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Flexibility in Aluminium Production

A Real Option Model

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Sammendrag

Denne oppgaven undersøker verdien av å ha fleksibilitet i produksjon av aluminium. Ved hjelp av realopsjonsanalyse presenterer vi en opsjonsmodell som gir muligheten til å bytte mellom to ulike driftsmoduser; full drift eller midlertidig nedstenging. Metoden vi bruker er utledet fra Longstaff og Schwartz og er kjent som least square Monte Carlo simulation. Ved å modellere stokastiske prosesser for underliggende risikofaktorer i aluminiumsproduksjon, samt ta høyde for kostnader knyttet til å bytte mellom driftsmoduser, finner vi en optimal driftsstrategi. Det fremkommer av analysen at fleksibilitetene som foreligger i en realopsjon tilfører store verdier til aluminiumsproduksjon. Vi finner at dette er gjeldene for flere endringer av parametre i modellen. Videre viser analysen at verdien på realopsjonen avhenger betydelig av hvilken type kraftskontrakt som blir valgt. Velger man å binde seg til en fast kontrakt for hele levetidet til aluminiumsanlegget, reduserer dette verdien på opsjonen betraktelig til tilnærmet null.

Abstract

This thesis investigates the value of flexibility in production of aluminium. By the use of real option theory we present a switching option model that measures the value of switching between two modes of production, open and closed. The value of the option is derived through a Least Square Monte Carlo Simulation, by modelling a stochastic process of the underlying risk factors of production, as suggested by Longstaff and Schwartz. Accounting for costs related to switching between modes of production the model derives at an optimal strategy of production. The analysis reveals that the flexibility embedded in real options adds a significant value to the aluminium plant. The result is consistent for several changes in underlying parameters of the model. Further analysis reveals that the value is heavily dependent on the sourcing of power. When including a fixed contract of power for the lifetime of the aluminium plant, we find that the value of a flexible production is significantly reduced.

Keywords

Real options analysis, Monte Carlo simulation, Aluminium production, Switching option, Value of flexibility, Stochastic process, Risk neutral valuation

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Preface

This thesis is written as a part of the Master of Business and Administration program at Høgskolen i Oslo og Akershus, within the major Finance and Financial Management.

The motivation for the thesis is based on a real options case presented by Hydro ASA. Real options analysis caught our attention as an alternative to other traditional valuation methods and we found the topic intriguing. The framework of real options incorporates flexibilities present in investment projects and is especially suitable for industries exposed for high uncertainties. We perceived the case as a great opportunity to broaden our knowledge within the discipline of valuation.

Writing this thesis has been a great educational experience. While it has been challenging at times, requiring extensive programming in R, we have increased our knowledge of real option analysis as well as computer programming.

Hydro ASA has contributed with historical data material and insightful information into the aluminium industry. We would like to thank Odd Arne Fossan and Håvard Haukdal from Hydro ASA for valuable contributions and good discussions during the process.

Finally, we would like to give a special thank to our supervisor Helge Nordahl for his great support and invaluable feedback during this period.

Oslo, May 2014

Karoline Ask Kristiansen and Marte Bore Tesaker

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List of Acronyms

AR	Autoregressive
ARIMA	Autoregressive Integrated Moving Average
CAPM	Capital Asset Pricing Model
EMM	Equivalent Martingale Measure
EUR	Euro
OU	Ornstein-Uhlenbeck
GBM	Geometric Brownian Motion
LME	London Metal Exchange
LSMC	Least Square Monte Carlo
MCS	Monte Carlo Simulation
MT	Metric Tons
NOK	Norwegian Krone
USD	United States Dollar

1 Introduction

In this thesis we investigate whether real option theory can be modeled to analyze the value of aluminium production. By allowing for flexibility in production we examine the effect on the overall value of an aluminium plant. The model takes into account a stochastic process of underlying risk factors and incorporates a switching flexibility in production, allowing for a shift between two production modes, open and closed. Through the method we seek to find the optimal strategy for operation and the value of the real option. In the real options analysis we will also investigate two power sourcing strategies; a flexible contract versus a fixed long-term contract.

1.1 Background

Today aluminum production is an industry faced with overproduction, consumer friendly prices and low profit margins. Under such circumstances it is difficult for producers of aluminium to remain profitable and continue production. Hydro, as one of the world's largest producers of aluminium, is able to endure low profit margins in the short run, more so than smaller producers. However, in the long run, higher margins are required in order to maintain profitable[1].

The price of aluminium is considered to be volatile, driven by the relationship between supply and demand. As such, the price of aluminium can be subject to large fluctuations from time to time. Only in 2013 the price of aluminium dropped by 6 %, and the World Bank anticipates a further reduction in 2014 (Finanstilsynet 2014). Companies producing aluminium faces a challenge in this case, as the fluctuations in aluminium prices affect the price of finished goods as well as the price on other aluminium products. This will further affect the profit margin required by the aluminum producers and might also impact their production and investment decisions.

There are several uncertainties existent in the aluminium industry. The industry is not only dependent on the price of aluminium, but it is also heavily dependent on the price of power. In

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[1] http://www.hydro.com/upload/Documents/Presentations/Capital%20Markets%20Day/2011/Hydro_CMD_13jan2011_AM.pdf (26.05.14)

fact the cost of power accounts for one third of the costs involved with production of aluminium[2]. As the price of power is considered to be highly volatile, this adds a second uncertainty factor to the aluminium production. In Norway power is mainly produced in hydropower plants and the price of power depend on the amount of precipitation. Precipitation can be stored in reservoirs for later production, but in times with less rain, the price of power will increase. Hydro, as the second biggest power producer in Norway, is able to use captive power in production of aluminium. However, as the prices of power are in constant change, an alternative is to buy power in the market, either through a flexible one-year front order contract or alternatively rely on long-term contracts of power. This will reduce some of the uncertainties faced in aluminum production. For further details concerning the aluminium industry and production, we refer to Breivik and Carlsen (2011).

In order to remain profitable in an industry with high uncertainties, such as the production of aluminium, it may be optimal to consider different operating modes of production. Moreover, allowing for production to be closed down when profits are low, and open when profits are high, one may reduce some of the risk related to fluctuations in power and aluminium prices. Flexibility in production could therefore add a considerable value allowing for margins to be kept at a minimum.

1.2 Purpose

The purpose of the thesis is to construct a model that can be used to value the production phase of an aluminium plant. The aim is to identify when it will be profitable to keep the aluminium plant open based on underlying risk factors of uncertainty. There are two underlying risk factors in the model, the price of aluminium and the price of power. Standard valuation methods such as Net Present Value (NPV) lack the flexibility embedded in real options, thus NPV can contribute to poor and incorrect investment decisions for investment projects with high uncertainty. However, valuation methods based on real options theory is expected to incorporate more flexibility and as such give a more accurate description of the production value.

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[2] http://www.hydro.com/en/Products/Energy/ (18.05.14)

1.3 Structure of the Thesis

In Chapter 2 of the thesis we start by presenting the theoretical background of real options and how real options analysis can be modelled to determine the value of a flexibility production. Chapter 3 describes the construction of our real option model, which is the model we base our analysis on. The subsequent chapter describes the datasets and parameters for the real option model. In Chapter 5 we present the results from the model analysis. Further, the following chapter includes implications of the model and suggestions for further research. Finally, we present the conclusion in Chapter 7.

2 Theory

In this section we give a short introduction to option theory and real options. We present a method by Longstaff and Schwartz for valuation of an American option, which is the method we base our model on. We also investigate the stochastic price processes of aluminium and power, as well as review Monte Carlo simulation (MCS) and risk neutral valuation in terms of Equivalent Martingale Measure (EMM).

2.1 Introduction to Option Theory and Real Options

An option is a financial instrument whose value is dependent on the value of other underlying variables such as the price of a stock (Hull 2012, p. 1). The option holds a right to buy or sell an asset for a given price, known as the strike or exercise price. The trade occurs at a certain date in the future, and gives the holder the right to carry out a transaction. If the transaction is not deemed as value adding to the holder, he or she is under no circumstances obligated to exercise the option. As such, the option is flexible, as opposed to forwards and future contracts[3].

Essentially, there are two types of options, call and put (Hull 2012, p. 7). A call option gives the holder the right to buy an asset, while a put option gives the holder the right to sell an asset. Furthermore, one distinguishes American options from European options. A European option may only be exercised at maturity, whereas an American option is more flexible in that it may be exercised at any time prior and up to maturity. In order to estimate the value of European call

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[3] A forward/future contract is a mandatory agreement between two parties to buy or sell an asset at a certain time in the future for a certain price (Hull 2012, p. 7).

and put options, one normally turns to Black-Scholes-Merton's formula for pricing of options (Black and Scholes 1973; Merton 1973). The same formula can be used to value an American call option at maturity. Valuing American options with early exercise has proven difficult and while several methods have been derived through the years, valuation of American options remains challenging.

There are several factors affecting the value of an option, such as the underlying stock price, the strike price and time to maturity. Additionally, changes in the volatility of the underlying stock price and risk free rate affect the value of an option. In particular, an increase in volatility of the underlying stock price increases the value, whereas a decrease in volatility decreases the value. This effect in options emerge from that there is a limited downside risk and an infinite upside potential (Hull 2012, p. 214-216).

With real options one applies the theory derived for financial options to value real investment projects related to real assets such as production, machinery, land, etc (Hull 2012, p. 765). As with financial options, the value of the real option depends on the risk and return of underlying variables.

Real options analysis is a method used to value investment projects and is an alternative to traditional valuation methods such as NPV. NPV is a widely used method for valuing investment projects even though it has several drawbacks and often undervalues projects (Copeland and Antikarov 2003, p. 5). The reason that NPV is seen unsuitable for projects with high uncertainty is that NPV requires that all information related to a project is known prior to making the investment decision (Hull 2012, p. 765). With real options on the other hand, one incorporates the value that flexibility may have on investment projects and accounts for management's ability to answer to market changes, by making changes throughout the life of the project. Copeland and Antikarov (2003, p. 5) state:

> *A real option is the right, but not the obligation, to take an action (e.g., deferring, expanding, contracting, or abandoning) at a predetermined cost called the exercise price, for a predetermined period of time - the life of the option.*

Under such circumstances options appears as different scenarios related to an investment and the option holds decisions that could affect the overall profitability of the investment. The value of a real option is related to the flexibility to evaluate each scenario and the possibilities to take action according to uncertainty about the future. However, when there is little uncertainty about the future and all possible scenarios can be identified prior to investing in a project, the positive effects of using real options diminishes (Schwartz 2012).

2.2 Least Square Monte Carlo

In 2001 Longstaff and Schwartz published a method known as the Least Square Monte Carlo approach (LSMC) for valuing an American put option with MCS. In their approach Longstaff and Schwartz use simulation to value and find the optimal exercise strategy for American options[4]. This approach is an alternative to traditional differential equations and binomial techniques, and allows the variables in the model to follow a stochastic process. The optimal exercise strategy is found by comparing the value of holding the option with the value obtained from early exercise. If the value of continuing to hold the option is higher than the payoff from immediate exercise, the holder will choose to keep the option. Ultimately, the method provides an approach in determining the value of holding the option, as opposed to the value obtained from early exercise. Longstaff and Schwartz (2001, p. 114) explains this as follow:

> *To understand the intuition behind this approach, (...), the holder of an American option optimally compares the payoff from immediate exercise with the expected payoff from continuation and then exercises if the immediate payoff is higher.*

The key to this approach lies in the method of starting at time T, at the end of the timeline, and working backwards until time zero. At each possible exercise node, the expected value of continuation at time t is found by regressing the expected cash flow received by continuing beyond time t, against the underlying risk factor of the option, the stock price. The method uses least squares regression. When the regression reaches time zero, the optimal exercise strategy at each node and for each simulated path is revealed. Implicitly in the model, an early exercise entails no further exercises at future nodes. Once the optimal strategy is determined, it is easy to find the cash flows obtained from exercising and consequently the value of the option. For every path the cash flow received at each node are averaged and discounted back to time zero. By

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[4] "A variable whose value changes over time in an uncertain" way is said to follow a stochastic process". (Hull 2012, p. 280)

multiplying the discounted values the value of the American option is obtained. For an American option at the final exercise date with no early exercise, the LSM approach should give the same result as Black- Scholes-Merton's formula for a European put option. As it is difficult to reach an infinite number of simulations, BSM will report a lower value of the option than the value reported by LSM. Nevertheless, BSM will give a good indication as to whether the LSM calculations report correct results.

The LSM approach to value an option can be transferred to the valuation of a real option. In Chapter 3 we will describe how this may be accomplished in practice as we look at the model of our real option.

2.3 Aluminium Production

Aluminium production is a complex process requiring several steps from bauxite mining, extraction of alumina from bauxite, and production of primary aluminium from alumina. In this thesis we will focus on the production phase of aluminium as an aluminium investment project for an aluminium plant. We include flexibility of production, meaning that we allow for decisions on production to be made based on changes in the value of underlying factors as new information is exposed throughout the lifetime of the investment project.

2.4 Profit Factors in Aluminium Production and Uncertainty

Hydro's profit function for aluminium production is given by:

$$\text{Profit } (\pi) = \text{aluminium (A) - alumina (Al) - carbon cost (C) - fixed cost (F) - power cost (P).} \quad (2.1)$$

In the profit function the only income factor is aluminium. Hence, the production profitability is first of all dependent on the price of aluminium. The production process is also heavily dependent on power, and it follows that the overall profit of production is affected by the stability in the price of power. In the thesis, we make the simplification of considering alumina and carbon as fixed. From the profit function it follows that the uncertainty in Hydro's profit is related to the price of aluminium and the price of power.

2.5 Price Process of Aluminium and Power

As the prices of aluminium and power are the two main sources of uncertainty in Hydro's production these variables become the underlying variables of the real option. Thus, in order to value the real option properly we need to determine the underlying process of the prices and how these factors should be modeled. Choosing the right underlying process is important and if the stochastic process of these variables is incorrect, the corresponding profits will be unrealistic.

Aluminium and power are both commodities and their prices are determined by supply and demand. Although both variables are considered similar goods, they also entail distinct features. As for aluminium it is easy to store and prices are unseasonal and unaffected by the weather. Further, the aluminium metal is an important factor used in constructing houses, cars, airplanes and other consumer goods. As such, the price of aluminium is determined by the trends in the production processes where aluminium is used. Power differs from aluminium in that it cannot easily be stored and must be consumed as soon as it is produced. In most cases the supply of power depend on local conditions. As a major use of electricity in Norway is for heating, power prices are unlike aluminium prices seasonal. As demand for power grow during the winter months the power price increase, as opposed to lower prices during the summer due to lower demand.

Geometric Brownian motion (GBM) and Ornstein-Uhlenbeck (OU), are stochastic processes often used to model the behavior of prices. GBM is a process whereby a variable follows a random walk with drift. The process assumes that small changes in the price, *dS*, can be described by the differential equation (Dixit and Pindyck, 1994, p.71):

$$dS = \mu S dt + \sigma S dz \ . \tag{2.2}$$

From Equation (2.2) μ denotes a constant drift rate and σ a constant volatility. The term, *dz*, is a basic Wiener process, which has a drift of zero and a variance rate of 1.0 [5].

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[5] A Wiener Process is "a stochastic process where the change in a variable during each short period of time length Δt has a normal distribution with a mean equal to zero and a variance equal to Δt." (Hull 2012, p. 817).

GBM has often been the preferred processed to describe the process of underlying variables of a derivative. However, recent literature indicates a tendency towards mean reversion for these variables. If a price is considered to follow OU, the assumption is that the variable will revert back to a long-term mean, i.e. follow a mean reversion process. This process can be described as per the differential Equation (2.3) (Dixit and Pindyck 1994, p. 74):

$$dS = \kappa(\theta - S)dt + \sigma dz \,. \tag{2.3}$$

Where a change in the price, dS, is described by the the speed of mean reversion by kappa, $\kappa < 1$. A high κ indicates a strong mean reversion, as opposed to weak mean reversion when κ is low. θ is the long-run value that the variable reverts to (Dixit and Pindyck 1994, p.74). The mean reversion processes presented by OU has been used in several terms to apply stochastic prices. Dixit and Pindyck (1994, p. 77) incorporates a volatility term, σSdz, in the stochastic process allowing the process in allowing for a growth in the variance rate. The process can be presented as in Equation 2.4:

$$dS = \kappa(\theta - S)dt + \sigma S dz \,. \tag{2.4}$$

This process is known as “geometric Ornstein-Uhlenbeck”, and it may be approximated to Equation 2.5 in discrete time (Campbell 2013)

$$S_t = S_{t-1} + \kappa(\theta - S_{t-1}) + \sigma S_{t-1}\epsilon_t \,. \tag{2.5}$$

Including an exponential growth term, g, the process can be represented by Equation 2.6

$$S_t = S_{t-1}e^g + \kappa(\theta - S_{t-1}) + \sigma S_{t-1}\epsilon_t \,, \tag{2.6}$$

where

$$\theta_t = S_0 e^{g(t-1)} \,. \tag{2.7}$$

The conditional forecast can be described as

$$\mathrm{E}[S_{t+1}|S_t] = \theta + e^{-\kappa}(S_t - \theta), \tag{2.8}$$

$$\mathrm{E}[S_{t+1}|S_t] = S_t e^{-\kappa} + \theta(1 - e^{-\kappa})\ . \tag{2.9}$$

This process can be described included exponential growth term as in Equations 2.10 and 2.11:

$$\mathrm{E}[S_{t+1}|S_t] = e^{g}(S_t e^{-\kappa} + \theta(1 - e^{-\kappa}))\ , \tag{2.10}$$

$$\mathrm{E}[S_{t+1}|S_t] = S_t e^{-\kappa+g} + \theta e^{g}(1 - e^{-\kappa})\ . \tag{2.11}$$

The equation above implies that if the current price, S_t, is lower than its long-run value θ, then the price, S_{t+1}, is expected to be higher than the current price for the next period (Campbell 2013).

Commodity prices in general are often considered to display mean reversion (Hull 2012, p. 748). Thus we believe that the prices of aluminium and power could follow a mean reverting process. In particular, when there is an increase in the price of aluminium, the commodity is likely to become less attractive to use in production processes and consequently the demand decreases. Due to less demand, there will be an overproduction causing a downward pressure on the prices and the price will fall and stabilize at an equilibrium level again. The same argument can be made for the consumption of power, where high prices will make the consumers search for other alternative solutions for heating, causing overproduction and a downward pressure on the price. On the other hand, if the price of the commodities decreases, it becomes more attractive to use the goods in production processes. For aluminium it follows that it will be less economical to extract. Similarly for power, a reduction in the power price is likely to result in higher consumption, while production becomes less economical. Hence, less extraction of aluminium and reduced production of power is likely to cause an upward pressure on the prices.

Based on the aforementioned mechanisms in commodity prices we assume that the prices of aluminium and power are likely to follow a mean reverting process. However, further analysis is required in order to determine the appropriate process for aluminium and power. It is out of scope of this thesis to determine the process that will fit the aluminium and power price the best. As the literature is divided in the view of how to model commodity prices, we use a mean reverting process to model the prices. This is consistent with Hull (2012, p. 753) in that commodity prices have a tendency to get pulled back to a central value.

2.6 Monte Carlo Simulation

Simulation techniques are often used to replicate the distribution paths of financial variables, and the simulation can either be generated randomly by MCS or by historical data, such as bootstrapping. Initially, MCS was developed for integration issues in statistical sampling (Ulam 1976), but was later introduced in finance to generate the distribution of financial variables to evaluate options (Boyle 1977).

A crucial first step when running a MCS is choosing a stochastic model for the behavior of the financial variables (Jorion 2007, p. 309). As determined earlier, aluminium and power will follow a mean reverting process. In addition to choosing the stochastic model there is also a need to specify the volatility and correlation of the financial variables before running the simulation. These parameters can be estimated by looking at historical data. Once the appropriate model has been chosen and the necessary parameters have been retrieved one can simulate price paths.

The method of MCS has several advantages. It can be used on path dependent options and it is easy to apply when the option value depends on multiple factors. Nevertheless, in order for MCS to bring value it is dependent on correct stochastic processes of the underlying variables. Early research indicates that MCS is not optimal for valuing an American option with early exercise features (Hull 2012, p. 626). The reason for this is that this type of option requires backward techniques in order to be valued properly, meanwhile MCS is forward looking. However, research done by Longstaff and Schwartz (2001) suggests otherwise. As mentioned in Section 2.2, Longstaff and Schwartz present how MCS can be used in order to value an American option with early exercise features.

2.7 Risk Neutral Valuation under Equivalent Martingale Measure

In order to value the aluminium production with real options analysis one needs to adjust for systematic risk. With traditional valuation methods such as NPV, the discount rate is adjusted to reflect the risk of the investment, and higher risk entails higher discount rate. Compared to traditional capital investment valuation, the risk adjusted discount rates for real options are difficult to calculate. One of the reasons for this is that these types of investment projects often contain several options. To exemplify, a company considering a new production facility will not only have one project, it may also have an option to expand the facility or abandon the project if new information occurs as time passes by. These types of options will consequently differ from the initial investment option and requires another discount rate. As the future is unknown, we cannot anticipate the options or cash flows that will occur. Estimating the appropriate discount rate therefore becomes difficult.

Risk Neutral Valuation is another method used to adjust for risk. The method assumes that the world is risk free and that investors on average require no extra return for higher risk. With this framework the adjustment for risk is made in the expected cash flows and not in the discount rate, avoiding estimating the risk adjusted discount rate. With continuous time, the principle of risk neutral valuation can be extended to adjust the underlying stochastic variables for risk. This entail simulating the underlying risk processes under the Equivalent Martingale Measure (EMM) and use the risk-free rate to discount the cash flow (Hull 2012, p. 634-636).

From the capital asset pricing model (CAPM) it follows that the expected rate of return of an asset x is

$$u = r + \emptyset \sigma_x \rho_{x,m} \ , \tag{2.12}$$

where u represents the risk adjusted discount rate and r is the risk free rate applicable to discount risk free cash flows. The coefficient $\rho_{x,m}$ denotes the correlation between an asset x and the market portfolio m, and σ_x is the variance of the asset x. The term Ø is the market price of risk and represents

$$\emptyset = \frac{(r_m - r_f)}{r_m}, \tag{2.13}$$

where σ_m is the volatility of the market (Dixit and Pindyck 1994, p. 115).

By adjusting the expected rate of return, i.e. the expected growth rate, we are able to discount expected cash flows by the risk free rate (Dixit and Pindyck 1994, p. 197). If g is the adjusted growth parameter, we get

$$g = u - \frac{\sigma_x \rho_{x,m}(r_m - r_f)}{\sigma_m}. \tag{2.14}$$

Equation 2.14 presents how the expected growth rate in an underlying variable of the real option can be adjusted in order to discount the cash flows by the risk free rate.

3 Real Option Model

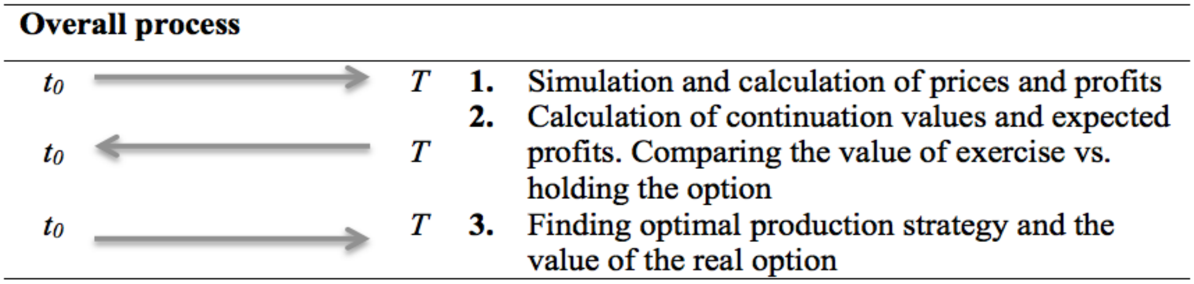
In order to value the aluminium plant as a real option we have constructed a model with the statistical analysis program R. The model is based on a switching option that entails operational flexibility and provides an opportunity to switch between different modes of operation. Thus, the switching option consists of a collection of options, whereby each option involve the right to shift production mode. For this aluminium plant in particular, we alternate between two modes of production; full operation or temporarily closed. For further reference we will refer to these states of production as being *open* and *closed*, respectively. The flexibility that lies within the model generates value in such a way that the aluminium plant will be closed at times of negative future expected profit, while it will be open when the value of future expected profits are positive. In the end, the value of the real option is compared to a situation where we consider no flexibility, equivalent to letting the aluminium plant always be operational.

3.1 Overall Process

There are two sources of uncertainty in the model, the aluminium price and the power cost. Subsequently, their respective prices will be referred to as A_t and P_t at time t. These variables are stochastic processes simulated with a drift and correlation. The simulations are executed by MCS and are simulated for N paths. The model is built based on a profit function consisting of the aluminium price as the source of income and the prices of alumina, carbon, fixed expenses and power as costs. The lifetime of the aluminium plant is denoted by T years and the model contains a discrete time interval of Δt. All prices in the model are risk adjusted with EMM as explained in Section 2.7, in order to be discounted by the risk free rate. When switching between modes of operation we assume that there are costs associated with opening and closing the aluminium plant, which is denoted as *oc* and *cc*, respectively.

The method we use to value the real option is an approach derived from Longstaff and Schwartz (Longstaff and Schwartz 2001). They use a method where they value an American put option by simulation as earlier mentioned in Section 2.2. Overall, the process of valuing the real option can be divided in three parts, which will be described in further detail later in this chapter. In general, the first part of the process involves simulations and calculations of the prices in the profit function from time 0 to T. In the second part, we start at the end of the timeline at time T, and conduct ordinary least square (OLS) regressions stepwise backwards from T to t_0. Here, we compare the value of continuation against exercising the option by either opening or closing the aluminium plant, depending on the state of production. In the last section, we find the optimal strategy of how the aluminium plant should operate and value the real option accordingly. The overall process is depicted in figure 3.1.

Figure 3. 1 Overall Procees



Overall process

$t_0 \longrightarrow T$ **1.** Simulation and calculation of prices and profits

$t_0 \longleftarrow T$ **2.** Calculation of continuation values and expected profits. Comparing the value of exercise vs. holding the option

$t_0 \longrightarrow T$ **3.** Finding optimal production strategy and the value of the real option

3.2 Step 1

By MCS we simulate the prices of aluminium and power over the chosen timeline of T years for the aluminium plant. The prices are set to follow an OU process with mean reverting drift, as described in Section 2.5. We use historical quarterly data to derive the parameters from the mean reverting process. It follows that the prices are simulated quarterly and converted into yearly figures. Following from Section 2.5, Equation 3.1 is the analytical solution used to simulate the prices for a given time interval Δt

$$S_{t+1} = S_t e^{-\kappa+g-\frac{1}{2}\sigma^2+\epsilon_t} + S_0 e^{gt}(1-e^{-\kappa}) \,. \tag{3.1}$$

After simulating the two uncertainty factors, the aluminium and power price, we calculate the expected prices of alumina, carbon, and fixed costs, with a fixed exponential growth.

From the generated prices we construct the profit function, π, referred to in Section 2.4, Equation 2.1. The profits expected in a given time interval Δt is calculated by taking the average of the start and end prices of that time interval.

3.3 Step 2

3.3.1 Regression

To find the optimal decision rules and exercise strategy for the aluminium plant we start at the end of the timeline at time T of the project and make calculations backwards in time. The computations are applied for all paths (N) and we work our way stepwise backwards at the interval of Δt. For every decision point we shift backwards in time we run a regression, which is based on three variables. The dependent variable is represented by the *value of continuation*, V, and consists of the simulated profits from time t until T, in which profits are dependent on the choices made regarding the operational activity. A more detailed description of the value of continuation is given later in this chapter in Section 3.3.3 and derived in Equations 3.3, 3.4 and 3.5. Furthermore, the simulated prices of aluminium and power are used as independent variables in the regression. The coefficients generated as output from the regression are then used to make the regression equation. The regression equation at time t can be written as:

$$V_n = \alpha + A_n\beta_A + P_n\beta_p + \epsilon \, . \tag{3.2}$$

By inserting the simulated prices of aluminium (A_n) and power (P_n) for each path, n, into the regression Equation 3.2, we get the predicted estimates of the value of continuation. With this predicted value as basis we make the optimal decision rule regarding whether to exercise or not for each path, dependent on the production mode of the aluminium plant in the previous period.

3.3.2 Mode of Production

As is known, there are two possible modes of production, which needs to be taken into consideration in the model. The aluminium plant could either be open or closed in the previous period and we have to find decision rules for both events. Henceforth, we will refer to a state in which the plant was open in the previous period as PO and a state where the plant was closed in the previous period as PC. The decision rules are illustrated in Figures 3.2 and 3.3.

Figure 3. 2 Operation mode open

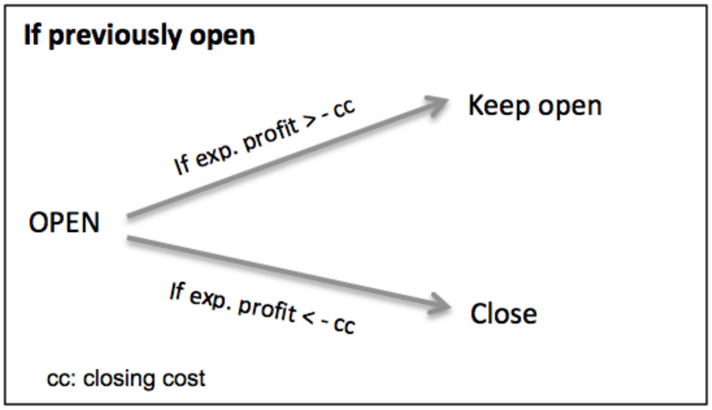
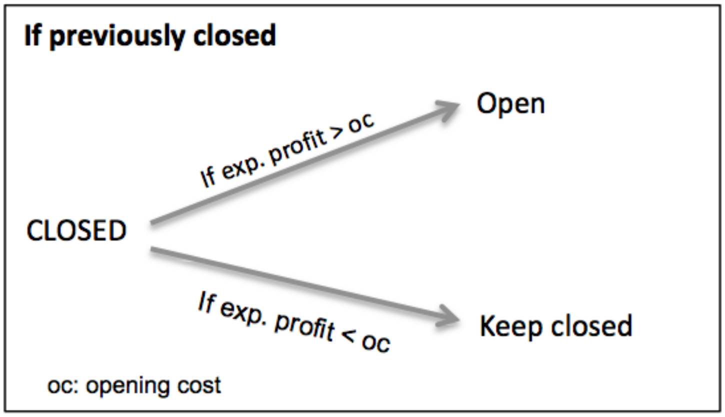


Figure 3. 3 Operation mode closed



If we are in PO, the aluminium plant was kept open in the previous period and we choose to stay open in cases where expected profits are higher than what it would cost the company to shut down production. If however, expected profits are lower than the costs of closing production, the decision would become to close the aluminium plant. In the opposite state, PC, where we assume that the aluminium plant was closed during the previous period, we compare expected profits with the cost of reopening. Hence, the production will open if expected profits at a given time are higher than what the company has to pay to open the plant, whereas it will stay closed if expected profits are lower than these costs.

3.3.3 Value of Continuation

In the second last time step of the lifetime of the aluminium plant, at time T-Δt*,* the value of continuation consists of simulated profits for the last remaining time period. However, when we move further time steps backwards, the value of continuation is more complicated to derive.

Based on the decisions rules and choices of operation made for the last time period, found at time T-Δt, we move another time step backwards to time T-2Δt, and calculate a new value of continuation. From this point (T-2Δt*),* the value of continuation applies to the final time period T of the aluminium plant, i.e. there are two time periods taken into consideration. In the first occurring time interval from T-2Δt to T-Δt, we account for simulated profit expected at this time period. For the last and second time period, the value of continuation is dependent on the choices regarding operational activity, thus accounts for profits and costs accordingly. Where it is optimal to reopen the aluminium plant from a period where it was previously closed, the cost of opening the aluminium plant is included. Consequently, we account for the opening cost (*oc*) in addition to the simulated profits from beeing open in this last period. In an opposite situation, where the aluminium plant is closed, the closing cost (*cc*) is applied as expenditure, and profits as zero, as there is no production in this period.

When we move even further back in time, the value of continuation at a given time consists of the simulated profits for the next forthcoming period, as well as profits from all subsequent periods dependent on the modes of operation and costs from opening and closing. The value of continuation in all time intervals from T-2Δt to t_0 for a given path, n, can be represented as follows:

$$E[V_t^{PO}] = PO_t * (profit_t + E[V_{t+1}^{PO}])e^{-r} + (1 - PO_t) * (E[V_{t+1}^{PC}] - cc_t)\,, \tag{3.3}$$

$$\begin{aligned} E[V_t^{PC}] = {} & PC_t * (profit_t + E[V_{t+1}^{PO}])e^{-r} - PC_t * oc_t + (1 - PC_t) * \\ & E[V_{t+1}^{PC}] * e^{-r}, \end{aligned} \tag{3.4}$$

$$E[V_{t-1}] = profit_t * e^{-r} + E[V_t^{PO}]e^{-r} + E[V_t^{PC}]e^{-r}PC_t\,, \tag{3.5}$$

where

$$cc_t = cc * e^{gt}, \tag{3.6}$$

$$oc_t = oc * e^{gt}. \tag{3.7}$$

When calculating the value of continuation we start by constructing two equations (3.3) and (3.4), which applies for n paths. $E[V_t^{PO}]$ is the expected value if we were open in the previous period and $E[V_t^{PC}]$ is the expected value if we were closed in the previous period. PO and PC are matrices containing N paths (rows) and T years (columns) and each cell is denoted by either 1 or 0 as we move stepwise backwards in time. PO is set to 1 at time t if it is optimal to operate after previously being open, whereas PO is set to zero if is optimal to shut down the aluminium plant after previously being open. PC is represented as 1 if it is optimal to operate after being closed in the last period, and it is zero if it is optimal to stay closed. $Profit_t$ is calculated profits in a given time interval. The value of continuation, $E[V_{t-1}]$, is represented in Equation 3.5 and consists of calculated profits in the first upcoming period and adds together the expected profits from being open and closed in the subsequent periods. All values are discounted with the risk free rate.

3.3.4 Completion of Step 2

After conducting the regressions and the procedures in the aforementioned sections for all time steps we reach time zero. At this point we are left with two completed matrices, PO and PC. The matrices comprise the optimal switching strategies for each operation mode. In the model we make the assumption that the aluminium plant always is operational in the first time interval from t_0 to t_1.

3.4 Step 3

3.4.1 Final Operation Matrix

In the third and final process of valuing the real option, we interweave the optimal switching strategies and find the coherent operational strategy for the aluminium plant. The final matrix is denoted as TS (Total strategy) and is made from the matrices PO and PC. In this step we start at

t_0 and work our way forward step by step until time T. As previously mentioned, we assume that the plant is operational for all paths in the first period. Based on this assumption we choose the operational strategy for the second time period based on the PO matrix, which contains the optimal strategy given that the aluminium plant was open in the preceding period. Hence, we proceed on to the third period and find the optimal strategy for this time interval. At this point, the strategy is found based on either PO or PC, depending on the strategy chosen in the previous period. This operation is then repeated for all time intervals until time T. Particularly, we follow PO if the aluminium plant was operational in the past period, while we look at PC if it was closed. The construction of TS can be derived as follows;

$$TS_t = PO \text{ if } TS_{t-1} = 1, \tag{3.8}$$

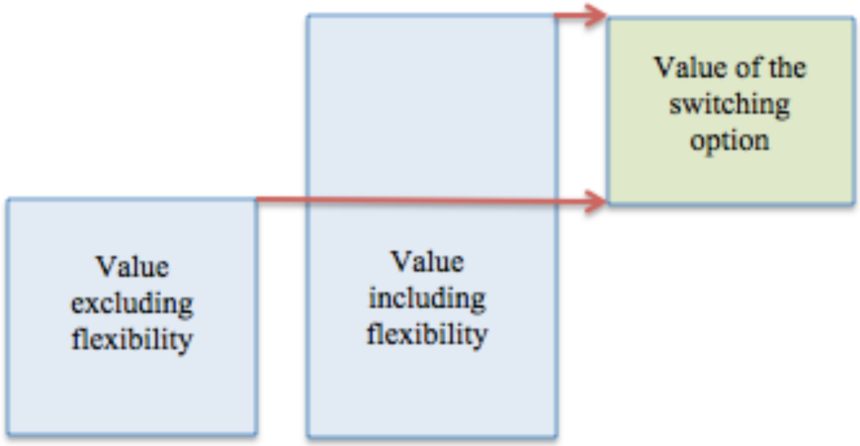
$$TS_t = PC \text{ if } TS_{t-1} = 0 \ . \tag{3.9}$$

3.4.2 Value of the Real Option

Finally, as the final strategy matrix TS is constructed we are able to calculate the value of the option. In order to do so, we start by accounting for simulated profits in the events of operation from the TS matrix. We will then take the average of this value and multiply it by T years. We also account for the mean of the opening and closing costs in accordance with the openings and closings of the plant. By subtracting the costs of reopening and closing from the profits during the lifetime of the aluminium plant, we reach the value of the real option.

Finally, we examine the value of the switching option. By comparing the value of the aluminum plant including flexibility to a case with no flexibility, in which we always operate, we get the value of the switching option. Illustrated in Figure 3.4, is an example of a situation where the value of the switching option is positive. As seen from the figure, the flexibility of being able to switch between modes of operation adds a positive value to the aluminium plant.

Figure 3. 4 Value of switching option



3. 5 Model with Fixed Power Contract

In this thesis we also want to investigate the impact of including a fixed power contract over the lifetime of the aluminium plant. In order to do so we adjust the model described in the previous sections and include a fixed power contract. The fixed power contract is taken into account in step 2 of the model (Section 3.3) with regards to the optimal exercise strategy based on the PO matrix, i.e. that the aluminium plant was open in the previous period.

We start by calculating a new profit function, which has the fixed power contract entailed. The cost of power with the fixed contract is estimated by taking the average of the simulated power prices for each time step, thus it becomes the *expected* power price. The expected power price estimated for a given time t, is applied for all paths, n, at that time.

The optimal exercise decision in the state of PO, in which the aluminium plant previously was open, is initialy based on expected variable profits, as explained earlier in Section 3.3.2. The new profit function is then applied as an additional condition. The new condition holds if profits including the fixed power contract are higher than a given value Q, the aluminium plant is kept open. This condition will overrule the previous choice based on expected variable profits in terms of the decision of keeping the aluminium plant open. This means that in some cases where the initial decision was to close the aluminium plant, it will now be kept open as profits with the fixed contract has a higher value than expected variable profits.

Finally, the fixed contract option is assessed against the base case option with flexible power sourcing, described in Section 3.1-3.4, and we value which type of sourcing that brings highest value to the aluminum plant.

4 Calibration

Finding appropriate market estimates is essential for making the model as realistic as possible. In the following we describe the historical dataset used to estimate the expected return, volatilities and the mean reverting process of the underlying risk factors. We further determine the parameters for growth, market risk and set the risk free rate. As we operate with prices denoted in USD it could be appropriate to apply expected inflation for the US market of 2 % (The Federal Reserve System 2014). However, as the aluminium plant is located in Norway, we have chosen to take into account an inflation of 2.5 %, which is line with the long-term expected growth in the Norwegian market (Gjedrem 2001).

4.1 Profit Function

The initial cash flow of production is based on prices denoted in NOK, USD and EUR. We base the profit function, π, and calculations in the model on USD and thus convert the costs denoted in NOK and EUR. The conversion rates are based on a dataset of the NOK/USD and EUR/USD exchange rate received from Hydro. For the base case in the profit function, we have chosen exchange rates equal to the last available observation per December 31, 2013.

The price of aluminium is set to an initial start price of $ 2400 per metric ton (MT). As alumina is the raw material required to produce aluminium, the initial start price is set to corresponds to 30% of the initial start price of aluminium. Carbon is a second material required in production and is set to an initial start price of $ 400 per MT. The fixed costs are reported as NOK 3000 per MT, while the expected power cost is reported to be 560 EUR/MT (14 mwht * 40EUR). The last observable exchange rates are NOK/USD 0.165 and EUR/USD 1.361, which corresponds to expected fixed costs of $495 and power costs of $762. As reported in Section 2.4, the expected profit function is given by:

Profit(π) = aluminium (A) - alumina(Al) - carbon cost (C) - fixed cost (F) - power cost (P). (4.1)

Given all costs in USD, the profit basis for the model is set to:

π = $2400 - $731 - $400 - $495 - $762 = $ 12

4.2 Aluminum and Power Prices

The aluminium price (LME) used in our analysis is retrieved from Datastream[6] and it is denoted in US dollars per metric ton. The dataset spans over 40 years and includes quarterly prices from March 1974 to December 2013. Today's market for aluminium have changed significantly since the first historical observations, however, by including a historical dataset of the last four decades we get a stronger basis for simulation of future aluminium prices.

With regards to the power price, we apply historical data collected from Hydro, initially obtained from Nordpool. Historically the Norwegian power market has been heavily regulated, and it was first in 1991 that the market opened for free trade of power contracts. Following the new energy law in 1991 it took several years until significant trading in power contracts, and as such the historical dataset available is fairly short (Koekebakker and Ollmar 2001). The dataset has a time frame of approximately 12 years, from March 2001 to December 2013. The observations are reported quarterly and are initially denoted in EUR. As previously mentioned, we convert all power prices to USD based on the historical dataset of the EUR/USD exchange rate in order to quote all data in USD. The data sets are summarized in Table 4.1.

Table 4. 1 Data sets

	LME (USD/MT)	Power (USD/MT)
Period	1974-2013	2001-2013
Frequency of data	Quarterly	Quarterly
Number of observations	160	51
Mean price	1586	47
Annual rate of return (log)	2.48 %	7.13 %
Standard deviation	22.80 %	18.39 %
ϕ	0.9492932	0.9801563

As seen in Table 4.1, LME has had an annual rate of return of 2.48 %, whereas the power price has had a rate of return of 7.13 % per annum. The volatilities of aluminium and power are 22.80

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[6] Datastream is an economic research and strategy tool, which deliver historical and daily market information including macroeconomic data.

% and 18.39 %, respectively. These are considered to be critically important in determining the value of the option (Hull 2012, p. 290).

In order to model the mean reversion process of the underlying risk factors we use an ARIMA(1, 0, 0), which correspond to an autoregressive process of order 1, i.e. AR(1) process. The AR(1) coefficient, ϕ, for aluminium and power is estimated from the historical datasets. For aluminium, ϕ is estimated as 0.94, whereas it is 0.98 for power. ϕ for aluminium gives a kappa of 0.06 ($\kappa = 1 - \phi$), which indicate a greater degree of mean reversion in aluminium prices compared to the power price with a kappa of 0.02.

4.3 Correlation between the Aluminium and Power Price

To describe the stochastic features of the variables and correctly conduct simulations, we estimate the correlation between the aluminium and power price. The correlation estimate is found by fitting both data series to the same time frame, March 2001 to December 2013. Our analysis shows that the prices are correlated by a correlation coefficient of 0.87 (Table 4.2) and hence follow a correlated stochastic process.

Table 4. 2 Correlation between Aluminium and Power Prices

$\rho_{a,p}$	0.87*

**Based on quarterly data*

Figure 4.1 displays the logarithmic movements in the variables. As the correlation is nonzero, the random samples, $\epsilon 1$ and $\epsilon 2$, which is used to obtain movements in the prices, are sampled from a bivariate normal distribution (Hull 2012, p. 290). This is carried out with the MASS package in R. In addition, we conduct a MCS, which generates the simulated prices with a correlation coefficient of 0.87.

Figure 4. 1 Logarithmic Aluminium and Power Prices

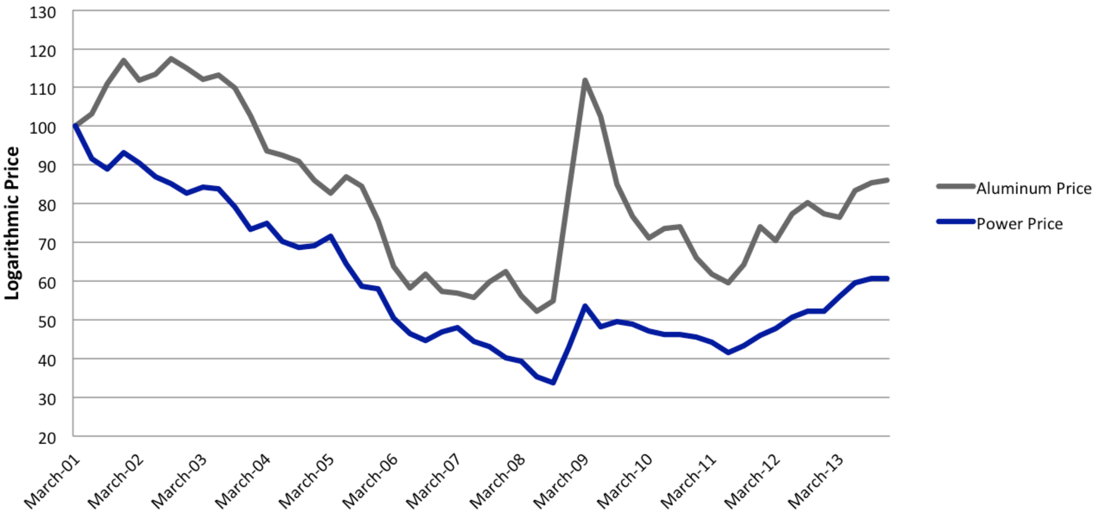

Figure 4.1 displays the logarithmic movements in the prices of aluminium and power from 2001-2013. The prices are normalized to start at the same value and are based on quarterly data.

4.4 Long-term Growth in Aluminium

To set the expected long-term growth in the aluminium price we research long-term growth projections for the world economy. The market for aluminium consists of a variety of industries and include commercial, consumer and end markets such as construction, building, packaging and transportation (Wikinvest 2012). Thus, there is reason to believe that the growth in aluminium price is linked to the growth in the world economy.

Among others, we look at OECDs growth prospects for the world. In the next half century, until 2060, OECD predicts that average unweight GDP per capita will grow by 1.7 % in real terms for the OECD countries, against approximately 3 % in the non-OECD area (OECD 2011). A report executed by PwC anticipates the average real growth in the world economy to be about 3 % in the period from 2011 to 2050 (PwC 2013). In the report it is estimated that the growth in emerging economies will be approximately 4 %, whereas it will be 2 % or less for advanced economies.

The consumption and price of aluminium is considered highly related to economic development and as emerging markets continue to develop the demand for aluminium is likely to increase (Wikinvest 2012). The World Bank especially points to the development of the Chinese economy as an important factor for development in aluminum prices. China as a part of BRIC, is considered to be one of the four major emerging economies[7], and accounts for 45% of world consumption of metal (Finanstilsynet 2014). Based on the foregoing, there are reasons to believe that growth in aluminium prices is linked to growth in emerging markets such as China. Considering that the average growth in aluminium prices has been 2.48 % in nominal terms over the last 40 years (Table 4.1), linking them to the high expected growth in emerging markets alone could be an aggrandizement. As such, we therefore take a conservative view, as we believe that it is reasonable to expect a more moderate growth in aluminium than the rates expected for emerging markets. Hence, we set the growth rate for the aluminium price at a moderate nominal rate of 4.5%.

4.5 Long-term Growth in Power

To determine the long-term growth in power prices we turn to other sources of energy, such as oil and natural gas. The prices for these commodities are considered to respond similarly to changes in the energy market, which is mainly driven by supply and demand. This is consistent with Gjølberg (2001) who assumes a correlation between oil and power as these sources of energy may to some extent be regarded as substitutes. Asche, Osmudsen and Sandsmark (2006) also report a cointegration between natural gas, oil and power prices, when investigating the dynamics of these commodities.

Based on the foregoing it is reasonable to assume that power prices to some extent follow the same drift as the oil price. The International Energy Agency indicates that the world oil and supply demand is expected to grow by 1 % in real terms in the period 2008 to 2030 (International Energy Agency 2009). Based on these figures we set the growth rate for the price of power at 3.5 % nominal.

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[7] BRIC is a group of the four developing countries, Brazil, Russia, India and China. These countries are considered to have promising economic markets and economies.

4.6 Long-term Growth in Deterministic Costs

In the model we have constructed, we make the simplification of assuming a deterministic price processes for alumina, carbon and fixed costs. Consequently, there is an implicit assumption that the volatility in these variables is zero and we set them to grow at a fixed rate. Fixed cost in production is primarily related to maintenance and labor costs, and we find it reasonable to link the growth in this cost to inflation at 2.5 %. For convenience purposes we also set the inflation rate of 2.5 % as the growth rate for the prices of alumina and carbon.

4.7 Risk Free Rate

The risk free rate is the rate of return expected for a risk free investment and it is often set equal to the rate of long-term government bonds. Currently, the average yield to maturity for a 10-year government bond is 2.58 % p.a. as of 29.04.14[8]. This interest rate level is historically low, which is underpinned by examining the average yield to maturity between 1984 and 2013, reported as 6.88 %. The Norwegian government predicts that a reasonable estimate for the real risk free rate is 2.5 % in 40 years (NOU 2012:16). As the government has an inflation target of 2.5%, this corresponds to a nominal rate of 5 % (Gjedrem 2001).

Based on these figures we think that it is reasonable to set the risk free rate at 5% and use this in our analysis.

4.8 Market Risk Premium

The market risk premium is the expected return on a market portfolio in excess of the risk free rate. Consequently, it is a compensation given to investors for taking higher risk by investing in a risky asset. There are several considerations to take into account when determining the market risk premium. The rate will differ dependent on the time frame, benchmark index and predictions by experts.

One alternative when determining the risk premium is to look at historical average return at Oslo Stock Exchange (OSE). The OBX index, which is a portfolio of the 25 most traded securities at

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[8] http://www.norges-bank.no/no/prisstabilitet/rentestatistikk/statsobligasjoner-rente-arsgjennomsnitt-av-daglige-noteringer/ (20.04.14).

OSE, shows that the annual geometric return since 1995 has been 10.9 %. Based on government bonds, the risk free rate in the same timeframe was 4.8 %, which results in a risk premium of 6.1 % (Oslo Stock Exchange 2013). A study made by PricewaterhouseCoopers AS (PwC) in cooperation with Norske Finansanalytikeres Forening (NFF), found that the risk premium expected by the private sector in the Norwegian market is 5 %, as of 2014 (PwC 2014). The survey also found that the observable market risk premium at OSE for 2010-2013 was 5.6 %.

Another source, such as Damoderan (2012), has calculated yearly risk premiums for several economies. From his estimates he finds that the Norwegian risk premium is 5 %, based on the period from 1900 to 2014[9].

Based on the aforementioned sources, we believe that a risk premium of 5 % is reasonable.

4.9 Risk Adjustment by Martingale Measure

In order to discount the cash flows in our analysis by the risk free rate, we adjust the growth rate of the aluminium and power price as explained in chapter 3. Table 4.3 and 4.4 show the numbers in which the adjustment is based on.

Table 4. 3 Risk Adjustment Market Parameters

Market Index	MSCI World
Risk free rate	5 %
Market risk premium	5 %
Market, std div (1974-2013)	16.89 %
Market, std div (2001-2013)	19.22 %

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[9] http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ctryprem.html (18.05.14).

Table 4. 4 Risk Adjustment Parameters

	Aluminium Price	**Power Price**
Rate of return	2.48 %	7.13 %
Standard deviation	22.80 %	18.39 %
Correlation with market	35.38 %	36.91 %
Risk adjustment	2.39 %	1.77 %
Expected growth after adjustment	2.11 %	1.73 %

As the market index we have chosen MSCI World as reference. For the risk adjustment of the aluminum price we use the market index in the period from 1974 to 2013. In regards to the power price we apply a timeframe from 2001 to 2013, due to the different time series available. When we take into account the longest time frame the market index has a standard deviation of 16.89 %, as opposed to 19.22 % in the shortest data set. The correlation between MSCI World and the aluminium price has a correlation coefficient of 0.3558. As for the power price, the correlation coefficient with the market index is estimates to 0.3691. The correlations are estimated from log returns.

5 Model Analysis

In order to evaluate the production phase of an aluminium plant we have constructed a real option model in the form of a switching option. The real option represents the basis of our analysis and we investigate how different parameters and flexibilities affect the value of the switching option. The analysis is applied for a hypothetical aluminium plant and we examine how flexibility can contribute in the valuation to aluminium production.

Specifically, our analysis starts by looking at how changes in the aluminium price affect the value of the switching option. Furthermore, we investigate the impact of changes in volatility and correlation between the underlying uncertainty factors, the prices of aluminium and power. Our analysis also examines the effect of different risk free rates and inflation rates on the option value. Furthermore, we present a valuation of the flexibility to open the aluminum plant after previously being closed. Finally, we investigate how the choice of power contract affects the value of the switching option.

5.1 Model assumptions

In the analysis we assume that the aluminium plant is in Norway, thus we base our model on Norwegian conditions. All prices are denoted in USD per MT and values are nominal. There are no taxes considered in the model. All interest rates are considered as fixed and we assume a nominal risk free rate of 5 %.

Aluminium and power prices are regarded as underlying risk factors of the switching option and are uncertainty factors of the model. It is assumed that aluminium and power prices follow an OU process. Growth rates for all factors are risk-adjusted in accordance with EMM, as described in Section 2.7. For simplicity, movements in the price processes of alumina, carbon and fixed costs are set as fixed. We assume that the simulated prices are yearly prices. Prices in the profit function, as described in Section 4.1, are seen as reasonable estimates and as a general example for an aluminum plant. Parameter assumptions for the base case with regards to the price variables in the model are displayed in Table 5.1

Table 5. 1 Base Case Parameters Assumption

	Aluminium	**Power**	**Alumina**	**Carbon**	**Fixed costs**
S_0	2400	762	731	400	495
$g_{risk\ adjusted}$	2.11 %	1.73%	2.5%	2.5%	2.5%
σ	22.80 %	18.39 %			

We assume that the aluminium plant has a lifetime of 40 years and that 1 time interval is equivalent to 1 year. The number of simulations for each underlying variable at every time step is set to 100 000. The company can make exercise decisions with regards to opening and closing the aluminium plant once a year. The switching option will be compared against a case where the aluminium plant always operates, i.e. a case with no flexibility.

In order to investigate the effects of different contracts on power, we consider a 40 year fixed contract up against flexible one-year contracts. The one-year contracts follow a process equal to the OU process for the price of power, whereas the 40-year contract follows a deterministic price process and is the expected value of the stochastic process. As earlier, prices are risk-adjusted according to EMM.

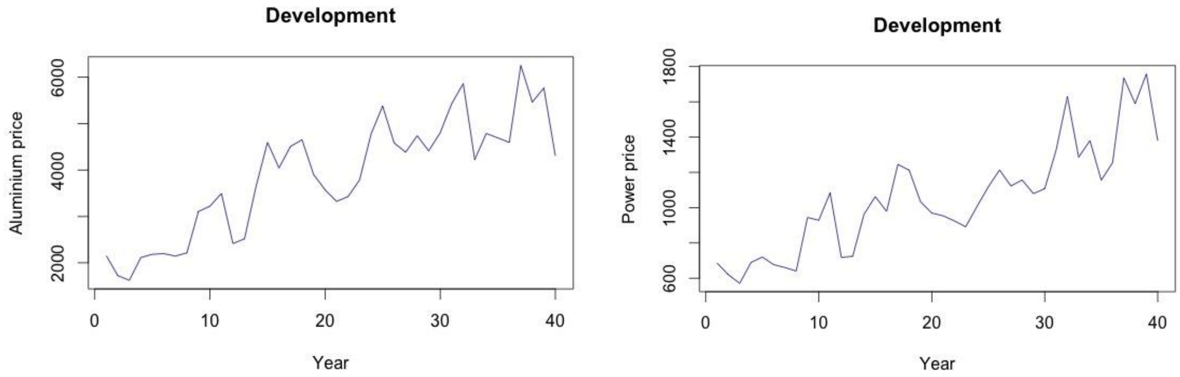
Table 5. 2 Base Case Model Assumptions

Name	**Base case value**		**Explanation**
T	40	years	Time of production
I.p	12	USD/MT	Initial profit at time zero
oc	500	USD/MT	Cost of opening production after temporarily closed
cc	1500	USD/MT	Cost of closing production

5.2 Sample Paths for Prices of Aluminium, Power and Profit

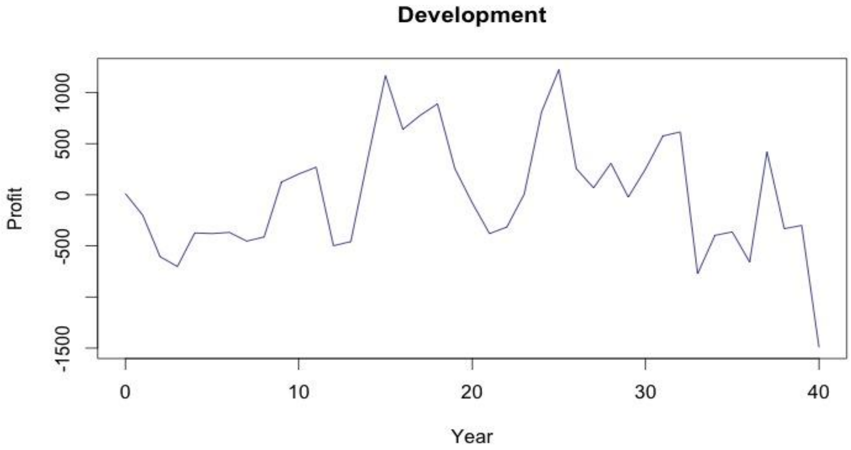
In this section we present sample paths of the simulated prices for aluminium and power, in addition to a sample path for profits. Figure 5.1 illustrates some random selected paths for aluminium and power prices over the lifetime of the aluminium plant of 40 years. We see that the prices start out at their initial values of \$2400 and \$762, and move in accordance with the mean reversion process.

Figure 5. 1 Sample Paths for Aluminium and Power Prices



The expected profits of the aluminium plant is based on the simulated prices for aluminium and power, as well as estimated deterministic costs of alumina, carbon and fixed costs. A sample path for profits is illustrated in Figure 5.2 below. We see that the initial profit is $12 and that it moves according to the calculated prices.

Figure 5. 2 Sample Paths for Profit



5.3 Mean Reversion

The simulated aluminium and power prices are considered to follow a mean reverting process, as explained in Section 2.5. The prices fluctuate but revert back to their long-term average of 2.11 % and 1.73 % for aluminium for power, respectively, as illustrated in Figure 5.3.

Figure 5. 3 Mean Reversion Process of Aluminium and Power Prices

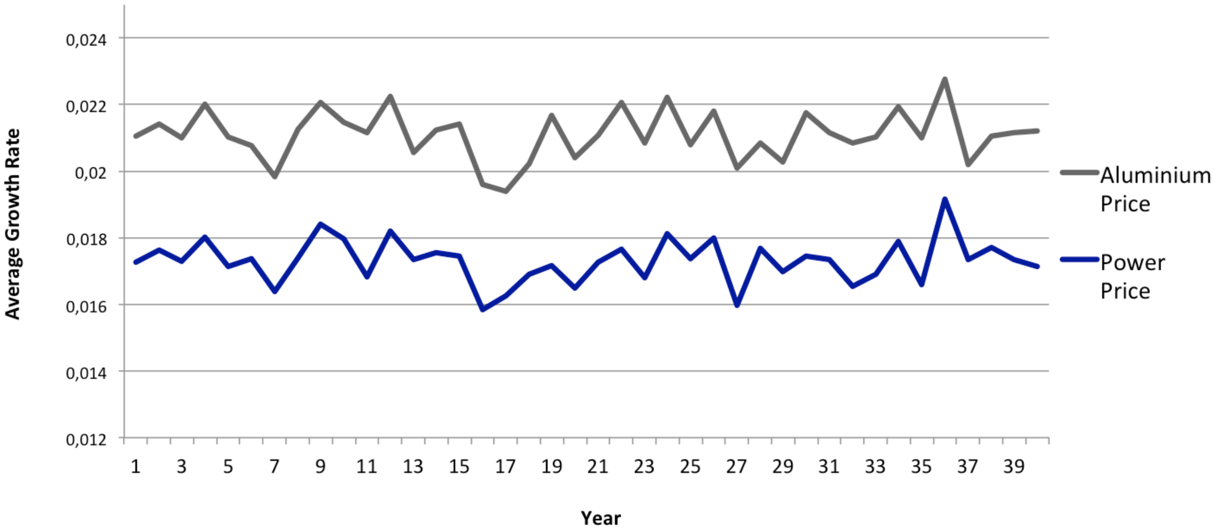


Figure 5.3 illustrates the mean reversion of the aluminium and power prices according to risk adjusted growth rates.

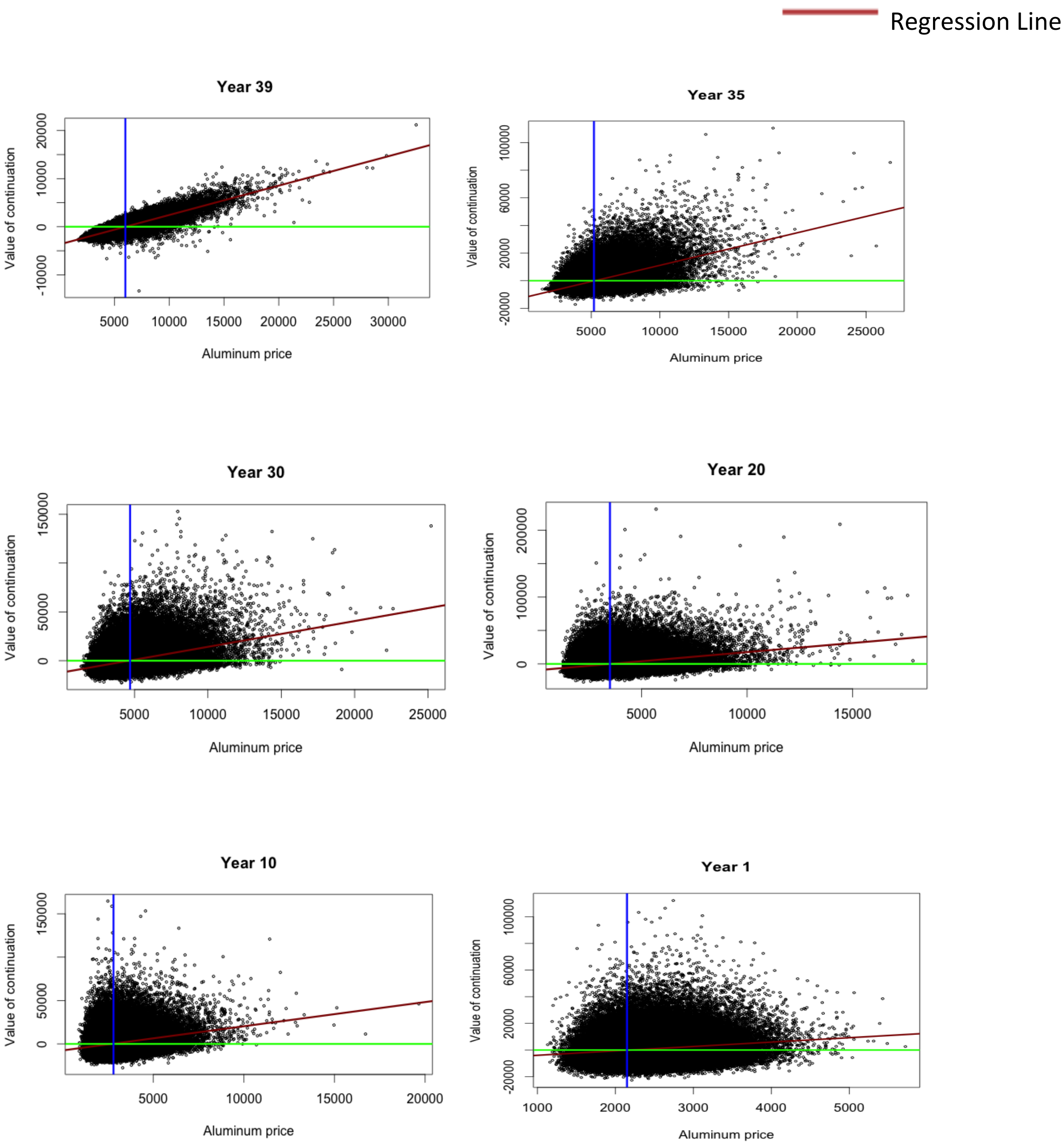
5.4 Ordinary Least Square Regression

In this section we illustrate regression plots containing the value of continuation and the price of aluminium. For simplicity the price of power is omitted from the illustrations. As explained in Section 3.2, the ordinary least square regression is conducted by comparing the value of continuation against the stochastic prices. The plots illustrate the basis for the optimal decision where we compare the value of holding the option against the aluminium price. Figure 5.4 depicts 6 plots for 6 different years.

All observations to the right of the intercept, illustrated by the blue line, where the regression line and the green line intersect, are observations for where we choose to hold the option, i.e. continue production if production was previously held open. For observations to the left of the blue line we choose to exercise and close production. The observations above the green line has a positive value of continuation, as opposed to negative values below the green line. It follows from the plots that there are some cases where we choose to hold the option, even though the value of continuation is negative. Conversely, there are some cases where we exercise the option, even though the value of holding the option is positive.

In year 39 we observe an approximately linear relationship between the value of continuation and the price of aluminium, as illustrated in Figure 5.4. This correlation is caused by low uncertainty about the expected value of continuation, as there is only one year remaining before maturity. When we move backwards in time, the relationship becomes more dispersed due to increased uncertainty about the value of continuation. For year 1, we observe that the value of continuation becomes highly uncertain as it depends on prices simulated 39 years into the future. This is illustrated by the spread in the observations at year 1.

Figure 5. 4 Ordinary Least Square Regressions



Year 39

Year 35

Year 30

Year 20

Year 10

Year 1

The plots display the value of continuation for different simulated prices of aluminium at year 1, 10, 20, 30, 35 and 39.

5.5 Model Results for Base Case

As per the base case, the model estimates a positive value of $354 for the aluminium plant including flexibility. In the case where the flexibility of switching is omitted and the aluminium plant maintains open for 40 years, the value of the plant becomes negative by $-1189. This results in a value of the switching option of $1544. The results are summarized in Table 5.5

Table 5. 3 Base Case Model

Name	Results	
% Operation	61	%
Value incl. flexibility	354	USD/MT
Value excl. flexibility	-1189	USD/MT
Value of switching option	1544	USD/MT

In the base case, the ability of switching between operational modes reduces the average operation rate of production from 100 % to 61 %, as illustrated in figure 5.5. An average operation rate of 61 % means that the aluminium plant is expected to operate 61 % of the time during its lifespan of 40 years. It needs to be taken into consideration that these calculations are made under risk adjusted probabilities, thus they are not entirely accurate.

Figure 5. 5 Average Expected Operation

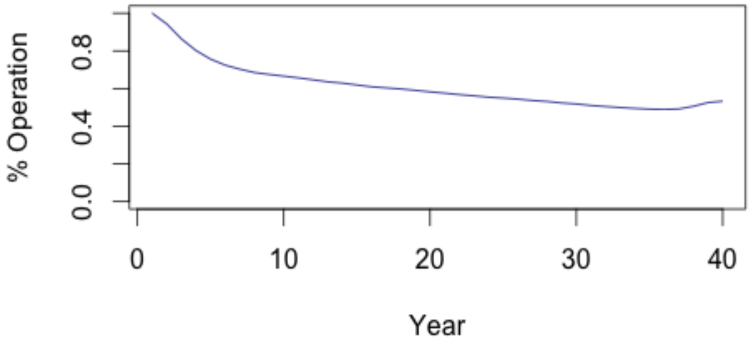


Figure 5.5 illustrates the average expected operation rate under risk adjusted probabilities.

5.6 Aluminium Price Analysis

We test the model by applying different initial prices of aluminium with various volatility rates and compare the values of the switching option. From the model we observe a positive correlation between the value of the aluminium plant including flexibility and the initial price of aluminium, as illustrated in Figure 5.6. The value of the option is highest for low initial prices of aluminium. In the case of high volatility rates the option value in general is higher than in the other two cases. However, once the price of aluminium reaches $2900, the value of the switching option is approximately worthless regardless of the volatility rates.

Figure 5. 6 Aluminium Price

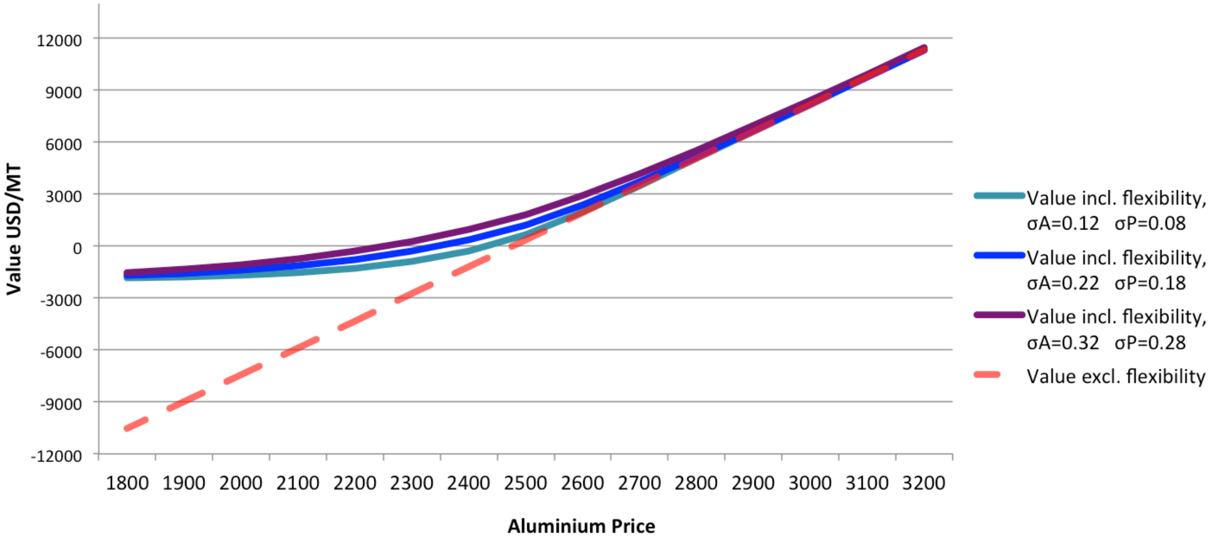


Figure 5.6 illustrates the value of the switching option with changes in the price of aluminium. The value of the switching option is the difference between the dotted and solid lines. The blue line and an initial start price of $2400 depict the base case.

When the initial price of aluminium declines and tends towards zero we observe that the value of the aluminium plant including flexibility declines asymptotically towards approximately -$1500. This equals the closing cost and a possibly negative profit, as production is closed after the first period when the initial price gets too low. The value including flexibility stays negative until the initial price of aluminium reaches $2400. This is in line with the initial profit function, which reports a moderate profit of $12 with an initial price of aluminium of $2400. From Figure 5.7 we observe that under these circumstances, production is often considered to be unprofitable as the operation rate is below 50 %. When the initial price of aluminum is $2600 and higher, there is a positive linear relationship between the value of the plant including flexibility and the initial aluminium price. It follows that an increase in the aluminum price leads to an increase in the value including flexibility.

A low initial aluminium price causes production to be less profitable leading to a lower rate of operation. To reduce downside risk when profits are low, the average operation rate of the base case declines from 61 % to 8 % when initial price of aluminium declines from $2400 to $1800, as illustrated in Figure 5.7.

Figure 5. 7 Aluminium Price and Operational Sensitivity

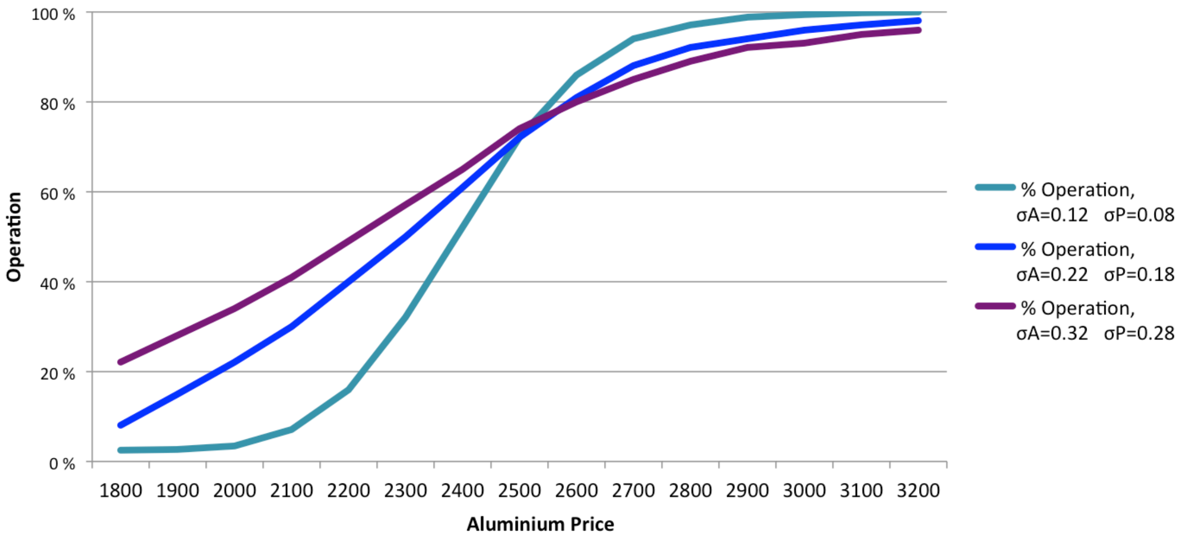


Figure 5.7 displays average operation rate for changes in the start price of aluminium. The operation rate is based on risk neutral probabilities. The blue line and an initial price of $2400 represent the base case.

For the case with volatilities of 0.12 and 0.08, the average reported operation is 2.5 % at an initial price of $1800. As expected, low volatilities along with a low aluminium price contribute to an overall low profitability and average production declines. On the other hand, when the price of aluminium increases, the profitability increase and the average operation rate goes up. In the case of high volatilities there are larger fluctuations in the prices, causing situations where production is considered unprofitable, even though the initial aluminium price is high. Under such circumstances the scenario with reduced volatilities will have a higher average operation rate. This is due to the fact that the price of aluminium and power fluctuate less, and the aluminium plant is seldom closed down. As expected, with an initial aluminium price of $3000 we report an average operation rate of 99.4 % for the low volatility case, compared to 96 % and 93 % for the two other cases.

5.7 Correlation Analysis

We analyze the effect of correlation between aluminium and power prices on the value of the aluminium plant. From Figure 5.8 we observe that there is a negative relationship between the value including flexibility and the correlation coefficient of the variables.

Figure 5. 8 Correlation between Aluminium and Power Prices

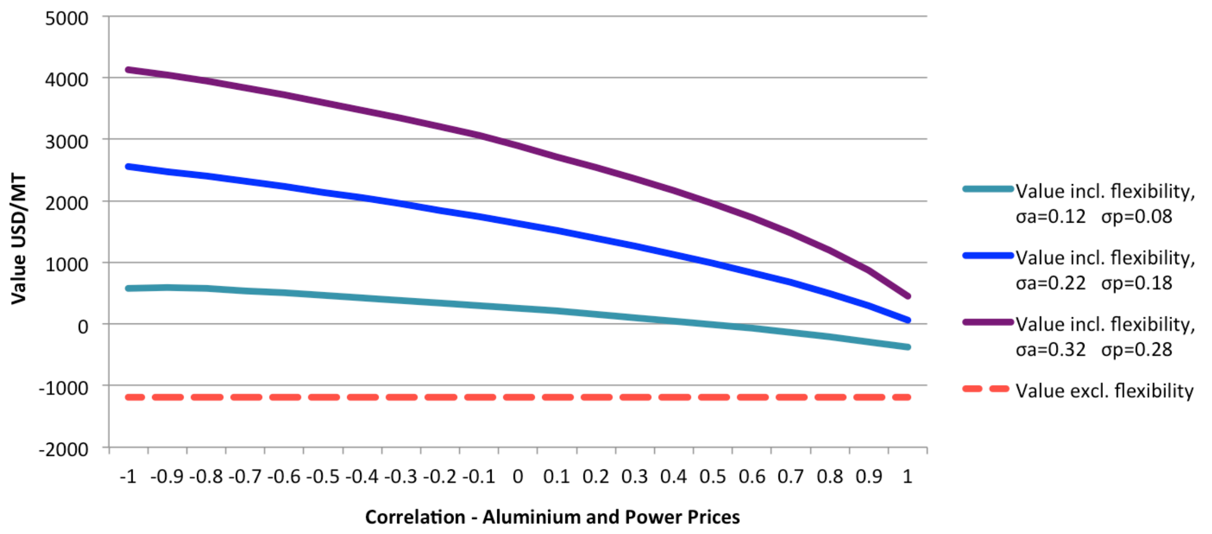


Figure 5.8 displays the value including flexibility for various correlations between aluminium and power prices. The value of the switching option is the difference between the dotted and solid lines. The base case is depicted by the blue line and has an initial start price of $2400.

In the base case the correlation coefficient between aluminium and power prices is 0.87, which results in a value of the switching option of \$1543. When the correlation coefficient is 1 the value of the option with volatilities as in the base case becomes \$1248. This is a decline of \$295 from \$1543, constituting a reduction of 19 %. In this case there are fewer situations with profits lower than the closing cost, due to lower fluctuation in prices. It follows that there are less events where it is optimal to close production. However, if we look at the other extreme where correlation between the prices is -1, there is a substantial increase in the value of the switching option of \$2202, relative from the base case. In this case the switching option benefits from being able to close production in times of negative profits, as well as gaining value in times of high profits.

The results show that the negative relationship between the correlation coefficient and the value of the switching option has greatest impact when volatilities in the prices are high. For the case with volatilities of 0.32 and 0.28, the value decreases from \$5328 to \$1639 when correlation changes from -1 to 1. This is a decline in value of \$3689. Correspondingly, the base case value falls with an amount of \$2497, whereas for low volatilities it decreases with \$954. As higher volatilities causes the level of profits to fluctuate more, the switching option becomes more valuable for high volatilities. This is because operation closes for situations with low profits while the aluminium plant is open in times of high profits. Thus we secure the downside and keep the upside.

5.8 Growth Rate in Aluminium Price Analysis

In this section we analyse the sensitivity of changes in the risk adjusted growth rate for the price of aluminium. As illustrated in Figure 5.9, the value of the switching option is sensitive for changes in the growth rate of the aluminium price. When the growth rate in the base case becomes lower than 1.9 % the value of the option becomes positive, all other things held fixed. Further, we observe that the lower the growth rate, the higher the value of the switching option.

Figure 5. 9 Adjusted Growth Rate

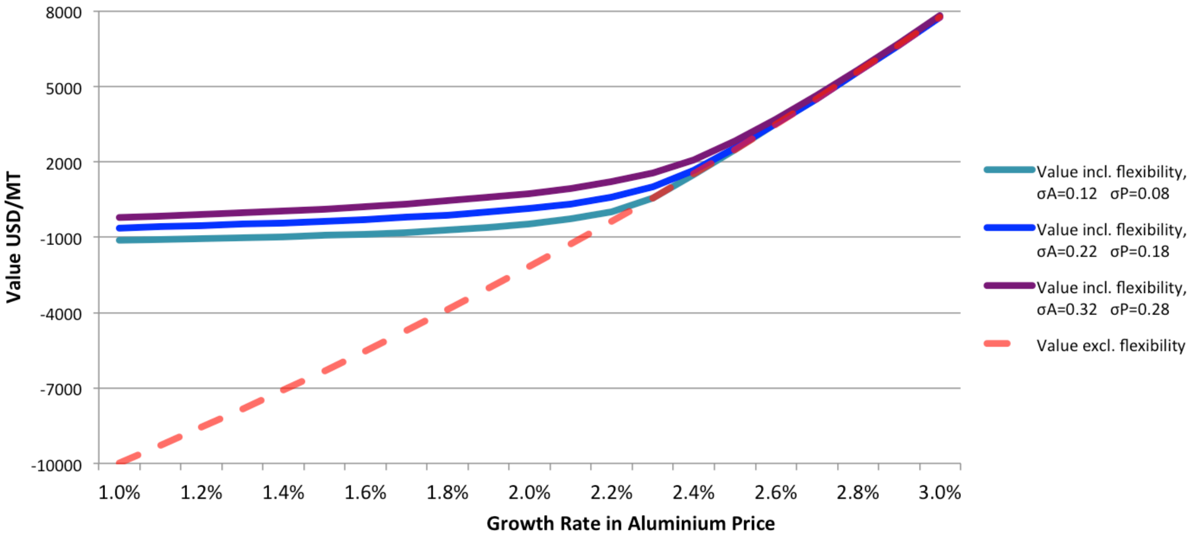


Figure 5.9 displays the value of the switching option with changes in the risk adjusted growth rate for the price of aluminium. The blue line depicts the base case, which has an initial risk adjusted growth rate of 2.11 %.

The level of volatility in the prices of aluminium and power has an important impact on the value of the switching option when growth rates are low. The three scenarios with various volatilities differ in option value with a value of approximately $400 to $600 as the growth rate range from 1 % to 2 %. The value of the switching option is highest for the case with high volatilities, and conversely lowest when volatilities are low. When volatilities are high it causes profitability to fluctuate more compared to the other two cases. This makes the switching option more valuable in that it gains the upside from high profits, but secures the downside by closing production when profits are low.

As the growth rate move from 2.5 % to 3 %, the switching option in the three cases becomes approximately worthless. This is because in the event of large growth rates there are fewer situations of low profits, thus fewer situations where it is optimal to close production, which contributes to a limited downside. Further, the upside gained from high profitability is not significantly higher than earned profit in the other two cases.

Figure 5.10 illustrates the relationship between the growth rate in the price of aluminium and the average operation rate of production for the aluminium plant. With a growth rate of 1% in the low volatility case, the plant is operating 9 % on average over its lifetime. In the base case the operation rate is 15 % for a 1 % growth rate, whereas it is 24 % in the case with high volatilities. When the growth rate increases to 2.5 %, the average operation rate is over 90 % for all three cases. This illustrates a positive relationship between the growth rate in the price of aluminium and the average operating rate. The result is expected as profits become more positive for higher growth rates and it is therefore optimal to keep the plant open more.

Figure 5. 10 Growth Rate and Operational Sensitivity

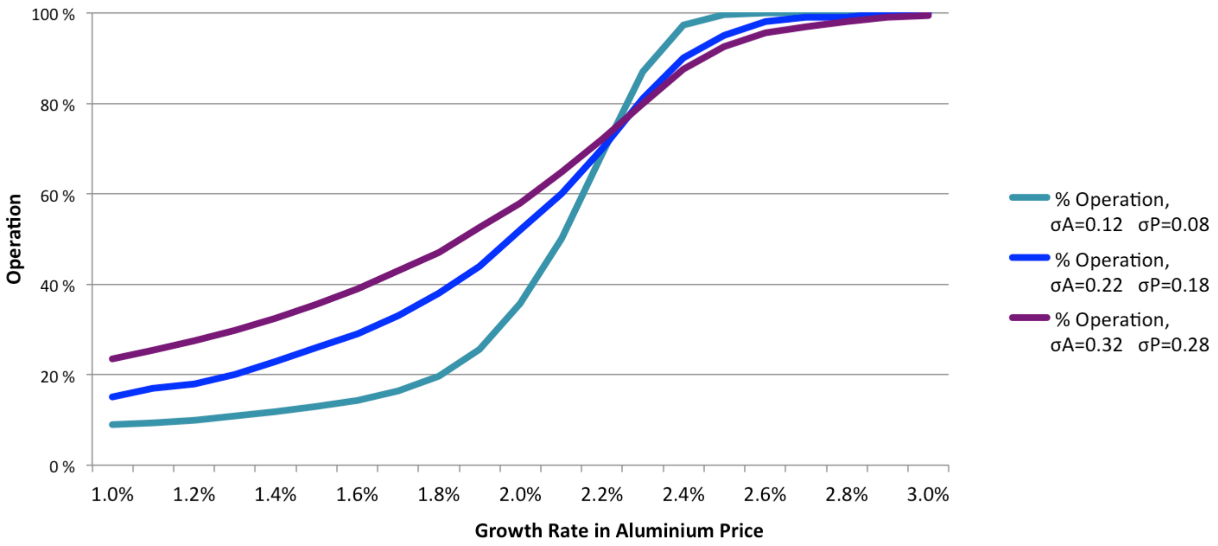


Figure 5.10 displays average operation rate for changes in risk adjusted growth rate of the aluminium price. The operation rate is based on risk neutral probabilities. The blue line represents the base case for different growth rates in aluminium price.

Considering the three cases, average operation rate is lowest as the growth rate range from 1 % to 2.2 % and when the volatilities are low. Conversely, average operation is largest when the interval ranges from 2.4 % to 3 %. In general, low growth rates results in less profitable situations, in turn closing production more often. For high growth rates, profits are generally at a more stable and acceptable level, which leads to a higher operation rate. In the case of high volatilities and high growth rates profits in general are positive. However, bigger fluctuations in prices induce scenarios where profits sometimes are negative, which leads to a reduced rate of operation.

5.9 Risk Free Rate Analysis

When we apply risk adjusted growth rates for the aluminium and power prices we are able to discount expected cash flows in the model by the risk free rate. In this part of the analysis we look at how a change in the risk free rate, i.e. the discount rate, affects the value of the aluminium plant including flexibility, as well as the case of excluding flexibility, i.e. always letting the aluminium plant operate.

As illustrated in Figure 5.11, there is a negative relationship between the value of the plant including flexibility and the risk free rate. This is expected as higher discount rates reduce the net present value of expected cash flows.

Figure 5. 11 Risk Free Rate

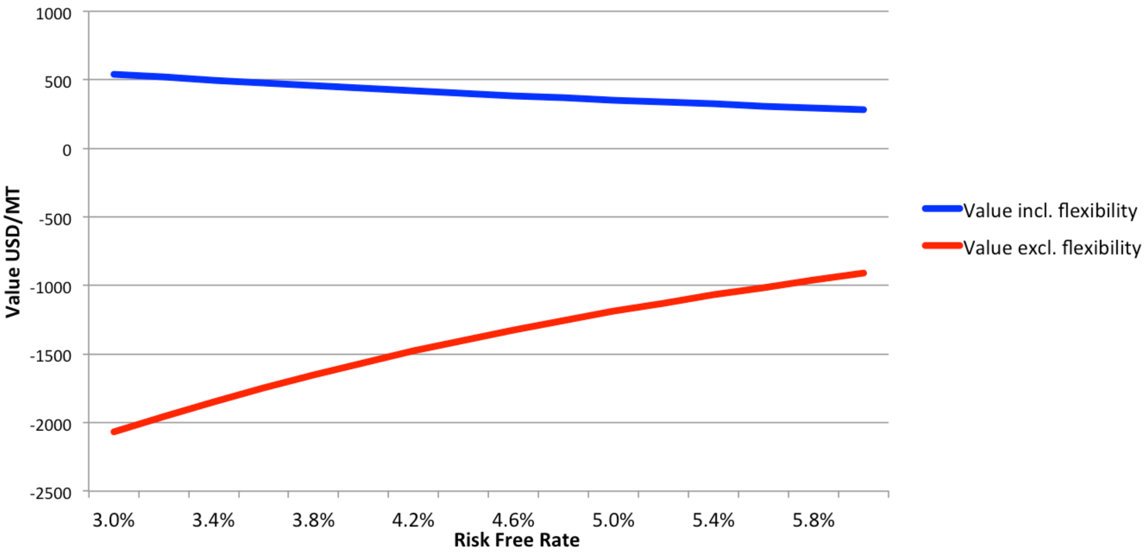


Figure 5.11 displays the value including flexibility versus the value excluding flexibility for changes in risk free rate. The value of the switching option is the difference between the blue and red line. The base case is depicted by the blue line and has an initial risk free rate of 5 %.

If we decrease the risk free rate in the base case from 5 % to 4 %, the value including flexibility increases by 24 % to $438. When the risk free rate is 3 %, the value including flexibility almost doubles with an increase of 53 % to $542. At the other end of the scale, with a relatively high risk free rate of 6 %, the value including flexibility declines by 20 % relative to the base case. This is as expected, due to the increased discount rate.

In the case where the aluminium plant is always operating, there is a positive correlation between the risk free rate and the value excluding flexibility. This follows from the value excluding flexibility being negative and that an increased discount rate causes the value of production to become more valuable, i.e. less negative. To recap, we find that the value of the switching option is largest for low risk free rates, and conversely lowest for high risk free rates.

5.10 Inflation Rate of Deterministic Costs Analysis

We analyze the effect of different inflation rates on the value of the aluminium plant including flexibility and the value of always operating the aluminium plant. As mentioned in Section 4.6 the deterministic costs is set to grow at the rate of inflation. Hence, when all other factors are held constant, an increase in inflation from 2.5 % to 3 % reduces the value including flexibility from $354 to -$139. Similarly the value of always operating is reduced from $-1189 to -$4757. Although the values of the aluminum plant with or without flexibility declines, the value of the switching option itself increases, as illustrated in Figure 5.12 by the difference between the blue and red line. This is because in the event of high inflation rates there are more situations of low profits, and the option benefits from being able to close production.

Figure 5. 12 Inflation Rate

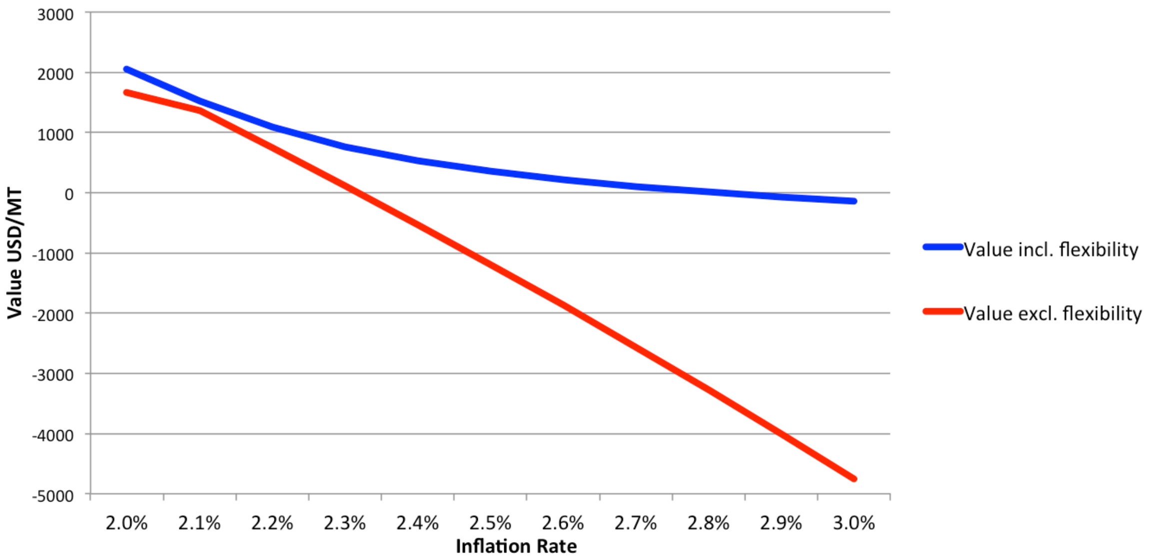


Figure 5.12 illustrates the value including flexibility versus the value excluding flexibility for changes in inflation rate. The value of the switching option is the difference between the blue and red line. The base case is depicted by the blue line and has an initial inflation rate of 2.5 %.

The base case has an inflation rate of 2.5 %, corresponding to the long-term inflation target in Norway. However, when the inflation rate is set to 2 % according to the long-term inflation target in the US, we observe a great increase in the value including flexibility and the value of always operating the aluminium plant. The value including flexibility increases by 82 % to $2048, while the value of always operating increases from -$1189 to $1670. It follows that the value of the switching option decreases for lower inflation rates. Decreases in the inflation rate from 2.5 % to 2.0 % makes the option value decrease from $1542 to $378, constituting a decline in value of $1164.

5.11 Switching Costs Analysis

In order to test the impact of switching costs on the value of the switching option, we analyse different scenarios of costs related to open and close the aluminium plant.

Figure 5. 13 Swicthing Costs

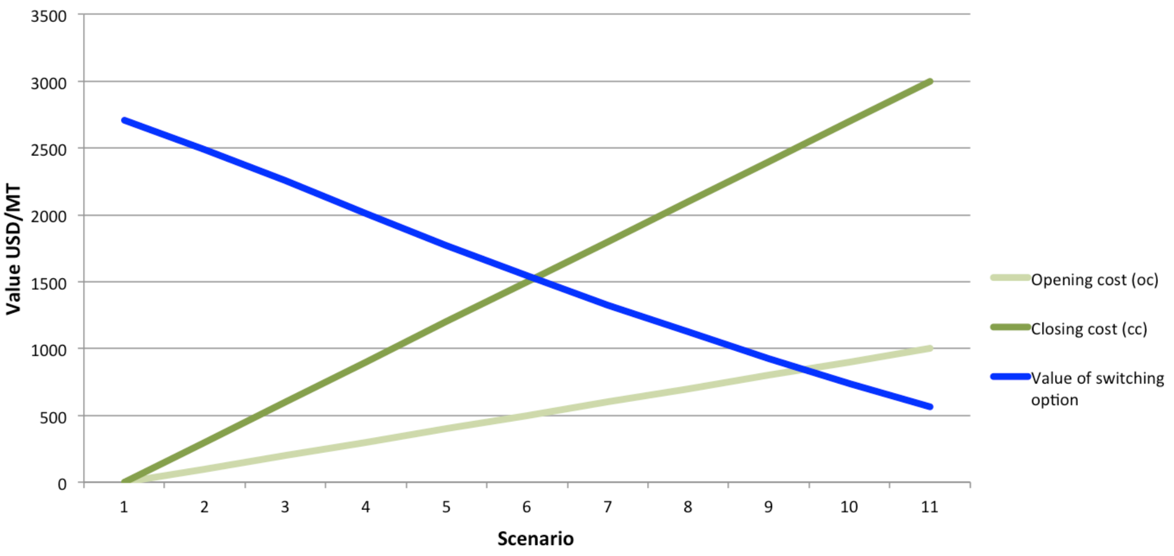


Figure 5.13 illustrate the option value for changes in switching costs. The blue line depicts the base case, with initial costs corresponding to Scenario 6.

Scenario 6 represents the base case, with opening and closing cost of $500 and $1500, respectively. This corresponds to a value of the switching option of $1543. From Figure 5.13 we observe a negative correlation between the value of the switching option and the opening and closing costs. Low costs will increase the value of the switching option, while high costs involve a decrease in value. When the switching costs are set to zero, as in Scenario 1, we observe a significant increase in the option value to $2708, relative to the base case. Thus, reduced costs entail increased profitability and increased operation rate.

When the costs related to opening and closing production increases to $700 and $2100, as in Scenario 8, the value of the option decreases by $419, relative to the base case. This is a decline in the value from $1543 to $1124. Increased switching costs contribute to reduced value of switching option as production is more often held open for unfavourable profits. The finding is that the opening and closing costs has a significant impact on the switching option.

5.12 Value of Flexibility Analysis

The switching option holds the flexibility of being able to close and reopen production in conjunction with expected profits. In this section we test what happens to the option value when we exclude the flexibility of being able to reopen the aluminium plant.

As presented in Table 5.6, the value of the switching option declines by 5.4 % from $1543 to $1460, when we shift from full to reduced flexibility. The operation rate falls by 31 %, meaning that the average operating time is 30 %. From these results we find that the flexibility of being able to reopen the plant adds value to the switching option and has a significant impact on the operation rate.

Tabell 5. 4 Value of Flexibility

	Full flexibility	Excluding flexibility to reopen	Δ from full flexibility
Value of switching option	$1543	$1460	- 5.4 %
Operation rate	61 %	30 %	- 31 %

** Base case (A0=2400), 100 000 simulations*

5.13 Fixed Contract Analysis

In this part of the analysis we apply a fixed contract as the source of power into the model. This type of contract is an extreme case in which we assume that it is valid for the entire lifetime of the aluminium plant, i.e. 40 years. As described in Section 3.5, the optimal exercise decision in the model in the case of fixed power sourcing is made from the value Q. In the base case for this analysis Q is set to -$1500. This means that if the value of continuation when including the fixed contract is higher than -$1500, the aluminium plant will be kept open given that we were open in the previous period.

From Figure 5.14, we observe that the value of the switching option with a fixed contract and Q equal to -$1500 is zero. Under these circumstances the average operation rate increases from 61 %, as reported in the base case, to 99.9 %. As such, including a fixed contract of power makes the value of the aluminium plant including flexibility approximately equal to the value of always operating the aluminium plant. In this case the operation rate is 100% and the value of always operation is -$1189. As can be observed from Figure 5.14 the value of the option with a fixed contract is asymptotic to zero.

Figure 5. 14 Power Sourcing

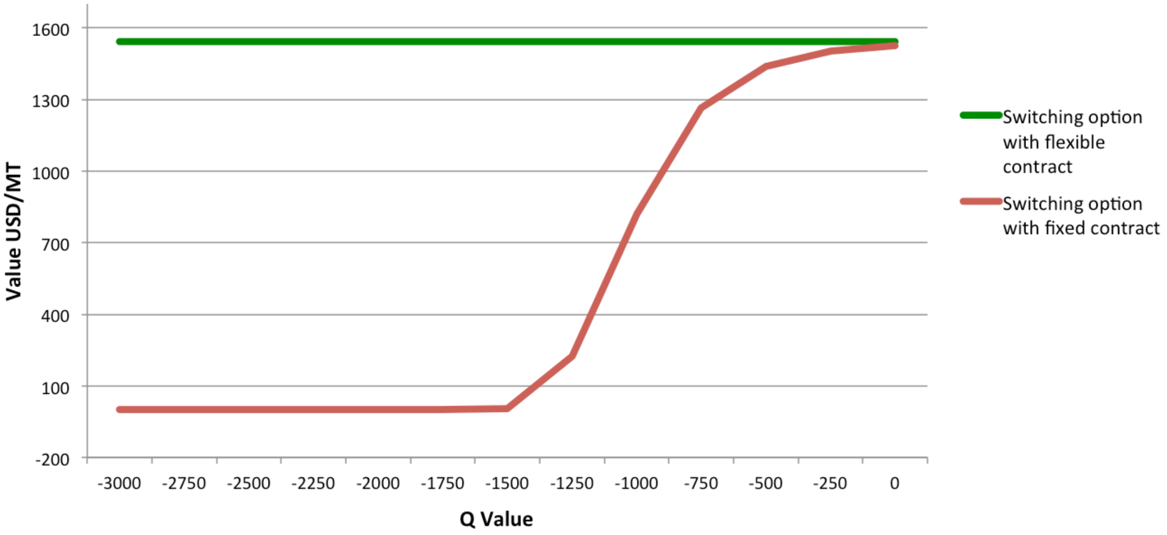


Figure 5.14 illustrates the value including flexibility versus the value of excluding flexibility for different sources of power. The light green line depicts the base case, with a valueof the switching option of $1543.

With Q set to zero the value of the switching option with the fixed contract increases to $1526 and results in an average operation rate of 61.9 %. Hence, the value of the option with a fixed contract increases as Q becomes closer to zero and production closes for unfavourable profits.

In general, the fixed contract reduces uncertainty and fluctuations in the expected cash flows. This means that in some situations where the initial decision was to close the aluminium plant, the aluminium plant will now be kept open. This follows from fixed expected profits having a higher value than expected variable profits. In these circumstances, we will keep the aluminium plant open more often, even though it would be more profitable to close production and alternatively sell the power contract in the market.

An aluminium producer might consider shutting down production in order to make a higher profit on the power contract. However, such actions are likely to leads to layoffs, which may contribute to political questions concerning the company's ethical standards.

6 Implications and Further Reaserch

In this chapter we review implications of the model and presents how the results from the analysis can be interpreted. Limitations of the model are discussed as the value of the switching option is affected by assumptions embedded in the model. Finally, some further research is suggested.

6.1 Implications of the Model

In our analysis we have investigated how changes in different parameters affect the value of the switching option. Overall, we find that the real option model is quite sensitive in terms of changes in various parameters.

The value of the switching option is highest in events of low profit margins. When the initial price of aluminium and the growth rate in the aluminium price are low, the flexibility in the switching option allows for production to be closed, thus securing the downside. Correspondingly, when the growth rates in deterministic costs are high production is closed for unfavorable profits.
However, if the aluminum plant is always operational, i.e. the flexibility of switching is omitted, operation continues regardless of negative profits and the value of the aluminum plant becomes negative for low profit margins.

Correlation between the stochastic prices has a significant effect on the value of the switching option. A positive correlation between the prices of aluminium and power reduces the value of the option. Conversely, a negative correlation adds value to the option in that revenues and costs moves in opposite directions, increasing the upside. The effect is amplified when high volatilities are included in the model. In general, high volatilities adds a significant value to the switching option, whereby it creates high fluctuations in profits, thus allowing for operation in times of high profits and closing in times of low profits.

The cost of being flexible is represented by the opening and closing costs generated by switching between modes of operation. The value of the switching option declines as switching costs increases. The costs of being flexible adds value to the option as long as a marginal increase in the switching costs is lower than the reduced value of the option. If the option value reaches zero, the flexibility to switch do not provide any value.

The flexibility of being able to reopen contributes to a moderate value to the switching option. From a situation of full flexibility to a situation of reduced flexibility, excluding the flexibility to reopen, leads to a reduction in the value of the option. Reduced flexibility means that production of aluminium declines and that if production closes it is never reopened. With permanently abandonment the option to take advantage of high profits in the future is lost. Thus, it is apparent that the flexibility of being able to reopen reduces the operation rate significantly.

Alternative sources of power contribute to different option values. A fixed contract reduces the value of the switching option as production is maintained at times where closing of production is considered to be more profitable. With a flexible contract production is closed at times when production is considered to be unprofitable, thus it contributes to a significant value of the switching option.

Overall, a fixed contract reduces uncertainty and fluctuation in expected cash flows. In the case of currency exposure, a fixed contract hedges currency risk in the price of power. However, a fixed contract reduces flexibility and consequently the value of the option. An alternative at times of low profit margins in the industry of aluminium is to sell the fixed contract in the market and close production of aluminium. This way the company gains a higher value by selling the power, rather than operate. It is questionable if this is an alternative for a company that primarily operates to sell aluminium. Closing production will affect several stakeholders, and the company would need to make a trade-off between maximizing the value of shareholders versus other stakeholders, such as employees, local communities and customers. This underpins that to close production is not as trivial as presumed by the model.

The analysis indicates that there is substantial value to valuing aluminium production through real option modelling. In industries faced with high uncertainties and volatile prices, such as aluminium production, there are considerable fluctuations in the profit margins. This type of industry benefits from the ability to be flexible in operation by responding to changes in underlying risk factors. By switching between modes of operation, the real option features makes it possible to gain an upside from high fluctuations, as well as reduce downside risk by closing of production.

Real option modelling is not only useful for aluminium production, but may also be transferred to other industries, especially those subject to low profit margins and high uncertainty. Inparticular, real option modelling may be useful in the oil and gas industry due to high fluctuations in the oil price. Industries such as producers of cars or airplanes may also benefit from the flexibility provided in a real option model. Utilizing the flexibility embedded in real options not only allows for switching between different operational modes, but also enables switching between various factors of input or suppliers.

6.2 Limitations and Further Research

The model constructed for analysing the value of the switching option rests on several assumptions. This may to some degree affect the accuracy of the model, which weakens the results. In this section we review some limitations of the model and make some suggestions on further research.

In the real option we model prices of aluminium and power as underlying risk factors of the model. The prices are modelled stochastically and are set to follow a mean reverting process. The other cost factors, alumina, carbon and fixed costs is set to follow a deterministic process. Optimally, all factors in the profit function should follow a stochastic process. In regards to choosing the appropriate process of the underlying risk factors, the literature is divided, making it difficult to determine the right process. Different processes are expected to have a great impact on the value of the option.

The model assumes that several parameters are fixed over the lifetime of the option. In reality, this is not realistic as the parameters are not constant and will vary over time. Ideally the interest rates should be based on interest rate models. In addition, the aluminium plant has an expected lifetime of 40 years, which makes determining the parameters problematic. The prices in the profit function are reported in various currencies, which contributes to an exchange rate exposure. The foreign exchange exposure should have been taken into account in the model. Furthermore, as taxes are excluded from the model, the value of the option may be somewhat amplified.

The model assumes that reopening can happen at any time during the lifetime of the option. It does not take into account that reopening should take place within the following 4-5 years in order to ensure that production can be restored completely. In the event of temporarily closing the aluminum plant, maintenance costs that may occur are not considered in the model. Limited closing of production and maintenance costs are factors to be considered in further research. Furthermore, it could be of interest to add an option to switch between different production capacities. A lower production capacity could be a good alternative as opposed to a closedown of production. This is especially in consideration of difficulties the company may encounter with stakeholders in closing of production.

7 Conclusion

In this thesis we have investigated whether real option analysis can be applied for valuing aluminium production and the value that this valuation method provides. We have constructed a real option model in the form of a switching option. The switching option holds the opportunity to switch between two modes of operation; full operation and temporarily closed. In the model we have simulated stochastic processes of the aluminum and power price, which are the underlying factors of the model. Our analysis has also taken into account the effect of alternative power sourcing by including a fixed long-term contract.

The conclusion from our analysis is that valuing aluminium production through real option analysis adds substantial value to the aluminium plant. As the production of aluminium is encountered with volatile prices and high uncertainties, switching between modes of operation in accordance with expected profits is beneficial. The flexibility features that lies within the real option makes it possible to gain an upside from high fluctuations and reduce downside risk by closing.

We find that the value of the switching option is largest when profit margins are low. In the event of a low initial aluminum price and low growth rates in the price of aluminium the value it adds to the aluminium plant is of greatest significance. Equivalent the value of the switching option has greatest value when deterministic costs are set to grow with high rates. Our analysis shows that the effect of correlation between the stochastic prices proves to have a significant impact on the value of the switching option. A negative correlation increases risk and adds a positive value to the option, whereas a positive correlation reduces uncertainty, thus reduces the value of the option.

Volatilities in the prices of aluminium and power have a significant effect on the value of the option. We find that high volatilities increase the value of the option, while low volatilities induce a reduction. When we investigated the flexibility of being able to reopen in isolation, we find that excluding the flexibility to reopen leads to a reduction in the value of the option. This underpins the value of being able to switch between modes of operation.

Introducing an alternative source of power also has a significant impact on the switching option value. A fixed power contract hedges the risk embedded in the power price and reduces some of the uncertainty in the expected profit. Although the contract contributes to more stable profits, it reduces the upside and downside of the option and consequently its value. In conclusion, a flexible contract is considered to be more value adding.

The model we have constructed is based on a method by Longstaff and Schwartz and applies MCS to value the real option. This method provides a framework to value a real option with American exercise features and allow the underlying variables in the model to follow a stochastic process. As risk-adjusted discount rates are difficult to apply for real options, we risk adjust expected growth rates under EMM and use the risk free-rate to discount the cash flows. Valuing a real option with the method derived from Longstaff and Schwartz is a complex process requiring high computer capacity. Nevertheless, we find this approach to be valuable in the process of valuing a real option.

The model we have constructed is subjected to simplifications and assumptions that may be improved in further research. We suggest that further research incorporates a restriction of permanent closing if the aluminum plant has been closed in a period of more than 4-5 years. This would be an appropriate assumption as a reopening after 4-5 years is difficult to implement in reality. It could also be of interest to add different production capacities and include maintenance cost while production is closed. Finally, we would suggest that currency exposure is incorporated into the model.

Data

Datastream: LME-Aluminium 99.7% Cash U$/MT. Code: S74108(P) (10.04.14)

MSCI: MSCI WORLD U$ - PRICE INDEX (~U$) Code: MSWRLD$(PI)~U$ (10.04.14)

References

Asche, F., P. Osmundsen and M. Sandsmark. 2006. “The UK Market for Natural Gas, Oil and Electricity: Are the Prices Decoupled?” *The Energy Journal* 27(2): 27-40.

Black, F. and Scholes, M. 1973. “The pricing of options and corporate liabilities.” *Journal of Political Economy* 81:637-659.

Boyle, P. 1977. “Options: A Monte Carlo Approach.” *Journal of Financial Economics* 4: 323-338.

Breivik, Ø. and Carlsen, C.L. 2011. “Prosjektanalyse av aluminiumsindustri.” http://brage.bibsys.no/xmlui/bitstream/handle/11250/168945/1/breivik%20carlsen%202011.pdf (18.05.14)

Campbell, W.A. 2013. “Valuation of Timberland under Price Uncertainty.” Dissertation, Clemson University. Available at Clemson University.

Copeland, T. and Antikarov, V. 2003 “Real Options a Practitioner's Guide.” 1st ed. New York: Cengage Learning

Dixit, A.K. and Pindyck, R.S. 1994. “Investment Under Uncertainty.” 1st ed. 41 William Street, NJ:Princeton University Press

Finanstilsynet. 2014. *Finansielt utsyn 2014*. http://www.finanstilsynet.no/Global/Venstremeny/Rapport/2014/Finansielt_utsyn_2014.pdf?epslanguage=no (07.05.2014)

Gjedrem, S. 2001. *Inflation target - how interest rates are set*. http://www.norges-bank.no/en/about/published/articles-and-chronicles/art-2001-05-29-enhtml/ (20.04.14)

Gjølberg, O. 2001. “When (and how) will the markets for oil and electricity become integrated? Econometric evidence and trends 1993-99” *Discussion paper, D-20* .

Hull, J. 2012 “Options, Futures And Other Derivatives.” 8th ed. Upper Saddle River, NJ: Prentice-Hall.

International Energy Agency. 2009. *World Energy Outlook 2009* 1: 81-84. http://www.worldenergyoutlook.org/media/weowebsite/2009/WEO2009.pdf (08.05.14)

Jorion, P. 2007 " Value at Risk." 3rd ed. New York: McGraw-Hill.

Koekebakker, S. and Ollmar, F. 2001, "Forward curve dynamics in the Nordic electricity market". *Discussion paper, D-21.*

Longstaff, F.A. and Schwartz, E.S. 2001. "Valuing American Options by Simulation: A Simple Least-Squares Approach." *The Review of Financial Studies* 14(1): 113-147.

Merton, R. 1973 "Theory of Rational Option Pricing." *Bell Journal of Economics and Management Science* 4: 141-183.

NOU 2012:16. *Economic Analysis.* Ministry of Finance

OECD. 2011. *Looking to 2016: long-term growth prospects of the world.* http://www.oecd.org/eco/lookingto2060.htm (01.05.14)

Oslo Stock Exchange. 2013. *Strongest Growth in Share Prices since 2009* http://www.oslobors.no/ob_eng/Oslo-Boers/About-us/Press-room/News-from-Oslo-Boers/Strongest-growth-in-share-prices-since-2009 (30.04.14)

PwC. 2013. *World in 2050.* https://www.pwc.com/en_GX/gx/world-2050/assets/pwc-world-in-2050-report-january-2013.pdf (01.05.14).

PwC. 2014. *Risikopremie i det norske markedet 2013 og 2014.* http://www.pwc.no/no/publikasjoner/deals/risikopremien-2013-2014.pdf (30.04.14).

Schwartz, E. 2012. *The Real Options Approach to Valuation: Challenges and Opportunities* http://www.nhh.no/Files/Filer/institutter/for/borch_lecture/Eduardo_Schwartz-KBL-12-Slides.pdf (16.05.2014).

The Federal Reserve System. 2014.*"Why are interest rates being kept at a low level?"* http://www.federalreserve.gov/faqs/money_12849.htm (19.05.14)

Ulam, S. 1976. *"Adventures of a Mathematician."* New York: Charles Scribner`s Sons.

Wikinvest. 2012. *Wiki analysis.* http://www.wikinvest.com/commodity/Aluminum (18.05.2014).