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Article 1:
A Description of Precision Teaching Used to Assess Skill Proficiency and Pinpoint Intervention Level

Article 2:
Examining the Relationship Between Component and Composite Frequency and Natural Growth in Basic Multiplication: A systematic replication of Lin and Kubina

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Abstract

Mathematic proficiency is considered to be a fundamental basic skill, and poor mathematic skills may impact further education, which further could affect national wealth and growth. Results from international tests such as PISA and TIMSS shows that Norwegian students are not doing that well when it comes to do basic skills in arithmetic. Strategies such as *the Knowledge promotion* and *Close to science studies* have been implemented in the Norwegian education system in order to heighten the mathematical competence of the students, but these have not yet seen to be effective. Further are assessment of student proficiency level, and possible learning disabilities traditionally done through norm- and criterion-referenced achievement tests. These tests will rarely show what the potential learning disabilities are, nor will they point to interventions likely to be effective. One system that may facilitate a more preventive approach is Response to Intervention (RtI). RtI aims to be a preventive model through frequent monitoring of student performance, that generates information on potential problems in need of change in the academic environment. Curriculum Based Measurements (CBM) has been found to be highly effective in assessing student performance in reading and writing, and is the preferred measurement procedure in the RtI framework. Precision Teaching (PT) has also been recommended as a supplementary procedure given its frequent monitoring and focus on frequency building, and could as such have the function of both a frequent measurement procedure and as an effective intervention. For an RtI framework to implement effective interventions in mathematics, there is a need for establishing cutpoints or benchmark standards, which could predict further development in the transition from basic to more complex skills and over grade levels. The current study has examined the relationship between component and composite frequency and natural growth in basic multiplication in the grade levels 5–9, and is a systematic replication of Lin and Kubina (2005) with variations.

Keywords: frequency, Curriculum Based Measurement, Precision Teaching, Response to Intervention, component and composite performance, instructional design, skill proficiency, progress monitoring, pinpointing

A Description of Precision Teaching Used to Assess Skill Proficiency and Pinpoint Intervention Level

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Abstract

Results from international tests such as PISA and TIMSS shows that Norwegian students are not doing that well when it comes to do basic skills in arithmetic. 13% of the students fail their final math exam at the end of the 10th grade level, and additional 24% of the students achieve the second lowest grade there is. Basic academic skills are necessary prerequisites for further learning and development within school, work and participation in the society. Strategies such as *the Knowledge promotion* and *Close to science studies* have been implemented in the Norwegian education system in order to heighten the mathematical competence of the students, but these have not yet seen to be effective. One system that may facilitate teachers' efforts in subjects like mathematics is *Precision Teaching* (PT), especially if the principles of PT are combined with elements such as an understanding of learning taxonomy and instructional design. Frequent progress monitoring and pinpointing learning objectives are essentials key concepts in PT, and may be helpful in increasing students' academic performance in mathematics. This article provides a description of PT, including the four guidelines of PT, and how this kind of measurement system can be used in a Response to Intervention framework.

Keywords: instructional design, Precision Teaching, skill proficiency, progress monitoring, pinpointing, Response to Intervention

There is an international consensus that basic skills should attain more focus in primary and middle schools (Lin & Kubina, 2005; Johnson & Street, 2013; Grønmo et al., 2012). In fact, *The Norwegian Directorate for Education and Training* states that some basic academic skills are necessary prerequisites for further learning and development within school, work and participation in the society. Those basic skills are reading, writing, doing math, oral skills, and digital skills (Utdanningsdirektoratet, 2012).

There has been an impetus to improve learning outcomes in especially basic skills. One strategy implemented by *The Norwegian Ministry of Education and Research* is *The Knowledge Promotion* (Kunnskapsdepartementet, 2013), a school structure altering reform, which was implemented in the fall of 2006. However, results from international tests such as *Programme for International Student Assessment* (PISA) (Kjærnsli & Olsen, 2013) and *Trends in International Mathematics and Science Study* (TIMSS) (Grønmo et al., 2012) have shown that Norwegian 4th grade level students are improving their mathematical skills, but the Norwegian 8th grade level students are declining.

PISA and TIMSS states that Norwegian students are not doing that well when it comes to basic skills in arithmetic (Grønmo et al., 2012; Kjærnsli & Olsen, 2013). Also, *The Norwegian Directorate of Education and Training* presents an annual analysis of the basic education in Norway, showing that as many as 13% of the students fail their final math exam at the end of the 10th grade level, this represents 2600 students, and an additional 24% of 10th grade level students achieves the second lowest grade (Utdanningsdirektoratet, 2015).

According to Norwegian legislation, students in official schools have the right to individually facilitated education. This means that the educational training shall be organized and adapted to the individual student’s skillset and academic environment (Opplæringsloven, 2008). Regulations in the *Norwegian Education Act* § 3–4 express that those students who get the lowest grade, 1, “has very low competence in the subject”. When such a high percentage of students show this lack of mathematic skills and gets the lowest grade possible it may be fair to say that society has not completed its task (Lovdata, 2009).

The Knowledge Promotion (Kunnskapsdepartementet, 2013) is, as earlier mentioned, implemented in order to alter the situation where a large proportion of students who graduate from 13 years of education with low proficiency levels. Core elements of the strategy were to alter the curriculum structure in schools and thereby induce more focus on increasing students' basic skills. This was implemented in 2006, today ten years later, Norwegian students still perform poorly within for instance basic math skills. Thus, *The Ministry of Education and Research* is once more implementing an intervention to alter the structure of the schools to improve student achievement.

This initiative is called *The science studies strategy - close to science studies*, and is a four year strategy where the aim is to increase children's competence level in science studies (Kunnskapsdepartementet, 2015). In this strategy the goals and aims are described, but very little is described about how to get there. The aim is to (1) increase teachers' competence, (2) focus on early interventions for students falling behind, and (3) ensure access to resources of learning. *The science studies strategy* is going to alter school structure by renewing, and possibly simplify the curricula in science studies, and with these measures its presumed that there might be a change in students competence level (Kunnskapsdepartementet, 2015).

There have been developed several educational tools in the new strategy, including creating *Supportive-learning tests*, which are voluntary tests that should be completed several times in a school year. These tests can yield information about the students' performance level (Kunnskapsdepartementet, 2015).

Increasing students' competence level in mathematics would benefit our society if it were achieved, a precise description on how this is to be reached is needed. Thus, several interventions have been tried out in Norwegian schools to improve basic mathematic skills, but significant results have not yet been seen. According to Hattie (2009) policy change initiatives such as *The science studies strategy - close to science studies* will not suffice by and of itself. They may set the stage for new practices in each school and every classroom. New practices, in terms of teaching and learning principles should be empirically supported. Some are according to Hattie (2009), and apart from focusing upon particular practices only, it is necessary to (1) attend to each student in order to personalize the student's learning, (2) ensure precise knowledge about each student's progress, and (3) provide teachers with knowledge and skills on how to deliver effective strategies for learning and progress.

One system that may facilitate teachers' efforts in subject like mathematics is *Precision Teaching* (PT), especially if the principles of PT are combined with elements such as an understanding of learning taxonomy and instructional design, this would combine into a generative model of instruction and learning (Johnson & Street, 2013). An overview of the generative model will be described. Also a description of PT, the four guidelines of PT, and how this kind of measurement system can be used in a Response to Intervention framework will be described. Furthermore, the description will lean towards mathematical instruction and learning.

A Model of Generative Instruction and Learning

Taxonomy of Learning and Instructional Design

Looking in to the taxonomy of learning and instructional design may be helpful in order to achieve the target of heightened mathematic competence of students. Kubina, Kostewicz and Lin (2009) define instructional design as how one will structure learning for success. Further they describe the many factors that affect the learning process, such as needs assessment, goal analysis, and performance assessment. When creating learning materials it will be helpful to understand the many factors of instructional design in order to be able to design effective learning programs.

Kubina et al. (2009) gives a description of Tiemann and Markle's taxonomy of learning which has four basic types of learning; psychomotor, simple cognitive and complex cognitive, the fourth and last one is emotional. Emotional learning is included in the three other types of learning to be a reminder that the learner will always have a physiological response that co-occur with the new learning material. All four types of learning are closely related, and function as building blocks.

In mathematics the learning types that the teacher can most readily influence are simple and complex cognitive learning. Simple cognitive learning consists of (1) associations (e.g., discrimination), (2) paired associations and multiple discriminations (e.g., basic math facts), (3) algorithms (e.g., multi-digit multiplication), and (4) sequences (e.g., following an instruction or recipe). Simple cognitive skills lays the foundation for complex cognitive skills, (1) concepts (e.g., classifying stimuli in categories), (2) principles (e.g., grouping concepts by applying rules), (3) and strategies (e.g., combining concepts and principles) (Kubina et al., 2009).

It is a prerequisite to have learned the simple behaviors to be able to learn complex behaviors. The classification system also gives definitions of the subcategories to the basic types of learning. Understanding the taxonomy of learning gives the teacher the ability to create more effective learning strategies for the students (Kubina et al., 2009).

Stages of Learning

All types of learning, simple or complex, proceed through different stages. According to Kubina et al. (2009) there are six different stages; the first being the entry level where the behavior occurs at very low frequency or not at all. The second stage is the acquisition level where the goal is to have an accuracy of 90–100% of the new learned behavior. The third stage is proficiency; where the learner will learn both speed and high accuracy. This stage correlates with the definition of fluency, which will be described later in this article.

Maintenance is the fourth stage in learning, where the learner still has a high degree of accuracy and speed and the goal is retention. Stage five, generalization, refers to the ability to transfer the newly acquired skill to new settings and responses. The sixth and last stage refers to the application of the acquired skill, and is labeled adaption.

The stages of learning show a process which all learned behaviors have been through. Learning is a complex process where practicing a behavior to a certain level of frequency can result in a behavior maintained over a longer period of time, and the generalization of that behavior.

Precision Teaching

Understanding both the taxonomy of learning and the description of the stages of learning can give the teacher the ability to design instruction, and with that structure for success. Precision Teaching (PT) is one system that gives some insight into instructional design. Kubina et al. (2009) refers to White (2005) and define PT "… as a system for defining instructional targets, monitoring daily performance, and organizing and presenting performance data in an uniform manner to facilitate timely and effective instructional decisions" (p.17).

PT has established a set of tools, methods, rules and procedures for making educational decisions based on student's own self-monitored performance. The system has "… high performance aims and custom-tailored prescriptions to maximize learning" (Lindsley, 1992, p.51). The two most important, and essential, key concepts in PT are pinpointing and progress monitoring. Pinpointing refers to a learning objective, an understandable, distinct and measureable target (Johnson &Street, 2013; Kubina & Yurich, 2012).

According to Johnson and Street (2013) the purpose of progress monitoring is to find out which of the students who are not meeting the criteria for the current learning material.

That way the teacher can provide the students with extra support if needed. PT is an intervention that can be used when increasing basic academic skills like mathematics. It is a designed measurement system, which can measure students' performance and register the outcome on the Standard Celeration Chart (SCC) (Lindsley, 1992). It is a way to systematically observe and register the growth and frequency of the trained skill.

The SCC is a chart with four distinct features; (1) it is standardized for easy comparison, (2) it is calendar-based, (3) focused upon frequency, not percent correct, and (4) focused on learning, not performance (Johnson & Street, 2013). There are several separate SCC's where a student's performance progress is charted on a daily basis. Two examples are; (1) the Daily per minute Standard Celeration Chart, where the student's day-to-day progress is monitored, and the student's best score each day is charted, and (2) the Timings Standard Celeration Chart, which is for making decisions within a program session, it is a sort of electron microscope of learning. That way student progression is monitored and eventual problems are found at an early stage. PT as a system can be used in all subjects and with every skill trying to be mastered. As long as performance is measured and charted teachers can know whether their teaching approach is working or needs to be revised, because they monitor every step of the progress of their students' performance (Kubina & Yurich, 2012).

PT is not necessarily an intervention that needs to be implemented for every student in a whole class, but it is a system for those students who need to increase their performance in subjects like mathematics, and need extra learning support to catch up with their classmates. With successively completed practice they can be a part of the normal progression on the same level as their classmates (Johnson & Street, 2013).

One assumption of why so many Norwegian students fail their math exams could be the lack of progress monitoring in schools. The teachers may lack potential effective strategies or resources for finding out or pinpointing where in the math curriculum the students do not meet the criteria of frequency levels. Assessments administered annually in Norwegian school today are *National tests* that yield, not only, comparable information about Norwegian schools, but also individual data about performance levels for each student participating (Haugberg, 2014; Utdanningsdirektoratet, 2015). It is positive that assessments are being completed, but annually assessments offer very little specific data, and is not enough to diagnose or pinpoint where a student might have learning disabilities. PT as a system has tools that comprise this.

Guiding Principles of Precision Teaching

PT is a measurement system designed for teachers to track the learning and performance of their students. Ogden Lindsley, who was a psychologist and professor of educational administration developed PT in the early 1960's, which includes observing and graphing performance to help teachers to be much more precise in their work with students (Johnson & Street, 2013).

PT has four guiding principles, which have contributed to PT's effectiveness; (1) the learner knows best, (2) focus on observable behavior, (3) use frequency as a universal measure of behavior and (4) display the data on a SCC (Kubina & Yurich, 2012). The principles directs the what, why and how of PT.

The first principle, the learner knows best, refers to that it always should be remembered that despite the predictions made in science, paying attention to the data is always important. Scientists or teachers often have a hypothesis of what the outcome of an intervention is going to be, but to discover the variables responsible for unusual performance data it is important to remember that the learner knows best. The learner is the one who gives responses to the reinforcement contingencies, and the outcome could be different than the original prediction facilitated by the teacher (Kubina &Yurich, 2012).

According to Kubina and Yurich (2012) do the second and third guiding principles emphasis the focus PT has on observable behavior and frequency as a measure of performance. Without a good measurement practice teachers are not able to evaluate the effects of teaching on students' performance. PT uses frequency as its standard behavioral measurement, which offers possibilities to compare results across examples, and celeration as its standard change expression.

Johnson and Street (2013) hold that frequency is a fundamental dimension of behavior, a skill performed at a high frequency is observably different than the same skill performed in a low frequency. Frequency is a sensitive way to discover changes in students' performance even with slight modifications in interventions. Unlike percentage correct, which is the common way to evaluate a student's progress. Percentage correct is very insensitive to individual differences in performance. An example is when a whole class of students can complete a mathematical test where everybody scored 100% accuracy, but the number of math problems completed in the time interval they had at their disposal can vary (Johnson & Street, 2013).

The most important feature of frequency might be that it can very precisely say something about the probability of future action (Johnson & Street, 2013). If the measurement unit is correct responses per minute, then frequency equals the number of correct responses made within that minute, and this answer gets charted on a SCC. One example is; 10 correct math fact answers in one minute will yield that 10 will be charted on the SCC. If 10 correct math fact answers are completed in a time interval of 30 seconds, then multiply the number times two to get the frequency per minute, 10 x 2 = 20, and 20 is the number of correct responses and will be charted. That is the fourth guiding principle: Using the SCC. The SCC brings the advantages of standardized measurement to education and teachers, and according to Kubina and Yurich (2012) is the SCC the driving force of PT. The chart focuses on learning, growth, change and progress over time and decision-making about a students progress is based on the registrations done in the chart. Johnson and Street (2013) states that "… frequency the best measure of performance, growth in rate of performance over time is the best definition of learning".

The Fluency Concept

Fluency is a concept much discussed within the PT literature. When a skill is fluent we often say it is carried out effortlessly, the skill is automatic. Fluency is often defined as accuracy plus speed (Binder, 1996), an accurate skill performed in a perfect speed to maintain the frequency at a certain level. The question is what level a trained skill has to be performed at to be fluent. There are several suggested fluency aims defined to which skills are being trained.

Johnson and Street (2013) writes that "… fluent performances are more likely to occur in the future than nonfluent behaviors. As behaviors become fluent, they take on a new dimension. They are fun, energetic and energizing, and naturally reinforcing"(p.23). Fluent behavior is a term that is used in different ways in psychology and pedagogy, and there is usually agreed that most behaviors performed on a daily basis are fluent, people often do not think about doing them when they do them. However, a more precise definition is necessary when identifying and training a skill to become fluent. To say that a skill is fluent within the PT literature is to refer to a certain level of performance frequency along with a certain level of accuracy. The teacher sets up performance standards/frequency aims for the particular skill being practiced, and these frequency aims are predicting fluency.

At the current time there are a few performance standards in mathematics being used. Morningside Academy is a behaviorally based laboratory school, which helps students who do not perform at performance criteria to catch up and get ahead (Johnson & Street, 2012). According to Johnson and Street (2013) Morningside Academy operate with a frequency aim of 80–100 correct 1–2 digits math answers per minute without any errors before the skill is fluent. Similar frequency aim have been described in several articles, an aim of 80 digits written correctly per minute (or 40–50 problems per minute) seem to be the appropriate level for basic math computation.

If a student is able to do 30–40 problems per minute he/she will continue to accelerate even though the curriculum in mathematics become more complex. But if a student whose basic skill performance is below 30 per minute he/she will show progressive decelerating frequencies as he/she advances through the curriculum (Haughton, 1972; Strømgren, Berg-Mortensen & Tangen, 2014). The empirical basis for these performance standards is unclear though, it appears to be set by experience, and we cannot say with certainty how many correct math answers a student has to complete to say that the trained skill is fluent. But it is possible to make an assumption that a student's ability to perform a skill is changing if the frequency level of the performed skill is over 30 correctly written math problems per minute, and that we need more research on the subject.

Defining Fluency with Precision

Within the PT literature there has been defined several learning outcomes to examine if the trained skill is at a frequency where it can be called fluent. And there have been several developments through the years when it comes to what should be included in the definition.

The first definition emphasized (1) retention and (2) application, Retention and Application Performance Standards (RA/PS). A student had achieved fluency in a skill being practiced if he/she (1) could keep the same performance frequency without practice the skill in a period of time, one month or over a summer vacation, and (2) the student had the ability to use the skill in a real world context where it is required (Johnson & Street, 2013).

A few years later another learning outcome was added, Endurance, REA/PS. Endurance refers to the students ability to maintain their performance frequency over an increasingly longer period of time. Subsequently two more outcomes were added; Stability and Adduction. Stability refers to the ability to perform a skill at the same frequency level even when faced with distractions, and Adduction refers to the students' ability to use new skills without instruction, thus the acronym RESAA/PS (Lindsley, 1992; Kubina & Yurich, 2012; Johnson & Street, 2013).

Morningside Academy has recently changed the acronym in two ways. They have replaced Retention with Maintenance. This because to test for Retention requires that the student is not exposed to the trained stimulus condition under where the skill is practiced for a period of time, before introducing the relevant stimulus conditions again and observe the behavior. When teaching math for instance it would not be possible to restrain the student from being exposed to mathematical stimuli, which makes it difficult to administer a retention test.

Thus, Maintenance refers to the ability to perform a task in the future, on demand, but without additional practice, but the individual can be in contact with the trained skill, which is preferable when it comes to academic achievement (Johnson & Street, 2013). An example will be when you have not played a song on the piano for several years, but when you sit down at the piano you still remember how it is played, the skill is fluent and maintained even if it is not practiced in a period of time. Many students have frequency levels in math skills that reach aim when practiced, but they cannot maintain the frequency when they do not practice it over a period of time like a natural break as the summer vacation. The second change Morningside Academy did to alter the learning outcome to better fit the real-world, which students would benefit from was to replace the often misunderstood outcome Adduction with the more easily understood concept Generativity. Generativity refers to the emergence of complex behavioral repertoires. Morningside Academy operates with the acronym MESAG/PS (Maintenance, Endurance, Stability, Application, Generativity/

Performance Standards) and they have found that the frequency level of 80–100 correct math facts, 1 to 2 digits, answers per minute, with no more than two errors pass the MESAG/PS test (Johnson & Street, 2013).

Instructional Design for PT

According to Johnson and Street (2013) is well-designed instruction and practice the keys to learner success. They further give a description of three instructional design technologies to maximize the effectiveness of a PT implementation; (1) component-composite analysis, (2) kinds of learning analysis, and (3) learning-channel analysis.

In a component-composite analysis is every skill analyzed and categorized into sets of tool skills, component skills and composite skills. Tool skills are considered those minimal responses that support many if not every other skill. For example, the tool skill for answering math facts would be numeral writing. Tool skills are prerequisites for teaching other skills (Kubina & Yurich, 2012; Haughton, 1972). Component skills are skills that depend upon one or more tool skills, for example as earlier mentioned writing the answer to a math fact problem would be a component skill. Component skills are the building blocks to learn composite repertoires.

Johnson and Street (2013) states that composite skills are "… the authentic, higher level performances that socially validate a learner's mastery of a content area" (p.41). In the previous example a composite skill would be completing multiplication problems, where a learner incorporate both component skills and tool skills that make up the composite skill completion. Component and Composite are dynamic concepts, and change place to fit each student and each skill being practiced. Component skill for one student might be a tool skill for another student, depending on the student and what is being practiced. Another example is if a student is fluent in multiplication and division, which are component skills, they will still struggle with computations if addition and subtraction as component skills are not mastered.

All four component skills have to be practiced to a certain frequency level before the student can complete computations fluently.

After identifying the different level of the particular skill, a content analysis could be done for each skill to find out the types of tasks and what kinds of procedures are required. Johnson and Street (2013) refers to the previously mentioned Tiemann and Markle's taxonomy of learning when they describe the kinds of learning analysis. The kinds of learning analysis is the preferred content analysis cause it is easily understood. The different tool skills, component and composite skills are divided into the different domains in kinds of learning, either pcychomotor, simple cognitive or complex cognitive. This way the teacher has identified which type of basic learning to use and develop procedures to teach accordingly.

Third, instructional design technology to maximize effectiveness of PT implementation is a learning-channel analysis, which describes how a student makes contact with instruction, and the student's response to the stimuli (Johnson & Street, 2013). This stimuli contact can be seeing, hearing, touching, smelling or tasting, and the responses can be saying, writing, pointing and so on. An example from mathematics can be that the stimuli be visual and the response be written, then the learning channel would be: see math problems/write answer.

To summarize; the teacher need to identify the teaching goals, complete a component-composite analysis for each goal, and then identify the kinds of learning for tool, component and composite skills, and last, describe the learning channels for each tool, component and composite skill (Johnson & Street, 2013).

Once the content the student is to be practicing is identified and analyzed, the teacher needs to define the instructional objectives. According to Johnson and Street (2013) an instructional objective has three parts; (1) it describes the conditions of the learning environment, (2) it specifies the performance, and (3) precisely states the criteria that need to be met to consider the instructional objective mastered. Lets examine an example: when seeing single-digit multiplication problems, the student will write 80–100 correct digits, within one minute. The example is based upon Morningside Academy's frequency aim of single-digit mathematical problems. This example offers a precise description of the conditions, which is when the student sees the single-multiplication problems. Secondly it describes the performance, the student is to write the answers, and third it describes the criteria of when the skill is mastered, the student is to write 80–100 correct digits.

When the instructional objective is precisely defined then the teacher teaches the student how to accomplish the performance specified be the objective, this is done by practicing the pinpoint set by the teacher. Kubina & Yurich (2012) writes that the pinpoint offers information about the learning channel, the performance, and the frequency aim. The pinpoint to the example above would be: see math problems/write answer, 80–100 answers/minute. The precise instructional objective and pinpointing makes teaching and learning more effective because the teacher and student simultaneous chart the progress in the current skill being trained (Kubina & Yurich, 2012).

But building performance frequency is not enough in PT, it is equally important that the student reaches the frequency aim in a timely manner. The celeration line drawn in the SCC indicates when the student is to achieve his/hers goal. If the chart shows that the student celeration is decelerating the teacher can if necessary initiate measures to help the student's further progress.

When the student completes this task, and reaches the frequency aim, he/she has mastered the skill, and it is time to test the skill further for maintenance, endurance, stability, application and generativity. Given that the frequency levels holds throughout testing then according to the MESAG/PS test described by Johnson and Street (2013) the skill is fluent.

Research have shown that practicing a component skill to a certain frequency level and then keep the frequency aim during MESAG/PS tests, are also beneficial for the student when learning composite skills later (Kubina & Morrison, 2000).

Johnson and Street (2013) summarizes the PT process in five different steps. The first step is to identify the pinpoints, this is done when the teacher defines the learning objective. Second, the teacher has to develop materials and procedures for practicing and learning the pinpoint. Third, the teacher and the student set a predetermined time interval, conduct the practice session, and count the performance frequency. Fourth step is to chart the performance, and last, number five, review celeration and make a decision about future progress.

Precision Teaching and Response to Intervention

Response to Intervention (RtI) is a framework of strategies that has one of its roots in PT, and it has been proven effective for improving academic and behavioral outcomes for students (Johnson & Street, 2013). Johnson and Street (2013) states that the primary purpose of RtI framework:

> … is to ensure that schools use skill specific screening to place learners in programs, measure the effects of the core curriculum or instructional practices on all learners' academic and behavioral responding, use measures at the earliest possible time to detect inadequate progress toward specified goals, select evidence-based interventions for a given learner or group of learners, monitor their progress on clearly identified learning objectives, and change instruction as needed to ensure a positive learning trajectory (p.4).

RtI is a prevention model that ideally is going to prevent academic and social failure by providing empirical supported programs and by intervening at an early stage in the learning process. The framework divides areas of emphasis in academic behavior systems and social behavior systems, and those two areas are divided into three tiers; a universal tier, a targeted group tier, and an individualized support tier. Research has shown that 80–90% of the students in a classroom are located at the bottom tier, the universal tier, these are the students who have adequate yearly progress in their education. Students who show inadequate growth will get extra support in Tier 2 (Kubina & Yurich, 2012).

Tier 2, the targeted group, often gets extra support within the regular classroom and the support is designed to strengthen some areas of weakness that students have developed. There is a goal to move students as quickly as possible back to ordinary curriculum, that is why supplementary instructional materials should be both effective and efficient. If the students do not show the expected progress in Tier 2, despite empirical supported instruction, the students can be moved to Tier 3; specialized instruction. Specialized instruction focuses on identifying specific skill deficits, and programs implemented are intensive and individualized (Johnson & Street, 2013; Kubina & Yurich, 2012).

Key components in RtI is progress monitoring and data based decision making, and Johnson & Street (2013) describes three levels of monitoring; macro, meta and micro level assessments. Macro level assessments are standardized test that are administered annually (e.g., *National tests*), and provide information about student progression. Meta level assessments are administered two to three times a month (e.g., *Supportive-learning tests*), and the micro level assessments are administered daily (e.g., SCC charts). The micro level assessments are assessments most sensitive to change. Due to space limitations the different RtI assessments will not be further elaborated.

Intervention programs that are used as supplemental curricula for the students in Tier 2 and 3 must be empirical supported and provide effective and efficient teaching. According to Johnson and Street (2013) is PT the perfect supplement to RtI for several reasons: its effective practice strategies, its use of frequency as the measure of performance, and its emphasis on celeration as an indicator for learning. For example can PT benefit RtI as a screening tool, RtI assessments measure only performance in a given moment, this often misrepresent the true current level of performance.

With PT performance is measured over a period of time, which gives a much more accurate picture of the student's present performance level. Some other benefits that Johnson and Street (2013) mention are; PT provides efficient intervention, which includes predetermined pinpoints for tool and component skills, which have been discussed earlier, and it provides visual analysis with the SCC; which offers progress monitoring and excellent data for decision making. PT can help and be the supplement some students need, it is effective and efficient, it teaches the determined skill to mastery within a predetermined timeframe with the use of frequency building and celeration line drawn in the SCC.

According to Kubina & Yurich (2012) PT can be a particular great intervention for those students in Tier 1, this because it can assist them to quickly achieve fluency in tool and component skills, and let them focus on composite skills. But students' performance in Tier 1 already show an adequate progression though the curriculum, because of this it might be more wise to use the resources to help students in Tier 2 and 3. It is similarly to when you take your temperature when feeing sick, is the thermometer showing 38–39 degrees Celsius we will act and do something about it, but if it is normal then you do not have to do anything.

Tier 2 interventions have its primary focus on increasing the students' performance to such a level that they can return to regular class teaching. It is important that interventions focus on building tool skills and component skills to support these students. The PT system can benefit these students through its screening measures, pinpointing, progress monitoring, and different procedures for increasing their performance levels (Kubina & Yurich, 2012).

The targeted group in Tier 2 can amongst other procedures for example benefit from peer-to-peer coaching. Peer-to-peer coaching has great benefits for students' success. One classroom teacher does not have the capacity to give feedback to every individual student all the time, but by pairing the students and give the coaching responsibilities to the students the feedback is frequent and powerful. Research has shown that by peer-to-peer coaching the student learn better and faster, learn the importance of self-evaluation, self-management and self-monitoring skills, as well as they learn from teaching others (Johnson & Street, 2013). PT can help Tier 2 students heighten their competence level, and such prevent students from further decline in performance and a possible reassigning to Tier 3.

Fewer than 10 % of the students in a classroom are located in Tier 3, these are the students whom are showing learning trajectories which are particularly slow, they are not meeting their frequency aims set in Tier 2, and might benefit more from individualized support to meet their performance goals. Tier 3 students have a great variance in the reasons why they struggle, but one common denominator may be that they lack tool skills, even tool skills that should have been taught prior to the general education classroom. PT technology can facilitate increase in performance levels on both academic subjects and social deficits that students may struggle with (Johnson & Street, 2013). PT technology has the instructional design to accommodate whole classrooms, targeted groups and even an individual student to reach their performance aims.

Relevance to Practice: PT, RtI and Norwegian Student Performance

According to PISA and TIMSS assessments, Norwegian 8th graders show a decline in mathematical proficiency, and the main problem is considered to be basic skills in arithmetic. This might illustrate a problem with early identification of struggling students and that schools needs more emphasis on frequent progress monitoring (Kjærnsli & Olsen, 2013; Grønmo et al., 2012). Several different strategies, for instance *The Knowledge Promotion* and *The science study strategy*, have been implemented in Norwegian school with the means of increasing student proficiency, but the results have failed to appear (Kunnskapsdepartementet, 2013/2015).

A potential problem faced by teachers in general education is the somewhat lacking information regarding student performance. International tests and national tests are generally measured towards a predetermined aim of proficiency, and the results yield how well a student performs in accordance to that aim, but give little or no information regarding what areas in a given subject that needs improvement. Essential key concepts within PT described in this article are progress monitoring and pinpointing relevant skills that students need to practice. Through developing learning objectives, practicing frequency building, and chart students' daily progress the teacher will be able to make databased decisions regarding students' performance (Kubina & Yurich, 2012; Johnson & Street, 2013; Binder, 1996).

This is also described in the PT literature; what the student presents as the problem, often turns out not to be the problem to solve (Johnson and Street, 2013). Example, if a student struggles with algebra problems, it might not be of any help to give that student many more algebra problems to solve. A possible solution to this problem may be to assess if the students attains the needed tool skills to complete the algebra problems, if not, then these tool skills should be mastered first. If the student have not learned the component skills needed in the composite skill we cannot expect the student to perform. This is sort of a mantra in PT: "The problem that a learner presents is not necessarily the problem to solve (Johnson & Street, 2013, p. 22).

Another potential problem to why students struggle with basic mathematical skills may be the lack of instructional design in the Norwegian education system. The teachers might not have the knowledge on how to deliver effective strategies for learning, or how to structure efficient material to secure student success. A possible solution is the implementation of empirical supported interventions. For instance can knowledge about the taxonomy of learning and the different stages in learning possibly make teachers and instructors more able to successively implement effective learning strategies, and find out where exactly an eventual intervention to increase certain skills is needed (Kubina et al., 2009).

Empirical supported intervention offers methods, strategies and rules to follow which further can increase a student's performance level. For instance can PT through component-composite analysis, kinds of learning analysis, and learning channel analysis identify individual teaching goals, which is essential for effective learning strategies (Johnson & Street, 2013). It is important in an education system with knowledge regarding how tool skills, component skills and composite skills are related to each other, and what to emphasize when teaching higher acquisition skills.

As illustrated above instructional design and progress monitoring may be supplementary in assessing student performance, and yield more accurate information about regarding students with potential learning disabilities.

Summary

Assessments such as national and international tests offer no information about students' daily progression in a particular subject. More frequent and effective assessment would yield better progress monitoring, and thus be an efficient tool to finding out where a student struggle in a curriculum. A possible solution is the implementation of effective measurement systems such as PT. PT offers assessments methods that pinpoints the exact skill that need improvement, for instance through daily charting in a SCC. Progress monitoring is the foundation that databased decisions are made upon (Kubina & Yurich, 2012; Johnson & Street, 2013).

Too many Norwegian students perform mathematical skills at an inadequate level (Kunnskapsdepartementet, 2015). Consequently, *The Norwegian Directorate for Research and Training* have stated that basic academic skills are a necessary prerequisites for further learning and development, and that it is a goal to improve basic skills on a national basis (Utdanningsdirektoratet, 2012). In order to accommodate this, helpful strategies may be PT strategies such as instructional design, component-composite analysis, and to heighten the teachers' knowledge on how to structure learning towards success. Such an emphasis on effective teaching strategies, by means of developing precise learning objectives, ensuring progress monitoring and pinpointing can contribute to increase students' basic skills performance (Johnson & Street, 2013; Kubina & Yurich, 2012; Kubina et al. 2009; Lindsley, 1992).

Furthermore, to help putting in place the PT-strategies on a regular basis, the implementation of RtI framework can be very useful as it can ensure that teachers acquire the strategies and tools they need to develop material and procedures. Also, through charting the teacher has a way to monitor the students' daily progress, and make decisions about students' academic performance based on data. If needed they can give the students more support or extra material so they can reach the criteria of frequency levels, and advance through the curricula together with their classmates (Kubina & Yurich, 2012; Johnson & Street, 2013).

PT with its key components can be both effective and efficient when used to assess skill proficiency and to pinpoint intervention level, and such might be a component between success and failure in academic performance.

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Examining the Relationship Between Component and Composite Frequency and Natural Growth in Basic Multiplication: A Systematic Replication of Lin and Kubina

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Distributions of tasks when writing article two.

The current article has been a collaboration from the beginning to the end. Both authors have contacted the participating schools, both have registered testpackets following the data collections, and analyzed the data.

In the writing process Cathrine has mainly written the introduction and discussion sections, while Linn has mainly written the method and results sections. But we both have contributed in the progress of each others written sections, and the progression of the latest edition, which is the current article.

Abstract

Mathematics is a universal problem, in Norway the most pressing concern is the evident decline in mathematic proficiency in high school, and the increased number of students in need of special education. Assessment of student proficiency level, and possible learning disabilities, are traditionally done through norm- and criterion-referenced achievement tests. These tests will rarely show what the potential learning disabilities are, nor will they point to interventions likely to be effective. In order to implement effective interventions there is a need for further research on natural frequency growth in the transition from component to composite skills, and over grade levels. Findings from these studies could establish frequency aims that could possibly predict future student outcome, and generate more effective interventions, that could enhance student proficiency in mathematics.

The current study has examined the relationship between component and composite frequency and natural growth in basic multiplication in the grade levels 5–9, and is a systematic replication of Lin and Kubina (2005) with variations; the inclusion of multiple grade levels (5th–9th), a second screening, and a simple linear regression analysis to incorporate prediction of direction of the variables. Findings from the current study shows that component frequency predicts composite frequency and as such reinforce the findings from Lin and Kubina (2005), and also support the notion that basic skills are prerequisites of more complex skills.

Keywords: frequency, Curriculum Based Measurement, Precision Teaching, Response to Intervention, component and composite performance

International tests such as *Program for International Student Assessment* (PISA) and *Trends in Mathematics and Science Studies* (TIMSS) shows that mathematics is a universal problem. In Norway, the most pressing concern is the evident decline in mathematic proficiency in high school (Grønmo et al., 2012; Kjærnsli & Olsen, 2013). It is also shown that the need for special education is growing with grade level. In the school year 2014/2015 students in need for special education increased from 3.6% in the first grade to 11.1% in the 10th grade (Utdanningsdirektoratet, 2015a). At the same time 14.5% of the Norwegian 10th graders failed their final exam in mathematics (Utdanningsdirektoratet, 2015b).

The currents state of mathematical proficiency has been an impetus for an increased national debate and effort aiming to find more effective interventions (Kunnskapsdepartementet, 2013/2015). This debate has mainly revolved around structural interventions such as, increased number of teachers and adults in schools, and demanding more specialized training of teachers in regard to core subjects (Kunnskapsdepartementet, 2013/2015). Structural interventions like these are all are ranged to have a low educational effect size; Cohen's $d = .40$ or less, if not combined with other empirically supported interventions (Hattie, 2009). The assessment of student proficiency level, and possible learning disabilities, is traditionally done through norm-and criterion-referenced achievement tests (Shapiro, 2011; VanDerHeyden & Burns, 2010). Norm-referenced achievement tests often refer to what is an expected level of proficiency at a certain age or grade level. The results will often show a narrow part of the total picture and primarily depict whether or not the student is at the expected proficiency level. These tests will rarely show what the potential learning disability is, nor will they point to interventions that would likely be effective, i.e., the "diagnostic value" may be questionable (Shapiro, 2011).

In Norway, *National tests* are implemented in order to screen students at the start of the 5th, 8th and 9th grade level. The main purpose of *National tests* is to assess students' basic skills in reading, mathematics and parts of the English course, and thereby be able to assure that the students have reached the expected level of academic proficiency, within the given subjects. *The National tests* results are further expected to guide the teachers in their instructional methods and the districts are also expected to use the results to assess the quality of the general education (Utdanningsdirektoratet, 2016). *Supportive-learning tests* have also been developed as an aim to assess student proficiency in mathematics in the grade levels 5–7 and 8–10. However, the schools themselves decide how often these tests are to be administered (Kunnskapsdepartementet, 2015). Both *National tests* and *Supportive learning tests* are examples of criterion-referenced tests and have the advantages of assessing student performance towards a standardized level based on the Norwegian standard curricula. However, these assessments are administered at a low frequency and, as such, show some limitations in detecting academic problems at an earlier time (Shapiro, 2011).

In spite of previously and current effort by the Norwegian government, student proficiency in mathematics have not improved since 2003 according to the international TIMSS and PISA tests (Grønmo et al., 2012; Kjærnsli & Olsen, 2013). Thus the need for more specific and effective screening procedures with improved “diagnostic value” seems to be desirable (VanDerHeyden & Burns 2010; Shapiro, 2011; Johnson & Street, 2013).

Effective Screening Procedures

More effective and brief assessments procedures were developed as an alternative approach to norm- and criterion-referenced tests, and aimed at being time efficient and highly predicable in regards to students’ future academic outcome.

A specific universal screening procedure is Curriculum based measurements (CBM) (VanDerHeyden & Burns, 2010; Shapiro, 2011). Deno (2003) writes that CBM originated as a tool for assessing the basic skill proficiency of students with learning disabilities. The generic CBM procedure is direct measurement of student proficiency with stimulus materials used by the teacher in the classroom, in order to assess the academic growth and outcome over time. In more recent years there has been an increase in the use of CBM as a standardized procedure for assessing students at risk for academic failure, in the four basic skill areas; reading (CBM-R), mathematics (CBM-M), written expression (CBM-W), and spelling (CBM-S). Student performance is repeatedly measured through the academic year, and each CBM procedure usually lasts from 1–3 minutes. Response frequency is scored by counting the number of correct and incorrect responses (VanDerHeyden & Burns 2009/2010; Shapiro, 2011).

CBM-M procedures are used to assess both basic and broader mathematical proficiency. When used to assess basic skills the duration of CBM-M is usually 2–8 minutes. Standard measurement unit is correct and incorrect responses with regard to the corresponding expected grade, goal, or instructional level during the timing interval (Christ et al., 2008). CBM-M is often referred to as a valid and reliable set of measurements, VanDerHeyden and Burns (2008) found that CBM-M as a measurement procedure applied to investigate fluency is most reliable to predict academic outcome in group-based administration of mathematics. However, CBM-M has been less researched than CBM-R and CBM-W, which makes it difficult to determine the growth or possibly the positive outcome for students in mathematics (Christ et al., 2008; VanDerHeyden & Burns, 2009). Further, it is important to notice that brief assessments of basic skills, are a measurement of one type of responses at a given point in time, and that the results from such assessments are to guide further interventions within the given domain (e.g. reading, mathematics etc.). The current recommendation is to use CBM-M as an assessment tool for specific subset skills of mathematics (Christ et al., 2008).

Another approach to brief and repeated universal screening procedures is *Brief Experimental Analysis* (BEA). BEA can be used as a tool for assessing where intervention is needed and further increase the probability for positive learning outcomes for students (VanDerHeyden & Burns, 2009).

Precision Teaching (PT) has also established effective screening procedures through the development of the Standard Celeration Charts. These charts monitor skill development at a frequent basis (day–day/session–session) and as such, offers crucial information on how a frequency changes as a curricula or intervention advances, and whether or not the current curricula or intervention is effective upon student learning (Johnson & Street, 2013; Kubina & Yurich, 2012).

PT, BEA and CBM have amongst some tried to investigate if a given response frequency can predict academic growth, from basic subset skills to more advanced mathematical skills. Frequency aims that could possible predict academic growth are in PT termed fluency, and refers to the ability to master a skill with accuracy plus speed (Binder, 1996) that produces the following outcomes, abbreviated as MESsAGe; (1) Maintenance, (2) Endurance, (3) Stability, (4) Application, and (5) Generativity. Given that a response frequency is fluent it also has to be (1) maintained in the future, (2) over longer period of times, (3) in other stimulus environments, (4) applied in a real world context, (5) and emerge in to more complex behavior repertoires (Johnson & Street, 2013). The concept of fluency is well described in most PT literature, but there is still a lack of evidence on what level of frequency that is necessary to reliably predict fluency (Binder, 1996; Haughton, 1972).

Haughton (1972) wrote that the best information one had at the time showed that writing single digits accurately at a frequency of 80 or above per minute, or computing 40–50 problems per minute was a good indicator for academic growth in mathematics. A frequency level at 30 or fewer math fact completions per minute would hence indicate deceleration as the curriculum advanced. Further studies have found the same frequency aim to be a predictor for future mathematical performance, but there is still a need for more research to establish a frequency aim that can predict positive learning outcomes for students in the total domain of mathematics (VanDerHeyden & Burns, 2009; Christ et al., 2008; Johnson & Street, 2013).

VanDerHeyden and Burns (2009) examined whether it is possible to develop cut-off scores in mathematics to identify students at risk for declining, and further if it is possible to identify a fluency criterion that can predict retention through BEA. Results indicated that the students mastering basic skills were more likely to reach the fluency criterion than the students that did not master basic skills. As a conclusive statement VanDerHeyden and Burns (2009) refers to that there is a need for developing cut-off scores in mathematics that can be used; to identify students at risk for academic failure, and as useful criteria to evaluate efficacy of different interventions, and further shows potential academic outcomes for the students in general. CBM-M shows the potential to be effective in assessing student proficiency but faces certain challenges (Christ et al., 2008). The main challenge is whether or not a certain level of frequency can be established as a potential cut-off score, or fluency aim that can reliably predict future academic outcome for the students (VanDerHeyden & Burns, 2009).

The notion of teaching academic skills in any area is that component skills are a prerequisite for more complex/composite skills, thus they are a necessity for the individual academic growth (Johnson & Street, 2013; Binder 1996; Kubina, Kostewich & Lin, 2009). The taxonomy of learning can yield information on how heightened competence can be achieved. Kubina et al. (2009) refers to the taxonomy of learning approach by Tiemann and Markle that comprises of four levels; (1) psychomotor, (2) simple cognitive, (3) complex cognitive and (4) emotional. In mathematics, the levels most susceptible to instruction are simple cognitive and complex cognitive, and the acquisition of knowledge from simple cognitive to complex cognitive is derived through the six stages of learning (Kubina et al., 2009). The six stages of learning are an illustration of pre-fluency skills (entry and acquisition) as well as the different fluency outcomes (Johnson & Street, 2013).

Shapiro (2011) writes that there is a compelling need for a methodology that can more accurately and directly assess basic academic skills, that can produce positive learning outcomes for the student.

Response to Intervention

A framework proven to be effective in improving academic outcomes for students is *Response to Intervention* (RtI) (Johnson & Street, 2013). RtI is founded on multiple methods and procedures developed in applied behavioral analysis, such as PT, CBM and BEA among others (VanDerHeyden & Burns, 2010; Johnson & Street, 2013).

RtI contains different strategies for assuring the quality and expected learning outcomes for the students. Johnson and Street (2013) mentions (1) early identification, (2) early interventions, (3) high-quality, empirical supported core curriculum and interventions, increasingly intensive interventions, (4) continuous and increasingly frequent progress monitoring and (5) data-based decision making as the core components of RtI. The RtI model is visualized, as a triangle with a wide bottom and narrow top, which illustrates the service delivery needs of the different tiers. The bottom level (Tier 1) refers to all students, where an empirical supported core curriculum is in effect and with the main purpose of having a preventive and proactive effect, 80–90% of students are found in Tier 1. Tier 2 illustrates students with similar error patterns who are in need for more specialized intervention, the interventions are usually group based and focuses on being efficient and create rapid responding. In Tier 3, students with individual learning disabilities with the need for more individual specific interventions are found (VanDerHeyden & Burns, 2010; Johnson & Street, 2013; Shapiro, 2011).

The triangle model is not a new phenomenon, in the medical domain, this have been an efficient way of screening patients for correct treatment. Tier 1 would refer to the general medical service, which is available for all to prevent potential disease outbreaks; an example is the vaccination program for newborns. Tier 2 and 3 would refer to groups or individuals in need for more specialized treatment to prevent diseases. Furthermore, the triangle is a known model of screening students for behavior problems, termed *Positive Behavior Interventions and Support* (PBIS). PBIS and RtI share many common features mainly that they both are preventive models (Johnson & Street, 2013).

Brief universal assessments, such as CBM, and progress monitoring are two of the five components of the RtI framework, and comprise the foundation for further decisions regarding potential effective interventions that could increase student learning. PT has also been proposed as a complementary measurement procedure within a RtI framework for assessing student proficiency at the different tiers, and also as an effective intervention for students in Tier 2 and 3 (Kubina & Yurich, 2012; Johnson & Street, 2013).

Universal screening procedures and progress monitoring can generate more accurate and reliable decisions for future interventions. However, for a RtI framework to prove effective in preventing student decelerating and enhancing student proficiency in mathematics, there is a compelling need for establishing frequency aims that could predict future mathematical growth with and without effective interventions.

Component and Composite Performance

Lin and Kubina (2005) conducted a study with 156 5th grade level students examining the relationship between the concepts of accuracy and fluency[1] in component (single-digit) and composite (multi-digit) multiplication. Their research questions were: “(1) How well will the participating 5th grade students perform basic multiplication facts and multi-digit multiplication problems in terms of accuracy and fluency,” and (2) “What is the relationship between basic multiplication fact performance and multi-digit problem completion" (Lin & Kubina, 2005, p.76).

The Lin and Kubina (2005) results for accuracy showed a mean of 98.12% for the component skill (single-digit) and a mean of 88.40% for the composite skill (multiple-digit). There was a medium but significant correlation (r = .371, *p* < .01[two-tailed]) between component and composite accuracy. Results for frequency showed a mean frequency of 57.01 for the component skill, and only 13.1% scored above the frequency aim of 80 correct digits per minute. For the component skill the mean frequency was 88.40, and only 3.2% scored above the frequency aim of 40 correct digits per minute. There was, however a strong and significant correlation between component and composite frequency (*r* = .745, *p* < .01[two-tailed]).

The correlation between component and composite frequency reported by Lin and Kubina (2005) may indicate that frequency building in component skills can improve frequency in composite skills. There is an uncertainty about this however, as the correlation measure used does not indicate the direction of the correlation. A more refined measure such as simple linear regression analysis (Yockey, 2011) could more accurately indicate whether component frequency levels predicts composite frequency levels. This kind of knowledge can lead to effective teaching by for instance, assuring that student's component skills are mastered to a certain degree or criteria before embarking upon more complex/composite skills.

The current study is a systematic replication of Lin and Kubina (2005). The systematic variations are: (1) inclusion of 6th–9th grade levels in addition to the original 5th grade level, (2) inclusion of a second screening for all grade levels (5th–9th), and (3) a simple linear regression statistical procedure to investigate whether component frequency and accuracy predicts composite frequency and accuracy.

The research questions for the current study were partly in accordance with Lin and Kubina (2005): (1) “How well will the participating 5th–9th grade students perform basic multiplication facts and multi-digit multiplication problems in terms of accuracy and fluency,” (p. 76). For the current study, the second research question is reformulated, and reads: (2) “To what degree do basic multiplication fact performance predict multi-digit problem completion.” An additional research questions was: (3) “What is the natural growth in numeral writing frequency, single and multi-digit frequency and accuracy for the different grade levels.”

Method

Participants

Three middle schools (5th to 7th grade level) and four high schools (8th to 9th grade level), all located in the Romerike region in Norway, participated in the current study. During the first data collection (T1) a total of 779 students were included: 66 from the 5th grade level, 60 from the 6th grade level, 63 from the 7th grade level, 243 from the 8th grade level and 347 from the 9th grade level. During the second data collection (T2) a total of 626 students were included, and the corresponding number of participants for each grade level were 22, 50, 37, 197, and 320 accordingly.

In order to recruit schools, the experimenters contacted principals in the designated school area, explained the study and asked if they were interested in participating. Seven schools accepted. All students attending class the day the experimenters arrived had the opportunity to participate. However, the students them self could refrain from participating. Since all the students’ answered test packets were anonymous (i.e., no traceable personal information) written parental consent were not needed.

Materials

The same test packets that Lin and Kubina (2005) constructed were used in the current study (Appendix), with approval. The test packet consisted of three different sections. Section one measured the students' number writing skills, a screening measure, where the students were told to write the Arabic numerals from 0 to 9 repeatedly on lined notebook paper. Section two presented the students with 156 random single-digit multiplication problems (e.g., 8 x 5) in order to assess their Component skill performance. Section three presented the students with 63 random multi-digit multiplication problems (e.g., 986 x 83) in order to assess their Composite skill performance. The iPhone 6 Clock application's Timer feature was used to start and end the 1-minute assessment intervals. Pens were preferred when answering the test packets and the students used their own.

Dependent Variables/Measurement Units

As in the Lin and Kubina (2005) study, the measurement unit of the current study was correctly written digits per minute. The total correct digits generated two variables—frequency and accuracy. Each correctly written digit was counted and included in the student's total frequency and accuracy score. The frequency variable was measured as the number of correctly written digits per minute, and the accuracy variable was measured by dividing correctly written digits on the total digits in each algorithm, which gave a percentage number correct digits. This yielded four variables to analyze: Component skill frequency, Component skill accuracy, Composite skill frequency and Composite skill accuracy. Also, a screening variable, Number writing frequency, was calculated.

The pre-constructed key described in Lin and Kubina (2005) was used. For example, the multiplication problem 8 x 5 = 40 gives two correct digits (frequency) and 100% accuracy. If the student made a mistake and wrote 41 as the final answer then it would be counted one correct digit (i.e., the 4 in the tens column is correct, but the 1 in the ones column is not), and 50% accuracy. In the case of 986 x 83, this includes an algorithm of 2958 + 78880 = 81838, which gives the total of 14 correct written digits (frequency) and 100% accuracy. But if a student only wrote 2958 then four correct digits was counted toward the students' total frequency, but the accuracy would still be 100% because all written digits are correct. However, if the student has calculated the whole algorithm (i.e. final answer: 2958 + 78880 = 81838) with all digits correctly placed, the accuracy will also be 100%.

The frequency aims in the current study equals those used by Lin and Kubina (2005). Component skill (single-digit) frequency aim was set at 80 correctly written digits per minute (the usual aim is 80–120 according to Lin and Kubina (2005)), and Composite skill (multiple-digit) frequency aim was set at 40 correctly written digits per minute (the usual aim is 40–60 according to Lin and Kubina (2005)). Both the Component and Composite accuracy aim was set at 100% correct.

Interscorer Agreement and Reliability

Both interscore agreement and Intraclass Correlation Coefficient will be assessed as there are some differences about the usability of the different reliability measures (Gast & Leford, 2014; Hallgren, 2012).

Interscore agreement was assessed for every tenth test packet, all measure sections were counted and correct and total digits were registered. The T1 mean interscorer agreement was 100% for all measures; Numeral writing, Component frequency, Component accuracy, Composite frequency and Composite accuracy respectively. The T2 mean interscorer agreement was 100% for the measures Numeral writing, Component frequency and Component accuracy respectively. For both Composite frequency and Composite Accuracy the mean interscorer agreement was 99.9% (Min. 91%, Max. 100%).

Reliability was also measured for every tenth test packet for the outcome variables Composite frequency and Composite Accuracy. The Intraclass Correlation Coefficient by means of a two-way mixed model with average measure, expressed as ICC (3,2), yielding a reliability measure expressed as Cronbach's alpha (α) with 95% Confidence Intervals reported in the format of [LL, UL]. Results for T2 showed that $\alpha = 1.00$ for Composite frequency, and Composite accuracy respectively.

Procedure

The procedure was a replication of the procedure used by Lin and Kubina (2005). All the tests were completed in the students' regular classroom at their school. The students were sitting at their regular desks and the experimenters presented themselves and gave a short description of the study and what was expected of the students. Test packets were handed out to all students with instruction of not turning them around until the experimenters said so. The experimenters informed that they should answer each of the three test sections in one-minute as quickly and accurately as possible. The experimenters used the same scripted message as Lin and Kubina (2005), only translated to Norwegian.

> You are going to do three short tests. Each test lasts one-minute. I will use this timer to set one-minute. For each test, when I say "ready set go," please start the test. When the timer starts to beep, please stop and put your pencil on the table (Lin & Kubina, 2005, p. 77).

The Norwegian translated message:

> Nå skal dere gjennomføre tre korte tester. Hver test har en varighet på ett-minutt. Jeg kommer til å bruke denne iPhone og sette på ett-minutt. For hver test, når jeg sier ”klar, ferdig, gå" så kan dere starte testen. Når klokka starter å pipe, vær snill å stoppe og legg ned blyanten på pulten.

Each section of the test had additional instructions. In section one of the test the students were told to repeatedly write 0–9 on lined paper. This was demonstrated by the experimenter using her hand writing from left to right in the air, simulating an answer. In both sections two (single-digit) and three (multiple-digit) of the test the students were told to cross out their answer if they did it wrong, and just to skip a problem they did not know the answer to. They were further informed that time is of the essence and that they should not waist the time to erase wrong answers. The sections two and three both included two pages each, and the students were told to start on page one and if they finished then go to page two. All students were told to write out their full answer, specifically on section three they were requested to write the total algorithm in addition to the final answer.

Questions from students were answered before the test was started, and all the students were informed that the tests contained a lot of problems, and they were not able to finish them all in one minute. When all three sections of the test were finished the students were told to write a code on the first page. The code was the first three letters in the students' mother's first name, then the first three letters in their own first name, then the first three letters of their father's first name. The code made it possible to compare students' test from both data collections (T1 and T2) and still preserve anonymity.

The first data collection was conducted in end of January 2015, and the second data collection was conducted in the beginning of June 2015. The days between the first and second data collection ranged from 126 to 143 days for the 7 schools. The same procedure was used on both data collections.

Statistical Analysis Plan

In order to answer the first research question, on how well will participating students perform single-digit multiplication problems and multi-digit multiplication problems in terms of accuracy and frequency aim, descriptive statistics were used to analyze the data collected (Tables 1, 2, and 3).

The second research question was examined using inferential statistics, more specifically a simple linear regression analysis, in order to examine to what degree a score on one (independent) variable predict a score on another (dependent) variable (Table 4). This analysis gives information about the direction of the data, not merely the correlation, and yields a predicting value termed beta (β). Beta is the average amount by which the dependent variable increases when the independent variable increases one standard deviation and other variables are held constant (Yockey, 2011), an expression of the predictive importance of the independent variable. Beta is equal to Pearsons r in simple linear regression. The effect size expression in simple linear regression is R^2 with the values.01, .09 and .24 corresponding to small, medium and large effect sizes respectively. R^2 is also an expression for «explained variance», how many percent of the variance in the dependent variable that is explained by the independent variable. Section one of the test packet measured the students' writing speed, and it was a way to find out which student who might have slow handwriting skills. If a student's Numeral writing performance was lower than the Composite frequency aim, 40 digits per minute, the student was excluded from the regression analysis.

In addition to the research questions that Lin and Kubina (2005) studied, the current study also looked at the students' growth from data collection one (T1) to data collection two (T2) in the different grade levels. For those students who participated on both occasions (T1 and T2), a repeated measures t-test was calculated for all outcome variables (Table 5) for each grade level. The Cohen's *d* effect size was calculated with the formula Mean difference/Standard deviation of the difference scores, and *d* conventions for small, medium and large effect sizes are .20, .50, and .80 accordingly (Yockey, 2011).

Results

779 completed test packets from data collection on (T1) were analyzed. Due to space limitations only 5th grade results will be written out, this because it is the comparison with Lin and Kubina (2005) results that is most relevant to examine. Results for the other grade levels are depicted in Tables 1, 2 and 3. All frequencies represent correct digits per minute.

Mean, Min., and Max. Comparisons

Results for all grade levels are depicted in Table 1. Fifth grade students had a Mean numeral writing frequency of 60.58 (Min. 25, Max. 92), a Mean Component frequency of 17.53 (min. 0, max. 45), and a Mean Composite frequency of 2.03 (Min. 0, Max. 8). Mean Component accuracy was 68.21% (Min. 0%, Max 100%), and Mean Composite accuracy 57.97% (Min 0%, Max. 100%).

Fifth grade students performed poorer than the other grade levels on both Component and Composite frequency, but performed better or equal to the other grade levels on Component and Composite accuracy.

Comparison with Performance Aims

Results for all grade levels are depicted in Table 2. 88% of 5th grade level student performed at or above a frequency of 40 on Numeral writing, 12% did not. None of the 5th grade level students performed at the frequency aim (≥ 80) on Component frequency, whereas 62% performed at at the aim (100%) on Component accuracy. None of the 5th grade level students performed at the frequency aim (≥ 40) at Composite frequency, but 43% performed at the aim (100%) on Composite accuracy.

In total only three students performed at the Component skill frequency aim (≥ 80), two from the 8th grade level and one from the 9th grade level. Furthermore, the Composite skill frequency aim (≥ 40) was not reached by any of the students.

Variability among Accuracy and Fluency

Results for all grade levels are depicted in Table 3. 62% of 5th grade level students had 100% Component accuracy, whereas 38% had below 100%. For those with 100% Component accuracy the Mean Component frequency was 20.32 (SD = 9.32) (Min. 4, Max. 45). For those with less than 100% Component accuracy the Mean Component frequency was 12.96 (SD = 8.42) (Min. 0, Max. 37).

44% of 5th grade level students had 100% Composite accuracy, whereas 56% had below 100%. For those with 100% Composite accuracy the Mean Composite frequency was 2.66 (SD = .90) (Min.1, Max. 4). For those with less than 100% Composite accuracy the Mean Composite frequency was 1.54 (SD = 2.05) (Min. 0, Max. 8).

Regression Analysis Between Component and Composite Frequency and Accuracy

Results for all grade levels are depicted in Table 4. Eight students from the 5th grade level, two from 6th grade level, four students from 7th grade level, 12 from 8th grade level, and nine from 9th grade level performed below the frequency aim of 40 written digits per minute on numeral writing (see Table 2). These 35 students' test packets were excluded from the simple linear regression analysis.

When examining if Component frequency predicts Composite frequency in 5th grade level students, there were no significant predicting value; $\beta = .136$, $p > .05$ and Component frequency accounts for only 1.8% ($R^2 = .018$) of the variance of Composite frequency. Furthermore, for 5th grade level students Component frequency predicted lower Composite accuracy, $\beta = -.284$, $p < .05$, with a small ES ($R^2 = .018$). Regarding Component accuracy for 5th grade level students, this variable did not predict Composite frequency ($\beta = .104$, $p > .05$, $R^2 = .011$) or Composite accuracy ($\beta = -.07$, $p > .05$, $R^2 = .005$).

Further results show no significant relationship examining the variables Component frequency and how it predicts Composite frequency, but 6th, 7th, 8th and 9th grade levels show medium ES.

For most of the participating students the analysis shows no effect at all, the only variable showing an effect is the Component frequency predicting Composite frequency. For the other three variables in Table 4, Component frequency predicting Composite accuracy, Component accuracy predicting Composite frequency, and Component accuracy predicting

Composite accuracy, the predicting levels are to low and not significant for any of the grade levels.

Repeated Measures t-test for Numeral Writing, Component Frequency, Component Accuracy, Composite Frequency and Composite Accuracy.

Results for all grade levels are depicted in Table 5. There were 17 5th grade level students who completed both data collections. Numeral writing showed a significant growth from T1 ($M = 57.88$, $SD = 14.80$) to T2 ($M = 65.00$, $SD = 15.13$), $p < .05$, with a medium ES ($d = .69$). Component frequency also showed a significant growth from T1 ($M = 11.65$, $SD = 7.049$) to T2 ($M = 19.41$, $SD = 9.23$), $p < .01$, and the ES is large ($d = 1.11$). Component accuracy showed a significant change from T1 ($M = 86.77$, $SD = 18.54$) to T2 ($M = 96.28$, $SD = 7.17$), $p < .05$, and the ES was medium ($d = .53$). Composite frequency and Composite accuracy showed no significant change from T1 to T2.

Overall, the significant changes from T1 to T2 are found in Numeral writing and Component frequency. A few grade levels achieve significant results in Component accuracy. For Composite frequency only 9th grade level showed a significant improvement from T1 to T2, and for Composite accuracy none did.

Discussion

The current study aimed to investigate (1) “How well will the participating 5th–9th grade students perform basic multiplication facts and multi-digit multiplication problems in terms of accuracy and fluency”, (2) “To what degree do basic multiplication fact performance predict multi-digit problem completion” and (3) “What is the natural growth in numeral writing frequency, single and multi-digit frequency and accuracy for the different grade levels”. A total of 779 test packets from T1 and 626 test packets from T2 were analyzed. Due to space limitations the results from the 5th grade will be emphasized, the remaining grade levels will mainly be discussed in regards to the third research question.

The data from the current study illustrates that overall most students reached the frequency aim for numeral writing, only 4.5% did not. None but three of the students (grades 5–9) reached the component frequency aim, and none reached the composite frequency aim (grades 5–9).

For component accuracy 62% of 5th graders reached the aim, and 43% in composite accuracy, the picture being similar in grades 6–9 for component accuracy but not for composite accuracy. Results indicate that component frequency predicted composite frequency with a medium effect for 6th–9th grade, but not for 5th grade. Component frequency did not predict accuracy (component or composite).

The natural growth from January to May 2015 showed an increase in mean frequency in component frequency for all grade levels with medium to large effect sizes. Only 9th grade showed a significant increase in composite frequency with a small effect size. In component accuracy only 5th and 9th grade showed a significant increase with a medium and small effect sizes. None of the grade levels had a significant increase in composite accuracy.

General Student Performance

Numeral Writing

Results from the current study illustrates that the majority of students reached the frequency aim of writing single-digit numbers from 0–9 at a frequency at 40 digits or more per minute. The results showed however that the participating 5th grade students in the current study had a lower frequency in numeral writing than did the 5th grade students in the Lin and Kubina (2005) study. Overall the Norwegian 5th graders held a lower minimum and maximum frequency, which gives a lower mean frequency than the American 5th graders. In general, none of the participating grade levels in the current study reached the same mean frequency as the American 5th graders. These results indicate a trend of lower frequency through the rest of the test packages among the Norwegian participants in comparison to the Lin and Kubina

(2005) study. A slower frequency among the Norwegian participants in numeral writing may indicate overall slower performance through the test-packets than the American 5th graders in component and composite skills.

Potential causes for the difference in numeral writing is difficult to determine, but there is a possibility that the curricula, teaching strategies, and emphasis on tool skills, such as handwriting, differs among the two countries and that could serve as a potential variable for the numeral frequency differences. Differences between the countries and the time passed since the Lin and Kubina (2005) study can indicate that students in the current study more often use keyboards for writing and calculators in mathematics, thus, the time spent practicing tool skills is reduced.

Component Skill Performance

The results in component skills in the current study illustrates that the majority of students in all grade levels reached the accuracy aim, but none except 3 students reached the frequency aim in component skills. The Norwegian 5th graders showed none students reaching the frequency aim, which illustrates a difference from the American 5th graders, where 22 students reached the component frequency aim. Which further strengthens the assumption that Norwegian students hold a lower frequency through the test-packets than the American 5th graders did.

When the accuracy aim in component skills was held consistent at 100%, the results show a greater variance in the 5th grade component skills, than the American 5th graders. The 5th grade students in the current study who scored below accuracy aim, also illustrates a lower frequency level. Which indicate that higher accuracy levels to a degree corresponds with higher frequency levels. However, they still performed below the frequency aim at 80 correct digits per minute. The variance among accuracy and fluency in the 6th grade is more equal to the level of variance displayed in the Lin and Kubina (2005) study. However, the general trend shows a larger variance between component frequency and accuracy scores through the grade levels. The overall low frequency in component skill could indicate poorer results in composite skills. Given that high frequency scores with accuracy would yield further celeration as the curricula advances, and that low frequency scores with accuracy would yield a deceleration as the curricula advances (Binder, 1996; Haughton, 1972).

Reasons for the overall low frequency scores may be due to a lack of proficiency in tool skills, but may also reflect an acquisition level in component skills even in the higher grade levels. This may be a function of too rapid progress through the curriculum, i.e., the students do not get a chance to move through the different stages of learning before a new skill is introduced (Kubina et al., 2009).

Composite Skill Performance

In composite skill performance 43% of the participating 5th graders reached the accuracy aim, but none reached the frequency aim of 40 correct digits per minute. In comparison; 3.2% of the 5th grade students reached the frequency aim, and 48.1% reached the accuracy aim in the Lin and Kubina (2005) study. In comparison with component skills, the assumed trend of deceleration in frequency and accuracy seems to hold true.

None of the participating students in the current study reached the frequency aim, and the majority of students did not reach the accuracy aim. The variability among frequency and accuracy is greater in composite skills than component skills, and in general, except the 5th grade, the amount of students scoring at the accuracy aim holds a higher frequency than students that score below the accuracy aim. Which is similar to the trend in component skills.

A factor worth noting in regards to the composite accuracy results is how correct digits were scored towards the accuracy percentage. The study by Lin and Kubina (2005) counted every correct digit in correct place as correct. As the current study is a systematic replication, the identical scoring method were applied, but was during the study discovered to be an unclear scoring method. Unfortunately, this was discovered too late. A more precise method could be; counting percentage of correctly placed digits for the whole algorithm. If the algorithm in total contains 13 digits, then 13 are the required number to achieve 100% correct (Kubina et al., 2009, p.23).

Component and Composite Frequency and Accuracy

In the current study results illustrates that component frequency predicted composite frequency with a medium effect, but not composite accuracy. Lin and Kubina (2005) found similar results; a strong correlation between component and composite frequency, but not between component frequency and composite accuracy. The correlation between component and composite frequency is consistent with previous research on fluency, which shows that frequency levels (high or low) in component skills correlates with frequency levels (high or low) in composite skills (Haughton, 1972; Lin & Kubina, 2005; Johnson & Street, 2013; VanDerHeyden & Burns, 2009).

Explanations for why component frequency did not predict composite accuracy could be a lack of proficiency in other component skills needed to compute the composite math facts other than single-digit multiplication, such as addition. Other factors could be that the students used the timing interval searching for math facts they would be able to compute.

Component accuracy did not predict composite frequency or accuracy, which is similar to the results from Lin and Kubina (2005). Explanation for this could be a lack of proficiency in tool skills such as place value, or component skills such as addition and single-digit multiplication (Johnson & Street, 2013). Such tool and component skills are needed for completing the composite math facts. Proficiency in tool skills are in general crucial prerequisites for more advanced mathematical problems. Which could be a possible explanation for the low frequency and accuracy score attained by the students in component and composite skills in the current study.

Component and Composite skill performance in the current study shows that students in the 5th grade hold a lower frequency than the other grade levels, but scores higher or equal to the other grade levels in regards to accuracy, both in component and composite skills. The current findings indicate that the students are more focused on getting the mathematical problems correct than they are sensitive to the time spent on each problem. Working slow may at times be functional, but in the case of student performance, there is in most cases a timeframe to be sensitive to. Exams and tests usually have a time limit, and given that one works slowly it may indicate an incomplete assignment, which could reflect the grade in mathematics. Which could impact further educational decisions, and in a long time scale impact national wealth and growth (Hanushek & Kimko, 2000).

Natural Growth

Results from T1 show that the mean frequency in numeral writing and component frequency increases with grade. There is a significant growth in all grade levels in component frequency. Although all grade levels illustrate growth, individual differences among grade levels in component frequency depict a drop in T1 from students in the 8th grade to 9th grade students in T2. In component accuracy only the 5th and 9th grade showed significant growth, but there is a decrease in component accuracy after the 7th grade in both T1 and T2.

In composite frequency only the 9th grade students illustrate a significant growth, but there is a drop in component frequency after 7th grade, which is illustrated in both T1 and T2. In composite accuracy none of the grade levels had significant growth, but the same trend is seen, i.e. a drop in accuracy after the 7th grade.

Overall the grade levels are all well below the frequency aims in component and composite skills, which could indicate that they might need a higher frequency to be able to maintain the required skill. The students' natural growth in the current study also illustrates that even though they hold a low frequency, they increase through the school year, but at the same time there is a drop in frequency between grade levels at certain points.

The results show that general education is effective to a degree. Hattie (2009) states that all educational effects below a benchmark at Cohens´s d = 0.40. is in need for further analysis, that is not saying that the educational method in effect is worthless, but that they need more investigation. At the same time the results correspond with international tests, such as PISA and TIMSS which report that mathematical proficiency among Norwegian 8th grade students are in decline. Given that these results only display a given point in time, makes it difficult to determine if the results is a representative picture of natural growth in mathematics. A more comprehensive approach would be to follow grade levels over time.

General Discussion and Conclusion

The current study has a couple of strengths in comparison with the Lin and Kubina (2005) study. First, multiple grade levels were included and thus offered the opportunity to examine the frequency and accuracy variables across grade levels. This made it possible to discover that low component and composite frequency is relative stable over the grade levels. This strengthens the notion that students may not attain a proficient level in tool or component skills before they are required to progress to more complex/composite skills.

Second, the current study performed two assessments in contrary to one. The first one was conducted during the first month of the spring semester, and the second one was administered in the last month of the semester. Performing two assessments made it possible to investigate natural growth in component and composite skill under the general teaching curriculum. The findings from T1 and T2 reinforce the notion that low frequency is relative stable over the grade levels, and that the students do not seem to reach the acquisition level in basic skills, by the fact that component skill improved from January to May/June but seem to decrease again in January for the nest grade level.

A third strength of the current study is the inclusion of a simple linear regression analysis. By including this statistical analysis, we could examine both the correlation and the direction of the correlation between the investigated variables, which is an expansion of the correlation analysis applied by Lin and Kubina (2005).

However, conclusion derived from the findings could be weakened by the fact that the assessments performed are snap-shot pictures from each classroom, and the question of growth over grade levels and/or months can thus only be presumed. Future research should make efforts to follow a group of students in a grade level, for instance 5th grade, over several years and perform CBM three times each school year, as proposed in the RtI framework (Johnson & Street, 2013; VanDerHeyden & Burns, 2010; Shapiro, 2011). This could make it possible to assess whether the same component and composite skills grow across the academic year, and also whether there is a decline from the end of one school year to the beginning of the next school year, in the same student group. This would make assumptions about possible frequency growth or decline and possible explanations for the academic changes, more sound.

Other potential weaknesses could be that one school from the grade levels 5–7 dropped out at T2. This is unfortunate and could serve as a weakness, even if it was uncontrollable. This may have affected the results in natural growth from T1 to T2, due to a lower number of participants in T2. However, it did not affect the analyses that were a direct replication of Lin and Kubina (2005), as that certain school participated in T1, and T2 data only refers to the third research question. Further, did the study not include a gender variable when the test packets were performed and collected. This may have provided less nuanced data and hence conclusions from the study. Since we do not have a gender variable we do not know whether there is a gender difference in skills. Further studies are recommended to include gender.

Another weakness could be the accuracy calculation applied in the current study, as it may not represent true accuracy. The accuracy calculation described by Kubina et al. (2009) might reflect a more accurate accuracy calculation, and are recommended in further studies. The lay out of the test packet (Appendix) also serve as a potential weakness, further studies should consider to arrange a different layout of the test-packet where place-value are provided in to visualizes where to write the digits for completion of the multi-digit math facts. Future research should also add new assessment packages each year in addition to some stable core packages. These additions should contain skills targeted as end of year goals/learning objectives for each grade level. This will make it possible to assess development of more complex skills and their relation to for instance critical simpler, or component, skills. Another weakness worth mentioning is that the data registration of the test-packet after screening, where done manually by the researchers on a Mac-computer which could have resulted in human error.

The current study illustrates that there is a difference in frequency level in tool, component and composite skills, from the Lin and Kubina (2005) study. A continuous lower performance among the Norwegian participants is heavily illustrated. A similar trend is also seen in international tests such as PISA (Kjærnsli & Olsen, 2013) and TIMSS (Grønmo et al., 2012) were USA shows a higher proportion of 4th graders reaching international benchmarks than Norwegian 4th graders (International Study Center, 2011). These results may reflect differences in the core-curricula and teaching strategies between the two countries.

Efforts conducted by the Norwegian government to increase student proficiency in mathematics, might not have the desired effect upon student learning, as they focus mainly on structural issues, such as more money, increase number of teachers and adults in schools and specializing teachers in core subjects (Kunnskapsdepartementet, 2013/2015). These are interventions that are ranged as having a small effect upon student learning (Hattie, 2009). A possible cause for implementing interventions that show little effect, could be the lack of adequate information regarding student performance and growth, due to infrequent and unprecise assessments.

International and national tests are examples of norm-and-criterion referenced tests. Which measure student proficiency towards a predetermined standardized level, and are administered at a low or infrequent basis. As norm-and-criterion referenced tests proposes some advantages (yield information on general and individual student performance) they have certain limitations in regards to pinpointing the specific area in the academic environment in need of change (Shapiro, 2009). This may lead to inaccurate decisions regarding potential effective interventions and thus develop ineffective teaching strategies or curricula. Brief universal screening procedures, such as CBM, are found to be more reliable to predict future student outcome. As they are aimed to assess student proficiency within a current curriculum, and as such could yield more accurate information of the academic environment and potential changes that could enhance student proficiency (VanDerHeyden & Burns, 2009/2010; Shapiro, 2011; Christ et al, 2008).

Overall the findings replicated the findings by Lin and Kubina (2005). Both studies found that component frequency predicted composite frequency, and illustrated a higher percentage of students scoring within the component/composite accuracy aim than in the component/composite frequency aim. Which strengthens the assumption that students work slow, but accurate. Results from natural growth indicate that the low frequency is stable over grade levels, but there is drop in frequency from T2-T1 at certain grade levels. This indicate that frequency building in component skills may be a necessity for further growth through more, advanced curricula. It is possible to assume that a higher frequency in combination with accuracy would be more functional as several studies have found that skill frequency in lower grades and basic skills, corresponds to acquisition in higher grades and complex/composite skills (VanDerHeyden & Burns, 2008/2009; Johnson & Street, 2013; Haugthon, 1972). Previous research has also found that frequency building in basic skills, can develop skills that are more likely to be maintained over longer period of times and as necessary key-components for more complex skills (Johnson & Street, 2013; Binder, 1996; Haughton, 1972; VanDerHeyden & Burns, 2009).

The current findings show that there might be a need for more effective interventions aimed at increasing student proficiency in basic skills. Possible solutions for establishing more specialized and effective interventions in schools, could be to apply empirically supported methods that incorporates both the taxonomy of learning and the six stages of learning (Kubina et al., 2009). A procedure found to comprise how to teach, how learning occurs and frequency building in combination with frequent progress monitoring is PT. Kubina and Yurich (2012) states that PT could be a good procedure to implement both for students with specific learning disabilities as well as whole classes, given its frequent monitoring of student progress and its focus on rapid frequency building of basic skills.

As previously discussed, too rapid advancement through the curricula may be a cause for the lack of proficiency in tool and component skills. In order to secure proficiency in one skill level before advancing to the next, there is a need for more frequent monitoring. Universal screenings procedures such as CBM is regarded as highly reliable in predicting future academic outcome in reading and writing, and show promise in assessing student proficiency in mathematics. However, there is a need for more research on natural growth frequency in mathematics, before a certain level of frequency could be established as predictable for future student academic outcome (Christ et al., 2008). By following a lower grade over grade levels and conducting CBM approximately three times a year it might be possible to establish more reliable frequency aims, and as such generate earlier and more effective interventions, such as PT (VanDerHeyden and Burns, 2009/2010; Christ et al, 2008).

Frequent monitoring of a skill development over grade levels through brief universal screening procedures has the potential to establish predictable frequency aim for future students' performance (VanDerHeyden & Burns, 2008). Establishing frequency aims could generate more accurate decisions regarding student proficiency, and the effectiveness of the current curricula and hence, lead to earlier and more effective interventions be set in place. Which again could enhance the probability of positive academic outcome for all students.

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Footnotes

[1] The Lin and Kubina (2005) term “fluency” is actually better described as “frequency”, as fluency in the PT-literature is conceptualized as both speed (frequency) and accuracy, and furthermore; a certain frequency with accuracy, only indicates fluency (Binder, 1996). Aside from referring to the Lin and Kubina (2005) study, the term frequency will be used throughout the current study.

Tables

Table 1.
Mean (M), Standard Deviation (SD), Min. and Max. Comparisons.

Variable	Grade level	N	M	SD	Min.	Max.
Numeral writing	5	66	60.58	15.37	25	92
	6	60	69.58	16.39	28	104
	7	63	75.92	21.64	9	108
	8	243	82.55	22.49	0	150
	9	347	84.59	19.65	6	130
Component frequency (digit)	5	66	17.53	9.62	0	45
	6	60	25.58	11.70	0	50
	7	63	33.57	11.68	11	52
	8	243	33.98	15.91	0	106
	9	347	30.24	15.50	0	89
Component accuracy (percent)	5	66	68.21	45.08	0	100
	6	60	62.93	48.36	0	100
	7	63	66.59	47.47	0	100
	8	243	64.95	47.36	0	100
	9	347	56.79	49.18	0	100
Composite frequency (digit)	5	66	2.03	1.73	0	8
	6	60	3.10	2.83	0	10
	7	63	4.38	3.94	0	20
	8	243	4.59	3.73	0	19
	9	347	4.19	4.01	0	33
Composite accuracy (percent)	5	66	57.97	42.26	0	100
	6	60	47.90	35.45	0	100
	7	63	59.77	35.30	0	100
	8	243	62.42	36.39	0	100
	9	347	61.84	37.45	0	100

Table 2.
Comparison with Performance Aim

Variables	Grade level	Aim	Frequency [a]	Percent [b]	M	SD	Min.	Max.
Numeral writing	5	< 40	8	12	33.00	4.96	25	38
		≥ 40	58	88	64.38	12.04	40	92
	6	< 40	2	3	29.00	1.41	28	30
		≥ 40	58	97	70.99	14.77	40	104
	7	< 40	4	6	14.00	7.57	9	25
		≥ 40	59	94	80.12	14.68	42	108
	8	< 40	12	5	17.83	14.99	0	37
		≥ 40	231	95	85.91	17.08	45	150
	9	< 40	9	3	29.11	12.77	6	39
		≥ 40	338	97	86.07	17.55	47	130
Component frequency	5	< 80	66	100	17.53	9.62	0	45
		≥ 80	0	0	–	–	–	–
	6	< 80	60	100	25.58	11.70	0	50
		≥ 80	0	0	–	–	–	–
	7	< 80	63	100	33.57	11.68	11	52
		≥ 80	0	0	–	–	–	–
	8	< 80	241	99	33.48	14.95	0	79
		≥ 80	2	1	94.50	16.26	83	106
	9	< 80	346	100	30.07	15.25	0	73
		≥ 80	1	0	89.00	–	89	89
Component accuracy	5	< 100	25	38	16.08	30.75	0	92
		= 100	41	62	–	–	–	–
	6	< 100	24	40	7.33	24.96	0	96
		= 100	36	60	–	–	–	–
	7	< 100	22	35	4.32	20.25	0	95
		= 100	41	65	–	–	–	–
	8	< 100	97	40	12.19	31.20	0	96
		= 100	146	60	–	–	–	–
	9	< 100	162	47	7.44	24.53	0	98
		= 100	185	53	–	–	–	–
Composite frequency	5	< 40	66	100	2.03	1.72	0	8
		≥ 40	–	–	–	–	–	–
	6	< 40	60	100	3.10	2.83	0	10
		≥ 40	–	–	–	–	–	–
	7	< 40	63	100	4.38	3.94	0	20
		≥ 40	–	–	–	–	–	–
	8	< 40	243	100	4.59	3.73	0	19
		≥ 40	–	–	–	–	–	–
	9	< 40	347	100	4.19	4.01	0	33
		≥ 40	–	–	–	–	–	–
Composite accuracy	5	< 100	37	56	25.03	26.20	0	80
		= 100	29	43	–	–	–	–
	6	< 100	47	78	33.49	25.15	0	80
		= 100	13	21	–	–	–	–
	7	< 100	42	67	39.65	25.31	0	83.33
		= 100	21	33	–	–	–	–
	8	< 100	161	66	43.28	30.17	0	91.66
		= 100	82	34	–	–	–	–
	9	< 100	223	64	40.62	30.34	0	94.11
		= 100	124	36	–	–	–	–

Note: [a] Frequency refers to number of participants in each category, [b] Percent refers to percent of participants in each category.

Table 3.
Variability among Accuracy and Fluency

	Grade level	Component accuracy	Frequency [a]	Percent [b]	M	SD	Min.	Max.
Component frequency	5	< 100 %	25	38	12.96	8.42	0	37
		= 100 %	41	62	20.32	9.32	4	45
	6	< 100 %	24	40	23.54	10.80	0	41
		= 100 %	36	60	26.94	12.21	2	50
	7	< 100 %	22	35	28.73	8.42	14	44
		= 100 %	41	65	36.17	12.43	11	52
	8	< 100 %	97	40	30.75	16.13	0	83
		= 100 %	146	60	36.13	15.44	3	106
	9	< 100 %	162	47	26.86	14.88	0	65
		= 100 %	185	53	33.19	15.56	2	85
	Grade level	**Composite accurancy**	**Frequency [a]**	**Percent [b]**	**M**	**SD**	**Min.**	**Max.**
Composite frequency	5	< 100 %	37	56	1.54	2.05	0	8
		= 100 %	29	44	2.66	0.90	1	4
	6	< 100 %	47	78	2.49	2.45	0	9
		= 100 %	13	22	5.31	3.12	1	10
	7	< 100 %	42	67	3.48	3.22	0	11
		= 100 %	21	33	6.19	4.65	2	20
	8	< 100 %	161	66	4.06	3.65	0	17
		= 100 %	82	34	5.62	3.68	1	19
	9	< 100 %	223	64	3.65	3.54	0	16
		= 100 %	124	36	5.15	4.60	1	33

Note: [a] Frequency refers to number of participants in each category, [b] Percent refers to percent of participants in each category.

Table 4.

Regression analysis between Component and Composite frequency and accuracy.

Component frequency predicting Composite frequency

Grade level	N	β	*p*	*R²*
5	58	.136	.309	.018
6	58	.382	.003	.146*
7	59	.315	.015	.100*
8	231	.427	.000	.182*
9	338	.447	.000	.200*

Component frequency predicting Composite accuracy

Grade level	N	β	*p*	*R²*
5	58	− .284	.031	.080
6	58	.297	.023	.088
7	59	.268	.040	.072
8	231	.117	.076	.014
9	338	.196	.000	.038

Component accuracy predicting Composite frequency

Grade level	N	β	*p*	*R²*
5	58	.104	.436	.011
6	58	.068	.614	.005
7	59	.069	.603	.005
8	231	.064	.336	.004
9	338	.039	.470	.002

Component accuracy predicting Composite accuracy

Grade level	N	β	*p*	*R²*
5	58	−.070	.602	.005
6	58	.119	.374	.014
7	59	.214	.104	.046
8	231	.098	.137	.010
9	338	.019	.730	.000

Note: * = medium effect size

Table 5.
Repeated measures t-test for Numeral writing, Component frequency, Compunent accuracy, Composite frequency and Composite accuracy.

	Grade		**January (T1)**		**May (T2)**			
Variable	**level**	**N**	**Mean**	**(SD)**	**Mean**	**(SD)**	t^a	***d***
Numeral writing	5	17	57.88	(14.80)	65.00	(15.13)	2.84*	.69^
	6	21	73.43	(15.24)	80.14	(15.63)	2.45*	.53^
	7	22	83.41	(13.65)	87.55	(18.13	2.20*	.47^
	8	131	81.71	(21.67)	88.70	(20.56)	4.74**	.41^
	9	186	85.39	(17.34)	90.61	(20.20)	4.48**	.34
Component frequency	5	17	11.65	(7.049	19.41	(9.23)	4.57**	1.11^^
	6	21	28.52	(12.26)	40.43	(14.02)	5.76**	1.26^^
	7	22	34.09	(13.61)	39.05	(13.69)	2.47*	.53^
	8	131	34.29	(15.87)	36.80	(14.74)	3.20**	.28
	9	186	29.90	(15.60)	36.00	(15.81)	10.71**	.79^^
Component accuracy	5	17	86.77	(18.54)	96.28	(7.17)	2.18*	.53^
	6	21	93.05	(12.63)	97.93	(3.39)	1.65	–
	7	22	98.79	(2.44)	98.58	(3.41)	.21	–
	8	131	95.52	(11.07)	96.11	(9.64)	.51	–
	9	186	93.73	(15.29)	97.74	(4.05)	3.55**	.26
Composite frequency	5	17	1.94	(1.60)	2.18	(1.74)	.51	–
	6	21	3.52	(3.14)	3.81	(3.20)	.33	–
	7	22	5.05	(3.48)	5.55	(3.20)	.56	–
	8	131	4.83	(4.00)	5.42	(4.53)	1.51	–
	9	186	4.31	(4.30)	5.23	(4.65)	2.81**	.21
Composite accuracy	5	17	59.18	(43.40)	61.70	(41.91)	.22	–
	6	21	44.22	(28.97)	50.53	(37.34)	.84	–
	7	22	63.88	(29.70)	66.73	(30.31)	.36	–
	8	131	63.71	(36.28)	62.15	(36.33)	.39	–
	9	186	61.05	(35.86)	66.91	(36.52)	1.77	–

Note: ^{a}t-test statistics: the degrees of freedom (df) is always N - 1. ES (*d*) are calculated for significant differences only.
* = p < .05, ** = p < .01
^ = medium ES, ^^= large ES

Appendix

Tool Skill Test (Write down 0-9 numerals)

Please use the back of this page if you need extra space.

Component Skill (1 digit x 1 digit multiplication)

8	8	1	2	4	3	8	5	6	1	6	4
x 5	x 4	x 1	x 7	x 8	x 7	x 8	x 6	x 8	x 1	x 0	x 5
2	9	4	0	8	8	2	7	5	9	6	9
x 2	x 5	x 3	x 0	x 6	x 1	x 8	x 1	x 6	x 2	x 2	x 9
1	1	8	1	6	2	8	8	9	7	8	4
x 5	x 6	x 8	x 6	x 9	x 3	x 0	x 3	x 6	x 4	x 3	x 4
4	2	7	1	9	4	1	9	0	6	7	6
x 5	x 4	x 7	x 2	x 5	x 4	x 8	x 9	x 2	x 8	x 4	x 1
0	8	6	1	2	3	8	8	1	9	6	7
x 6	x 2	x 8	x 8	x 1	x 3	x 9	x 3	x 4	x 8	x 3	x 6
8	8	7	4	7	7	6	2	3	3	8	1
x 0	x 4	x 5	x 6	x 8	x 7	x 7	x 1	x 4	x 5	x 4	x 5
1	8	5	2	8	8	4	2	4	4	2	7
x 1	x 8	x 7	x 1	x 8	x 6	x 2	x 4	x 2	x 6	x 4	x 6

0 x 4	3 x 8	6 x 6	9 x 2	0 x 6	4 x 2	7 x 2	4 x 1	4 x 6	7 x 3	2 x 8	7 x 8
6 x 7	7 x 0	7 x 9	4 x 3	9 x 0	8 x 2	7 x 8	5 x 8	4 x 5	9 x 4	7 x 4	6 x 8
1 x 7	5 x 9	8 x 3	2 x 4	6 x 5	9 x 8	1 x 4	9 x 2	8 x 6	4 x 5	0 x 4	0 x 2
2 x 5	5 x 3	2 x 5	7 x 3	4 x 8	2 x 7	8 x 3	5 x 9	1 x 1	6 x 2	4 x 6	7 x 1
2 x 3	1 x 4	6 x 8	6 x 7	5 x 8	3 x 8	1 x 5	7 x 8	5 x 4	8 x 3	0 x 1	1 x 2
9 x 2	2 x 1	7 x 1	4 x 7	8 x 4	6 x 0	4 x 1	8 x 5	7 x 8	1 x 0	3 x 6	4 x 1

3

Composite Skill Test (2 or 3 digits x 1 or 2 digits Multiplication)

986 x 83	171 x 59	503 x 9	619 x 71	184 x 88	166 x 48	86 x 9
51 x 68	342 x 76	218 x 96	893 x 17	693 x 75	948 x 14	140 x 59
90 x 77	353 x 49	373 x 27	145 x 66	964 x 81	111 x 63	642 x 55
474 x 46	981 x 1	295 x 20	127 x 41	921 x 79	567 x 47	973 x 55
269 x 31	261 x 52	409 x 94	69 x 4	748 x 49	274 x 6	290 x 92

808 x 51	57 x 88	977 x 1	821 x 7	773 x 67	163 x 39	181 x 8
515 x 37	430 x 22	930 x 61	453 x 32	625 x 82	268 x 89	819 x 56
953 x 64	906 x 76	729 x 27	466 x 21	312 x 74	991 x 70	268 x 96
605 x 22	545 x 42	411 x 8	958 x 8	534 x 67	280 x 97	42 x 95

HØGSKOLEN I OSLO
OG AKERSHUS

Leveres sammen med oppgaven i eksamensinnleveringsrommet i Fronter

KLAUSULERING AV OPPGAVE AV ANDRE GRUNNER ENN LOVBESTEMT TAUSHETSPLIKT

AVTALE INNGÅTT MELLOM HØGSKOLEN I OSLO OG AKERSHUS, FAKULTET FOR HELSEFAG OG

STUDENT Linn Tangen **FØDT** 09.03.84

Studentnummer: s292387 **Studieprogram:** Læring i komplekse systemer

Der master- eller bacheloroppgaver ikke inneholder opplysninger undergitt lovbestemt taushetsplikt, kan Høgskolen i Oslo og Akershus med hjemmel i åndsverksloven § 27, jf. offentlighetsloven § 2 og § 6 nr. 6, gi forskere og studenter *innsyn* i oppgaven, men uten at disse kan sitere fra oppgaven.

Studenten kan reservere seg mot at oppgaven gjøres tilgjengelig i høgskolens åpne vitenarkiv, ODA. Dersom denne adgangen til klausulering benyttes, må varighet og begrunnelse angis fra studentens side. Oppgaven vil da bli oppbevart i et lukket arkiv i den oppgitte perioden.

Jeg reserverer meg herved mot at oppgaven gjøres tilgjengelig i høgskolens åpne vitenarkiv, ODA før embargotidens utløp (1-5 år).

Oppgavens tittel:
Main title: On the Relationship Between Component and Composite Multiplication skills and Natural Frequency Growth

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Jeg tilrår at oppgaven klausuleres	Sted: Kjeller	Dato: 12.05.2016
	Veileders underskrift: Børge Strømgren	

Maksimumstid for denne type klausulering er 5 – fem år (embargotid).

Klausuleringen gjelder for 5 år og settes på grunn av:
Planlagte innsendinger til fagfellevurderte tidsskrifter.

Linn Tangen
studentens underskrift