An interesting consequence of the general principle of relativity

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We show that Einstein's general theory of relativity, together with the assumption that the principle of relativity encompasses rotational motion, predicts that in a flat Friedmann-Lemaitre-Robertson-Walker (FLRW) universe model with dust and Lorentz Invariant Vacuum Energy (LIVE), the density parameter of vacuum energy must have the value $\Omega_{\Lambda 0} = 0.737$. The physical mechanism connecting the relativity of rotational motion with the energy density of dark energy is the inertial dragging effect. The predicted value is necessary in order to have perfect inertial dragging, which is required for rotational motion to be relative. If one accepts that due to the impossibility of defining motion for a single particle in an otherwise empty universe, the universe must be constructed so that all types of motion are relative, then this solves the so-called cosmological constant problem.

Introduction

Celebrating that it is now a hundred years since Einstein completed the general theory of relativity, we shall here investigate a consequence of this theory in the spirit of Einstein when he presented the theory in his great article that appeared in the spring 1916 [1].

The extension of the principle of relativity from rectilinear motion with constant velocity to accelerated and rotational motion was an important motivating factor for Einstein when he constructed the general theory of relativity. He was inspired by the point of view of E. Mach who wrote in 1872: "It does not matter whether we think of the Earth rotating around its axis, or if we imagine a static Earth with the celestial bodies rotating around it." Mach further wrote: "Newton's experiment with the rotating vessel of water simply informs us that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative rotation with respect to the mass of the Earth and the other celestial bodies."

The second paragraph of Einstein's great 1916-article is titled: "The need for an extension of the postulate of relativity". He starts by writing that the restriction of the postulate of relativity to uniform translational motion is an inherent epistemological defect. Then he writes: "*The laws of physics must be of such a nature that they apply to systems of reference in any kind of motion*. Along this road we arrive at an extension of the postulate of relativity".

This point of view was inspired by the conception that if a particle is alone in the universe one cannot decide whether it moves or not. Hence all motion should be relative, meaning that an observer cannot decide whether he is in rest or in motion. An arbitrary observer may consider himself as at rest.

In our universe this means that it should be a valid point of view for an observer to consider himself as at rest and the rest of the universe as moving. This requires that the observer must be able to explain all his experiments under the assumption that he is at rest.

P Kerzberg [2] has given a very interesting discussion of "The relativity of rotation in the early foundations of general relativity". In particular he reviews and comments an article published in 1917 titled "On the relativity of rotation in Einstein's theory" by W. De Sitter [3]. Kerzberg writes "De Sitter thus maintains that rotation is relative in Einstein's theory, and even as relative as linear translation. Both rotation and translation are susceptible to being transformed away. Nonetheless a difference persists". He then cites De Sitter: "If a linear translation is transformed away (by a Lorentz transformation), it is utterly gone; no trace of it remains. Not so in the case of rotation. The transformation which does away with rotation, at the same time alters the equation of relative motion in a definite manner. This shows that rotation is not a purely kinematical fact, but an essential physical reality."

In the present article we shall consider rotational motion and use the general theory of relativity to deduce a cosmic consequence of assuming that rotational motion is relative, meaning that an observer with a Fouceault pendulum on the North pole of the Earth, for example, may consider the Earth as at rest and the swinging plane of the pendulum as rotating together with the starry sky.

The significance of inertial dragging for the relativity of rotation

As pointed out by Pfister [4] inertial dragging inside a rotating shell of matter was described already in 1913 by Einstein and Besso in a manuscript that was not published. This work was based on Einstein's so-called Entwurf theory of gravity which Einstein soon discovered had some serious weaknesses. The first published paper on inertial dragging inside a rotating shell based on the general theory of relativity was published by H. Thirring [5] in 1918. He calculated the angular velocity of a ZAMO inside a shell with Schwarzschild radius and radius rotating slowly with angular velocity , in the weak field approximation, and found the inertial dragging angular velocity,

$$\omega_d = \frac{8R_S}{3r_0}\omega \qquad (1)$$

This calculation does not, however, remove the difficulty with absolute rotation in an asymptotically empty Minkowski space. Both the angular velocity of the shell and that of the ZAMO are defined with respect to a system that is non-rotating in the far away region. There is nothing that determines this system. The absolute character of rotational motion associated with the asymptotically empty Minkowski spacetime, has appeared.

Let us think about what happens, according to the theory of relativity if the cosmic mass is made less and then removed. The solution of Brill and Cohen [6] gives the dragging angular velocity inside a massive shell with radius r_0, Schwarzschild radius R_S, and which is observed to rotate with an angular velocity ,

$$\omega_d = \frac{4R_S\left(2r_0 - R_S\right)}{\left(r_0 + R_S\right)\left(3r_0 - R_S\right)}\omega \qquad (2)$$

Assume that initially the Schwarzschild radius of the shell is equal to its radius, so that there is perfect dragging inside it. The surface of the water would be flat if the water is at rest in an inertial Zero Angular Momentum (ZAMO) frame. The shape depends upon the angular velocity of the water relative to the ZAMO inertial frame. An observer at rest in the water would observe that the cosmic shell has an angular velocity, and the ZAMO inertial frame has an angular velocity . Initially and the surface of the mass is maximally curved. The Brill-Cohen formula shows that if the cosmic mass is made less, the angular velocity of the ZAMO inertial frames would decrease relative to the water, and when there was no cosmic mass they would be at rest relative to the water. Hence the shape of the water would flatten out as the mass of the cosmic mass decreased. The conclusion is that the general theory of relativity predicts that there would not be any change of the surface of the water if one tried to put it into rotation in an empty universe. Such an effort would not succeed. The water would not begin to rotate because there is nothing it can rotate relative to.

In 1966 D. R. Brill and J. M. Cohen [6] presented a calculation of the ZAMO angular velocity inside a rotating shell valid for arbitrarily strong gravitational fields, but still restricted to slow rotation, giving the expression (1). For weak fields, i.e. for $r_0 >> R_S$, this expression reduces to that of Thirring. But if the shell has a radius equal to its own Schwarzschild radius, $r_0 = R_S$, the expression above gives $\omega_d = \omega$. Then there is *perfect dragging*. In this case the inertial properties of space inside the shell no longer depend on the properties of the ZAMO at infinity, but are completely determined by the shell itself. Brill and Cohen further write that a shell of matter with radius equal to its Schwarzschild radius together with the space inside it can be taken as an idealized cosmological model, and proceeds: "Our result shows that in such a model there cannot be a rotation of the local inertial frame in the center relative to the large masses in the universe. In this sense our result explains why the "fixed stars" are indeed fixed in our inertial frame.

A cosmic consequence of assuming that rotational motion is relative.

The distance that light and the effect of gravity have moved since the Big Bang is called the lookback distance, $R_0 = ct_0$, where t_0 is the age of the universe. *In order to have perfect inertial dragging in our universe, which is necessary in order that rotational motion shall be relative,* the Brill-Cohen condition implies that the Schwarzschild radius of the mass inside the lookback distance should be equal to the lookback distance.

At the present time the Schwarzschild radius of the cosmic mass inside the lookback distance is

$$R_S = \frac{2GM}{c^2} = \frac{8\pi G\rho_0}{3c^2} R_0^3 , \tag{3}$$

where ρ_0 is the present density of all the mass and energy contained in the universe. Requiring that $R_S = R_0$ gives

$$t_0^2 = \frac{3}{8\pi G\rho_0} . \tag{4}$$

The present value of the critical density, corresponding to a universe with Euclidean spatial geometry, is

$$\rho_{cr0} = \frac{3H_0^2}{8\pi G} . \tag{5}$$

where H_0 is the present value of the Hubble parameter. Hence

$$t_0^2 = \frac{\rho_{cr0}}{\rho_0} \frac{1}{H_0^2} . \tag{6}$$

The present value of the density parameter and the Hubble age of the universe are defined by

$$\Omega_0 = \frac{\rho_0}{\rho_{cr0}} \quad , \quad t_H = \frac{1}{H_0} , \tag{7}$$

The assumption that rotational motion is relative thus leads to the relationship

$$t_0 = \frac{1}{\sqrt{\Omega_0}} t_H , \tag{8}$$

A large amount of different observations indicate that the density of the universe is very close to the critical density. Equation (8) shows that for a flat universe, *the assumption that rotational motion is relative, leads to the simple relationship*

$$t_0 = t_H , \tag{9}$$

i.e. that *the age of the universe is equal to its Hubble age*.

The standard model of the universe is a flat universe with dust and Lorentz Invariant Vacuum Energy, LIVE, with a constant density that may be represented by the cosmological constant, Λ. The present density parameter of the dust is Ω_{M0}, and of the LIVE is $\Omega_{\Lambda 0}$. The age of such a universe is given in terms of its Hubble age by [7]

$$t_0 = \frac{2}{3} t_H \frac{\arctan\left(\sqrt{\Omega_{\Lambda 0}}\right)}{\sqrt{\Omega_{\Lambda 0}}}, \tag{10}$$

Equations (9) and (10) lead to

$$\tanh\left(\frac{3}{2}\sqrt{\Omega_{\Lambda 0}}\right) = \sqrt{\Omega_{\Lambda 0}}. \tag{11}$$

The positive, real solution of this equation gives the following present value of the density parameter of dark energy, $\Omega_{\Lambda 0} = 0.737$. This is the prediction of the general theory of relativity together with the assumption that rotational motion is relative in our universe and that the universe is flat and contains dust and LIVE.

Conclusion

The prediction that $\Omega_{\Lambda 0} = 0.737$ might of course be falsified by observations. It is remarkable that the WMAP and Planck measurements have given a present value of the density parameter of dark energy which is 0.73±0.03 [8] in agreement with the predicted value.

This shows that relativity of rotation may be a physical reality in our universe – namely: This implies that the fact that the swinging plane at the North Pole rotates together with the stars, is no coincidence. It shows that the rotation of the plane is a gravitational effect from the cosmic masses upon the pendulum. The rotation of its swinging plane is due to perfect dragging from the rotating cosmic mass.

This means that every gyroscope is also acted upon by the inertial dragging effect due to the cosmic masses. The cosmic perfect inertial effect lines up the axis of a gyroscope so that it keeps on pointing at a fixed star even as the night sky rotates around it. This was used as a reference in the Lageos I and II experiments and the gravity probe B experiment that confirmed the existence of inertial dragging. The agreement of the predicted present value of the density parameter of dark energy and observations, confirms the existence of perfect cosmic dragging and implies that rotational motion is relative in our universe.

This solves the so called cosmological constant problem. The density of vacuum energy is 120 orders of magnitude less that the quantum mechanical cut off value, which is the Planck energy density. This is considered to be the quantum mechanical prediction for the energy density of the vacuum energy, leading to the largest known conflict between theory and observation.

We have shown that the general theory of relativity tells another story. It is not possible to define any type of motion for a particle that is alone in the universe. Motion can only be defined relative to other particles. One may therefore argue that the universe must be constructed so that if we have the correct theory, then this theory has to contain the principle of relativity for all types of motion. This was the way Einstein argued a hundred years ago inspired by Ernst Mach.

In this letter we have investigated the assumption that the general theory of relativity is just such a theory, and in particular that the principle of relativity is valid for rotational motion in our universe according to Einstein's theory. We have shown that if this is the case, and if the universe is flat and dominated by dust and LIVE, then the universe must be constructed so that the present value of the density parameter of the vacuum energy is $\Omega_{\Lambda 0} = 0.737$.

This solves the cosmological constant problem. The solution is that the universe has to be constructed so that all types of motion are relative, since the opposite would imply that it should be possible to define the motion of a particle which is alone in the universe, which is not regarded as meaningful.

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