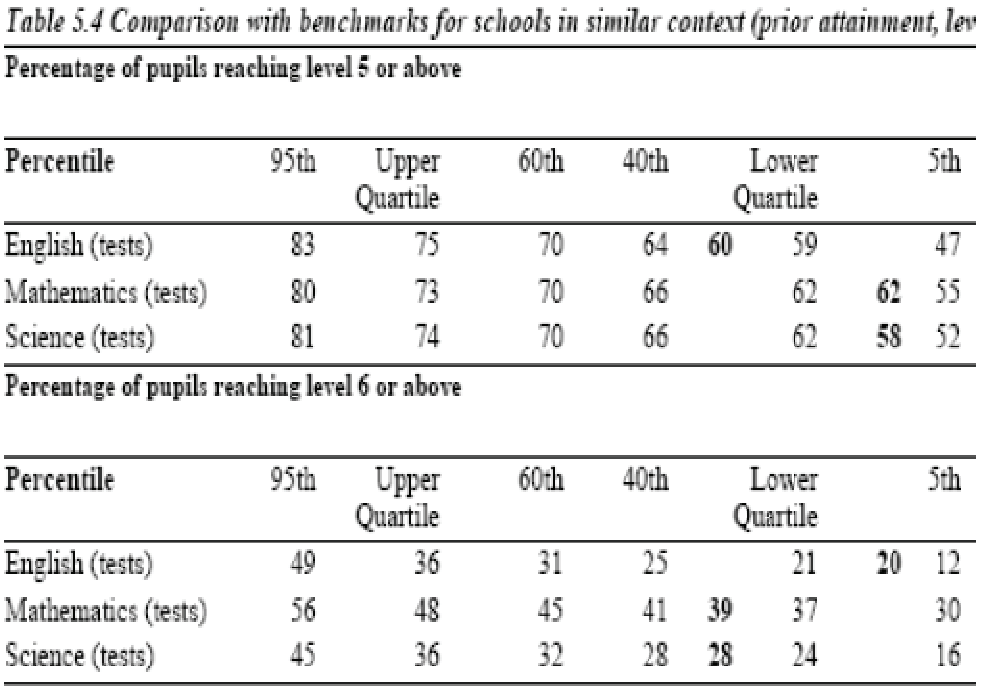


Table 5.4 Comparison with benchmarks for schools in similar context (prior attainment, lev

Percentage of pupils reaching level 5 or above

Percentile	95th	Upper Quartile	60th	40th		Lower Quartile		5th
English (tests)	83	75	70	64	**60**	59		47
Mathematics (tests)	80	73	70	66		62	**62**	55
Science (tests)	81	74	70	66		62	**58**	52

Percentage of pupils reaching level 6 or above

Percentile	95th	Upper Quartile	60th	40th		Lower Quartile		5th
English (tests)	49	36	31	25		21	**20**	12
Mathematics (tests)	56	48	45	41	**39**	37		30
Science (tests)	45	36	32	28	**28**	24		16

D/3

Remember, you do not need to solve the problems.

C29
Using calculations involving volume and different measures to solve problems, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

The table below shows how the circumference of a glass varies with height. By modelling the glass using a series of concentric cylinders, find the height to which you would need to fill the glass so that you have half a glass of beer.

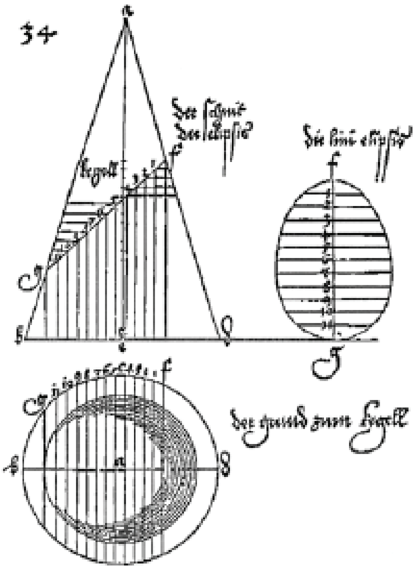
Height cm	Circumference cm
0.0	0.0
0.3	8.7
1.0	16.6
2.0	22.6
3.0	26.4
4.0	29.8
5.0	32.0
6.0	33.3
7.0	32.0
8.0	30.0
8.7	27.3

M/3

C30
Understanding and interpreting mathematical diagrams, such as:

not confident at all	Not very confident	fairly confident	Very confident
1	2	3	4

A cone can be cut by a plane to from different shaped figures. The diagram below can be found in a work by the German artist and mathematician Durer showing how an elliptical cross-section can be formed.
Explain how the ellipse can be accurately drawn by taking measurements taken from the front elevation and plan shown in the diagram below.



MD/3

Forespørsel om deltagelse i forskningsprosjekt

NOKUT evalueringen av allmennlærerutdanningen i 2006 viser at lærerstudenter opplever «lite sammenheng og systematikk i forholdet mellom høyskolene og praksisskolene» og etterlyser «en sterkere kobling mellom utdanningens teoretiske komponenter og det som skjer i praksisperiodene» (Hansén, 2006). Dette er bakgrunnen for forskningsprosjektet *Matematikk i praksisopplæring* (MAPO).

I tilknytning til MAPO prosjektet ble det utlyst en stipendiatstilling for PhD-graden. Datainnsamlingen som skal foregå høst 2013 til vår 2015 skal inngå i PhD-prosjektet *Pre-service teachers' developing theorizing about mathematics teaching during school placement*.

Målet er å samle inn datamateriale for å forstå utfordringene som møter studenter fra grunnskolelærerutdanningen 1-7 i forbindelse med matematikkfaget ved høyskolen og matematikkfaget i praksisopplæringen. Det er studentene selv som kan hjelpe med å forstå disse utfordringene ved at de meddeler sitt forhold til matematikk, sine forventninger til praksisopplæringen, og sine refleksjoner underveis og etter praksisopplæringen. Stipendiaten ber derfor om å få intervjue studentene i forkant og etterkant av de 3 praksisperiodene høst 2013, vår 2014 og høst 2014, samt om å dele sine refleksjoner gjort underveis i praksisen. Deltakerne vil oppfordres til ikke å bruke navn i intervjuene og refleksjonene.

Intervjuene/ samtalene vil ledes av stipendiaten som ikke samtidig innehar rollen som faglærer for studentene. Stipendiaten fra HiOA kommer til å ta lydopptak av disse intervjuene. Intervjuene vil bli transkribert og anonymisert. Kun stipendiaten og veiledere får tilgang til lydfilene.

I forbindelse med eventuell publiseringen vil alle data bli anonymisert slik at de ikke kan spores tilbake til studenter, lærere eller skole. Stipendiaten er underlagt taushetsplikt og alle opplysninger vil bli behandlet konfidensielt. Lydfiler og koblingsnøkkel som er oppbevart konfidensielt av HiOA vil bli slettet 1. august 2017. Anonymiserte transkripsjoner og notater vil bli passordbeskyttet, og kun stipendiat og veiledere vil ha tilgang til disse filene.

Prosjektet er godkjent av NSD (Norsk samfunnsvitenskapelige datatjeneste). Det er frivillig å delta og du kan når som helst trekke deg uten begrunnelse, og uten at det påvirker ditt forhold til HiOA.

Stipendiat: Annette Hessen Bjerke ***Veiledere:*** Prof. Yvette Solomon og Claire Berg

Samtykkeerklæring

Jeg har lest ovennevnte og ønsker å delta i datainnsamlingen til PhD-prosjektet *Pre-service teachers' developing theorizing about mathematics teaching during school placement*.

Jeg er innforstått med at forskerne får tilgang til lydopptak av intervjuene. Dette tillater jeg.

Navn: ____________________________

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Underskrift ***Dato***

Invitation to participate in a research project

In 2006, an external evaluation of teacher education programmes claimed that pre-service teachers experienced "a lack of coherence and system in the relationship between schools and practice schools", and called for a "stronger link between the theoretical components of the programme and what happens in the school placement" (Hansén, 2006). This is the background for the research project Mathematics in school placement (Matematikk i praksisopplæringen, MAPO).

A PhD position was established in connection with the MAPO project. Data collection within the PhD project *Pre-service teachers' developing theorizing about mathematics teaching during school placement* will take place between the autumn of 2013 and the spring of 2015.

The goal is to collect data in order to understand the challenges that meet preservice teachers from the teacher education programme for grades 1-7 in connection with the mathematics courses at the UC and the subject mathematics in school placement. The preservice teachers can shed light on the challenges by expressing their relationship to mathematics, their expectations from school placement, and their reflections during and after school placement. The candidate asks therefore to be allowed to interview the participants before and after the three school placements in the autumn of 2014, and the spring and autumn of 2015, and asks the participants to share their reflections during school placement. The participants are asked not to use names in the interviews and reflections.

The interviews will be led by the PhD candidate who does not hold a double role as a teacher educator for the participants. The candidate will take audio recordings of the interviews. These will be transcribed and anonymized. Only the candidate and the supervisors will have access to the audio files.

In connection with potential dissemination of findings all data will be anonymised so that it cannot be connected to the preservice teacher, the teacher or the school. The PhD candidate has a duty of ensuring confidentiality and all information will be handled accordingly. Audio recordings and records connecting those to the individuals will be stored confidentially at Oslo University College and will be deleted on August 1st, 2017. Anonymised transcriptions and notes will be stored protected by passwords and only the PhD candidate and the supervisors will have access to these files.

The project is approved by NSD (Norwegian Centre for Research Data). Participation is voluntary and you may withdraw at any time without giving an explanation, and without this affecting your relationship to the Oslo and Akershus University College.

Stipendiat: Annette Hessen Bjerke ***Supervisors:*** Prof. Yvette Solomon and Claire Berg

Consent form

I have read the information given above and would like to participate in the PhD-project *Pre-service teachers' developing theorizing about mathematics teaching during school placement*.

I understand that the researches have access to the recorded interviews and I allow this.

Name: ___________________________

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Signature ***Date***

Intervju 1

Bakgrunn

- Hva er ditt forhold til matematikk?
- Hva har påvirket ditt forhold til matematikk?
- Hva tenker du om å bli matematikklærer?
- Hadde du valgt matematikk som fag dersom det ikke var obligatorisk?
- Hvordan lærer du matematikk best?
- Tror du at lærerutdanningen vil støtte opp om din foretrukne måte å lære matematikk på?

Med utgangspunkt i svarene på det testen som hele trinnet tok

- Er det noe du har lyst å kommentere når det gjelder testen?
- Vil du si at du har god selvtillit når det gjelder matematikk?

Jeg ber dem om å utdype og kommentere noen av sine egne svar, samt tendenser i datamaterialet samlet. Når jeg viser til min oppfatning av dem gjennom analyser av testen, spør jeg

- Stemmer det bildet jeg har laget meg av deg?

Kommende praksisperiode

- Hvilke forventninger har du til praksis?
- Hva tror du blir viktig for at praksisperioden skal bli bra?
- Har du lyst å undervise matematikk i første praksisperiode dersom du får muligheten?

Intervju 2

Om praksis

- Hvordan er det å være tilbake fra praksis?
- Hvordan opplevde du praksis?
- Var det noe som overrasket deg i praksis?
- Kan du beskrive din opplevelse av matematikk som fag i praksis?
- Kommer du på en spesiell hendelse i forbindelse med matematikk og praksis som av en eller annen grunn gjorde inntrykk på deg?
- Fikk du bruk av noe av det du har lært/ noe av det dere har snakket om i matematikkundervisningen på HiOA i praksis?
- Har undervisningen på HiOA på noen måte forberedt deg på møtet med praksisfeltet?

Tar opp tråden fra forrige intervju

- Før praksis beskrev du [sett inn individuelle utsagn] som «suksesskriterier» for din første praksis. Stemte dette? Eller sett i etterkant: Hva bidro til at dette ble en god praksis for deg? Hvis det var det da? Hvis ikke: Hva ble feil?
- Har det skjedd noe i praksis som du mener har påvirket ditt forhold til matematikk? Noe som har påvirket din tro på deg selv om fremtidig matematikklærer?

Intervju 3

Nå har dere hatt en lengre periode på HiOA etter den første praksisperioden

- Hva har du lært i denne perioden?
- Har du lært noe matematikk? Synes du det var nyttig?
- Har du lært noe didaktikk? Synes du det var nyttig?

Om den kommende praksisperioden

- Blir den kommende praksisperioden annerledes enn den første? Hvorfor/Hvorfor ikke?
- Føler du deg bedre forberedt? Hvorfor/ hvorfor ikke? Hvordan?
- Er det noe som gjør at du føler deg usikker?
- Er det noe du ser spesialt fram til med tanke på praksis?
- Hva tror du at du kommer til å klare helt fint?
- Hva tror du at du ikke kommer til å klare?

Generelt

- Har det skjedd noe siden vi snakket sist som har påvirka ditt forhold til matematikk?
- Hvordan er det å være matematikklærer? Kan du gi din beskrivelse av det?

Intervju 4

Om praksis

- Hva har du lært?
- Hva var forskjellig fra forrige praksisperiode?
- Er det viktigst med tilbakemelding og etterveiledning når du synes det har gått bra med undervisningen din eller når du synes det har gått dårlig med undervisningen din?
- Hva bidrar kontaktlærer med når han/hun er på besøk? Er disse besøkene viktige? Hvorfor/ hvorfor ikke?

- Hvordan er det å være matematikklærer? Kan du gi din beskrivelse av det nå?
- Hva er fokuset ditt når du er på HiOA, og hva er fokuset ditt når du er i praksis?

Studentene ble bedt om å beskrive en god og en dårlig opplevelse på epost mens de var i praksis

- Kan du fortelle om hva som gjorde den gode opplevelsen ‘god’, og den dårlige opplevelsen ‘dårlig’?

Eksamen og innspurt av dette studieåret

- Har du kontroll? Hvordan planlegger du å jobbe med matematikkfaget fremover?

Intervju 5

Nytt studieår

- Er du klar for nytt semester og andre år på lærerutdanningen?
- Hva gleder du deg mest til?
- Er det noe du ikke gleder deg like mye til?
- Hvordan gikk det på eksamenen i vår? Hvordan har resultatet påvirket deg?
- Hvis du tenker tilbake på forrige studieår, hva er annerledes med studiestart i år i forhold til i fjor?
- Har ditt forhold til matematikkfaget endret seg i løpet av det første studieåret?
- Kan du beskrive forskjellen på matematikk som fag i videregående og på lærerutdanningen?
- Har du større tro nå på at du skal kunne trives som matematikklærer enn det du hadde i fjor høst?

Om praksis

- Hvordan blir det å komme ut i praksis igjen?
- Hvordan er følelsen rundt praksis sett i forhold til høstpraksisen i fjor?

Om å være matematikklærer

- Hvordan er det å være matematikklærer? Kan du gi din beskrivelse av det nå?
- Hva er fokuset ditt når du er på HiOA, og hva er fokuset ditt når du er i praksis?

Intervju 6

Om praksis

- Hvordan var det å komme ut i praksis igjen?
- Var det som forventet?
- Hvordan er følelsen rundt praksis sett i forhold til høstpraksisen i fjor?
- Hva var annerledes i forhold til andre praksisperioder?
- Ville du gjerne hatt meg på besøk i praksis? Hvorfor/hvorfor ikke?

Du har nå gjennomført nesten 3 semestre på lærerutdanningen og 3 praksisperioder

- Hva har endret seg?
- Hvilken 'karakter' vil du gi lærerutdanninga? Med tanke på matematikkfaget, kobling av praksis og teori?
- Kommer du til å velge matematikk videre?
- Vil du si at din tro på deg selv har som matematikklærer har vokst i løpet av disse semestrene? Hva har i så fall bidratt til det?

Litt mer overordnet

- Hvorfor valgte du lærerutdanning?
- Har forventningene blitt innfridd?
- Hva tenker du ellers om veien videre?

Om å være matematikklærer

- Hvordan er det å være matematikklærer? Kan du gi din beskrivelse av det nå?

Interview 1

Background

- What is your relationship with mathematics?
- What has influenced your relationship with mathematics?
- What are your thoughts on becoming a mathematics teacher?
- Would you have chosen mathematics as a subject if it had not been compulsory?
- How do you learn mathematics best?
- Do you think teacher education will support your preferred way of learning mathematics?

Regarding the survey given to the cohort

- Is there anything about the test you would like to comment?
- Would you say that your confidence is high when it comes to mathematics?

I ask participants to elaborate some of their answers on the survey and comment on trends in the cohort-data. When presenting my perception of the interviewee based on my analysis of the test, I ask:

- Does this give a good description of you?

The upcoming school placement

- What are your expectations for the upcoming school placement?
- What do you think is important for the placement to be good?
- If you get the chance, would you want to teach mathematics in your first school placement?

Interview 2

On the school placement

- How is it to be back at UC after school placement?
- How was the placement?
- Was there anything that surprised you during placement?
- Can you describe your experience of mathematics as a school subject in school placement?
- When it comes to mathematics in school placement, can you think of a special incident that made an impression on you?

- Did any of what you have learned or discussed in the mathematics classes at UC come in handy during placement?
- Have the courses at UC prepared you for your first meeting with the practicum?

Following up on interview 1, I ask

- Before school placement, you gave some “success criteria” for your first placement (I repeat what he/she said). Where they right? In hindsight, what made this a good placement, if it was a good placement? If not, what went wrong?
- Has anything that happened during placement that changed your relationship with mathematics? Is there anything that has changed your belief in yourself as a future mathematics teacher?

Interview 3

You have just had a long period at UC following your first school placement

- What have you learned during this period?
- Have you learned any mathematics? Anything you find useful?
- What about mathematics pedagogy, have you learned any? Anything useful?

About the upcoming placement

- Do you expect the upcoming placement to be different from the first? Why/ Why not?
- Do you feel better prepared? Why/Why not? How?
- Is there anything that makes you feel insecure?
- Is there anything special that you look forward to in placement?
- What do you expect that you will be able to manage just fine?
- What do you expect will be hard to master?

General

- Has anything happened since the last time we spoke that has influenced your relationship with mathematics?
- How is it to be a mathematics teacher? How would you describe it?

Interview 4

About school placement

- What have you learned?
- What was different from the first placement?
- When is it more important to get feedback, when you feel your teaching was good, or when you feel it did not go well?
- When your UC teacher visits you in placement, what is his/hers contribution? Are these visits important? Why/ Why not?
- What is it like to be a mathematics teacher? How would you describe it now?
- What is your focus when you are at UC? And when you are in placement?

When in placement, the PSTs were asked to give a description of a good and a bad experience

- What made the good experience ‘good’, and the bad ‘bad’?

The exam and the rest of this academic year

- Do you have it under control? How do you plan to work with mathematics the rest of this semester?

Interview 5

A new academic year

- Are you ready for a new semester and your second year in teacher education?
- About what are you most excited?
- Is there anything you are not looking forward to?
- How did the exam go? How did the result affected you?
- What is different at the start of this academic year compared to the previous one?
- Has your relationship with mathematics changed during the first academic year in teacher education?
- Can you describe the difference between mathematics as a subject in upper secondary school and in teacher education?
- Are you more confident that you will enjoy being a mathematics teacher now than you were in your first semester?

About school placement

- How will it be to go back into school placement again?

- How are your feelings on school placement now compared to your first semester?

About being a mathematics teacher

- What is it like to be a mathematics teacher? How would you describe it now?
- What is your focus when you are at UC? And when you are in placement?

Interview 6

About school placement

- How was it to get into school placement again?
- Was it as you expected it to be?
- What are your feelings about placement this semester compared to your first placement last autumn?
- What was different in this placement compared to the previous placements?
- Would you have liked for me to visit you in school placement? Why/ Why not?

You have now completed nearly three semesters in teacher education and three periods of school placement

- What has changed?
- What 'grade' would you give the teacher education you are attending? Especially when you think about mathematics and the connections between theory at UC and practice in schools?
- Will you choose to study more mathematics?
- Would you say that your belief in yourself as a mathematics teacher has grown through these three semesters? If yes, what has contributed to this growth?

More overarching

- Why did you choose teacher education?
- Have your expectations been fulfilled?
- What are your thoughts on the way ahead?

About being a mathematics teacher

- What is it like to be a mathematics teacher? How would you describe it now?

Errata list

p.16, line 28: ‘by’ instead of ‘be’

p.18, line 30: ‘Biesta (2012a)’ instead of ‘(Biesta, 2012a)’

p.23, line 13: ‘programmes’ instead of ‘programs’

p.30, line 25: delete the misplaced comma after ‘- the SETcPM-instrument –’

p.41, line 22: ‘0.7’ instead of ‘0,7’

p.51, line 19: ‘programme’ instead of ‘program’

p.72, line 25: ‘into’ instead of ‘in to’

p.75, line 9: add ‘an’ in front of ‘intervention’

p.75, line 26: The sentence starting with “Teacher education demands that she…” should be replaced with: “Teacher education demands that she engage in *understanding* mathematics, and that she is able to explain *why* it makes sense to use those rules, not only *how* to use them.”

p.76, line 30: ‘it is’ instead of ‘is it’

p.78, line 9: ‘programmes’ instead of ‘programs’

Paper 3, p.2, line 37: ‘programmes’ instead of ‘programs’

Paper 3, p.4, line 1: ‘socialisation’ instead of ‘socialization’

Paper 3, p.4, line 10: ‘*socialisation*’ instead of ‘*socialization*’

Paper 3, p.6, line 15: ‘programme’ instead of ‘program’

Paper 3, p.7, line 13: ‘operationalisation’ instead of ‘operationalization’

Paper 3, p.9, line 32: ‘(PST4, I4)’ instead of ‘(PST4. I4)’

Paper 3, p.11, line 20: ‘Biesta (2012)’ instead of ‘(Biesta, 2012)’

Paper 4, p.1, line 7: ‘grades 1 – 7’ instead of ‘grade 1 – 7’

Paper 4, p.3, line 31: ‘ages 6 – 13’ instead of ‘ages 6 – 14’

Paper 4, p.8, Figure 1: In the right half of the figure, three consecutive bars are named 9’. Their correct naming is 8’, 9 and 9’.

Paper 4, p.10, line 12: ‘is’ instead of ‘are’

Paper 4, p.10, line 25: insert ‘the’ in front of ‘pre-test’